POWER CORRECTIONS FROM SHORT DISTANCES*

G. GRUNBERG†
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

(grunberg@pth.polytechnique.fr)

ABSTRACT

It is argued that power contributions of short distance origin naturally arise in the infrared finite coupling approach. A phenomenology of $1/Q^2$ power corrections is sketched.

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†On leave of absence from Centre de Physique Théorique de l'Ecole Polytechnique, 91128 Palaiseau Cedex, France.

1. Introduction

Power-behaved contributions to hard processes not amenable to operator product expansion (OPE) have been derived [1] in recent years through various techniques (renormalons, finite gluon mass, dispersive approach), which all share the assumption that these contributions are of essentially *infrared* (IR) origin. In this talk, I point out that the IR finite coupling approach [2,3] naturally suggests the existence of additionnal non-standard contributions of *ultraviolet* (UV) origin, hence not related to renormalons (but which may be connected [4,5] to the removal of the Landau pole from the perturbative coupling).

2. Power corrections and IR regular coupling

Consider the contribution to an Euclidean (quark dominated) observable arising from dressed virtual single gluon exchange, which takes the generic form (after subtraction of the Born term):

$$D(Q^2)=\int_0^\infty \frac{dk^2}{k^2}\ \bar{\alpha}_s(k^2)\ \varphi\left(\frac{k^2}{Q^2}\right) \tag{1}$$

The "physical" coupling $\bar{\alpha}_s(k^2)$ is assumed to be IR regular, and thus must differ from the perturbative coupling $\bar{\alpha}_s^{PT}(k^2)$ (which is assumed to contain a Landau pole) by a non-perturbative piece $\delta\bar{\alpha}_s(k^2)$:

$$\bar{\alpha}_s=\bar{\alpha}_s^{PT}+\delta\bar{\alpha}_s \tag{2}$$

To determine the various types of power contributions, it is appropriate [2,6] to disentangle long from short distances "a la SVZ" with an IR cutoff Λ_I:

$$D(Q^2)=\int_0^{\Lambda_I^2} \frac{dk^2}{k^2}\ \bar{\alpha}_s(k^2)\ \varphi\left(\frac{k^2}{Q^2}\right)+\int_{\Lambda_I^2}^\infty \frac{dk^2}{k^2}\ \bar{\alpha}_s^{PT}(k^2)\ \varphi\left(\frac{k^2}{Q^2}\right)+\int_{\Lambda_I^2}^\infty \frac{dk^2}{k^2}\ \delta\bar{\alpha}_s(k^2)\ \varphi\left(\frac{k^2}{Q^2}\right) \tag{3}$$

The first integral yields, for large Q^2, "long distance " power contributions which correspond to the standard OPE "condensates". If the Feynman diagram kernel $\varphi\left(\frac{k^2}{Q^2}\right)$ is $\mathcal{O}\left((k^2/Q^2)^n\right)$ at small k^2, this piece contributes an $\mathcal{O}\left((\Lambda^2/Q^2)^n\right)$ term from a dimension n condensate, with the normalization given by a low energy average of the IR regular coupling $\bar{\alpha}_s$. The integral over the perturbative coupling in the short distance part represents a form of "regularized perturbation theory " (choosing the IR cut-off Λ_I above the Landau pole). The last integral in eq.(3) usually yields (unless $\delta\bar{\alpha}_s(k^2)$ is exponentially supressed) new power contributions at large Q^2 of short distance origin , unrelated to the OPE. Assume for instance a power law decrease:

$$\delta\bar{\alpha}_s(k^2)\simeq c\left(\frac{\Lambda^2}{k^2}\right)^p \tag{4}$$

The short distance integral will then contribute a piece:

$$\int_{Q^2}^{\infty} \frac{dk^2}{k^2}\ \delta\bar{\alpha}_s(k^2)\ \varphi\left(\frac{k^2}{Q^2}\right) \simeq A\ c\left(\frac{\Lambda^2}{Q^2}\right)^p \tag{5}$$

where $A \equiv \int_{Q^2}^{\infty} \frac{dk^2}{k^2}\ \left(\frac{Q^2}{k^2}\right)^p\ \varphi\left(\frac{k^2}{Q^2}\right)$ is a number. If one assumes moreover that $p < n$, the lower integration limit in eq.(5) and in A can actually be set to zero [7], and one gets a *parametrically leading* $\mathcal{O}\left((\Lambda^2/Q^2)^p\right)$ power contribution of UV origin, unrelated to the OPE.

3. Power contributions to the running coupling

A power law decrease of $\delta\bar{\alpha}_s(k^2)$ is a natural expectation for a coupling which is assumed to be defined at the non-perturbative level, and could eventually be derived from the OPE itself as the following QED analogy shows. In QED, the coupling $\bar{\alpha}_s(k^2)$ should be identified, in the present dressed single gluon exchange context, to the Gell-Mann-Low effective charge $\bar{\alpha}$, related to the photon vacuum polarisation $\Pi(k^2)$ by:

$$\bar{\alpha}(k^2) = \frac{\alpha}{1+\alpha\ \Pi(k^2/\mu^2, \alpha)} \tag{6}$$

One expects $\Pi(k^2)$, hence $\bar{\alpha}(k^2)$, to receive power contributions from the OPE. Of course, this cannot happen in QED itself, which is an IR trivial theory, but might occur in the "large β_0", $N_f = -\infty$ limit of QCD . Instead of $\Pi(k^2)$, it is convenient to introduce the related (properly normalized) renormalisation group invariant "Adler function" (with the Born term removed):

$$R(k^2) = \frac{1}{\beta_0}\left(\frac{d\Pi}{d\log k^2} - \frac{d\Pi}{d\log k^2}|\alpha = 0\right) \tag{7}$$

which contributes the higher order terms in the renormalisation group equation:

$$\frac{d\bar{\alpha}_s}{d\ln k^2} = -\beta_0(\bar{\alpha}_s)^2\,(1+R) \tag{8}$$

where β_0 is (minus) the one loop beta function coefficient. Consider now the $N_f = -\infty$ limit in QCD. Then $R(k^2)$ is expected to be purely non-perturbative, since in this limit the perturbative part of $\bar{\alpha}_s$ is just the one-loop coupling $\bar{\alpha}_s^{PT}(k^2) = 1/\beta_0\ln(k^2/\Lambda^2)$. Indeed, OPE-renormalons type arguments suggest the general structure [8] at large k^2:

$$R(k^2) = \sum_{p=1}^{\infty}\left(a_p\log\frac{k^2}{\Lambda^2} + b_p\right)\left(\frac{\Lambda^2}{k^2}\right)^p \tag{9}$$

where the log enhanced power corrections reflect the presence of double IR renormalons poles [9]. Eq.(8) with R as in eq.(9) can be easily integrated to give:

$$\bar{\alpha}_s(k^2) = \bar{\alpha}_s^{PT}(k^2) + \frac{\Lambda^2}{k^2}\left[a_1\ \bar{\alpha}_s^{PT}(k^2) + \beta_0\ (a_1+b_1)\left(\bar{\alpha}_s^{PT}(k^2)\right)^2\right] + \mathcal{O}\left((\Lambda^4/k^4)\right) \tag{10}$$

One actually expects : $a_1 = b_1 = 0$ (corresponding to the absence of $d = 2$ gauge invariant operator), and $a_2 = 0$ (corresponding to the gluon condensate which yields only a single renormalon pole). It is amusing to note that keeping only the $p = 2$ (gluon condensate) contribution in eq.(9) with $a_2 = 0$, eq.(8) yields :

$$\bar{\alpha}_s(k^2) = \frac{1}{\beta_0\left(\ln\frac{k^2}{\Lambda^2} + \frac{b_2}{2}\frac{\Lambda^4}{k^4}\right)} \tag{11}$$

which coincides with a previously suggested ansatz [10] based on different arguments.

4. $1/Q^2$ power corrections

The previous QED - inspired model (with $a_1 = b_1 = 0$) remains of academic interest, since it is clear that the short distance power corrections induced in QCD observables by the OPE-generated corrections in $\bar{\alpha}_s$ are then parametrically consistent with those expected from the OPE, and are actually probably numerically small compared to those originating directly from the long distance piece in eq.(3) (although it is still an interesting question whether such short distance contributions will not mismatch the expected OPE result for the coefficient functions). The situation changes if one assumes [4,5] the existence of $1/k^2$ power corrections of non-OPE origin in $\bar{\alpha}_s$. Evidence for such corrections has been found in a lattice calculation [11] of the gluon condensate, and physical arguments have been given [5,12] for their actual occurence. For instance, consider the case where $n = 2$ in the low energy behavior of the kernel $\varphi\left(\frac{k^2}{Q^2}\right)$, i.e. where the leading OPE contribution has dimension 4 (the gluon condensate). Then, setting $p = 1$ in eq.(4), the parametrically leading power contribution will be a $1/Q^2$ correction of short distance origin given by the right-hand side eq.(5) with:

$$A \equiv \int_0^\infty \frac{dk^2}{k^2}\,\frac{Q^2}{k^2}\,\varphi\left(\frac{k^2}{Q^2}\right) \tag{12}$$

Assuming further the physical coupling $\bar{\alpha}_s$ is *universal* [2,3], so is the *non-perturbative* parameter c in eq.(4). Then the process dependance of the strength of the $1/Q^2$ correction is entirely contained in the *computable* parameter A. In particular, it is interesting to check [13] whether A in the pseudoscalar channel is substantially larger then in the vector channel, which could help resolve [14] a long standing QCD sum rule puzzle [15], in addition to provide further evidence for $1/Q^2$ corrections. Note that the proposed mechanism is different from the (in essence purely perturbative) one based on UV renormalons [14,5]. The latter yields [16] an enhancement factor of 18 already in the single renormalon chain approximation (consistent with the present dressed single gluon exchange picture), but is subject to unknown arbitrarily large corrections from multiple renormalons chains, at the difference of the present argument.

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