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Decoherent Scattering of Light Particles in a D -Brane Background

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Abstract

We discuss the scattering of two light particles in a D -brane background. It is known that, if one light particle strikes the D brane at small impact parameter, quantum recoil effects induce entanglement entropy in both the excited D brane and the scattered particle. In this paper we compute the asymptotic ‘out’ state of a second light particle scattering off the D brane at large impact parameter, showing that it also becomes mixed as a consequence of quantum D -brane recoil effects. We interpret this as a non-factorizing contribution to the superscattering operator \mathcal{S} for the two light particles in a Liouville D -brane background, that appears when quantum D -brane excitations are taken into account.

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1 Introduction and Summary

The fate of quantum coherence in the presence of topologically non-trivial space-time backgrounds is currently the subject of intensive research. The problem may be posed at two levels: that of quantum fields in semi-classical gravitational backgrounds such as macroscopic black holes, and in the presence of non-perturbative microscopic quantum fluctuations in the space-time background. At the semi-classical level, the pioneering work of Hawking and Bekenstein demonstrated that a description in terms of pure quantum-mechanical states could not be maintained. Proceeding to the microscopic quantum level, Hawking [1] has argued that the conventional rules of quantum field theory require some revision, and that particle scattering should be described by a scattering operator \mathcal{S} that does not factorize as a product of S and S^\dagger matrix elements:

$$\rho_{out} = \mathcal{S}\rho_{in} : \mathcal{S} \neq SS^\dagger \quad (1)$$

If this is the case, there should be a modification [2] of the conventional Liouville equation for the time evolution of the density matrix ρ :

$$\partial_t \rho = i[\rho, H] + \mathcal{J}H\rho \quad (2)$$

where H is the conventional Hamiltonian. This formulation is reminiscent of open systems, with Planck-scale degrees of freedom playing the rôle of the unobserved ‘environment’ through which light low-energy particles move.

This possibility has been analyzed from various points of view in recent years. In particular, it has been argued that behaviours of the form (1) and (2) are inevitable in the non-critical Liouville string approach, with the non-factorizability of \mathcal{S} and the magnitude of $\mathcal{J}H$ related explicitly to departures from the criticality that defines classical string vacua [3]. Pursuing a more conventional quantum-gravity approach, divergences in the partition function of a scalar field in a semi-classical Einstein-Yang-Mills background have been exhibited, which have been interpreted [4] as leading to a modified quantum Liouville equation of the type (2).

Recent advances in D -brane technology [5] have provided more powerful tools for analyzing these issues. In particular, it has been shown that D branes provide *in principle* an exact accounting for the quantum states of black holes [6]. On the other hand, it has been argued that not all these states are *in practice* measurable in feasible low-energy experiments, and specific calculations [7] exhibit a loss of information and corresponding gain in entropy in the scattering of a light low-energy particle off a massive D brane, associated with quantum recoil effects.

The purpose of this paper is to link these different approaches to the quantum coherence problem. We first build on the work of [7] by computing the Riemann,

Ricci and curvature tensors associated with the singular metric that describes quantum D -brane recoil. This metric had previously been derived using conformal field theory techniques, and has many of the properties argued for independently in [8]. We then proceed to compute the effective action that characterizes long-wavelength physics in the presence of this D -brane recoil. We find singularities associated with the quantum recoil of the D brane induced by the impact of the first scalar particle, and compute properties of a low-energy scalar field in the presence of such a recoiling D brane, exhibiting a non-trivial Bogolubov transformation and particle production. These effects are sensitive to quantum fluctuations in the D -brane recoil, which may be treated as a stochastic source. This induces a non-trivial influence functional, which in turn makes a non-trivial contribution to the evolution of the density matrix for the external light particle, as foreseen in (1,2) [2, 3] ¹. This calculation may be regarded as a contribution to low-energy scalar-scalar scattering in a D -brane black-hole background, incorporating the effects of quantum D -brane recoil, whose magnitude we estimate. Our approach opens the way towards a representation of the associated non-factorizing terms in the \mathcal{S} matrix description as a spectral integral over D -brane states weighted by their recoil properties.

2 Space-Time Curvature Induced by Quantum D -Brane Recoil

We consider the collision of two low-energy, light closed-string particles in a D -brane background, examining the possibility of a loss of quantum coherence that cannot be accommodated in a conventional S -matrix description. We consider explicitly a configuration in which one closed-string state impacts the D brane and has a ‘hard’ scattering off it, while the second light state passes by at large impact parameter. We have shown previously that the colliding particle loses coherence as a result of the necessary sum over D -brane states excited by the collision [7]. Our interest here is whether the more distant light particle also loses some coherence during the scattering process.

A necessary preamble to this discussion of light-particle scattering in a D -brane background is a brief review of relevant aspects of our previous analysis of quantum effects in the recoil of the D brane struck by the first low-energy light particle [7]. We have demonstrated that this quantum recoil problem is described in conformal field theory language by a pair of logarithmic operators [10] C, D , which are conjugate to the D -brane position y_i and velocity u_i . The ‘kick’ provided by the incident low-energy particle requires the introduction of a Heaviside step function, for which we

¹The quantum origin of this effect is manifest in the associated annulus topology of the corresponding world sheet, which in turn corresponds to a loop diagram in field theory, as also argued in [8]. There is no decoherence at tree level, as also argued in [9].

adopt the following integral representation:

$$\Theta_\epsilon(t) = -i \int_{-\infty}^{\infty} \frac{dq}{q - i\epsilon} e^{iqt} : \epsilon \rightarrow 0^+ \quad (3)$$

where ϵ is an infrared regulator parameter. The quantum treatment of D -brane recoil necessitates the introduction of world-sheet annulus diagrams, whose large-size limit is characterized by an infrared regulator size parameter L that, together with a conventional ultraviolet world-sheet regulator parameter a , specifies the value of ϵ [10]:

$$\frac{1}{\epsilon^2} \simeq 2 \log|L/a|^2 \quad (4)$$

Further, in our interpretation, we identify the Liouville field ϕ with the renormalization scale on the world sheet [3, 11], and its zero mode, ϕ_0 , is further identified with the time variable

$$\phi_0 = t \simeq \log|L/a| \quad (5)$$

Note that there is no absolute time in this approach, since physical quantities are described by the renormalization group, which relates different scales L, L' that correspond to time differences $\delta t \simeq \log|L/L'|$.

These identifications of the renormalization scale, the zero mode of the Liouville field and the time variable are supported by various consistency checks, notably of momentum and energy conservation, which are documented in [7] and references therein. On the basis of this analysis, we derived the following form for the singular part of the target-space metric G_{MN} around the moment of the collision:

$$G_{00} = -1, G_{ij} = \delta_{ij}, G_{0i} = G_{i0} = \epsilon(\epsilon y_i + u_i t) \Theta_\epsilon(t) \quad (6)$$

It is to be understood that, in addition to (6), there is also a static, spherically-symmetric part of the metric, which is $\mathcal{O}(M/R)$ at large distances R from the struck D brane of mass M . We will consider the scattering of a second low-energy light particle at large impact parameter R , so that we may neglect this spherically-symmetric part in a first approximation, and consider the asymptotic metric as flat to zeroth order in M/R . The physics that interests us is that associated with the ϵ -dependent singularity in (6).

We have already pointed out that (6) implies a singularity in the Riemann curvature scalar as $t \rightarrow 0$ [7]. Our first priority in this paper is to compute this more explicitly. To do this, we consider the following more general form of D -dimensional metric:

$$G_{00} = -1, G_{ij} = \delta_{ij}, G_{0i} = G_{i0} = f_i(y_i, t), \quad i, j = 1, \dots, D-1 \quad (7)$$

It is easy to check that, for the above metric, the only non-vanishing components of the fully covariant form of the Riemann curvature tensor $R_{\mu\nu\rho\sigma}$ are the following:

$$R_{0i0i} = \frac{\partial^2 f_i}{\partial y_i \partial t} - \frac{1}{(1 + \sum_{k=1}^{D-1} f_k^2)} \frac{\partial f_i}{\partial y_i} \left(\sum_{k=1}^{D-1} f_k \frac{\partial f_k}{\partial t} \right) \quad (8)$$

$$R_{ijij} = \frac{1}{(1 + \sum_{k=1}^{D-1} f_k^2)} \frac{\partial f_i}{\partial y_i} \frac{\partial f_j}{\partial y_j} \quad (9)$$

Correspondingly, the different components of the Ricci tensor $R_{\mu\nu}$ take the following forms:

$$R_{00} = -\frac{1}{(1 + \sum_{i=1}^{D-1} f_i^2)^2} \left(\sum_{i=1}^{D-1} f_i \frac{\partial f_i}{\partial t} \right) \left[\sum_{j=1}^{D-1} \frac{\partial f_j}{\partial y_j} \left(1 + \sum_{k=1, k \neq j}^{D-1} f_k^2 \right) \right] \quad (10)$$

$$+ \frac{1}{(1 + \sum_{i=1}^{D-1} f_i^2)} \left[\sum_{i=1}^{D-1} \frac{\partial^2 f_i}{\partial y_i \partial t} \left(1 + \sum_{j=1, j \neq i}^{D-1} f_j^2 \right) \right] \quad (11)$$

$$R_{ii} = \frac{1}{(1 + \sum_{k=1}^{D-1} f_k^2)^2} \left\{ \frac{\partial f_i}{\partial y_i} \left(\sum_{j=1}^{D-1} f_j \frac{\partial f_j}{\partial t} \right) - \left(1 + \sum_{k=1}^{D-1} f_k^2 \right) \frac{\partial^2 f_i}{\partial y_i \partial t} \right. \\ \left. + \frac{\partial f_i}{\partial y_i} \left[\sum_{j=1, j \neq i}^{D-1} \frac{\partial f_j}{\partial y_j} \left(1 + \sum_{k=1, k \neq j}^{D-1} f_k^2 \right) \right] \right\} \quad (12)$$

$$R_{0i} = \frac{f_i}{(1 + \sum_{k=1}^{D-1} f_k^2)^2} \left\{ \frac{\partial f_i}{\partial y_i} \left(\sum_{j=1}^{D-1} f_j \frac{\partial f_j}{\partial t} \right) - \left(1 + \sum_{k=1}^{D-1} f_k^2 \right) \frac{\partial^2 f_i}{\partial y_i \partial t} \right\} \quad (13)$$

$$R_{ij} = \frac{1}{(1 + \sum_{k=1}^{D-1} f_k^2)^2} f_i f_j \frac{\partial f_i}{\partial y_i} \frac{\partial f_j}{\partial y_j} \quad (14)$$

Finally, the Riemann curvature scalar R is given by

$$R = \frac{2}{(1 + \sum_{i=1}^{D-1} f_i^2)^2} \left\{ - \left(1 + \sum_{i=1}^{D-1} f_i^2 \right) \left(\sum_{i=1}^{D-1} \frac{\partial^2 f_i}{\partial y_i \partial t} \right) + \left(\sum_{i=1}^{D-1} f_i \frac{\partial f_i}{\partial t} \right) \left(\sum_{i=1}^{D-1} \frac{\partial f_i}{\partial y_i} \right) \right. \\ \left. + \sum_{i,j=1, i \neq j}^{D-1} \frac{\partial f_i}{\partial y_i} \frac{\partial f_j}{\partial y_j} \left(1 + \sum_{k=1, k \neq i, j}^{D-1} f_k^2 \right) \right\} \quad (15)$$

It is a trivial matter to insert into the above expressions the specific form of the metric (6) found in the previous quantum D -brane recoil calculation. We find the following expressions for curvature scalars

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 4(D-1) \epsilon^4 [\delta_\epsilon(t)]^2 + \mathcal{O}(\epsilon^6)$$

$$\begin{aligned}
R_{\mu\nu} R^{\mu\nu} &= D(D-1) \epsilon^4 [\delta_\epsilon(t)]^2 + \mathcal{O}(\epsilon^6) \\
R^2 &= 4(D-1)^2 \epsilon^4 [\delta_\epsilon(t)]^2 + \mathcal{O}(\epsilon^6)
\end{aligned} \tag{16}$$

where $\delta_\epsilon(t)$ is the appropriate derivative of $\Theta_\epsilon(t)$ (3), that will be used in the next section. The reader should not be alarmed by the appearance of the $[\delta_\epsilon(t)]^2$ factors, which do not make ϵ -dependent divergent contributions to the physical quantities of interest to us, such as the integrated central charge deficit Q , etc., even in the limit $\epsilon \rightarrow 0^+$. In our regularization, it can be easily shown that in this limit $\Theta_\epsilon(0) \rightarrow \pi$ whilst $\delta_\epsilon(0)$ becomes formally a linearly-divergent integral that is independent of ϵ .

We note that the derivation of the space-time discussed above was essentially non-relativistic, since we worked in the approximation of a very heavy D brane. This is the reason why the singularity in the geometry (15) is space-like, and why the consequent change in the quantum state also occurs on a space-like surface. This apparent non-causality is merely an artifact of our approximation. We expect that, in a fully relativistic D -brane approach, the space-time singularity would travel along a light-cone, and that the quantum state would also change causally. Here we are interested in the difference between early- and late-time quantum states, and this apparent non-causal behaviour is irrelevant.

3 Derivation from an Effective Action

We expect that the metric (7) can be derived using the equations of motion for a suitable singular effective action induced by quantum D -brane effects. The most naive guess for a possible form of the action is

$$S = \int d^D x \sqrt{-G} \alpha' e^{-2\phi} \hat{R}_{GB}^2 \tag{17}$$

where ϕ is the dilaton field and \hat{R}_{GB}^2 is the ghost-free Gauss-Bonnet quadratic combination of the Riemann tensor, Ricci tensor and curvature scalar:

$$\hat{R}_{GB}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \tag{18}$$

This is indeed the case, as we now show.

The field equations derived from the action (17) and (18) are

$$\alpha' \hat{R}_{GB}^2 = \alpha' (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2) = 0 \tag{19}$$

$$\begin{aligned}
\alpha' K_{\mu\nu} &\equiv \alpha' (R \phi_{;\mu\nu} - R \phi_{;\rho}^{\rho} G_{\mu\nu} - 2 R_{\mu\rho} \phi_{;\nu}^{\rho} - 2 R_{\nu\rho} \phi_{;\mu}^{\rho} \\
&+ 2 R^{\rho\sigma} \phi_{;\rho\sigma} G_{\mu\nu} + 2 R_{\mu\nu} \phi_{;\rho}^{\rho} - 2 R_{\mu\rho\nu\sigma} \phi_{;\rho\sigma}^{\rho\sigma}) \\
&- 2\alpha' (R \phi_{;\mu} \phi_{;\nu} - R \phi_{;\rho}^{\rho} \phi_{;\rho} G_{\mu\nu} - 2 R_{\mu\rho} \phi_{;\nu} \phi_{;\rho}^{\rho} - 2 R_{\nu\rho} \phi_{;\mu} \phi_{;\rho}^{\rho} \\
&+ 2 R^{\rho\sigma} \phi_{;\rho} \phi_{;\sigma} G_{\mu\nu} + 2 R_{\mu\nu} \phi_{;\rho}^{\rho} \phi_{;\rho} - 2 R_{\mu\rho\nu\sigma} \phi_{;\rho}^{\rho} \phi_{;\sigma}^{\rho}) = 0
\end{aligned} \tag{20}$$

where ; denotes covariant differentiation. In the case of the first equation of motion, we see immediately from the expressions (16) that

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 = \mathcal{O}(\epsilon^6) \quad (21)$$

Therefore, the first equation of motion (20) is satisfied to leading order in the small parameter ϵ .

Motivated by the Liouville string theory approach of ref. [7], we use the linear-dilaton configuration [12, 3]

$$\phi = Qt \quad (22)$$

where Q^2 is the central-charge deficit of the Liouville string, and $Q = \mathcal{O}(\epsilon^2)$, as we shall argue below. This linear dependence of the dilaton field on the time t is expected to be valid approximately at large times $t \sim 1/\epsilon^2 \gg 0$ after the collision, where the system is close to its non-trivial world-sheet fixed point [7, 3]. This is sufficient for our purposes to demonstrate particle production and decohering effects, as we shall argue in the next sections. Using (22), we obtain for the components of the second (Einstein) equation of motion (20):

$$K_{00} = \mathcal{O}(\epsilon^8) \quad (23)$$

$$\begin{aligned} K_{0i} &= 2Q f_i \sum_{j,k=1, j \neq k \neq i}^{D-1} \frac{\partial f_j}{\partial y_j} \frac{\partial^2 f_k}{\partial y_k \partial t} + \mathcal{O}(\epsilon^8) \\ &= 2(D-2)(D-3)Q \epsilon^5 u_i t \Theta_\epsilon^2(t) \delta_\epsilon(t) + \mathcal{O}(\epsilon^8) \end{aligned} \quad (24)$$

$$\begin{aligned} K_{ii} &= 2Q \sum_{j,k=1, j \neq k \neq i}^{D-1} \frac{\partial f_j}{\partial y_j} \frac{\partial^2 f_k}{\partial y_k \partial t} + \mathcal{O}(\epsilon^8) \\ &= 2(D-2)(D-3)Q \epsilon^4 \Theta_\epsilon(t) \delta_\epsilon(t) + \mathcal{O}(\epsilon^8) \end{aligned} \quad (25)$$

$$K_{ij} = \mathcal{O}(\epsilon^{10}) \quad (26)$$

Thus, we again conclude that the equations of motion are satisfied at least through $\mathcal{O}(\epsilon^6)$.

To see that the central charge deficit $Q = \mathcal{O}(\epsilon^2)$, we proceed as follows. We recall that Q^2 is the change in the Zamolodchikov C function over the entire world-sheet renormalization-group trajectory from the Gaussian to the non-trivial fixed point. In our interpretation of target time as the world-sheet Liouville scale, this results for our purposes in the following space-time integrated expression:

$$Q^2 \sim \int dt \beta^i \mathcal{G}_{ij} \beta^j \quad (27)$$

where β^i is the β function of a generic string background characterized by couplings g^i , and \mathcal{G}_{ij} is the corresponding Zamolodchikov metric. In our case, to lowest non-trivial order in α' , the only background that contributes to \mathcal{G} is the target metric G_{ij} ², and the corresponding Zamolodchikov metric is given by

$$\mathcal{G}_{\mu\nu\alpha\beta} = a[G_{\mu\nu}G_{\alpha\beta} - G_{\mu\alpha}G_{\beta\nu} - G_{\mu\beta}G_{\alpha\nu}] + \mathcal{O}(\alpha') \quad (28)$$

where a is an arbitrary constant [13]. The lowest-order graviton σ -model β -function is:

$$\beta_{\mu\nu}^G = \alpha' \{R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi\} + \mathcal{O}[(\alpha')^2] \quad (29)$$

so that the integrand in (27) simplifies to

$$\beta^i \mathcal{G}_{ij} \beta^j = (\alpha')^2 a \epsilon^4 [2(D-1)(D-2)[\delta_\epsilon(t)]^2] + \mathcal{O}(\epsilon^6). \quad (30)$$

to leading order in ϵ .

The above expression (30) is understood to be multiplied by an overall normalization factor of the form $W = [1 + \sum_{i=1}^{D-1} \epsilon^2 (\epsilon y_i + u_i t)^2 \Theta^2(t)]^{-1}$, which ensures the finiteness of all spatial integrals. At leading order in ϵ , this factor may be approximated by unity. The temporal integration over t yields a divergent constant normalization factor $\delta(0)$, which is irrelevant because a is arbitrary [13]. We conclude that

$$Q^2 \sim (\alpha')^2 a \epsilon^4 [2(D-1)(D-2)\delta_\epsilon(0)] + \mathcal{O}(\epsilon^6), \quad (31)$$

which implies that Q is of order ϵ^2 for $D > 2$, as assumed above.

4 Particle Creation in the D -Brane Background

In this section we show that an observer who scatters a scalar field off the space-time whose geometry is described by (6) sees particle creation in the singular background metric of the struck D brane. To this end, we first note that an on-mass-shell scalar field ϕ of mass μ in the background (6) may be expanded in terms of ordinary flat-space Minkowski modes which play the rôle of the “in” modes, since our background space (6) can be mapped, for $t > 0$, to a flat space to $\mathcal{O}(\epsilon^2)$ by means of a simple coordinate transformation:

$$\tilde{t} = t \quad , \quad \tilde{X}^i = X^i + \frac{1}{2}\epsilon u_i t^2 \quad (32)$$

which may be represented by the Penrose diagram shown in Fig. 1.

²The dilaton β function is already $\mathcal{O}(\alpha')$ compared to the graviton.

Each of the four diamonds corresponds in a conventional Rindler space to one of the causally-separated regions [14]. Our space-time corresponds to the right-hand diamond. We see that it is flat Minkowski for $t < 0$, and that for $t \gg 0$ the shaded region of the space-time resembles, to $\mathcal{O}(\epsilon)$, a Rindler wedge, with ‘acceleration’ ϵu_i . The event horizons are indicated by the straight lines separating the diamond-shaped regions of the diagram. The dashed line corresponds to the curvature singularity $\epsilon^2 \delta_\epsilon(t)$, which may be ignored in order $\mathcal{O}(\epsilon)$. As a consequence of the non-relativistic nature of the D brane, this singularity appears to violate causality, lying outside the light cone. This is merely an artifact of taking the limit as the velocity of light $c \rightarrow \infty$, as is appropriate for a non-relativistic (heavy) D particle. One expects causality to be restored, with a rotation of the singular surface so as to become light-like, in a fully relativistic treatment of the problem, which lies beyond the scope of the present work.

In view of this figure, we expect [14] to find a non-trivial Bogolubov transformation between the “in” and “out” vacua at $t < 0$ and $t \gg 0$, which we now exhibit. The Minkowski mode expansion for the scalar field takes the standard form:

$$\phi = \int dE d^{D-1} K [a_{EK} \hat{u}_{EK} + a_{EK}^\dagger \hat{u}_{EK}^*] \quad (33)$$

where

$$\hat{u}_{EK} = \exp(-iEt + i\underline{K} \cdot \underline{X}) \quad E^2 - \underline{K}^2 = \mu^2 \quad (34)$$

are the usual Minkowski “in” modes. Such an expansion is valid for $t < 0$. For $t \gg 0$ one may expand the scalar field in terms of “out” modes,

$$\phi = \int dE d^{D-1} K [b_{E,K} \tilde{u}_{E,K} + b_{E,K}^\dagger \tilde{u}_{E,K}^*] \quad (35)$$

with

$$\tilde{u}_{EK} = \exp(-iEt + i\underline{K} \cdot \underline{X} + i\frac{1}{2}\epsilon \underline{u} \cdot \underline{K} t^2) + \mathcal{O}(\epsilon^2) \quad (36)$$

These modes are related to the “in” ones by the Bogolubov coefficients α, β :

$$\tilde{u}_{E,K} = \int dE' d^{D-1} K' [\alpha_{EE'KK'} \hat{u}_{E'K'} + \beta_{EE'KK'} \hat{u}_{E'K'}^*] \quad (37)$$

Extending appropriately the definitions of the “in” and “out” modes to the entire space-time, we find that the Bogolubov coefficients are given by:

$$\begin{aligned} \alpha_{EE'KK'} &= \int dt d^{D-1} X \tilde{u}_{E,K} \hat{u}_{E'K'}^* = \\ &\delta^{(D-1)}(K - K') [\delta(E - E') + \frac{i}{2}\epsilon(\underline{u} \cdot \underline{K}) \int_0^\infty dt e^{-i(E-E')t} t^2] + \mathcal{O}(\epsilon^2) \\ \beta_{EE'KK'} &= \int dt d^{D-1} X \tilde{u}_{E,K} \hat{u}_{E'K'} = \\ &\delta^{(D-1)}(K + K') \frac{i}{2}\epsilon(\underline{u} \cdot \underline{K}) \int_0^\infty dt e^{-i(E+E')t} t^2 + \mathcal{O}(\epsilon^2) \end{aligned} \quad (38)$$

which satisfy $|\alpha|^2 - |\beta|^2 = 1$ in a distribution-theoretic sense. The t integrals are divergent, but they are viewed as irrelevant constants.

Bearing in mind the analogy with Rindler space, we expect particle creation, which is controlled by the Bogolubov β coefficient, since it is this term that contributes to positive frequency mixing [14]. Indeed, we find that the number of particles created in the mode labelled by E and K is given by:

$$\begin{aligned}
n_{EK} &= {}_{IN}\langle 0|b_{EK}^\dagger b_{EK}|0\rangle_{IN} = |\beta_{EK EK}^2| \\
&\simeq (\epsilon \underline{u} \cdot \underline{K})^2 \left| \left(\int_0^\infty dt e^{-i2Et} t^2 \right) \right|^2 \\
&\propto \left(\frac{\epsilon^2 (\underline{u} \cdot \underline{K})^2}{E^6} \right)
\end{aligned} \tag{39}$$

The formula (39) describes particle creation with a non-trivial angular distribution around the direction of the velocity vector u_i . The particle spectrum is *not thermal* as it would have been in the case of a uniform acceleration [15].

This particle creation (39) may be interpreted as non-thermal Hawking radiation from the recoiling D-brane. An analogous effect was suggested in the context of non-critical string theory [3] on the basis of an analysis of two-dimensional stringy black holes [16], the latter being viewed as ‘massive’ string particles. It was argued in [16] that the scattering of light matter off the black hole excites the black hole to a higher string level. Quantum instabilities would then cause the decay of this excited state, with the inevitable emission of non-thermal radiation. In a similar context, we mention a related work [17], where the presence of non-thermal radiation has been demonstrated when a massive scalar field is quantized in a two-dimensional dilaton-gravity black-hole background. In a large-mass $1/m$ expansion for the scalar field, such non-thermal radiation effects appear, associated with R^2/m^2 -corrected terms in the target-space effective action. These describe effects due to the back reaction of the massive matter on the two-dimensional space-time geometry, and are very analogous to our higher-dimensional case, in which we also obtained an effective Gauss-Bonnet R^2 target-space action term (17),(18), describing the leading effects of the back reaction of the heavy recoiling D brane on the surrounding space-time geometry.

5 Quantum Background Fluctuations

Despite this particle production, we still need to demonstrate the appearance of entropy, and thus decoherence, in the spectator-particle system. According to our interpretation of the entropy generation as being associated with quantum effects in D -brane recoil, any such decoherence effects should be related to the quantum excitation of the D brane, and the resulting sum over internal states. This is not apparent from the above discussion, which relates the spectator particle number to the velocity u_i . Particle production could in principle occur in a coherent way, so how do we know that is not the case here? The answer is that the quantum summation over world-sheet genera results in fluctuations of the velocity u_i about

the classical value which would be expected on the basis of momentum conservation in an elastic collision [18, 7].

Using generic arguments [18] based on the quantization of σ -model backgrounds in string theory associated with the summation over genera [3], it has been shown that the D -brane velocity fluctuates about u_i with a Gaussian distribution

$$\mathcal{P} = e^{-\frac{(\delta u_i)^2}{\Gamma_u^2}} \quad \Gamma_u^2 \sim (g_s^2 + \delta g_s^2) \ln \delta, \quad \delta \rightarrow 0 \quad (40)$$

where δg_s^2 is the change in the square of the coupling constant for the string, g_s^2 , induced by the matter deformation of the σ model describing the D brane that is caused by the scattering of the first string particle. The appearance of a $\ln \delta$ dependence is due to world-sheet divergences, arising from world-sheet loops down to a size $\delta \ll 1$, which were argued in [18] to dominate the description of velocity fluctuations around a recoiling heavy- D -brane background.

It is this divergence related to a pair of logarithmic operators [10] that explains in conformal-field-theory language why decoherence appears in scattering off a D brane, but would not appear in scattering off a conventional massive string state. Physically, the dispersion in recoil velocity u_i implies that at large times the D -brane state must be described by an ensemble of widely-separated configurations. In addition to the mixed nature of the D -brane state itself, this distributed ensemble yields a superposition of macroscopically-distinct space-time backgrounds, leading to entropy growth also for the second light string state in the scattering process, as we discuss below.

The change δg_s in g_s due to the matter deformation is given in conventional σ -model perturbation theory by:

$$\delta g_s^2 = e^{-2\langle \Phi \rangle_g} - e^{-2\langle \Phi \rangle_0} \simeq g_s^2 \sum_N \langle \Phi V_{i_1} \dots V_{i_N} \rangle q^{i_1} \dots q^{i_N} \quad (41)$$

where the q^i denote background target-space couplings corresponding to vertex operators V_i , and $\langle \dots \rangle_g$ ($\langle \dots \rangle_0$) denotes σ -model correlators in the deformed (free) string. In our case, only correlators with even powers of D are non-vanishing, and an order-of-magnitude estimate of δg_s can be made using the two-point correlator of the recoil operator D_i on the disc, which diverges as $1/\epsilon$:

$$\delta g_s^2 = g_s^2 \langle \Phi \int \int_{\partial \Sigma} D_i D_j \rangle u^i u^j + \dots \sim g_s^2 \frac{1}{\epsilon^2} \times \mathcal{O}(E^2/M_D^2) \quad (42)$$

where $\partial \Sigma$ denotes a world-sheet boundary, and E is a typical energy of the light-string state that scatters off the D -brane background. Although the expression (42) appears formally divergent, it is in fact subleading for small $u_i \ll 1$, compared to the g_s^2 terms in the width (40), as we discuss later on. For now we note that higher-order correlators of D may be ignored in the dilute-gas approximation, which is valid

for weak string couplings $g_s \ll 1$. Using a generalized Fischler-Susskind mechanism [19], we may identify $1/\epsilon^2 = \ln\delta$, thereby implying the following expression for the dominant terms in the width (40):

$$\Gamma_u^2 \equiv (\Delta u)^2 \sim g_s^2 \ln\delta + g_s^2 u_i^2 \ln^2\delta \quad (43)$$

It is clear from this discussion that if the string is very weakly coupled: $g_s \rightarrow 0$, the quantum fluctuations Δu_i will be suppressed, as is appropriate for a semi-classical heavy D brane of mass $M \sim 1/g_s$.

As discussed above, here and in [7] we further identify $1/\epsilon^2$, and hence $\ln\delta$, with the target time t ³. Within this interpretation, we may interpret (43) as the spread in time t of the probability distribution $|\Psi^{st}(u_i, t)|^2$ for the string wave function. This interpretation of the summation over topologies in a σ -model path integral has previously been made in [20, 21], where it was shown that such probability distributions satisfy stochastic Fokker-Planck diffusion equations.

We now show how this effect leads to stochastic dynamics and decoherence for the spectator scalar field propagating in the background of a D brane with such a quantum-mechanical treatment of recoil. Using the initial coordinate system (y_i, t) and the metric (6), we find the following equation of motion for a massive scalar field φ , of mass μ ,

$$\begin{aligned} -\partial_i^2 \varphi + \sum_{i=1}^{D-1} \left(\partial_i^2 + 2f_i \partial_i \partial_t \right) \varphi + \sum_{i=1}^{D-1} \left[\sum_{i \neq j} f_j^2 \right] \partial_i^2 \varphi - \\ \sum_{i \neq j} f_i f_j \partial_i \partial_j \varphi + \rho \partial_t \varphi + \sum_{i=1}^{D-1} B_i \partial_i \varphi - \mu^2 W \varphi = 0 \end{aligned} \quad (44)$$

where $f_i = \epsilon(\epsilon y_i + u_i t) \Theta_\epsilon(t)$ and $W = 1 + \sum_{i=1}^{D-1} \epsilon^2 \Theta_\epsilon(t)^2 (\epsilon y_i + u_i t)^2$ as before, $\sqrt{-G} = \sqrt{W}$, and

$$\begin{aligned} \rho &= W^{-1} \left\{ \sum_{i=1}^{D-1} \left(\epsilon^2 (\epsilon y_i + u_i t) \Theta_\epsilon(t) [u_i \Theta_\epsilon(t) + \delta_\epsilon(t) (\epsilon y_i + u_i t)] \right) \right. \\ &\quad \left. + (D-2) \epsilon^2 \Theta_\epsilon(t) W + \epsilon^2 \Theta_\epsilon(t) \right\}, \\ B_i &= \epsilon u_i \Theta_\epsilon(t) + \epsilon (\epsilon y_i + u_i t) W^{-1} \delta_\epsilon(t) - f_i \Theta_\epsilon(t) W^{-1} \sum_{j=1}^{D-1} \epsilon u_j f_j \\ &\quad - (D-2) f_i \epsilon^2 \Theta_\epsilon(t) - \epsilon^2 W^{-1} f_i \Theta_\epsilon(t) \end{aligned} \quad (45)$$

We see that (44) contains a friction term, proportional to the quantity $\rho = \mathcal{O}(\epsilon^2)$ in (45), which is of the same order as the curvature in our space time, and depends on the $\delta_\epsilon(t)$ singularity structure.

³For an alternative interpretation, see [18].

To illustrate our basic argument for the stochastic nature of the problem, we consider the simplest possible configuration for the massive scalar field φ , namely that of a field that is constant in space: $\varphi(t)$. In this case (44) simplifies to:

$$-\partial_t^2 \varphi(t) + \rho \partial_t \varphi = \mu^2 W \varphi \quad (46)$$

Consistency with the approximation of spatial independence can be maintained only up to order ϵ^2 , where the y_i dependent terms in ρ are ignored, and the latter may be approximated by

$$\rho \simeq \epsilon^2 u_i^2 t \Theta_\epsilon(t) (\Theta_\epsilon(t) + t \delta_\epsilon(t)) + (D-1) \epsilon^2 \Theta_\epsilon(t) + \mathcal{O}(\epsilon^3) \quad (47)$$

For large times $t \sim 1/\epsilon^2$, the equation of motion (46) resembles that for an inflaton field to the order in ϵ studied, with ‘friction’ $\sim u_i^2$, and a time-dependent force that increases linearly with large times $t \gg 0$:

$$-\partial_t^2 \varphi(t) + u_i^2 \Theta_\epsilon(t) \partial_t \varphi = \mu^2 \varphi - \mu^2 u_i^2 t \Theta_\epsilon(t) \varphi \quad (48)$$

Thus there are two possible sources of decoherence in our problem. One is related to the friction term in (48), which for any finite time $t < 1/\epsilon^2$, is of the same order in ϵ as the space-time curvature, i.e., of $\mathcal{O}(\epsilon^2)$, and hence can be ignored in our present discussion. The other is due to the interaction of the scalar field φ with the gravitational background - recall that W is the determinant of the metric tensor - and therefore provides [21] an environmentally-induced stochastic source term in the inflation equation (48), associated with the quantum recoil degrees of freedom of the D brane, which survives to order ϵ in the flat space-time limit that we are considering here. It is this source that we now explore further, and use to estimate the size of the induced decoherence effects.

A general approach to the analysis of such a system, described by field variables a in interaction with some environmental degrees of freedom q with some action: $S[a, q] = S[a] + S_e[q] + S_{int}[a, q]$, is provided in [22, 23]. The system a is characterized by a reduced density matrix $\rho_r(a, a')$:

$$\rho_r(a, a') = \int_{-\infty}^{\infty} dq \int_{-\infty}^{\infty} dq' \rho(a, q; a', q') \delta(q - q') \quad (49)$$

whose time evolution is given by the evolution operator \mathcal{J}_r :

$$\rho_r(a, a', t) = \int \int_{-\infty}^{\infty} da_i da'_i \mathcal{J}_r(a, a', t | a_i, a'_i, t_i) \times \rho_r(a_i, a'_i, t_i) \quad (50)$$

with the subscript i denoting initial data at a time $t = t_i$, when the system and the environment are assumed to be uncorrelated: $\rho(t_i) = \rho_r(t_i) \times \hat{\rho}_{environment}(t_i)$. Using the path-integral formalism [22], it has been shown that the evolution operator can be expressed in terms of the influence functional $\mathcal{F}(a, a')$:

$$\mathcal{J}_r(a_f, a'_f, t | a_i, a'_i, t_i) = \int_{a_i}^{a_f} \int_{a'_i}^{a'_f} Da Da' e^{\frac{i}{\hbar} \{S[a] - S[a']\}} \mathcal{F}(a, a') \quad (51)$$

with the subscript f denoting the ‘final’ state at some large time t . A formal representation of the influence functional is given by [22, 23]:

$$\begin{aligned} \mathcal{F}(a, a') &\equiv e^{\frac{i}{\hbar}S_{IF}} = \int \int \int_{-\infty}^{\infty} dq_f dq_i dq'_i \int_{q_i}^{q_f} Dq \int_{q'_i}^{q'_f} Dq' \exp\left(\frac{i}{\hbar}\{S_e[q] + S_{int}(a, q)\}\right) \\ &\exp\left(-\frac{i}{\hbar}\{S_e[q'] + S_{int}(a, q')\}\right) \rho_e(q_i, q'_i, t_i) \end{aligned} \quad (52)$$

There is a simple representation⁴ of the influence functional in terms of the Bogolubov coefficients $\alpha_k(a), \beta_k(a)$ that appear in the transformation between the creation and annihilation operators of field amplitudes at different times:

$$\mathcal{F}(a, a') = \prod_k \frac{1}{\sqrt{\alpha_k(a')\alpha_k^*(a) - \beta_k(a')\beta_k^*(a)}} \quad (53)$$

Note that since the Bogolubov coefficients in general satisfy $|\alpha(a)|^2 - |\beta(a)|^2 = 1$, the influence functional reduces to the unit matrix when $a' = a$, as it should.

In the application of this formalism to our problem, the rôle of the field variables a at an initial time $t_i \leq 0$ is played by the Minkowski “in” modes \hat{u}_{EK} of the previous section, whilst the rôle of the field variables a' , at a later time $t \gg 0$, is played by the “out” modes \tilde{u}_{EK} (36), but corresponding to a velocity which is given by the quantum fluctuations (43). We shall consider decoherence between two field configurations that preserve energy and momentum. This is a feature which is respected on average by our Liouville approach to target time [3].

From the expressions (38) for the Bogolubov coefficients we find the following order of magnitude of the pertinent influence action S_{IF} (52),(53):

$$\begin{aligned} S_{IF} &\propto -i\hbar \ln \mathcal{F}(a, a') \sim i\hbar \left[\frac{1}{2} \ln(\alpha_{EK EK}(\tilde{u}_{EK})) + \dots \right] \\ &\sim i\hbar \left[\frac{1}{4} g_s^2 |\hat{u} \cdot \underline{K}|^2 t + \mathcal{O}(g_s^2 u_i^2 |\hat{u} \cdot \underline{K}|^2 t^2) \right] + \dots \end{aligned} \quad (54)$$

where \hat{u} denotes a unit vector in the direction of \underline{u} , and the \dots indicate real parts, which are not of interest to us here, as well as terms of higher order in ϵ^2 , which are suppressed as $\epsilon \rightarrow 0$. We see that the term in (54) proportional to t^2 , which owes its existence to the δg_s^2 term in (40), is subleading for small $u^2 \ll 1$, compared to the term with a linear t dependence. This follows from the fact that, for asymptotically large times $t \sim 1/\epsilon^2$, these terms are of order $(u_i^2/\epsilon^2)t$. It has been argued in [7, 18] that $u_i/\epsilon = u_i^R$ is a ‘renormalized’ velocity, corresponding to the σ -model coupling of an exactly-marginal velocity recoil operator D , which in our framework is also assumed small $u_i^R \ll 1$.

⁴The derivation of [23], which was based on some cosmological models, can easily be generalized to more general cases including the one we consider here, using a finite-volume regularization.

Thus, we see that the quantum fluctuations of the D -brane recoil velocity (43) induce decoherence for the second light particle that grow linearly in time, provided that the modular world-sheet infinities $\ln\delta$ have the target-time interpretation [7] assumed here. If only classical recoil velocity were present, there would be no $\ln\delta$ dependence and hence no time dependence and no decoherence. The imaginary coefficient in (54), and its linear dependence on t , imply that the quantum recoil of the D brane, which corresponds in conformal-field-theoretic language to a departure from criticality, induces a non-Hamiltonian contribution δH (2) into the generic evolution equation of the reduced density matrix for the scalar particle in this background. Since the momentum of the spectator particle, K , may be taken generically to be of the same order of magnitude as its energy E , one obtains from (54) the order-of-magnitude

$$\delta H \sim \mathcal{O}(E^2/M_D) \quad (55)$$

where M_D is the heavy D -brane mass, which is related to the conventional string scale $M_s = (\alpha')^{-1/2}$, with α' the Regge slope, via $g_s \sim M_s/M_D < 1$ ⁵. The formula (55) agrees with the generic estimates of [2, 3, 7], which indicated a suppression by a single power of the heavy mass scale, in this case M_D .

6 Conclusions and Outlook

We had previously shown explicitly using conformal field theory techniques [7] that a light particle that has a ‘hard’ scattering off a D brane finishes up in a mixed state, with an entanglement entropy corresponding to that of the excited struck D brane. The analysis in the previous section demonstrates that a light spectator particle passing by at large impact parameter is also converted into a mixed state, by virtue of the quantum fluctuations in the recoil of the struck D brane. This can be regarded as a decohering contribution to the scattering of light particles in a D -brane background, and hence a non-trivial contribution to the \mathcal{S} matrix, over and above the decohering effects of D -brane scattering on the propagation of a light particle that we discussed previously [7].

More specifically, we have demonstrated that this decoherence effect causes off-diagonal density matrix elements to decay exponentially with time, just as expected in [2], on the basis of the non-Hamiltonian term δH in (2). Moreover, this effect increases quadratically with the particle energy E , as also suggested previously [2, 3, 4].

In the future, it would clearly be interesting to extend this analysis to light-particle scattering in the absence of an initial-state D brane, but taking into account the

⁵Here we assume that the dimensionless time scale $t \sim \ln\delta$ is expressed in units of M_D^{-1} . Further analysis is needed to pin down the most appropriate scaling. If instead one assumes that the dimensionless time scale $\ln\delta$ is expressed in string units M_s^{-1} , then (55) has an extra factor g_s .

excitation of intermediate virtual D -brane states. Such a computation goes beyond the scope of this paper, but we expect it to share features with this calculation, in particular that it would lead to a contribution to $\delta H \sim E^2$.

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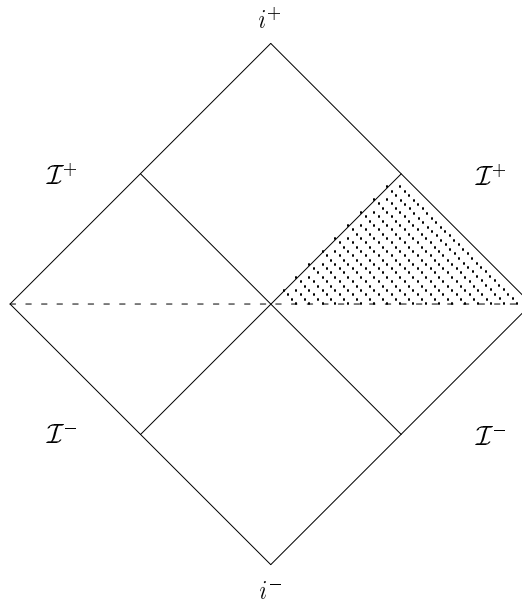


Figure 1: *Penrose diagram for the space-time environment derived from the quantum recoil of a heavy D brane induced by the scattering of a light closed-string state.*