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MBTR Simulations for Different Multibunch Structure Models.

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Abstract

This paper reports on simulations undertaken with the code MBTR to explore the emittance growth for various multibunch CLIC structures. Two simplified models of long-range dipole fields are considered and compared with that of a damped structure with frequency discriminated wave guide damping. The second model corresponds to a possible design of a multibunch structure, with damping and moderate detuning, recently studied and showing promising characteristics. The reported simulations carried out with the CLIC main beam parameters of the LC97 workshop corroborate this hope. They show that the vertical emittance growth of a 60-bunch train accelerated to 1 TeV is only a fraction larger than the single-bunch emittance growth, in the same conditions.

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1 Simplified model of field roll-off in a multibunch stucture

This simplified model is a kind of generic model [1], approximating the expected field roll-offs, and simple enough to be used in basic estimates of what would be the requirements for multibunch stability. The model is based on the following expression for the envelope amplitude of the long-range transverse wakefield, inspired by the shape of the wakefields calculated for multibunch structures (Fig.1):

$$W_{T,k} = W_{T,max} \cdot f_{rel} \cdot e^{[-(k-2)\cdot f_{dec}]} \quad k \geq 2 \tag{1}$$

where W_T, k is the wake amplitude at bunch number k, $W_{T,max}$ is the maximum amplitude of the short-range transverse field (in kV/pC/m/m), f_rel is the fraction of the maximum field seen by bunch number 2 after the roll-off, and f_{dec} is an exponent describing the exponential decay of the field in the train, for the bunches following bunch 2.

The equation (1) gives the variation of the dipole field envelope or amplitude, but says nothing about the phase. Therefore, in the code MBTR [1], one of the two following options can be selected:

1. Option 1.
   The long-range transverse wakefield, the amplitude of which is given by 1,is assumed to be locally constant across any single bunch of the whole train, with no phase considerations.
2. Option 2.
   Though the amplitude of the long-range transverse wakefield is still given by (1), a random phase is introduced according to a random selection around the phase which would correspond to the z-position of the bunch and the expected frequency f_{dip} of the dipole fundamental mode. The selection is done within one entire period of the dipole mode with a constant probability distribution. The resulting phase factor looks like:

   $$F_{ph} = \sin\left[\frac{2\pi f_{dip}}{cR_{ran}}\left(z + \frac{cR_{ran}}{f_{dip}}\right)\right] \tag{2}$$

   where R_{ran} is a random number between 0 and 1, taken from a uniform distribution.

The longitudinal wakefield is again taken equal to the long-range field calculated for a kind of damped-detuned structure.

This generic model can for instance be compared with the design calculations of a damped structure (TWG-DS) with frequency discriminated waveguide damping [2]. In the later case, the transverse wakefields are directly given in V/pC/m/m as a function of the position z behind the exciter-charge in m. For the TWG-DS structure studied at CLIC, the field values are given for $z \leq 2m$ and in steps of $0.13mm$. The corresponding exponential drop-off of the transverse wakefield is shown on Fig.1 over this distance of 2 m behind bunch 1. Beyond this distance, a persistent wake at 33.3 GHz, associated with the cut-off frequency of the waveguide, becomes dominant and decreases with $z^{-\frac{3}{2}}$. For $z \geq 2m$ the numerical TWG-DS model keeps only this persistent component, with a transverse field amplitude at this point equal to the value given by the initial drop-off. For the longitudinal wakefield, all modes beyond the fundamental have been assumed to be reduced by a factor 10, with respect to the modes of CIS (constant impedance structure). The first mode of the long-range longitudinal wakefield is put to zero in order to simulate the beam loading compensation, and this in all the models.

2 Wakefield model for a structure design with damping and weak detuning

The model considered here corresponds to a damped structure which has a certain amount of detuning, close to the one associated with a 'smoothed constant-gradient' condition, i.e. with a certain amount of a 'gaussian' distribution of detuned frequencies (nick-named here CG-DS). It is a structure specifically studied for CLIC [3] in an attempt to provide (with limited detuning) a sufficient attenuation of the long-range transverse wakefields along the bunch train, which was not really obtained in the damped-detuned structure (DDS), with strong detuning and low-level damping, previously analysed [4]. The corresponding and simplified field description includes parameters allowing to introduce different field roll-off exponents as a function of the distance z behind the exciter-bunch. If we assume that the bunch separation is equal to 20 RF-periods, it is possible to unequivocally relate the distance z and the bunch index number k. From this arise the following expressions for the envelope amplitude of the long-range transverse field, deduced from the estimates carried on with an equivalent circuit analysis (Fig.1):

$$
\begin{aligned}
W_{T,k} &= W_{T,max} \cdot f_{rel} \cdot f_{rel0} \qquad k = 2 && (3)\\
W_{T,k} &= W_{T,max} \cdot f_{rel0} \cdot e^{[-(k-2) \cdot f_{dec}]} \qquad 2 \leq k \leq 6 && (4)\\
W_{T,k} &= W_{T,max} \cdot f_{rel1} \cdot e^{[-(k-6) \cdot f_{dec1}]} \qquad 6 \leq k \leq 11 && (5)\\
W_{T,k} &= W_{T,max} \cdot f_{rel2} \qquad k \geq 11 && (6)
\end{aligned}
$$

where $W_{T,k}$ is the wake amplitude at bunch number k, $W_{T,max}$ is the maximum amplitude of the short-range transverse field (in kV/pC/m/m), f_{rel0} is the fraction of the maximum field (envelope value) seen by bunch number 2 after the roll-off, f_{dec} is an exponent describing the decay of the field envelope for the bunches number 3 to 6, f_{rel1} the maximum field seen by bunch 6, f_{dec1} is an exponent describing the decay of the field envelope for the bunches number 7 to 11, and f_{rel2} the field seen by all the bunches following bunch number 11 (envelope supposed to stay constant, at a low level). There is an additional coefficient f_{rel} that allows to adjust the field level at bunch 2, without touching at the transverse field envelope. It corresponds to the possibility of somehow simulating either the phase adjustment of the field for approaching a nod within $\pm 45°$ (amplitude reduction) or the effect of a field roll-off worse than expected (amplitude increase).

As in previous-Section model, the two options associated with the phase of the long-range dipole field can be selected, in order to either represent or not the effect of wakefield random-phases at the different positions of the following bunches.

The longitudinal wakefield is taken equal to the long-range field calculated for a kind of damped-detuned structure, as in Section 1.

Fig.1 shows the transverse, long-range dipole wakefield-envelope of the two simplified models of sections 1 and 2, as well as of the CLIC wave-guide damped TWG-DS, for comparison purposes. The curves indicate that the generic model with $f_{rel} = 0.27$ has a dipole wakefield drop-off slightly weaker than the one of the TWG-DS stucture, in the interval $0.2m \leq z \leq 2m$, while the damped, weakly detuned structure model CG-DS exhibits a dipole envelope with an amplitude lower than that of the TWG-DS for most z values, with a noticeable exception around $z = 0.4m$ where the two corresponding curves cross each other.

3 Results

Fig.1 shows that the model for the latest multibunch structure studied (CG-DS) exhibits a Green's function for the transverse wakefield that corresponds to the best roll-off expected from the analysis of various designs, except perhaps over a short z-interval around $z = 0.3m$.

Simulations have been carried out with the most recent CLIC main linac parameters (LC97) with a final energy of 500 GeV, a charge per bunch of $4 \cdot 10^9$ particles, 60 bunches, a bunch length of $50\mu m$, a loaded accelerating gradient of 100 MV/m and r.m.s. misalignments of $10\mu m$ for all the linac components. The goal was to compare in terms of beam stability, the CG-DS structure with the other structures or structure models considered for CLIC. Comparisons between DS and DDS have already been published [5], showing the need for a field attenuation by 100 at one bunch separation and a further continuous reduction of the wake behind the exciting bunch. Hereafter, the comparison is carried out between the CG-DS model on the one side and the simplified generic model on the other side, but also with the DS structure which is providing good stability.

Fig.2 gives for CG-DS the variation of the vertical emittance growth of the whole train as a function of the dipole mode amplitude at bunch 2, varied from 0 to 3 % (f_{rel} = 0 to 2) of the short-range peak value. Comparing with previous results obtained with the simplified generic model (Fig.3), this variation has a limited slope in the interval considered and a less steep rise, which indicates a more moderate risk of instability if f_{rel} is accidentally larger than expected.

In order to give an illustration of the emittance distributions with different random seeds or "machines", Figs 4 and 5 show the statistical results obtained respectively for a single bunch and a multibunch train, the later with the generic model using a coefficient f_{rel} of 1 % and f_{dec} of 0.22. These distributions are close to Poisson's functions [6] that have a long tail, a lot more stretched in the multibunch case of course than in the single bunch example. As a consequence, the average and r.m.s. emittance values do not provide the complete information about the risk to get an unacceptable growth, the worst case corresponding to an emittance 4 to 5 times larger than the average.

Coming back to the CG-DS simulation results, Figs.6 to 8 show the vertical emittance growth along the 500 GeV linac of the whole train, as compared to that of a single bunch, for three different values of the dipole field at bunch 2:

1. Field of 1 % of the maximum short-range amplitude $W_{T,max}$
2. Field of 1.5 % of the maximum short-range amplitude $W_{T,max}$
3. Field of 3 % of the maximum short-range amplitude $W_{T,max}$

The first two cases are stable over the total distance of the linac, as expected from the growth dependence shown in Fig.2. However, the last case, selected by purpose to generate a strong effect, shows that the train instability indeed begins to rise after half the linac length approximately.

Fig.9 is a tentative explanation of why the results with CG-DS are comparable but not really better than those with the TWG-DS (see wakefields on Fig.1). With TWG-DS, an amplitude at bunch 2 of about 1.5 % of $W_{T,max}$ gives indeed an absolute increase of the emittance of the train with respect to a single bunch by $2 \cdot 10^{-8} radm$, while it gives about $3 \cdot 10^{-8} radm$ with CG-DS. This can be related to the envelopes of Fig.1 that cross each other at small z for these two models. Hence at short distances of 2 or 3 bunch separations (one separation is 20 RF periods), the effect of CG-DS is stronger than that of TWG-DS. In the train, this shows up as an increase of the single bunch emittance for bunches 4 to 8 about, before a steady state takes place for the rest of the train, with CG-DS, while the individual emittances have essentially the same values with the TWG-DS.

These multibunch simulations were carried out with the pessimistic assumption that the dipole fields had locally constant values in all the bunches and were in phase. The question is then to get an idea of how conservative this assumption is with respect to reality. One way of trying to answer this question is to select randomly the dipole-field phase in each bunch and repeat the simulations with different seeds. Doing this with CG-DS and a wakefield at bunch 2 of 1 % of $W_{T,max}$ shows (Fig. 10) that the emittance distribution for random field phases is again of the Poisson type. Consequently, the average multibunch emittance growth is about 4 to 5 times smaller than in the pessimistic case, but the tail of the distribution expectedly approaches the value obtained in the conservative assumption. This does not seem to be an exagerated margin of security, since the real case can always be one of the worst in the distribution.

As a general conclusion from the numerical simulations, the CG-DS model of a multibunch structure looks as a good design candidate, that would give in addition to the single bunch blow-up a tolerable train-emittance growth in the presence of 60 bunches, without some of the engineering difficulties associated with TWG-DS.

4 References

1. G. Guignard and J. Hagel, MBTR Description and Users' Guide with examples of applications, report to be published.

2. M. Dehler, I. Wilson, W. Wuensch, CLIC Waveguide Damped Accelerating Structure Studies, CERN/PS 96-39 (LP), 1996.

3. M. Dehler, Private communication.

4. M. Dehler *et al.*, Design of a 30 GHz Damped Detuned Accelerating Stucture, 1997 Particle Accelerator Conference, Vancouver, B.C., Canada, 1997.

5. G. Guignard and J. Hagel, Beam Characteristics Versus Cavity Models in CLIC, 1997 Particle Accelerator Conference, Vancouver, B.C., Canada, 1997.

6. S. Fartouhk, A Statistical Approach to Analyse the Efficiency of BNS Damping and Correction Algorithms, CERN/PS 97-06 (LP), 1997.

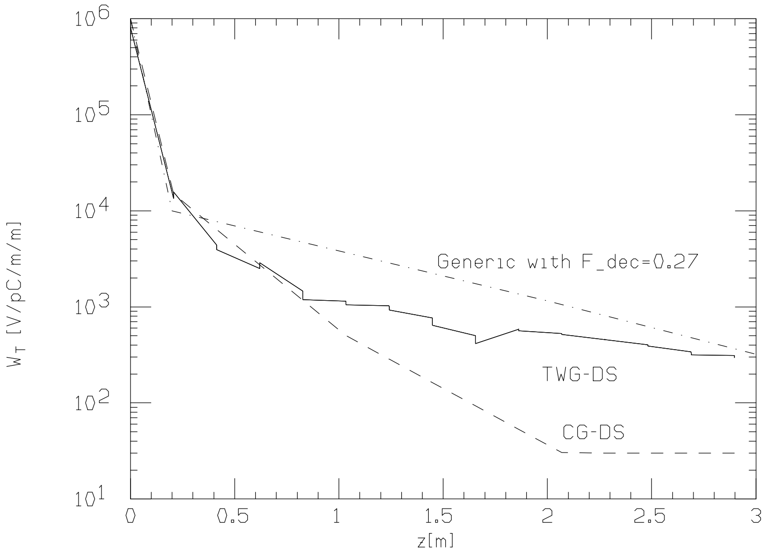

Figure 1: Comparison of long-range wakefield for various structure models

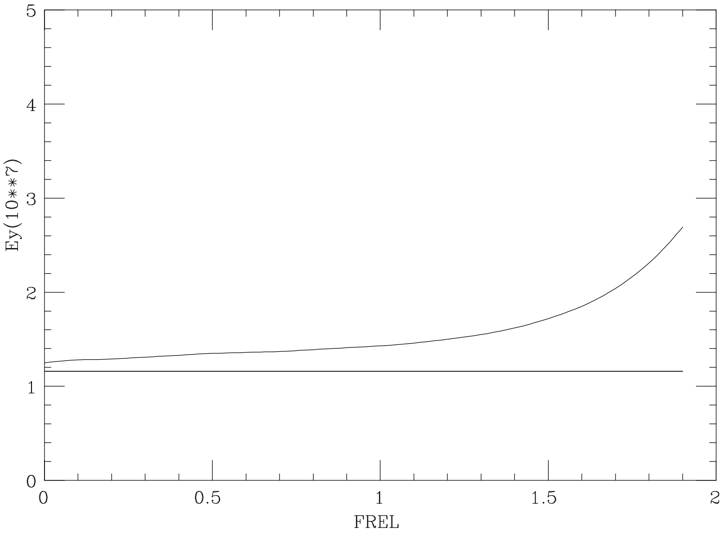

Figure 2: Vertical emittance dependence on wakefield level at bunch 2, in CG-DS model.

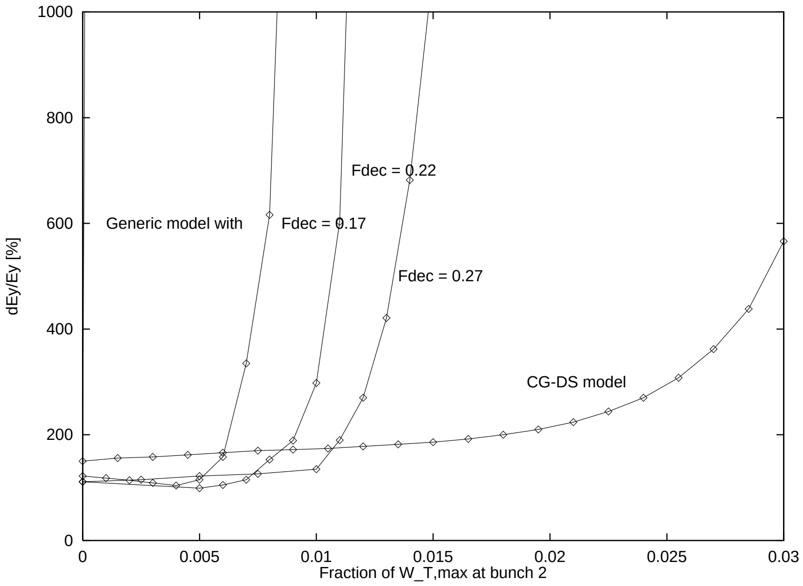

Figure 3: Comparison of emittance blow-up dependence on wakefield level at bunch 2, in both the generic and CG-DS models.

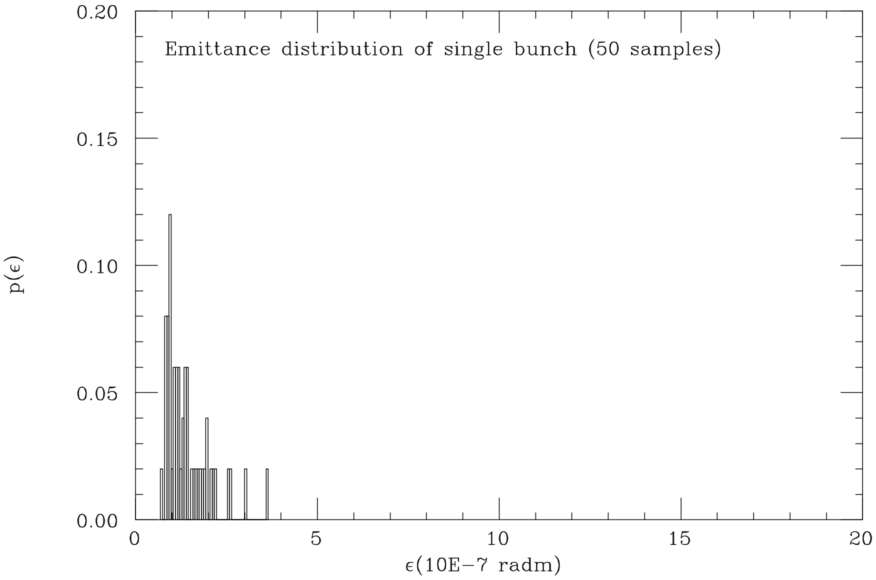

Figure 4: Histogram of the single bunch emittance.

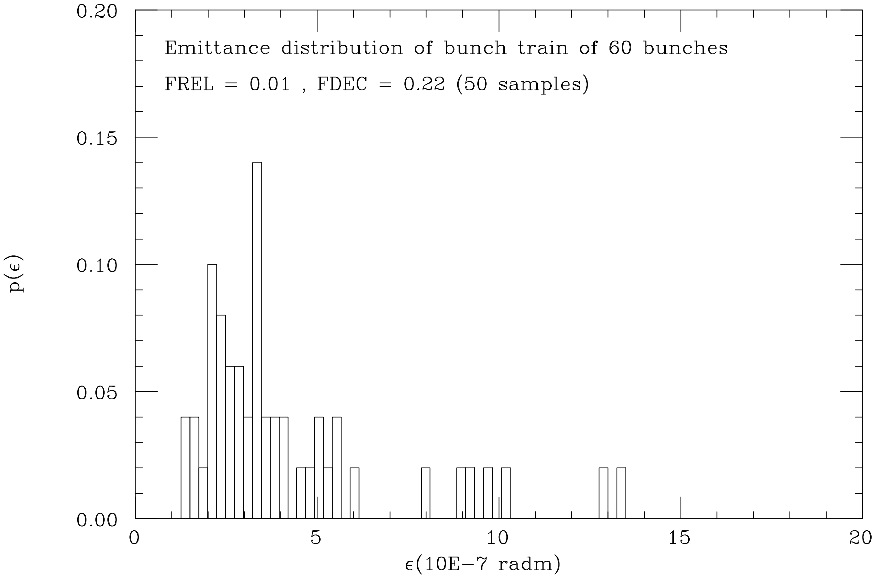

Figure 5: Histogram of the bunch-train emittance, in the simplified model.

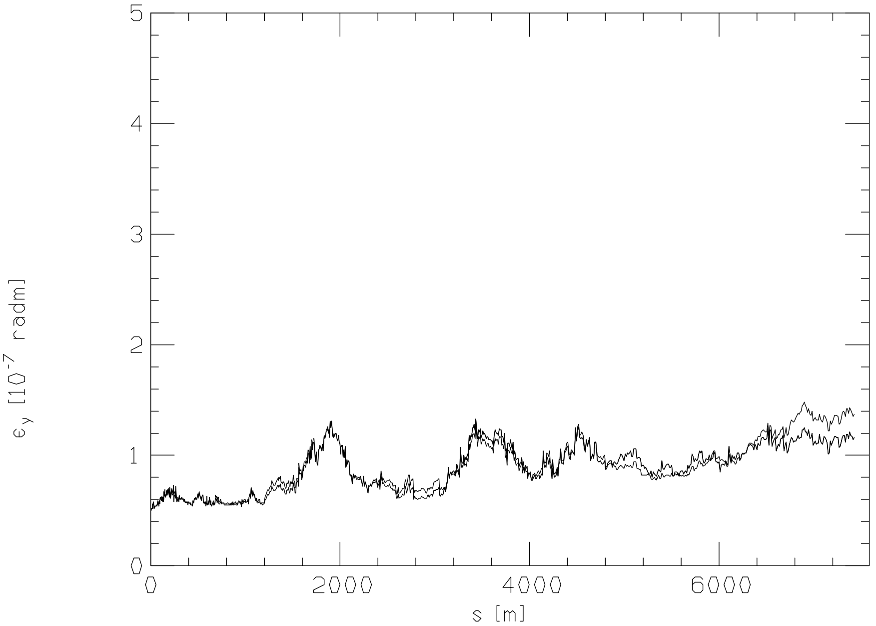

Figure 6: Emittance growth along the linac for bunch 1 and the train, with $f_{rel} = 0.7$ in the CG-DS model.

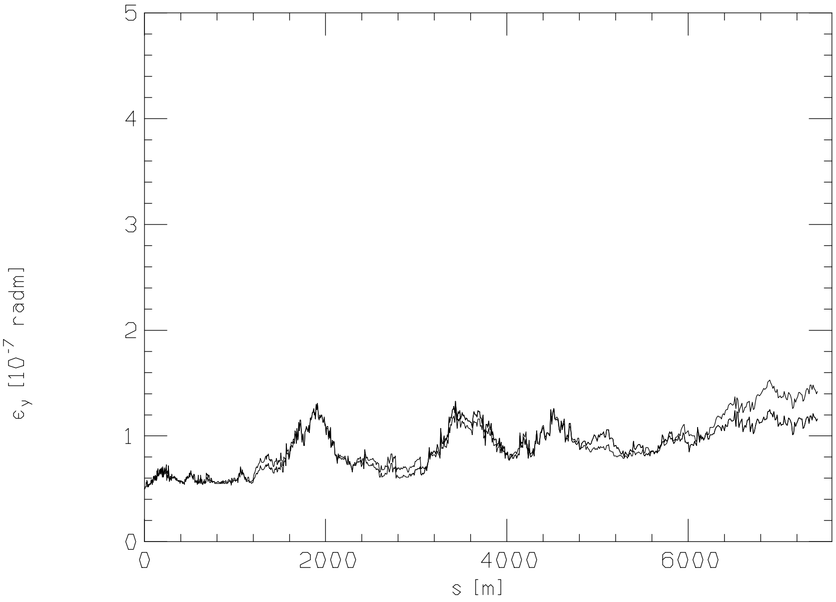

Figure 7: Emittance growth along the linac for bunch 1 and the train, with $f_{rel} = 1.0$ in the CG-DS model.

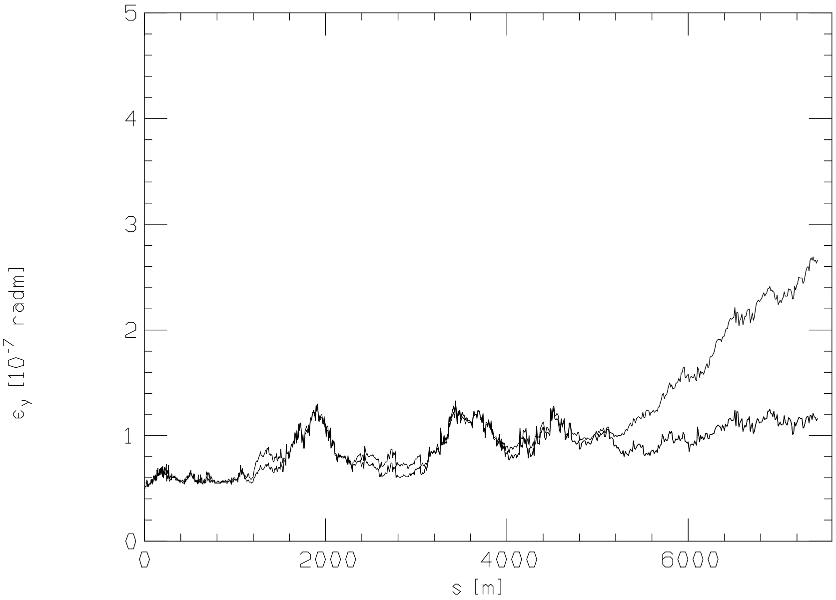

Figure 8: Emittance growth along the linac for bunch 1 and the train, with $f_{rel} = 2.0$ in the CG-DS model.

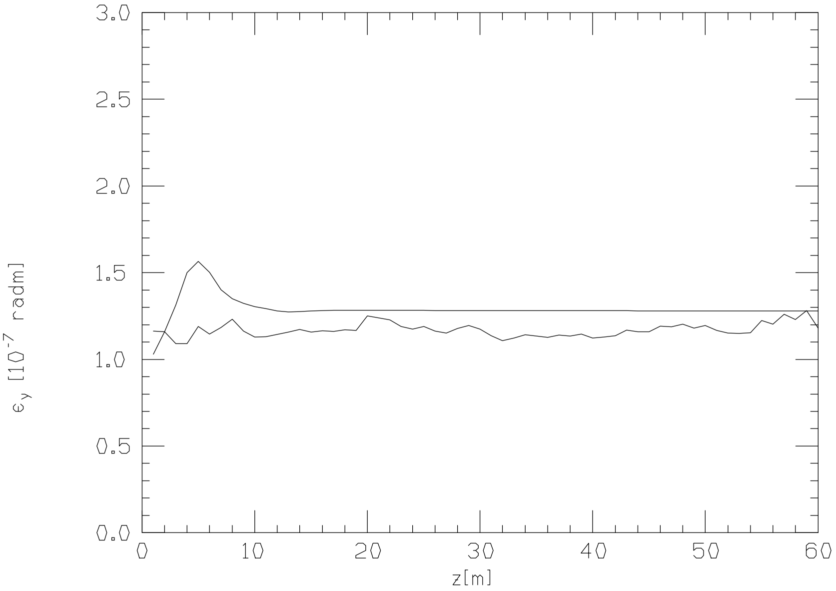

Figure 9: Emittance growth at the linac end for all the bunches in the train, for the structures TWG-DS and CG-DS, with $f_{rel} = 2.0$.

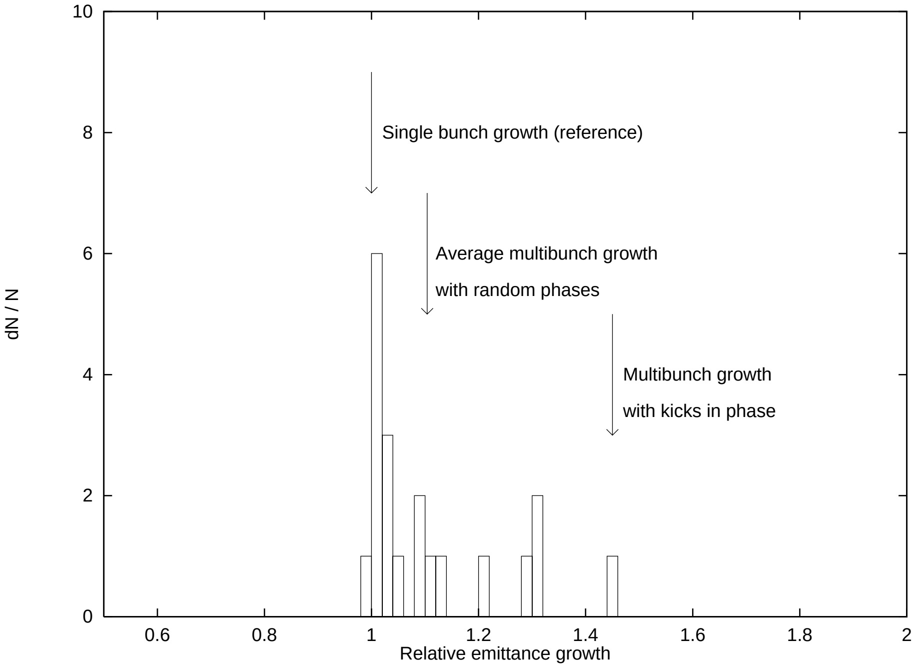

Figure 10: Emittance growth distribution with random phases of the long-range field, in the CG-DS model.