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Normal Form Analysis of the LHC Dynamic Aperture

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Abstract

Normal form is a well developed tool to study the nonlinear dynamics of even the most complicated structures. This can become particularly useful in the design of large nonlinear hadron colliders like the LHC. In this study the correlation of various quality factors derived through normal forms with the dynamic aperture are used to investigate a machine that is dominated by octupolar errors.

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1 Introduction

Almost ten years ago normal form [1–5] has been introduced to the accelerator community. Although this technique allows an in–depth analysis of complicated structures like the Large Hadron Collider (LHC), the acceptance of this tool has been rather slow. This is mainly due to the rather abstract character of the theory; indeed, we believe that this is also due to the lack of easy-to-use tools with a simple interface with the existing codes. Normal forms codes written by independent groups (i.e., the DaLie code by Forest [6] and the ARES code by Bazzani et al. [7, 8]) have been used at CERN to analyse the nonlinear dynamics in the LHC [9, 10]. Recently, the NERO program [11] has been developed on the basis of well-tested existing tools [8, 12], in order to allow an easy analysis of resonances (including contributions from higher orders) and a simple interface with tracking codes. The user-friendliness is ensured by allowing the analysis of internally or externally produced maps and by providing an elaborate user's manual [13]. For the time being the program reads maps as produced by SIXTRACK [14] which uses the differential algebra package of Berz [3] (on request the program can be adapted to any other map format).

The main advantage of the analysis based on maps and perturbative tools that can be carried out through the NERO code or similar tools is the possibility of analysing the correlations of the dynamic aperture with some quantities, called quality factors, that can be computed in a relatively easy way. It is well-known that in each lattice the dynamic aperture is dominated by some effects, and therefore some quality factors can be well-correlated and some not. Indeed, the correlation of a given quality factor indicates what is the dominant effect, and therefore suggests a way to correct it.

Both NERO and the DaLie codes have been used in this report to study 60 different realizations of the errors of the LHC version 4. In the first section the quality of the correlation of the dynamic aperture with two simple quality factors are tested. In the second section more sophisticated tools based on the computation of the detuning and on the resonance strength are used. Results about the correlation of resonance norms including higher order contributions are presented in the third section.

It has to be mentioned that for all studies with NERO the maximum amplitude where the quality factors are evaluated is fixed to a value of eight beam sigmas which is close to the lower end of the dynamic aperture. It must be pointed out that one has to select an amplitude in order to weight the contributions of the higher orders with respect to the first one: if one restricts to the analysis of the first order, no amplitude is necessary.

2 Simple quality factors

The dynamic aperture of LHC version 4 is dominated by just two values [15], i.e. the octupole bias b_4 and a_4 of the main dipoles which are caused by fabrication tolerances. In that report a correlation of the dynamic aperture has been found with a octupole strength norm

$$\sqrt{\frac{{b_4}^2 + {a_4}^2}{2}}. \tag{1}$$

In Tab.1 the correlation coefficient $\mathbf{R}^2$ of this case is denoted by 'I'. Although the correlation is not bad we have hoped to get an improvement by a more refined study. Moreover, one may need something else as the knowledge of the multipolar errors may not be available with sufficient precision to power the correction circuits.

A very simple quality factor can be defined as the norm of the nonlinear part of the map, that gives a rough indication about the strength of the nonlinearities in a given mapping. It is defined as the sum of the absolute values of all map coefficients $|F_{i,n}|$ relative to the order n, weighted with the power of the amplitude A (see Ref. [10]):

$$\sum_n A^n \cdot \left(\sum_i |F_{i,n}| \right). \tag{2}$$

Fig. 1 and 2 show the correlation of the map norm with the dynamic aperture. While at order 2 there is no correlation, at order three, where the first-order contribution of the octupoles is present, the correlation is good, and slightly increases up to order seven. Both map norms up to order three and up to order seven feature a correlation that is slightly better than the octupole strength norm (0.63 and 0.64 respectively, see Table 1). In fact, we also evaluated the correlation of the map norm up to order three with the octupole norm, finding a good correlation as expected (see Fig. 3). This also implies that the sextupoles which also feed the map coefficients of order three can be neglected.

After the large octupolar bias values are corrected, the map norm is practically not correlated any more ($\mathbf{R}^2 = 0.12$) with the improved dynamic aperture. This means that after the correction the dynamic aperture is not dominated by the large octupolar terms as before, but is determined by more intricate effects that become relevant when the octupole bias is wiped out.

As the map norm is excellently correlated with the octupole strength norm but also well correlated with the dynamic aperture it can be used to correct the octupole effect. To this end a map has been produced with the strength of each of the b_4 and a_4 spool piece correctors as a parameter. The third order coefficients and thereby the third order map norm are therefore know as a function of the two correctors. Using the first order terms only, the corrector strengths to zero the map norm can easily be predicted. In Fig. 4 the original bias of the b_4 coefficients of the 60 error representations is compared with the resulting b_4 bias after the map norm cancellation with spool pieces. The improvement of dynamic aperture after such a correction has not been tested but it will probably be very beneficial given the good correlations between the map norm and the dynamic aperture. The possibility to predict a correction for a dominant multipole coefficient after installation of the machine is very attractive, in particular as not all magnets may have been measured. The prerequisites are a sufficient large number of correctors installed in the machine and some means to measure the map. One possible candidate to measure those map coefficients is the recently proposed method to derive them from the spectrum lines of a kicked pencil beam [16].

3 Tune Norm and First Order Resonances

The previous quality factors are based on the simple inspection of the octupole strength and map coefficients respectively. Normal forms provide a perturbative way to compute both detuning and resonances. Nonresonant normal form provide the tune as a function of amplitude, and one can compute the average of the tune at a fixed amplitude A (that has the same meaning as before) in a analytical way, by using the perturbative series. We refer to [10] for more details.

It is now rather well understood that the tuneshift, as any other quality factor, is not *a priori* correlated with the dynamic aperture. Indeed, it strongly depends on the particular lattice if the dynamic aperture is dominated by the detuning, or by some resonance, or simply by the norm of the map, as it seems in this case. The correlation of the detuning with the dynamic aperture, even though it is increasing with the order, is always worse than the correlation with the norm of the map (case III in Tab. 1). This provides some relevant information: the detuning is not the driving mechanism of the dynamic aperture.

Even more interesting is the study of resonances. In parallel to this work, first order octupole resonances [1)] of the same case have been found to be correlated with the dynamic aperture [17]. The authors have discovered that the linear multi-variant fit of the three largest resonances, i.e. the (2 –2), the (3 1) resonance and the first order sub-resonance (1 –1) [18] lead to a correlation factor of $\mathbf{R} = 0.8$. With the DaLie program we have calculated the same cases except that we use the Hamiltonian H instead of the conjugating function F (both in resonance basis). Although the results should be equivalent to good approximation we prefer the Hamiltonian as it is the starting point for the higher order study to follow below. Moreover, the Hamiltonian reproduces the same phases that one expects from first order calculations à la Guignard [18]. For the linear multi-variant fit we use the $LINEST$ program of $EXCEl$ and of course the results agree with those of Ref. [17] (see part 'IV' in Tab. 1.

4 Higher Order Resonances

The NERO program mainly aims at resonance at higher orders. The resonant perturbative formalism is carried out at arbitrary order for generic mappings. A norm of the resonant part of the interpolating Hamiltonian is worked out. When only the first order contribution is considered, this corresponds to the standard first order perturbative theory based on Hamiltonians [18]. One of the main advantages of the code is that it allows to determine whether the higher orders are relevant or not, thus providing relevant information on the different effects that contribute to the determination of the dynamic aperture.

For example, the correlation between the norm of the (3 1) resonance and the dynamic aperture is shown in Fig. 5 up to order three, five and seven. The correlation coefficient $\mathbf{R}^2$ jumps by some factor of four comparing order seven with three: the higher orders seem to be rather relevant in this case. The figure shows that this improvement is due to a de–population of the lower left corner of the figure. This procedure has also been performed for the (2 –2) resonance. Also in this case we have a correlation improvement, even though it is less pronounced. Nevertheless, with two resonances only we have achieved a correlation coefficient better than 0.5 at order seven (see part 'V' in Tab. 1). Of course one would have to check if the addition of the the (1 –1) resonance would further improve the correlation quality. Unfortunately resonances of order two such as the (1 –1) are not yet considered in the code which analyses presently only resonances of order greater or equal to three.

5 Conclusion

Various norms have found to be correlated with the dynamic aperture of 60 error representations of the LHC lattice version 4. The octupole strength norm, the map norm and the linear multi-variant fit of three first order resonances all have a correlation coefficient $\mathbf{R}^2$ slightly above 0.6. There are also indications that the higher order analysis of resonances may improve the correlation quality. As the map norm is the simplest and may be obtained in the easiest way it appears to be the best choice for this lattice lay-out. Moreover, it has been shown that it can be used to predict the correction strength for the bias of large b_4 and a_4. The usefulness of this correction will have to be shown experimentally.

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[1)] Of course these resonance are also driven by sextupoles in second order. For our purpose here there is no need to separate the effects because the sextupole contributions are small following the argument of above.

References

[1] A. Bazzani et al. “Normal forms for Hamiltonian maps and nonlinear effects in a particle accelerator”, Nuovo Cim., B **102**, pp. 51–80 (1988).

[2] M. Berz, É. Forest and J. Irwin, “Normal form methods for complicated periodic systems: a complete solution using differential algebra and lie operators”, Part. Acc. **24**, pp. 91–107 (1989).

[3] M. Berz,“Differential-algebraic description of beam dynamics to very high orders”, Part. Acc. **24**, pp. 109–124 (1989).

[4] A. Bazzani et al. , “Resonant normal forms, interpolating Hamiltonians and stability analysis of area preserving maps”, Physica D **64**, pp. 66–93 (1993).

[5] A. Bazzani, E. Todesco, G. Turchetti and G. Servizi, “A normal form approach to the theory of nonlinear betatronic motion”, CERN 94–02.

[6] É. Forest, written in 1986, private communication.

[7] G. Servizi and G. Turchetti, “A computer program for the Birkhoff series for area preserving maps”, Comput. Phys. Commun., **32** (1984) 201–7.

[8] A. Bazzani, M. Giovannozzi, E. Todesco, “A program to compute Birkhoff normal forms of symplectic maps in $\mathbf{R}^4$”, Comp. Phys. Comm., **86** (1995) pp. 199–207.

[9] W. Scandale, F. Schmidt, E. Todesco, “Compensation of the tune shift in the LHC, using the normal forms techniques”, Part. Accel. **35** (1991) pp. 53-81.

[10] M. Giovannozzi, R. Grassi, W. Scandale, E. Todesco, “Sorting approach to magnetic random errors”, Phys. Rev. E **52**, pp. 3093–101 (1995).

[11] F. Schmidt and E. Todesco, “Evaluating high order resonances using resonant normal forms”, in the proceedings of the the Fifth EPAC conference, Sitges, June 1996, http://wwwslap.cern.ch/frs/report/MOP034L.ps.Z.

[12] M. Giovannozzi et al. “PLATO: a program library for the analysis of nonlinear betatronic motion”, CERN PS (PA) 96–12 (1996), submitted to Nuclear Instruments and Methods.

[13] E. Todesco, M. Gemmi and M. Giovannozzi, “NERO: a code for the Nonlinear Evaluation of Resonances in One-turn mappings”, Comp. Phys. Commun., in press; documentation is also available at: /afs/cern.ch/project/lhcnap/nero.

[14] F. Schmidt, “SIXTRACK, version 1.2, single particle tracking code treating transverse motion with synchrotron oscillations in a symplectic manner”, CERN SL/94–56 (AP) (1994), http://wwwslap.cern.ch/frs/Documentation/doc.html.

[15] M. Böge, H. Grote, Q. Qin and F. Schmidt, “Dynamic Aperture Studies for the LHC Version 4”, LHC Project Report 31, in the proceedings of the Fifth EPAC Conference, Sitges, June 1996, http://wwwslap.cern.ch/frs/report/MOP033L.ps.Z.

[16] R. Bartolini and F. Schmidt, “Evaluation of Non-Linear Phase Space Distortions via Frequency Analysis”, LHC Project Report 98 and in the proceedings of the workshop on: “Nonlinear and Collective Phenomena in Beam Physics”, Arcidosso, September 1996, http://wwwslap.cern.ch/frs/report/arcif.ps.Z.

[17] Y. Cai and J.P. Koutchouk, “Correlations of dynamic aperture and resonance strengths for the LHC version 4.3 lattice”, LHC Project Note 75.

[18] G. Guignard, “A general treatment of resonances in accelerators”, CERN 78–11 (1978).

Table 1: *Correlation coefficient* $\mathbf{R}^2$ *for all studied cases*

#	**Case**	**Description**	**Correlation Coefficient** R^2				
				Map Order			
				2	**3**	**5**	**7**
I	Octupole Strength Norm		0.61				
II				$3 \cdot 10^{-4}$			
IIa	Map				0.63		
IIb	Norm						0.64
IIc		after Correction					0.12
IIIa	Tune				0.25		
IIIb	Norm					0.42	
IIIc							0.41
IVa	First	(2 –2)			0.25		
IVb	Order	(3 1)			0.12		
IVs	Resonance	(1 –1)			0.22		
IVd	Strength	Combination			0.64		
Va	High	(2 –2)			0.25	0.25	0.39
Vb	Order	(3 1)			0.12	0.23	0.46
Vc	Resonances	Combination			0.33	0.42	0.52

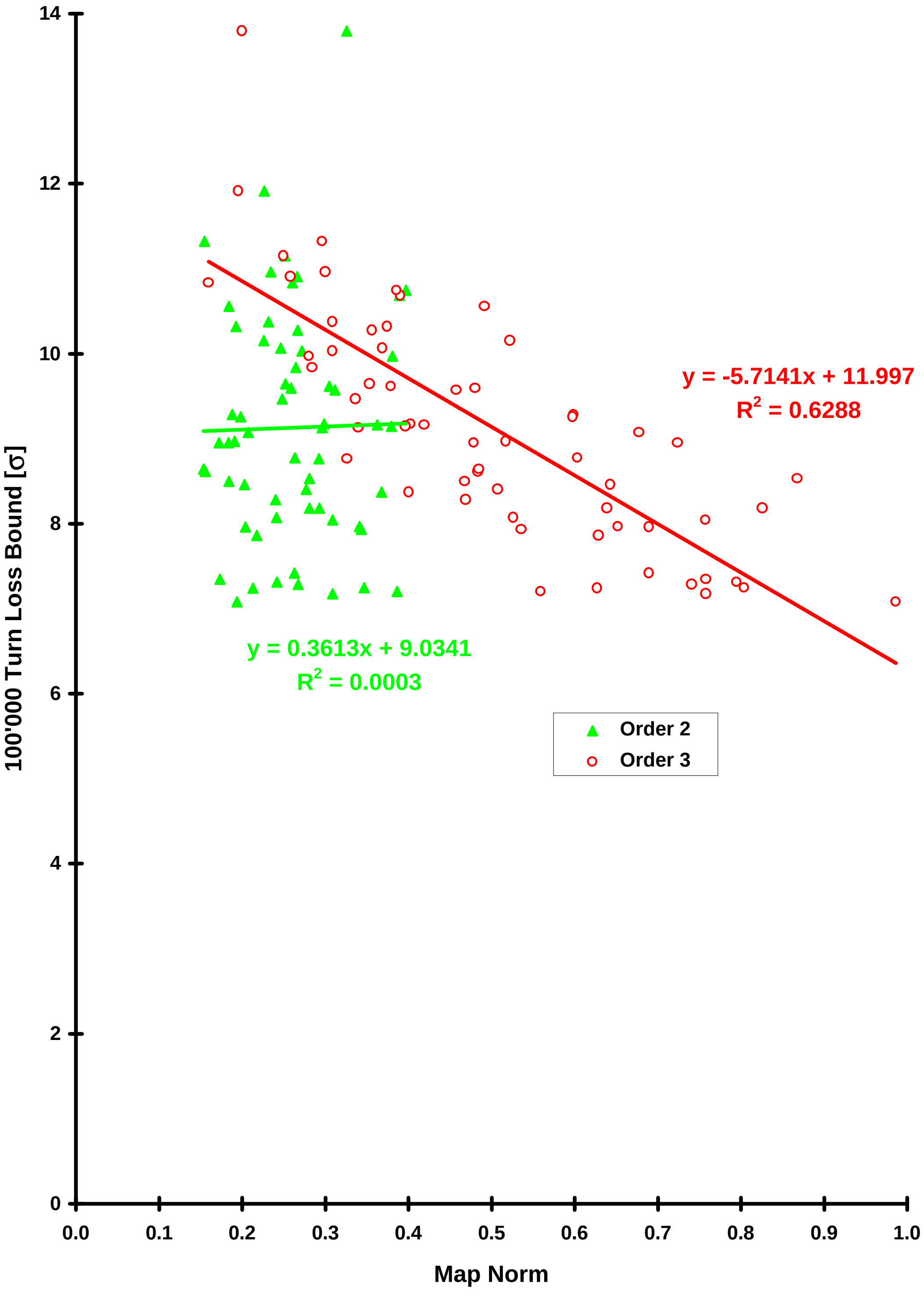

Figure 1: *Correlation of the dynamic aperture with the map norm of order two and three*

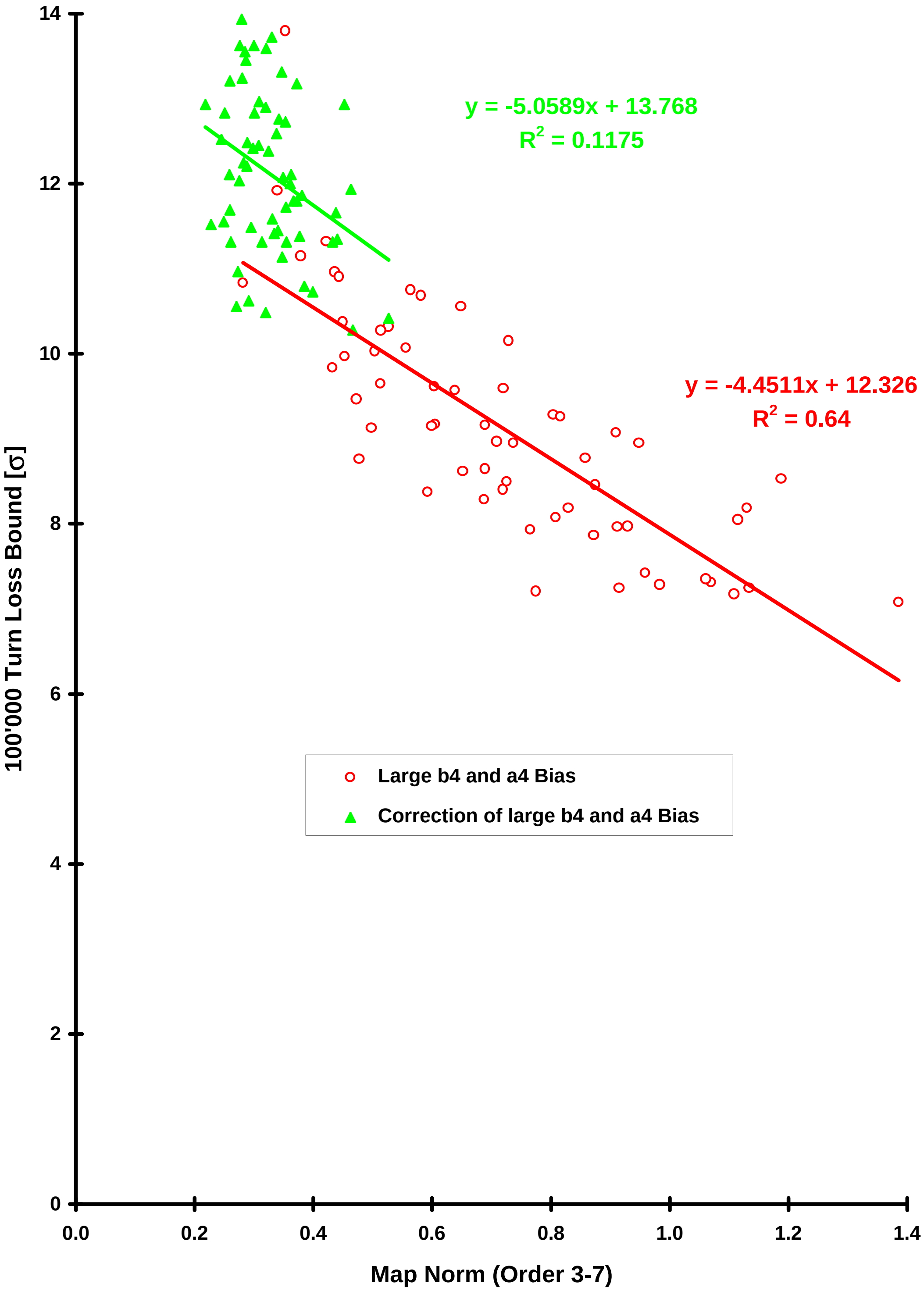

Figure 2: *Correlation of the dynamic aperture with the map norm (order three through seven) before and after the correction of the large* b_4 *and* a_4 *bias*

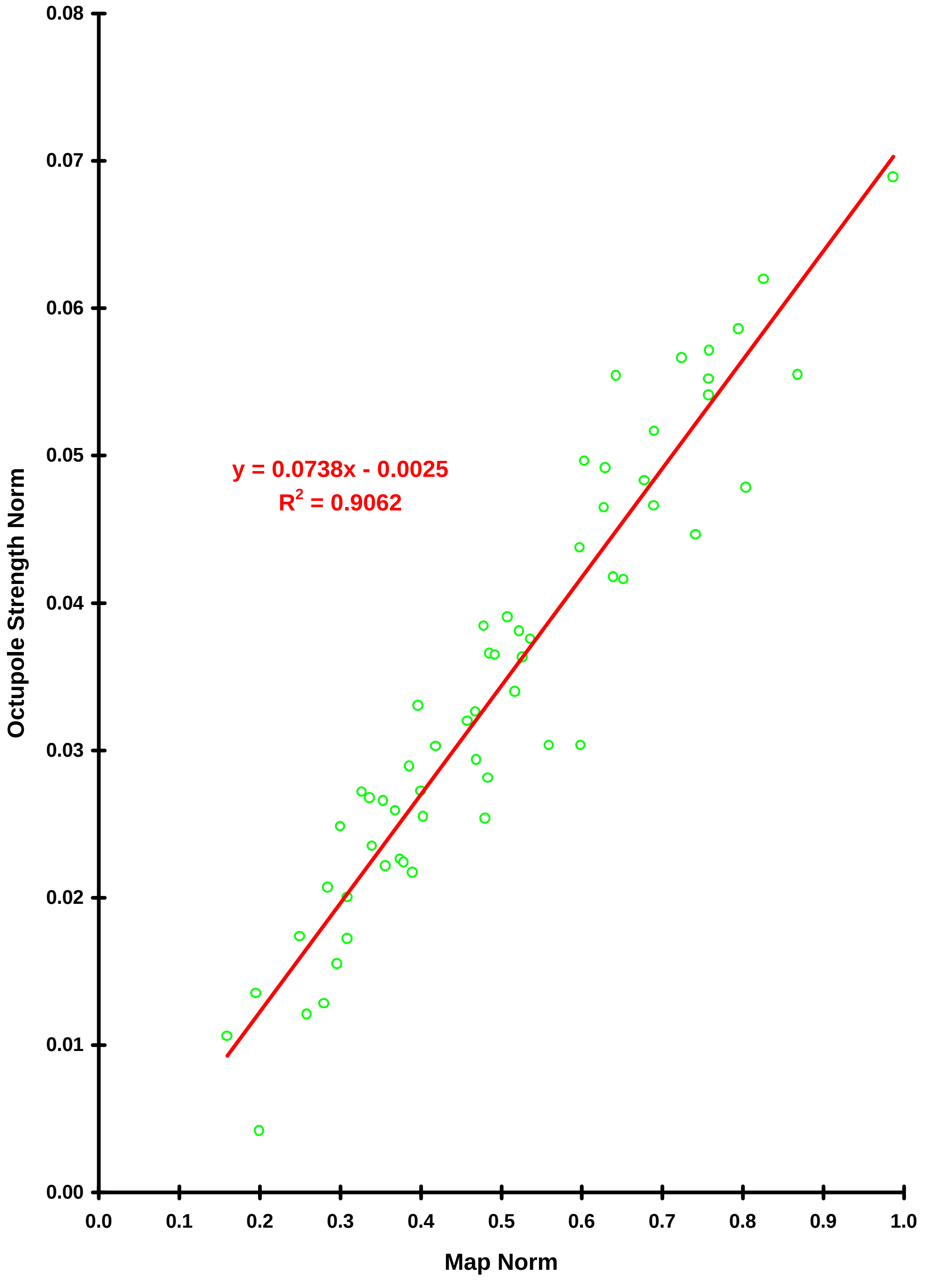

Figure 3: *Correlation of the octupole strength norm with the map norm (order three)*

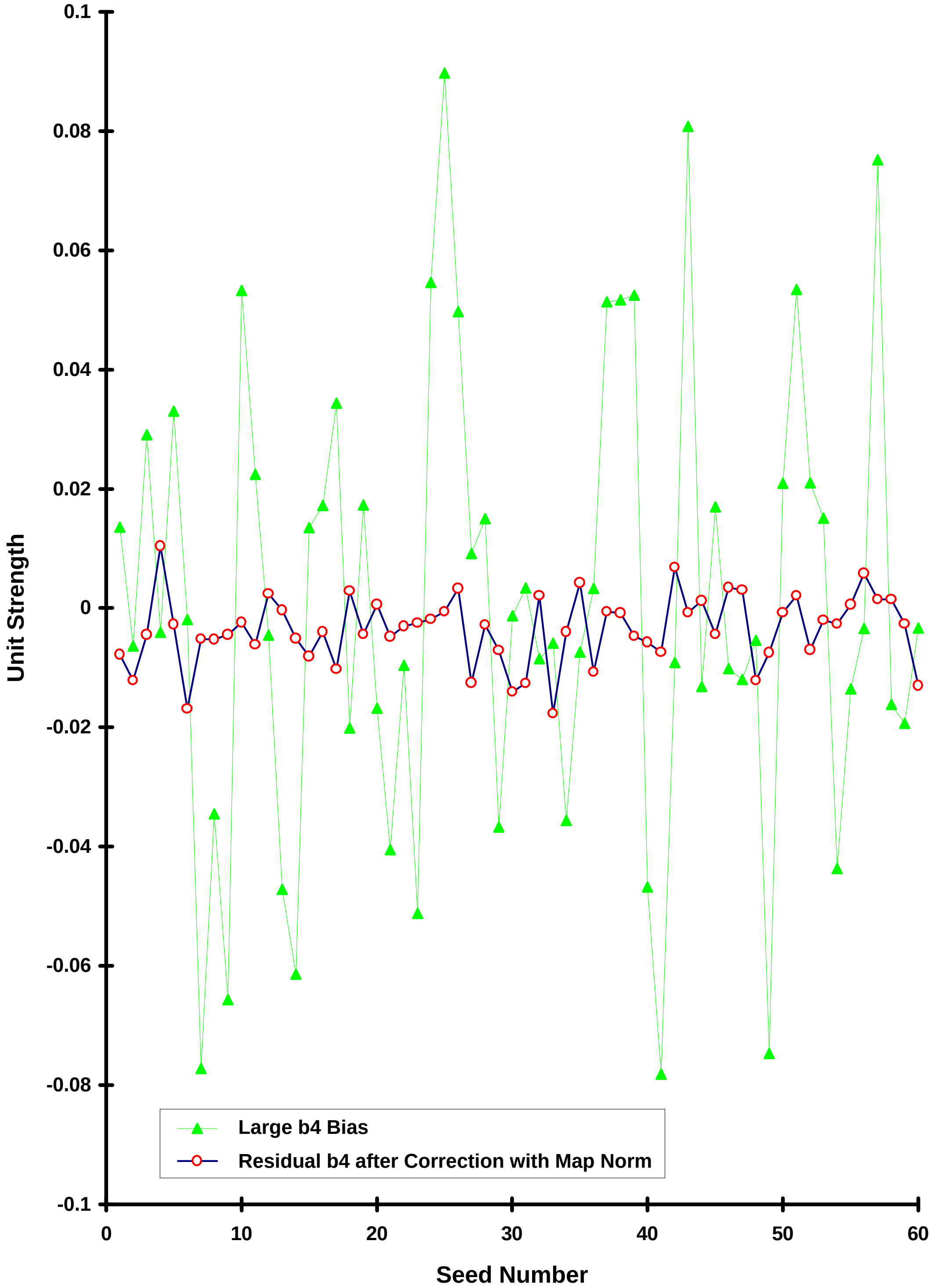

Figure 4: *Correction of the b4 bias using the map norm*

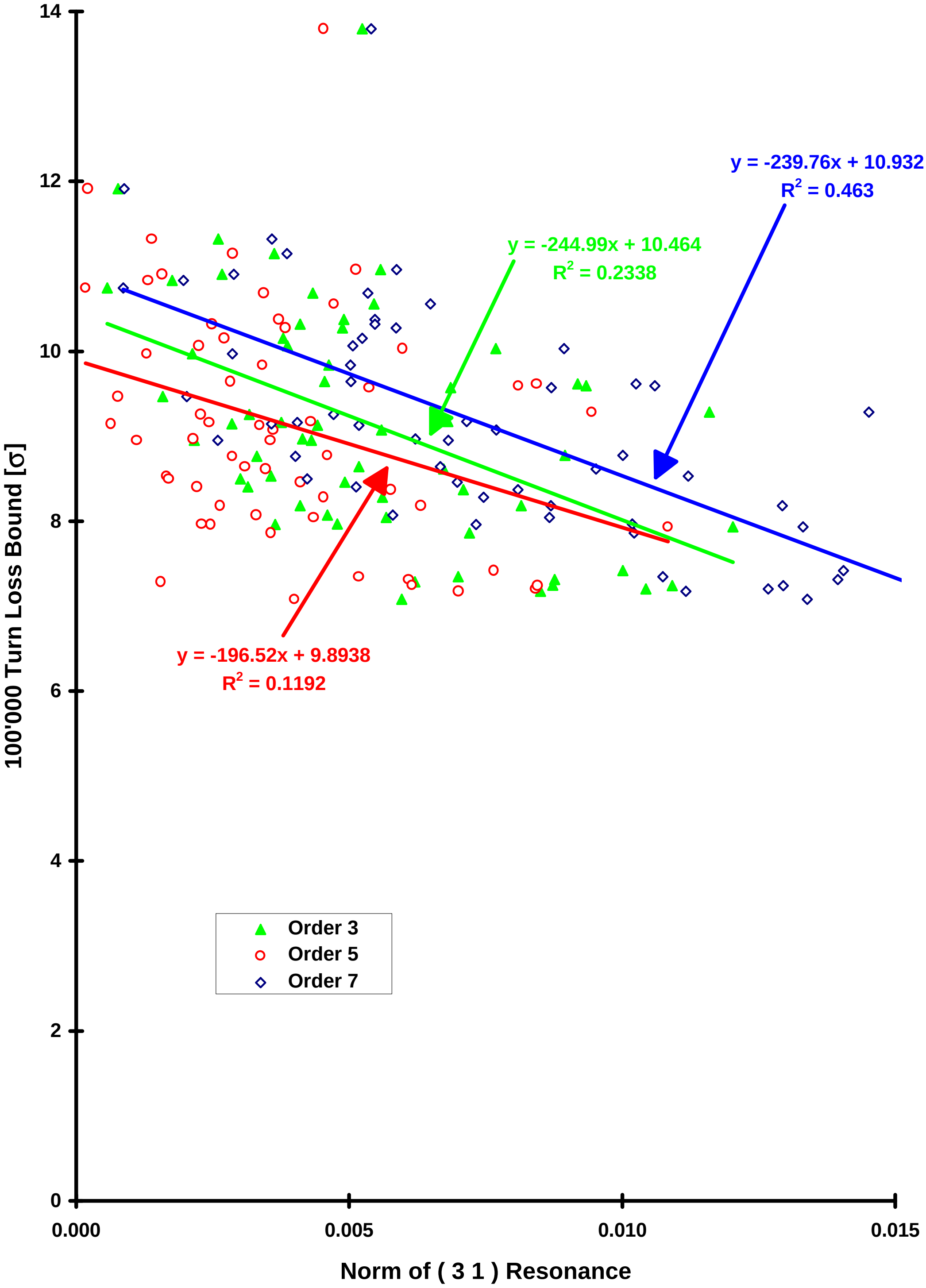

Figure 5: *Correlation of the dynamic aperture with the resonance norm for the (3 1) resonance between order three and seven*