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# Softly Broken MQCD and the Theta Angle

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**Abstract:** We consider a family of non-supersymmetric MQCD five-brane configurations introduced by Witten, and discuss the dependence of the curves on the microscopic theta angle and its relation with CP. We find evidence for a non-trivial spectral flow of the curves (vacua) and for the level-crossing of adjacent curves at a particular value of the theta angle, with spontaneous breaking of CP symmetry, providing an MQCD analogue of the phase transitions in theta proposed by 't Hooft.

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The modelling of gauge dynamics via brane configurations of weakly coupled string theory [1,2,3,4], or M-theory [5,6,7,8,9], provides a geometrical interpretation of various field theoretical strong coupling phenomena. In some cases, the geometrical viewpoint can be used to discover new exact solutions [5] of $N = 2$ theories. In the context of $N = 1$ theories, it provides a semiclassical approach to such thorny problems as confinement and chiral symmetry breaking [6,7]. These constructions are similar in spirit to previous work on geometric engineering of gauge theories (see for example [10]). Here the role of a non-trivial string compactification is played by some complicated brane configuration sitting in flat space at small string coupling, and carefully adjusted to provide a weakly coupled supersymmetric Yang–Mills theory at intermediate scales. Since the gauge theory lives on the branes world-volume, one can relate many classical and semiclassical features of gauge theories to some properties of brane dynamics. The real power of the method arises when taking the strong coupling limit of the brane configuration. If a strong coupling dual is available, one can describe many non-perturbative, infrared properties of the gauge theory, just by reading the tree-level data of the dual brane configuration.

In four-dimensional models one uses the duality between type IIA strings and M-theory [5]. Here one maps the type IIA brane configuration to a single smooth five-brane, whose world-volume is appropriately embedded in the flat eleven-dimensional background, as a product $M_4 \times \Sigma$, with $M_4$ the four-dimensional Minkowski space, and $\Sigma$ a holomorphic curve with respect to a given complex structure of the background. In general, the detailed physics of the resulting M-theory model differs from the Yang–Mills theory we are interested in; however, to the extent that some observables are protected by supersymmetry, we can calculate them in the deformed strong coupling model. This is the case, for example, of all holomorphic quantities determined by the Seiberg–Witten curve in $N = 2$ models [5], including BPS spectra [11]. For $N = 1$ models the agreement is only qualitative in principle, but some observables can be accurately matched, like superpotentials and gaugino condensates [6,8].

In the absence of some unbroken supersymmetry on the world-volume, we cannot use holomorphy to accurately match observables, and a working assumption must be made that no phase transitions occur in the way to strong coupling. Still, qualitative features based on topological properties can be established [6,7].

In this note we study some qualitative features of a class of non-supersymmetric brane configurations. In particular, using selection rules provided by the symmetries, we present evidence for a non-trivial structure of the spectral flow with respect to the microscopic theta parameter, $\theta$, with the associated phase transitions as suggested by 't Hooft in the field theory context [12]. Reviews of the relevant facts regarding the so-called "theta puzzle", and its connection with the Veneziano-Witten formula, can be found in [13], [14]. For related discussions of non-supersymmetric brane configurations, see [9,15,16].

In the non-supersymmetric case, the criterion of perturbative stability of the five-brane background becomes simply the extremality with respect to local deformations of the embedding. Namely, for a given set of asymptotic boundary conditions, the five-brane world-volume is embedded in eleven-dimensional space as $M_4 \times \Sigma$, where now $\Sigma$ is a two-dimensional surface of "minimal area". This surface being infinite in the flat eleven-dimensional metric, the requirement is really stability with respect to small local deformations.

Part of the asymptotic data are easily identifiable in the weak coupling limit of the brane configuration, in terms of intersecting branes of type IIA string theory. Starting with a type IIA $N = 2$ configuration consisting of a pair of parallel NS5-branes on the $x^4, x^5$ plane, and a set of $n$ D4-branes stretched in between along the $x^6$ direction, we shall consider arbitrary rotations of one of the NS5-branes into the four-dimensional space parametrized by $x^4, x^5, x^8, x^9$. The corresponding rotations form a coset $\frac{SO(4)}{SO(2)\times SO(2)}$, parametrized by four angles. In the complex structure $z = x^4 + ix^8$, $z' = x^5 + ix^9$, a particular $SU(2)$ rotation of the form $e^{i\alpha}z$, $e^{-i\alpha}z'$ preserves $N = 1$ supersymmetry in the $M_4$ space-time [17], the angle of rotation being related to the mass that splits the $N = 2$ vector multiplet into a massless $N = 1$ vector multiplet and a massive chiral multiplet in the adjoint representation. The rest of the parameters should be associated to supersymmetry-breaking masses for the gauginos, and supersymmetry-breaking mass splittings between the adjoint scalars and matter fermions. On the other hand, the bare Yang–Mills coupling is related to the separation of the NS5-branes along $x^6$. One coupling parameter, which is not easily identifiable in the type IIA picture, is the bare theta angle. One needs to switch on non-perturbative corrections to the classical brane geometry in order to be sensitive to the value of theta [18]. For applications involving $N = 2$ or $N = 1$ supersymmetric backgrounds, the actual value of the microscopic theta parameter is irrelevant, since there are anomalous $U(1)$ symmetries in those cases.

Witten has shown how these branes configurations can be lifted to M-theory. In the representation of the five-brane world-volume as $M_4 \times \Sigma$, one first identifies $\Sigma$ with the complex $z$ plane, with $z = 0$ and $z = \infty$ associated to the two asymptotic regions of the NS5-branes. Then, a "minimal area" embedding $\vec{X}(z, \overline{z})$ is characterized by harmonic functions with a vanishing two-dimensional energy-momentum tensor [1]

$$T_{zz} = g_{ij} \partial_z X^i \partial_z X^j = 0, \tag{1}$$

where $g_{ij}$ is the background metric in eleven dimensions, in M-theory units:

$$ds^2 = \sum_{i,j=0}^{9} \eta_{ij} dx^i dx^j + R^2 (dx^{10})^2. \tag{2}$$

Witten configurations in parametric form are, in the $\vec{X} = (x^4, x^5, x^8, x^9)$ space:

$$\vec{X}(z, \overline{z}) = \mathrm{Re}(\vec{p}\, z + \vec{q}\, z^{-1}). \tag{3}$$

The complex vectors $\vec{p}, \vec{q}$ define the asymptotic orientation of the NS5-branes in the weak coupling limit: the region with $z \to 0$ corresponds to the "left" NS5-brane, while the region with $z \to \infty$ leads to the "right" NS5-brane. In addition, the five-brane configuration wraps $n$ times the compact circle of radius $R$. Choosing an angular variable in which the five-brane wraps rigidly we have

$$x^{10} = -n \,\mathrm{Im}\, (\log z). \tag{4}$$

Finally, the profile of the five-brane in the $x^6$ direction is parametrized as

$$x^6 = -R\, n \,\mathrm{Re}\, (c \log z) \tag{5}$$

with $c$ a real constant.

The leading terms as $z \to 0$ and $z \to \infty$ of the vacuum equations (1) are $\vec{p}^{\,2} = 0$ and $\vec{q}^{\,2} = 0$, respectively. In addition, a subleading term in (1) relates $c$ to the asymptotic vectors:

$$\vec{p} \cdot \vec{q} = \frac{R^2 n^2}{2} (c^2 - 1). \tag{6}$$

Notice that (6) makes sense for complex $c$, since the vectors $\vec{p}$ and $\vec{q}$ are complex. However, if $\mathrm{Im}(c) \neq 0$ in (5), the embedding becomes multivalued in the $x^6$ direction. Since $x^6$ is non-compact, a continuous embedding of $\Sigma$ requires $\mathrm{Im}(c) = 0$. For notational convenience,

[1] The mixed components $T_{z\overline{z}}$ vanish because of the two-dimensional classical Weyl invariance.

we find it useful to work with the extended family of embeddings with a general complex $c$. The reader should consider, however, that only the $\mathrm{Im}(c) = 0$ family has a natural physical interpretation.

Using the isometries of the metric (2) we may rotate the left NS5-brane, given by $\vec{X} = \mathrm{Re}(\vec{p}\, z)$, such that it lies along the $x^4, x^5$ plane. Furthermore, rescaling $z$, we can bring $\mathrm{Re}\,\vec{p}$ and $\mathrm{Im}\,\vec{p}$ to unit vectors, so that $\vec{p}^{\,2} = 0$ implies $\mathrm{Re}\,\vec{p} \cdot \mathrm{Im}\,\vec{p} = 0$. Up to parity transformations in the $x^4, x^5$ plane, we may then completely fix the first vector to, say, $\vec{p} = (1, -i, 0, 0)$.

A convenient parametrization of supersymmetry breaking arises when using the complex structure $v = x^4 + ix^5$, $w = x^8 + ix^9$, $t = e^{-s}$, $s = R^{-1}x^6 + ix^{10}$, in terms of which the relevant part of the background metric becomes

$$ds^2 = |dv|^2 + |dw|^2 + R^2 \frac{|dt|^2}{|t|^2}. \tag{7}$$

In these variables, a subgroup $U(2) \times U(1)$ of the compact "internal" isometry group $O(5) \times U(1)$ is manifest in terms of complex rotations of $(v, w)$, and phase redefinitions of $t$. In addition we also have the discrete complex conjugation symmetries of all variables, and inversions of $t$. If we parametrize the "right" NS5-brane vector $\vec{q}$ as

$$\vec{q} = (\eta + \varepsilon, -i\eta + i\varepsilon, \zeta + \lambda, -i\zeta + i\lambda), \tag{8}$$

the null condition $\vec{q}^{\,2} = 0$ translates into

$$\eta\varepsilon + \zeta\lambda = 0. \tag{9}$$

Then, the most general $N = 0$ curve takes the simple form:

$$\begin{aligned} v &= z + \frac{\eta}{z} + \frac{\overline{\varepsilon}}{\overline{z}} \\ w &= \frac{\zeta}{z} + \frac{\overline{\lambda}}{\overline{z}} \\ t &= z^{n(c+1)/2}\, \overline{z}^{n(\overline{c}-1)/2} \end{aligned} \tag{10}$$

and is specified by three complex parameters (for example $\eta$, $\zeta$ and $\varepsilon$) since the remaining equation of motion, eq. (6), relates the constant $c$ with $\varepsilon$:

$$\vec{p} \cdot \vec{q} = 2\,\varepsilon = \frac{R^2 n^2}{2}(c^2 - 1) \tag{11}$$

In particular, reality of $c$ implies reality of $\varepsilon$.[2] The required reality condition on $\varepsilon$ or $c$ raises an interesting point. Notice that, from the point of view of the IIA brane configuration, there are four angles parametrizing the most general rotation within the $(v, w)$ hyperplane. This would correspond to the four degrees of freedom contained in $\vec{q}$, after subtracting the two degrees of freedom characterizing unrotated $N = 2$ curves (the Yang–Mills coupling and theta angle), and the constraints from the leading field equation at infinity: $\vec{q}^2 = 0$. However, the subleading equation (11), together with the reality condition on $c$, imposes one extra constraint on $\vec{q}$, and we find that only a subclass of the rotated IIA brane configurations, depending on three real parameters, can be lifted into M-theory in a smooth way.

Among the configurations described by the curve in eq. (10), the supersymmetric ones are holomorphic embeddings and, accordingly, correspond to $\varepsilon = 0$, which in turn implies $c^2 = 1$.

The "field equations" (9) and (11) have an obvious symmetry under the interchange of $\lambda$ and $\zeta$. This symmetry is translated into the curve (10) as the isometry $w \to \overline{w}$, and implies that, for fixed $\eta$ and fixed supersymmetry-breaking parameter $\varepsilon$, we can restrict the values of $\zeta$ as $|\zeta| \geq \sqrt{|\eta\varepsilon|}$. This is analogous to the effect of T-duality on the restriction of the moduli space of a string compactification.

Depending on the values of the (complex) parameters $\eta$, $\varepsilon$, $\zeta$ and $\lambda$ subject to eq. (9), and up to the replacements $\lambda \to \zeta$ and $w \to \overline{w}$, the curve in eq. (10) has the following limiting regimes:

*i.* The $N = 2$ curve at the singular points where $n - 1$ dyons become massless:

$$v = z + \frac{\eta}{z} , \qquad w = 0 , \qquad t = z^n . \tag{12}$$

This is the case in which $\varepsilon = \lambda = \zeta = 0$ (i.e. $\vec{q} = \eta\vec{p}$).

*ii.* The generic $N = 1$ curve, that is MQCD in presence of an adjoint chiral multiplet of mass proportional to $\mu = \zeta/\eta$:

$$v = z + \frac{\eta}{z} , \qquad w = \frac{\zeta}{z} , \qquad t = z^n \tag{13}$$

[2] The sign ambiguity of $c$ as a solution of eq. (6) is equivalent to the symmetry $t \to \bar{t}^{-1}$, i.e. $x^6 \to -x^6$.

(the $N = 2$ limit corresponds to $\mu \to 0$ at $\eta$ fixed).

*iii.* The $N = 1$ MQCD curve

$$v = z \ , \qquad w = \frac{\zeta}{z} \ , \qquad t = z^n , \tag{14}$$

which corresponds to the limit $\mu \to \infty$ at $\zeta$ fixed of the previous configuration.

*iv.* The $N = 1$ MQCD curve softly broken to $N = 0$ supersymmetry by the breaking term $\overline{\varepsilon}/\overline{z}$

$$v = z + \frac{\overline{\varepsilon}}{\overline{z}} \ , \qquad w = \frac{\zeta}{z} \ , \qquad t = z^{n(c+1)/2} \overline{z}^{n(\bar{c}-1)/2} , \tag{15}$$

where $\lambda = \eta = 0$.

Notice that the two-dimensional surface $\Sigma$, described by the curve (10), is embedded in the $N = 2$ case in a four-dimensional space spanned by $(x_4, x_5, x_6, x_{10})$. In the $N = 1$ case $\Sigma$ is embedded in a six-dimensional space $(x_4, x_5, x_6, x_8, x_9, x_{10})$. Going to $N = 0$, since we lose holomorphicity, there appear more cases. For generic values of the parameters in the curve (with $\varepsilon \neq 0$), $\Sigma$ is embedded in the same six-dimensional space as the $N = 1$ case. But if we choose $c = 0$ (i.e. $\varepsilon = R^2 n^2/4$) then $\Sigma$ is embedded in a five-dimensional space spanned by $(x_4, x_5, x_8, x_9, x_{10})$. Instead, for $\eta = \lambda = \zeta = 0$, i.e. $\vec{q} = \varepsilon \vec{p}^*$ (a particular case of *iv*, $N = 0$ pure MQCD), $\Sigma$ is embedded in a four-dimensional manifold spanned by $(x_4, x_5, x_6, x_{10})$. Finally if we set also $c = 0$ besides $\eta = \lambda = \zeta = 0$ (an even more particular case of *iv*), then $\Sigma$ is embedded in a three-dimensional space spanned by $(x_4, x_5, x_{10})$ [6].

The family of $N = 1$ curves parametrized by $\eta, \zeta$ (cases *ii* and *iii*) describes the soft breaking of the $N = 2$ model to $N = 1$ by the mass of the adjoint superfield, i.e.

$$\Delta \mathcal{L}_{N=1} = \int d^2\theta \, m \, \mathrm{Tr}\Phi^2 \ + \ \mathrm{h.c.} \tag{16}$$

Here, holomorphicity and global symmetries can be used to obtain a precise match of the parameters appearing in the curve at weak string coupling (recall that $R \sim (g_s)^{2/3}$) to the microscopic parameters of the effective low-energy field theory, i.e. the dynamical $\Lambda_{QCD}$ scales of the $N = 2$ or $N = 1$ models, $\Lambda_2$, $\Lambda_1$, and the adjoint mass $m$ from (16): taking $v$ and $w$ with mass dimension, we have (see [7]) $\eta = (\Lambda_2)^2$, $\zeta = C_\zeta \, \ell_P^2 (\Lambda_1)^3/R \equiv \mu \, \eta$. With the one-loop matching $(\Lambda_1)^3 = m \, (\Lambda_2)^2$, we are led to $\mu = C_\zeta \, \ell_P^2 \, m/R$ and $\eta = (\Lambda_1)^3/m$.

Notice that, with these matchings, the $N=2$ limit of the curves, eq. (12), does not exhibit explicit dependence on the compact radius $R$. This agrees with the expectations from $N=2$ supersymmetry: all data encoded in the Seiberg–Witten curve become protected against variations of the string coupling when $\Lambda_2$ is kept fixed.

In these $N=1$ models, both the microscopic theta angle $\theta = n\arg(\eta)$, and the phase of the mass of the adjoint chiral multiplet $\alpha_m = \arg(m)$, are physically irrelevant, as one may absorb them into phase redefinitions of $v, w, t$, which are allowed isometries of the background metric of eq. (7). In the field theory language, this is related to the existence of anomalous $U(1)$ symmetries. In addition, there are $n$ curves solving the "field equations" eq. (1) for each set of asymptotic data fixed at infinity. These solutions are related by $2\pi$ shifts of the microscopic theta angle: a redefinition $(\eta,\zeta) \to (e^{i\delta}\eta, e^{i\delta}\zeta)$ can be absorbed at infinity (i.e. in the region $z\sim 0$), by a reparametrization $z \to e^{i\delta}z$, leaving the embedding of the curve in target space completely invariant, precisely if $\delta = 2\pi k/n$, for $k=0,...,n-1$. This degeneracy is related to the existence of a symmetry at infinity: $t\to t$, $v\to v$, $w\to e^{2\pi i k/n}w$, which is broken at finite distances by the brane configuration. So, we have the usual picture of spontaneous symmetry breaking by the gaugino condensate. For each of the $n$ vacua we have $\arg(\zeta_k) = \arg(\langle \mathrm{Tr}\lambda\lambda\rangle_k) = (\theta+2\pi k)/n$. The fact that vacua related by a spontaneously broken symmetry are physically equivalent is realized on the curve by the fact that the redefinition $(\eta_k,\zeta_k)\to(\eta_{k+1},\zeta_{k+1})$ can be absorbed into a reparametrization $z\to e^{i\pi/n}z$, plus an isometry of the target $(v,w,t)\to(e^{i\pi/n}v, e^{i\pi/n}w, -t)$. This ensures the physical equivalence of the tree-level effective Lagrangians obtained by reducing the M-theory on each of the $n$ five-brane geometries labelled by $k$.

In summary, the asymptotic behaviour of the $N=1$ curves (13) depends only on the values of $\zeta^n$ and $\zeta/\eta$, leading to $n$ equivalent curves, which are each mapped into the next one by the phase transformation $\theta\to\theta+2\pi$. Each individual vacuum (curve) is mapped into itself by $\theta\to\theta+2\pi n$. Another interesting property of the family of curves (13) is the emergence of accidental additional symmetries in the MQCD limit $\mu\to\infty$, $\zeta$ fixed, i.e. curves (14). A new $U(1)$ symmetry $(v,w,t)\to(e^{i\delta}v, e^{-i\delta}w, e^{in\delta}t)$ appears only when $\eta\to 0$. Notice that this symmetry is not even present at infinity for $\eta\neq 0$.

After supersymmetry breaking, i.e. when $\varepsilon\neq 0$, we lose the holomorphy constraints both on the geometry of the curve and on the precise mapping between the microscopic parameters in the effective low-energy, weak-coupling ($R\ll 1$) Lagrangian, and the parameters of the curve. The only remaining constraints would follow from selection rules imposed by global symmetries. The family of $N = 1$ configurations has two natural $U(1)$ symmetries: $U(1)_v = U(1)_R$, associated to rotations in the $v$-plane, an anomalous R-symmetry in the field theory description, and $U(1)_w = U(1)_J$, the non-anomalous R-symmetry surviving the full $SU(2)_R$ of the $N = 2$ models. The charges of the relevant quantities under $U(1)_v \times U(1)_w$ are $Q(v) = (2, 0)$, $Q(w) = (0, 2)$, $Q(\mu) = Q(m) = (-2, 2)$, $Q(\eta) = (4, 0)$, $Q(\zeta) = (2, 2)$, with $R$ inert under the phase redefinitions.

For a non-zero supersymmetry breaking parameter, the symmetry mentioned above, $\lambda \to \zeta$, $w \to \overline{w}$, implies the effective bound $|\zeta| \geq \sqrt{|\eta\varepsilon|}$, which translates in the bound on the adjoint mass parameter: $|\mu| \geq \sqrt{|\varepsilon/\eta|}$. Thus the family of configurations we consider has a natural built-in hierarchy of soft breakings, since most of parameter space satisfies $|\mu|^2 \gg |\varepsilon/\eta|$. The analysis of the curves in eq. (10) becomes more tractable when $|\mu|^2 \gg |\varepsilon/\eta|$. Indeed in this case the adjoint mass parameter $\mu$ can be unambiguously associated to a term of the form (16) since, in units of the natural $N = 2$ scale $\eta$, the $N = 1$ SUSY breaking scale is much larger than the $N = 0$ SUSY breaking scale. Moreover, in this case the supersymmetry breaking effects at low energies in the effective field theory must be dominated by the gaugino mass since the adjoint chiral multiplet masses are much larger. As a result, the associated field theoretical models are generically the ones in refs. [19], where spurion superfields lie in $N = 1$ multiplets. So we choose to parametrize supersymmetry breaking in the microscopic effective Lagrangian in terms of the operator $m_\lambda \mathrm{Tr}\lambda\lambda$. These considerations do not hold when $|\mu|^2 \sim |\varepsilon/\eta|$, since the mass splitting between the ($N = 1$) vector superfield and the adjoint chiral superfield, i.e. the scale of the $N = 2 \to N = 1$ breaking, is of the same order as the supersymmetry breaking parameter and it is natural to expect an $\mathcal{O}(1)$ mixing of eq. (16) with the supersymmetry breaking operators. In this case we could have a supersymmetry breaking pattern of the type studied in refs. [20], with a full $N = 2$ multiplet of spurions; however, the analysis, based on global $U(1)$ selection rules, that we will pursue, is expected to be even less powerful, owing to the significant operator mixing expected at the microscopic level. Thus in the rest of the paper we will restrict ourselves to the case $|\mu|^2 \gg |\varepsilon/\eta|$.

Therefore, saturating the effects of supersymmetry breaking with a gaugino mass, selection rules from the $U(1)_v \times U(1)_w$ symmetry are easily derived, assigning the charges $Q(m_\lambda) = (-2, -2)$. Since eq. (10) fixes the charge of $\varepsilon$ to be $Q(\varepsilon) = (0, 0)$, the global continuous symmetries fix the dependence on the various CP-violating phases to be

$$
\begin{aligned}
\varepsilon &= f_\varepsilon(\xi) \\
\eta &= |\eta_{\rm susy}| e^{i\theta/n} \left(1 + f_\eta(\xi)\right) \\
\zeta &= |\zeta_{\rm susy}| e^{i(\alpha_m + \theta/n)} \left(1 + f_\zeta(\xi)\right).
\end{aligned}
\tag{17}
$$

In these equations, $\xi = e^{i\theta_{ph}/n}$, with $\theta_{ph} = \theta + n\arg(m) + n\arg(m_\lambda)$ the physical theta angle, is invariant under the anomalous $U(1)_R$ symmetry, i.e. it is the CP-violation parameter that cannot be rotated away by means of anomalous phase rotations. The functions $f_{\varepsilon,\eta,\zeta}(\xi)$ depend on any real combination of the couplings in the theory ($|m|$, $|m_\lambda|$, and $|\Lambda_2|$), as well as the eleven-dimensional Planck length $\ell_P$, and the compact radius $R$, and admit power expansions in the breaking parameter $m_\lambda$, such that they vanish in the supersymmetric limit $m_\lambda \to 0$.

In fact, the generalized functions $f(\xi)$ are real functions, in the sense that, under complex conjugation, $\overline{f(\xi)} = f(\bar{\xi})$. This is due to the fact that a CP transformation acts on the curve by complex conjugation,[3] but in the microscopic Lagrangian at weak string coupling, it acts simply by inverting all CP-violating phases. So, to the extent that eq. (17) is a smooth limit of infrared quantities defined at weak coupling, we should be able to characterize completely the CP transformation in terms of complex conjugation of microscopic phases.

This structure with respect to CP-violating phases can be used to get some insights in the theta-dependence of the $N = 0$ MQCD theory described by the curve of eq. (10). We have seen that, in the supersymmetric limit, $\varepsilon = 0 = c^2 - 1$, shifts of the theta angle by $2\pi$ lead to identical boundary conditions at infinity, leading to the appearance of $n$ degenerate vacua, whose curves $C_k$ are obtained by the replacement $\theta \to \theta_k = \theta + 2\pi k$, with $k = 0, ..., n-1$. Once we break supersymmetry, the $n$-fold degeneracy of the vacuum is lost. In field theory, a small supersymmetry breaking makes $n-1$ of the vacua metastable, and only one stable vacuum remains for generic values of the parameters. In the M-theory picture, we see that, as long as some of the functions in eq. (17) are non-trivial, the value of $\theta$ has physical effects, as it cannot be rotated away into an isometry. This is because the

[3] This can be seen by recalling the form of the generic $N = 2$ curve, where the $v$ plane is in fact the $\langle {\rm Tr}\Phi^2 \rangle$ plane of the effective field theory. Also, in the $N = 1$ models, a distance in the $w$ plane is related to the expectation value of the dyons (see [21]), so CP really acts by complex conjugation on the curve.

charge of all functions $f$ is trivial: $Q(f) = (0,0)$. A stronger statement is that, because of the structure of the third equation in (10) for $c^2 \neq 1$, no rescaling of $\varepsilon$ can be absorbed into an isometry, and therefore the first equation in (10) implies that no rescaling of $\eta$ is allowed either.

Thus, as we turn on a non-zero gaugino mass, each of the curves $C_k$ is deformed, and the associated parameters $(\eta_k, \zeta_k, \varepsilon_k)$ are given by eq. (17), with the substitutions $\theta \to \theta_k = \theta + 2\pi k$. The resulting $n$ vacua are no longer physically equivalent since, for generic values of the parameters, the transformation $\theta \to \theta+2\pi$ is not equivalent to a phase redefinition of $(\eta, \zeta)$, and therefore it cannot be absorbed into a background isometry. One cannot see directly in the M-theory picture the metastability of most of the vacua, because the supergravity equations for the five-brane only probe perturbative stability. Still, the euclidean bounce solution leading to a false vacuum decay via tunneling, could perhaps appear as a particular interpolating five-brane configuration, in analogy with the domain wall construction of ref. [6]. [4]

Now we face an apparent problem. The $\theta$ angle now has physical effects, but the parameters of each curve $C_k$ are periodic in $\theta$ with period $2\pi n$ instead of $2\pi$. Still we know that the physics should be invariant under a $2\pi$ periodicity of $\theta$, and not $2\pi n$. One might try to resolve the puzzle by invoking a special form of the functions $f_{\eta,\zeta,\varepsilon}(\xi)$ in (17). For example, if all supersymmetry breaking deformations are really functions of $\xi^n$, then the parameters $(\eta_k, \zeta_k, \varepsilon_k)$ become $2\pi$-periodic in $\theta$ up to a phase, which could be absorbed into a background isometry. Such a dependence could be justified by requiring that instantons completely saturate the supersymmetry breaking deformations. However, this does not seem very likely, because we have non-trivial branching in $\theta$ in the gaugino condensate already in the supersymmetric case. In addition, the analysis of field theory models of soft breaking [22] indicates that the puzzle of the "wrong" theta periodicity is resolved through a non-trivial spectral flow. That is, when $\theta \to \theta + 2\pi$, a metastable vacuum will become stable and take the place of the original one. Thus at a particular value of $\theta$ there must be a level crossing of two contiguous vacua, which are related by the redefinition $\theta \to \theta + 2\pi$. At this particular value of $\theta$ we should then find a spontaneously broken $\mathbf{Z}_2$ symmetry. According to standard lore, such a $\mathbf{Z}_2$ group is the action of CP at that particular value of $\theta$, i.e. an example of the Dashen phenomenon [23].

[4] We thank C. Bachas and M. Douglas for a discussion on this point.

We can then, even with our limited knowledge of the curve of eq. (10) given by eq. (17), try to see if a CP transformation, i.e. a complex conjugation of the curve, is a symmetry of the curve for a particular value of $\theta = \theta_c$.

Indeed let us consider two curves related by the redefinition $\theta \to \theta + 2\pi$, for example the curves $C_0$ and $C_{n-1}$. Now we will show that at $(\theta_{ph})_0 = \pi$, i.e. at $\theta_c = \pi - n(\arg(m) + \arg(m_\lambda))$, there is a symmetry between the $C_0$ and $C_{n-1}$ curves under a complex conjugation (CP) and a background isometry. Indeed since $\theta_k = \theta + 2\pi k$, $(\theta_{ph})_{n-1} = 2\pi n - \pi \simeq -\pi$ at the point where $(\theta_{ph})_0 = \pi$. This implies that at $\theta = \theta_c$, $\overline{\xi}_{n-1} = \xi_0$, and $\overline{\varepsilon}_{n-1} = \varepsilon_0$, $\overline{\eta}_{n-1} = e^{i\alpha_\eta}\, \eta_0$, $\overline{\zeta}_{n-1} = e^{i\alpha_\zeta}\, \zeta_0$, with $\alpha_\eta = 2\arg(m) + 2\arg(m_\lambda)$, and $\alpha_\zeta = 2\arg(m_\lambda)$. It is now trivial to check that such transformations can be absorbed into an isometry of the $(v, w, t)$ space consisting of complex conjugation and a phase redefinition, $(v, w, t) \to (e^{-i\alpha_v}\overline{v}, e^{-i\alpha_w}\overline{w}, e^{-i\alpha_t}\overline{t})$, with the values:[5]

$$\begin{aligned}
\alpha_v =& \frac{\alpha_\eta}{2} = \arg(m) + \arg(m_\lambda) \\
\alpha_w =& \alpha_\zeta - \frac{\alpha_\eta}{2} = \arg(m_\lambda) - \arg(m) \\
\alpha_t =& \alpha_v\, n = n\arg(m) + n\arg(m_\lambda)
\end{aligned} \tag{18}$$

This implies that, according to our interpretation (17) of the data of the curve in eq. (10), at $(\theta_{ph})_0 = \pi$, i.e. at $\theta_c = \pi - n(\arg(m) + \arg(m_\lambda))$, there is a level crossing between the two vacua described by the curves $C_0$ and $C_{n-1}$. Clearly, such level crossings occur for any pair of adjacent curves, so we can label as $C_0$ the absolutely stable curve at $(\theta_{ph})_0 = 0$, without any loss of generality. From the known behaviour of the softly broken $N = 1$ SQCD theories in field theory [22], we would then infer that for $-\pi < (\theta_{ph})_0 < \pi$ the $C_0$ curve describes the stable vacuum, but that for $\pi < (\theta_{ph})_0 < 3\pi$ the stable vacuum is described by the $C_{n-1}$ curve, while the $C_0$ curve now describes a metastable vacuum. This picture thus reconciles the $2\pi n$ periodicity in $\theta$ of each single curve, with the $2\pi$ periodicity in $\theta$ of the physics described by the $N = 0$ MQCD curves of eq. (10).

To conclude, we would like to comment on the physical interpretation of the discontinuous embeddings, i.e. the case of general complex $c$. The relation between the curves parameters and the microscopic couplings is encoded in the specific form of the functions in (17). To the extent that $\varepsilon$ is real, we can interpret supersymmetry breaking in terms

[5] Here we consider the physical situation with $\mathrm{Im}(c) = 0$. It is only for this case that the $t$-rescaling corresponds to a $U(1)_R$ transformation.

of a continuous rotation of the brane configuration. However, one cannot exclude that, for sufficiently strong supersymmetry breaking, the function $f_\varepsilon(\xi)$ develops an imaginary part at some point. This could be the geometrical interpretation of a phenomenon characteristic of softly broken models with $N = 2$ spurions [20]. In these models one finds a number of metastable vacua of order $n$ in the large-$n$ limit, but a particular metastable vacuum could disappear when transported around by a shift of $\theta$. Typically, the absolute vacuum at, say $\theta = 0$, becomes metastable at about $\theta = \pi$, and disappears completely at $\theta \sim n\pi$. We can understand this phenomenon in simple terms in a model with hierarchical supersymmetry breaking of the type discussed in [19] and [22]. The potential of the $N = 1$ model around a particular vacuum with condensation of $n - 1$ dyons has the structure $V(U) = m(\Lambda_2)^3 f(\sqrt{U}/\Lambda_2)$, with $U = \langle \mathrm{Tr}\phi^2 \rangle$, and $f(x)$ a function with $\mathcal{O}(1)$ coefficients in the series expansion. Defining the dimensionless axion field as $a = \arg(U)/2$, we have an axion potential with global scale of order $m(\Lambda_2)^3$, and period $2\pi$. The leading-order correction in the gaugino mass is of the form $\delta V \sim m_\lambda \langle \mathrm{Tr}\lambda\lambda \rangle \sim m_\lambda m(\Lambda_2)^2\, e^{ia/n}$. Therefore, we see that the local minimum of the axion at $\langle a \rangle \sim n\pi$ could be upset by the correction if $m_\lambda \sim \Lambda_2$. If this happens, the axion's vacuum expectation value at the local minimum becomes complex, and we interpret this as the disappearance of this vacuum at $\theta \sim n\pi$.

The previous discussion suggests that a complex value of $\varepsilon$ as a function of the microscopic parameters could be interpreted as the disappearance of a metastable vacuum. However, this phenomenon occurs at $\theta \sim n\pi$, and thus the picture of level crossing at $\theta = \pi$ presented here remains, at least for large enough $n$. The general conclusion that we can draw from this discussion is the robustness of the level-crossing solution to the "theta puzzle". Here we have derived it from very general geometric considerations involving no detailed analysis of dynamics.

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