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Recent Developments in String Theory†

W. Lerche

Theoretical Physics Division, CERN, Geneva, Switzerland

Abstract

After briefly reviewing basic concepts of perturbative string theory, we explain in simple terms some of the new findings that created excitement among the string physicists. These developments include non-perturbative dualities and a unified picture that embraces the so-far known theories.

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RECENT DEVELOPMENTS IN STRING THEORY

W.LERCHE
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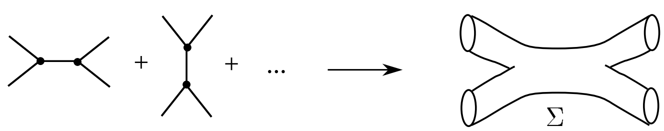
After briefly reviewing basic concepts of perturbative string theory, we explain in simple terms some of the new findings that created excitement among the string physicists. These developments include non-perturbative dualities and a unified picture that embraces the so-far known theories.

1 Perturbative String Theory

1.1 Basic features

Heuristically one may introduce string theory[1] by describing tiny loops sweeping through D-dimensional space-time. Being one-dimensional extended objects, they trace out two-dimensional "world-sheets" Σ that may be viewed as thickened Feynman diagrams:



Accordingly, perturbative string theories are defined by two-dimensional field theories living on such Riemann surfaces. Suitable two-dimensional field theories can be made out of a great variety of building blocks, the simplest possibilities being given by free two-dimensional bosons and fermions, eg., $X_i(z)$, $\psi_i(z)$.

By combining the two-dimensional building blocks according to certain rules, an infinite variety of operators that describe *space-time* fields can be constructed. Typically, the more complex the operators become that one builds, the more massive the associated space-time fields become. In total one obtains a finite number of massless fields, plus an infinite tower of string excitations with arbitrary high masses and spins. Schematically, among the most generic massless operators are:

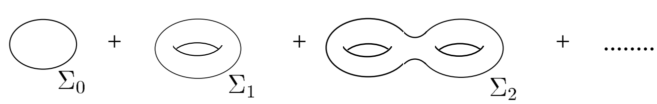
$$\begin{array}{rll} \text{Dilaton scalar} & \phi & \eta^{\mu\nu}\bar{\partial}X_\mu\partial X_\nu \qquad (\mu,\nu=1,...,D) \\ \text{Graviton} & g_{\mu\nu} & \bar{\partial}X_\mu\partial X_\nu \\ \text{Gauge field} & A^a_\mu & \bar{\partial}X_\mu\partial X^a \\ \text{Higgs field} & \Phi^{ab} & \bar{\partial}X^a\partial X^b \end{array} \tag{1}$$

We see here how simple combinatorics provide an intrinsic and profound unification of particles and their interactions: from the 2d point of view, a gauge field operator is very similar to a graviton operator, whereas the D-dimensional space-time properties of these operators are drastically different. An important point is that gravitons necessarily appear, which means that this kind of string theories *imply* gravity. This is one of the main motivations for studying string theory. Indeed, it is believed by many physicists that string theory provides the only way to formulate a consistent quantum theory of gravity.

Fermions ("electrons" and "quarks") are obtained by changing the boundary conditions of the two dimensional fermions ψ on Σ. These can be anti-periodic or periodic (the fields change sign or not when transported around Σ), giving rise to fermionic or bosonic fields in space-time, respectively. It turns out that whenever the theory contains space-time fermions, it must have two dimensional supersymmetry, and this is why such string theories are called *superstrings*. It is however not so that the D-dimensional space-time theory must be supersymmetric, although practically all models studied so far do have supersymmetry to some degree. But this is to simplify matters and keep perturbation theory under better control,[a] much like the study of supersymmetric Yang-Mills theory makes the analysis much easier as compared to non-supersymmetric gauge theories. String theory does not intrinsically predict space-time supersymmetry, although it arises quite naturally.

That very simple, even free two dimensional field theories generate highly non-trivial effective dynamics in D-dimensional space-time, can be visualized by looking to the perturbative effective action:

$$\begin{aligned} S^{(D)}_{\rm eff}(g_{\mu\nu}, A_\mu, ..) &= \sum_{\Sigma_\gamma} e^{-\phi\chi(\Sigma_\gamma)} \int_{M(\Sigma_\gamma)} \int d\psi dX ..\, e^{\overbrace{\int d^2z \mathcal{L}_{2d}(\psi, X, .., g_{\mu\nu}, A_\mu, ..)}^{\text{2d action}}} \\ &= \int d^D x \sqrt{g} e^{-2\phi} \underbrace{\left[R + {\rm Tr} F_{\mu\nu} F^{\mu\nu} + \ldots \right]}_{\text{general relativity, gauge theory etc}} + \underbrace{\mathcal{O}({m_{\rm planck}}^{-1})}_{\text{small string corrections}} \end{aligned} \tag{2}$$



Here, $g_{\mu\nu}, A_\mu, ...$ are space-time fields that provide the background in which the strings move, and $\mathcal{L}_{2d}(\psi, X, .., g_{\mu\nu}, A_\mu, ..)$ is the lagrangian containing the two-dimensional fields, as well as the background fields which are simply coupling constants from the two dimensional point of view. The 2d fields are integrated out, and one also sums over all the possible two-dimensional world-sheets Σ_γ (as well as over the boundary conditions of the $\psi(z)$). This corresponds to the well-known loop expansion of particle QFT. Note, however, that there is only one "diagram" at any given order in string perturbation theory.

The topological sum is weighted by $e^{-\phi\chi(\Sigma_\gamma)}$, where ϕ is the dilaton field and where the Euler number $\chi(\Sigma_\gamma) \equiv 2 - 2\gamma$ is the analog of the loop-counting parameter. The coupling constant for the perturbation series thus is

$$g = e^{\langle\phi\rangle},$$

which must be small in order for the perturbation series to make sense. We see here that a coupling constant is given by an *a priori* undetermined vacuum expectation value of some field, and this reflects a general principle of string theory.

In addition, the topological sum is augmented by integrals over the inequivalent shapes that the Riemann surfaces can have, and this corresponds to the momentum integrations in ordinary QFT. There may be divergences arising from degenerate shapes, but these divergences can always be interpreted as IR divergences in the space-time sense. In particular, the well-known logarithmic running of gauge couplings in four dimensional string theories arises from such IR divergences.

The absence of genuine UV divergences was another early motivation of string theory, and especially makes consistent graviton scattering possible: remember that ordinary gravity is not renormalizable and one cannot easily make sense out of graviton loop diagrams. A very important point

[a] An often-cited argument for low-energy supersymmetry is that perturbation theory of non-supersymmetric strings tends to be unstable due to vacuum tadpoles. However, even when starting with a supersymmetric theory, this problem will eventually appear, namely when supersymmetry is (e.g., at the weak scale) spontaneously broken. We therefore have at any rate to find mechanisms to stabilize the perturbation series, which means that vacuum tadpoles are no strong reason for beginning with a supersymmetric theory in the first place. See especially refs.[2,3] for a vision how strings could be more clever and avoid supersymmetry (similar ideas may apply to the "hierarchy problem" as well); for a different line of thoughts, see ref.[4].

is the origin of this well-behavedness of the string diagrams. It rests on *discrete reparametrizations* of the string-world sheets Σ_γ ("modular invariance"), which have no analog in particle theory. The string "Feynman rules" are very different, and cannot even be approximated by particle QFT. String theory is therefore *more* than simply combining infinitely many particle fields, and it is this what makes a crucial difference. The whole construction is very tightly constrained: modular invariance determines the whole massive spectrum, and taking any single state away from the infinite tower of states would immediately ruin the consistency of the theory.

So far, we described the ingredients that go into computing the perturbative effective action in D-dimensional space-time. Now we focus on the outcome. As indicated in (2), the effective action typically contains (besides matter fields) general relativity and non-abelian gauge theory, plus stringy corrections thereof. These corrections are very strongly suppressed by inverse powers of the Planck mass, $m_{\text{Planck}} \sim 10^{19}$GeV, which is the characteristic scale in the theory. The value of this scale is dictated by the strength of the gravitational coupling constant, and governs the size or tension of the strings, as well as the level spacing of the excited states. That strings are so difficult to observe, and characteristic corrections so hard to see, stems from the fact that the gravitational coupling is so small, and this is not at the string physicists' fault – they have no option to change that !

That general relativity, with all its complexity just pops out from the air (arising from eg., a free 2d field theory), may sound like a miracle. Of course this does not happen by accident, but is bound to come out. The important point here is that there is a special property that the relevant two dimensional theories must have for consistency, and this is *conformal invariance.* This is a quite powerful symmetry principle, which guarantees, via Ward identities, general coordinate and gauge invariance in space-time – however, only so if and only if $D = 10$.

As we will see momentarily, this does not imply that superstrings must live exclusively in ten dimensions, but it means that superstrings are most naturally formulated in ten dimensions.

1.2 10d Superstrings and their compactifications

Two-dimensional field theories can have two logically independent, namely holomorphic and anti-holomorphic pieces. For obtaining $D = 10$ string theories, each of these pieces can either be of type superstring "S" or of type bosonic string "B" (with extra $E_8 \times E_8$ or $SO(32)$ gauge symmetry). By combining these building blocks in various ways,[b] plus including an additional "open" string variant, one obtains the following exhaustive list of supersymmetric strings in $D = 10$:

Composition	Name	Gauge Group	Supersymmetry
$S \otimes \bar{S}^\dagger$	Type IIA	$U(1)$	non-chiral $N = 2$
$S \otimes \bar{S}$	Type IIB	-	chiral $N = 2$
$S \otimes \bar{B}$	Heterotic	$E_8 \times E_8$	chiral $N = 1$
$S \otimes \bar{B}'$	Heterotic'	$SO(32)$	chiral $N = 1$
$(S \otimes \bar{S})/Z_2$	Type I (open)	$SO(32)$	chiral $N = 1$

Since these theories are defined in terms of 2d world-sheet degrees of freedom, which is intrinsically a perturbative concept in terms of 10d space-time physics, all we can really say is that there are five theories in ten dimensions that are different in *perturbation theory.* Their perturbative spectra are indeed completely different, their number of supersymmetries varies, and also the gauge symmetries are mostly quite different.

Now, we would be more than glad if strings would remain in lower dimensions as simple as they are in $D = 10$. Especially string theories in $D = 4$ turn out to be much more complicated. Specifically, the simplest way to get down to four dimensions is to assume/postulate that the space-time manifold is not simply $\mathbb{R}^{10}$, but $\mathbb{R}^4 \times X_6$, where X_6 is some compact six-dimensional manifold. If it is small enough, then the theory looks at low energies, ie., at distances much larger than the

[b]There are further, but non-supersymmetric theories in $D = 10$.

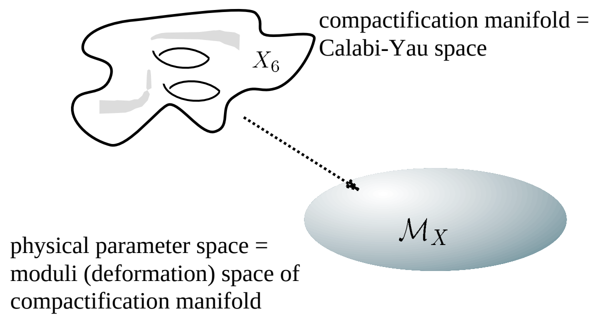


Figure 1: The parameter space of the low energy effective field theory of a string compactification is essentially given by the moduli space (deformation space) of the compactification manifold. This leads to a geometric interpretation of almost all of the parameters and couplings. Shown is that with each point in the moduli space $\mathcal{M}_X$, one associates a compactification manifold X of specific shape; the shape in turn determines particle masses, Yukawa couplings etc. The picture will be refined below.

size of X_6, effectively four-dimensional. This is like looking at a garden hose from a distance, where it looks one-dimensional, while upon closer inspection it turns out to be a three dimensional object.

The important good feature is that such kind of "compactified" theories are at low energies exactly of the type as the standard model of particle physics (or some supersymmetric variant thereof). That is, generically such theories will involve non-abelian gauge fields, *chiral* fermions and Higgs fields besides gravitation, and all of this coupled together in a (presumably) consistent and truly unified manner ! That theories that are able to do this have been found at all, is certainly reason for excitement.

But apart from this generic prediction, more detailed predictions, like the precise gauge groups or the massless matter spectrum, cannot be made at present − this is the dark side of the story. The reason is that the compactification mechanism makes the theories *enormously* more complicated and rich than they originally were in $D = 10$. This is intrinsically tied to properties of the possible compactification manifolds X_6:

i) There is a large number of choices for the compactification manifold. If we want to have $N = 1$ supersymmetry in $D = 4$ from e.g. a heterotic string, then X_6 must be a "Calabi-Yau" space[5], and the number of such spaces is perhaps 10^4. On top of that, one has to specify certain instanton configurations, which multiplies this number by a very large factor. *A priori*, each of these spaces (together with a choice of instanton configuration) leads to a different perturbative matter and gauge spectrum in four dimensions, and thus gives rise to a different four dimensional string theory.

ii) Each of these manifolds by itself has typically a large number of parameters; for a given Calabi-Yau space, the number of parameters can easily approach the order of several hundred.

These parameters, or "moduli" determine the shape of X_6, and correspond physically to vacuum expectation values of scalar ("Higgs") fields, similar to the string coupling g discussed above. Changing these VEVs changes physical observables at low energies, like mass parameters, Yukawa couplings and so on. They enter in the effective lagrangian as free parameters, and are not determined by any fundamental principles, at least as far is known in perturbation theory. Their values are determined by the choice of vacuum, much like the spontaneously chosen direction of the magnetization vector in a ferromagnet (that is not determined by any fundamental law either). The hope is that after breaking supersymmetry, the continuous vacuum degeneracy would be lifted by quantum corrections (which is typical for non-supersymmetric theories), so that ultimately there would be fewer accessible vacua.

The situation is actually worse than described so far, because we have on top of points i) and ii):

iii) There are five theories in $D = 10$ and not just one, and a priori each one yields a different theory in four dimensions for a given compactification manifold X_6. If one of them would be the fundamental theory, what is then the rôle of the others ?

iv) There is no known reason why a ten dimensional theory wants at all to compactify down to $D = 4$; many choices of space-time background vacua of the form $\mathbb{R}^{10-n} \times X_n$ appear to be on equal footing.

All these points together form what is called the *vacuum degeneracy* problem of string theory – indeed a very formidable problem, known since a decade or so. Summarizing, most of the physics that is observable at low energies seems to be governed by the vacuum structure and not by the microscopic theory, at least as far as we can see today. Still, it is not so that string theory would not make any low-energy predictions; the theories are very finely tuned and internal consistency still dramatically reduces the number of possible low energy spectra and independent couplings, as compared to ordinary field theory.

The recent progress in non-perturbative string theory does not solve the problem of the choice of vacuum state either. The progress is rather of conceptual nature and opens up completely new perspectives on the very nature of string theory.

2 Duality and non-perturbative equivalences

Duality is the main new concept that has been stimulating the recent advances in supersymmetric particle[6] and string theory. Roughly speaking, duality is a map between solitonic (non-perturbative, non-local, "magnetic") degrees of freedom, and elementary (perturbative, local, "electric") degrees of freedom. Typically, duality transformations exchange weak and strong-coupling physics and act on coupling constants like $g \to 1/g$. They are thus intrinsically of quantum nature.

Duality symmetries are most manifest in supersymmetric theories, because in such theories perturbative loop corrections tend to be suppressed, due to cancellations between bosonic and fermionic degrees of freedom. Otherwise, observable quantities get so much polluted by radiative corrections that the more interesting non-perturbative features cannot be easily made out.

More precisely, certain quantities (eg., Yukawa couplings) in a supersymmetric low energy effective action are protected by non-renormalization theorems, and those quantities are typically *holomorphic* functions of the massless fields. As a consequence, this allows to apply methods of complex analysis, and ultimately of algebraic geometry, to analyze the physical theory. Such methods (where they can be applied) turn out to be much more powerful then traditional techniques of quantum field theory, and this was the technical key to the recent developments.

A typical consequence of the holomorphic structure is a continuous vacuum degeneracy, arising from flat directions in the effective potential. The non-renormalization properties then guarantee that this vacuum degeneracy is not lifted by quantum corrections, so that supersymmetric theories often have continuous quantum moduli spaces $\mathcal{M}$ of vacua. In string theory, as mentioned above, these parameter spaces can be understood geometrically as moduli spaces of compactification manifolds.

2.1 A gauge theory example

One of the milestones in the past few years was the solution of the (low energy limit of) $N = 2$ supersymmetric gauge theory in four dimensions.[7,8] Important insights that go beyond conventional particle field theory have been gained by studying this model, and this is why we briefly touch it

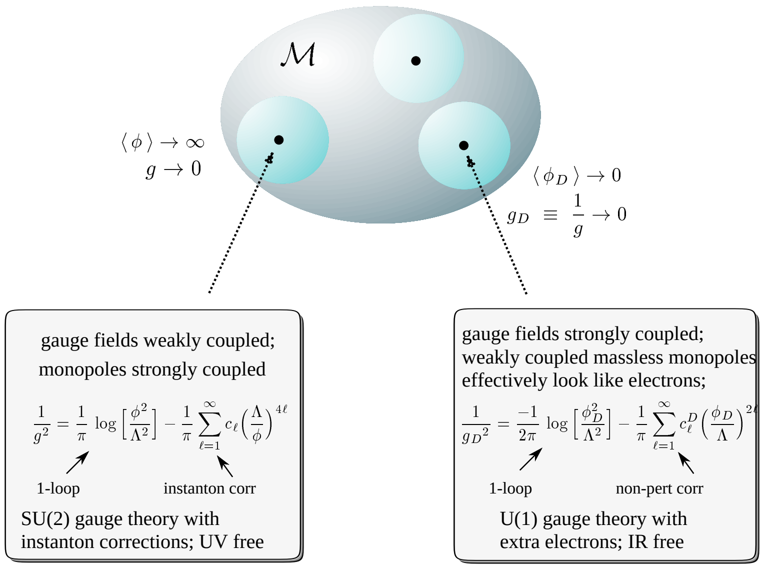


Figure 2: The exact quantum moduli space of $N = 2$ supersymmetric $SU(2)$ Yang-Mills theory has one singularity at weak coupling and two singularities in the strong coupling region. The latter are caused by magnetic monopoles becoming massless for the corresponding VEVs of the Higgs field. One can go between the various regions by analytic continuation, ie., by resumming the non-perturbative instanton corrections in terms of suitable variables.

here. In fact, even though this model is an ordinary gauge theory, the techniques stem from string theory, which demonstrates that string ideas can be useful also for ordinary field theory.

Without going too much into the details, simply note that $N = 2$ supersymmetric gauge theory (here for gauge group $G = SU(2)$) has a moduli space $\mathcal{M}$ that is spanned by roughly the vacuum expectation value of a complex Higgs field ϕ. The relevant holomorphic quantity is the effective, field dependent gauge coupling $g(\phi)$ (made complex by including the theta-angle in its definition). Each point in the moduli space corresponds to a particular choice of the vacuum state. Moving around in $\mathcal{M}$ will change many properties of the theory, like the value of the effective gauge coupling $g(\phi)$ or the mass spectrum of the physical states. An important aspect is that there are special regions in the moduli space, where the theory behaves specially, ie., becomes *singular*. This is depicted in Fig.2.

More precisely, there are two different types of such singular regions. Near $\langle\phi\rangle \to \infty$, the gauge theory is weakly coupled since the effective gauge coupling becomes arbitrarily small: $g(\phi) \to 0$. In this semi-classical region, non-perturbative effects are strongly suppressed and the perturbative definition of the theory is arbitrarily good. That is, the instanton correction sum to the left in Fig.2 gives a negligible contribution as compared to the logarithmic one-loop correction (further higher loop corrections are forbidden by the $N = 2$ supersymmetry). Furthermore, the solitonic magnetic monopoles that exist in the theory become very heavy and effectively decouple.

However, when we now start moving in the moduli space $\mathcal{M}$ away from the weak coupling region, the non-perturbative instanton sum to the left in Fig.2 will less and less well converge, and the original perturbative definition of the theory will become worse and worse. When we are finally close to one of the other two singularities in Fig.2, the original perturbative definition is blurred out and does not make sense any more. The problem is thus how to suitably *analytically continue* the complex gauge coupling outside the region of convergence of the instanton series. The way to do this is to resum the instanton series in terms of another variable, ϕ_D, to yield another expression for

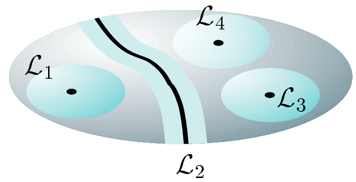


Figure 3: The moduli space of a generic supersymmetric theory is covered by coordinate patches, at the center of each of which the theory is weakly coupled when choosing suitable local variables. A local effective lagrangian exists in each patch, representing a particular perturbative approximation. None of such lagrangians is more fundamental than the other ones.

the gauge coupling that is well defined near $1/g \to 0$. That is, there is a "dual" Higgs field, ϕ_D, in terms of which the dual gauge coupling, $g_D(\phi_D)$, makes sense in the strong coupling region of the parameter space $\mathcal{M}$. Indeed, ϕ_D becomes small in this region, so that the infinite series for the dual coupling g_D to the right in Fig.2 converges very well.

The important point to realize here is that the perturbative physics in the strong coupling region is completely different as compared to the perturbative physics in the weak coupling region that we started with ! At weak coupling, we had a non-abelian $SU(2)$ gauge theory, while at strong coupling we have now an abelian $U(1)$ gauge theory plus some extra massless matter fields ("electrons"). But the latter only manifest themselves as elementary fields if we express the theory in terms of the appropriate dual variables; in the original variables, these "electrons" that become light at strong coupling, are nothing but some of the solitonic magnetic monopoles that are very massive in the weak coupling region.

All this said, we still do not know how to actually solve the theory and determine all the unknown non-perturbative instanton coefficients c_ℓ, c_ℓ^D in Fig.2. A direct computation would be beyond what is currently possible with ordinary field theory methods. The insight of Seiberg and Witten[7] was to realize that the *patching together of the known perturbative data in a globally consistent way* is so restrictive that it fixes the theory, and ultimately gives explicit numbers for the instanton coefficients c_ℓ and c_ℓ^D. This shows that sometimes much can be gained by not only looking to a theory at some given fixed background VEV, but rather by looking to a whole family of vacua, i.e., to global properties of the moduli space.

2.2 The message we can abstract from the field theory example

The lesson is that one and the same physical theory can have many perturbative descriptions. These can look completely different, and can involve different gauge groups and matter fields. There is in general no absolute notion of what would be weak or strong coupling; rather what we call weak coupling or strong coupling, or an elementary or a solitonic field, depends to some extent on the specific description that we use. Which description is most suitable, and which physical degrees of freedom will be light or weakly coupled (if any at all), depends on the region of the parameter space we are looking at.

More mathematically speaking, an effective lagrangian description makes sense only in *local coordinate patches* covering the parameter space $\mathcal{M}$ – see Fig.3. These describe different perturbative approximations of the same theory in terms of different weakly coupled physical local degrees of freedom (eg, electrons or monopoles). No particular effective lagrangian is more fundamental than any other one. In the same way that a topologically non-trivial manifold cannot be covered by just one set of coordinates, there is in general no globally valid description of a family of physical theories in terms of a single lagrangian.

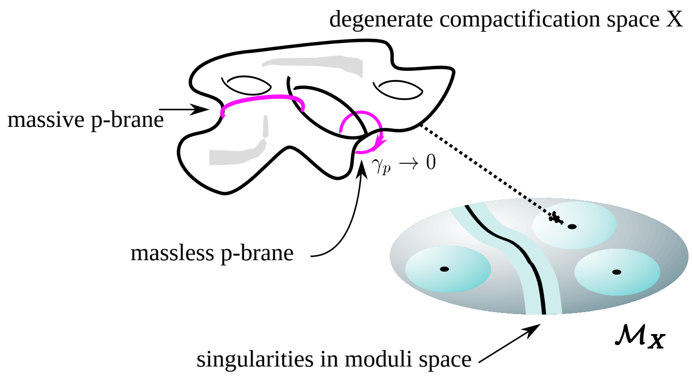


Figure 4: Non-perturbative particle-like states arise from wrapping of p-branes around p-dimensional cycles γ_p of the compactification manifold X. These are generically very massive, but can become massless in regions of the moduli space where the p-cycles shrink to zero size. The singularities in the moduli space are the analogs of the monopole singularities in Fig.2.

It is these ideas that carry over, in a refined manner, to string theory and thus to grand unification; in particular, they have us rethink concepts like "distinguished fundamental degrees of freedom". String moduli spaces will however be much more complex than those of field theories, due to the larger variety of possible non-perturbative states.

2.3 P-branes and non-perturbative states in string theory

String compactifications on manifolds X are not only complex because of the large moduli spaces they generically have, but also because the spectrum of physical states becomes vastly more complicated. In fact, when going down in the dimension by compactification, there is a dramatic *proliferation of non-perturbative states.*

The reason is that string theory is not simply a theory of strings: there exist also higher dimensional extended objects, so called "p-branes", which have p space and one time dimensions (in this language, strings are 1-branes, membranes 2-branes etc; generically, $p = 0, 1, ..., 9$). Besides historical reasons, string theory is only called so because strings are typically the lightest of such extended objects. In the light of duality, as discussed in the previous sections, we know that there is no absolute distinction between elementary or solitonic objects. We thus expect p-branes to play a more important rôle than originally thought, and not necessarily to just represent very heavy objects that decouple at low energies.

More specifically, such $(p+1)$ dimensional objects can wrap around p-dimensional cycles γ_p of a compactification manifold, to effectively become particle-like excitations in the lower dimensional (say, four dimensional) theory. These extra solitonic states are analogs of the magnetic monopoles that had played an important rôle in the $N = 2$ supersymmetric Yang-Mills theory. Since such monopoles can be light and even be the dominant degrees of freedom for certain parameter values, we expect something similar for the wrapped p-branes in string compactifications. In fact, the volumina of p-dimensional cycles γ_p of X_n depend on the deformation parameters, and there are singular regions in the moduli space where such cycles shrink to zero size ("vanishing cycles") – see Fig.4. Concretely, if a p-dimensional cycle γ_p collapses, then a p-brane wrapped around γ_p will give a massless state in $D = 10 - n$ dimensional space-time.[9] This is because the mass formula for the

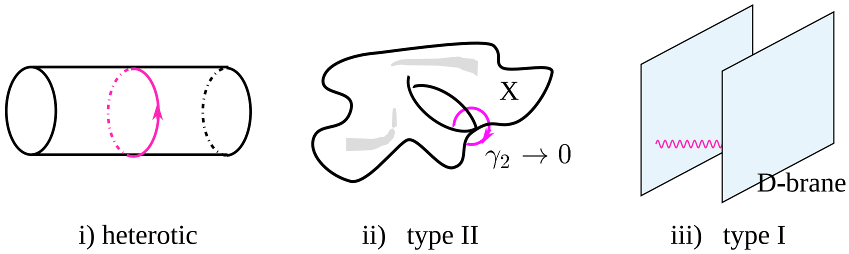


Figure 5: Different geometries can parametrize one and the same physical situation. Shown here are three dual ways to represent an $SU(2)$ gauge field and the associated Higgs mechanism.

wrapped brane involves an integration over γ_p:

$$m_{p-\text{brane}}{}^2 \;=\; |\int_{\gamma_p} \Omega\,|^2 \;\longrightarrow\; 0 \qquad \text{if } \gamma_p \to 0\ .$$

The larger the dimension of the compactification manifold X_n is (and the lower the space-time dimension $D = 10 - n$), the more complicated the topology of it will be, ie., the larger the number of "holes" around which the branes can wrap. For a six dimensional Calabi-Yau space X_6, there will be generically an abundance of extra non-perturbative states that are not seen in traditional string perturbation theory. These can show up in $D = 4$ as ordinary gauge or matter fields. It is the presence of such non-perturbative, potentially massless states that is the basis for many non-trivial dual equivalences of string theories.

When going back to ten dimensions by making the compactification manifold very large, these states become arbitrarily heavy and eventually decouple. In this sense, a string model has after compactification many more states that were not present in ten dimensions before. The non-perturbative spectrum is very finely tuned: in an analogous way that taking out a perturbative string state from the spectrum would destroy modular invariance (which is a global property of the 2d world-sheet) and thus would ruin perturbative consistency, taking out a non-perturbative state would destroy duality symmetries (which are a global property of the compactification manifolds): it would make the global behavior of the theory over the moduli space inconsistent.

2.4 Stringy geometry

We have seen in section 2.1 that $N = 2$ supersymmetric Yang-Mills theory is sometimes better described in terms of dual "magnetic" variables, namely when we are in a region of the moduli space where certain magnetic monopoles are light. In this dual formulation, the originally solitonic monopoles look like ordinary elementary, perturbative degrees of freedom ("electrons"). Analogously, dual formulations exist also for string theories, in which non-perturbative solitons are described in terms of weakly coupled "elementary" degrees of freedom. It was one of the breakthroughs in the field when it was realized how this exactly works: the relevant objects dual to certain solitonic states are special kinds of (p-)branes, so-called "D(p-)branes".[10] Due to the many types of p-branes on the one hand, and the large variety of possible singular geometries of the manifolds X on the other, the general situation is however very complicated. It is easiest to describe it in terms of typical examples.

For instance, massless or almost massless non-abelian gauge bosons $W^\pm$ belonging to $SU(2)$ can be equivalently described in a number of dual ways (see Fig.5):

i) In the heterotic string compactified on some higher dimensional torus, a massless gauge boson is represented by a fundamental heterotic string wrapped around part of the torus, with a certain

radius (say $R = 1$ in some units; changing the orientation of the string maps $W^+ \leftrightarrow W^-$).This is a perturbative description, since it involves an elementary string. If the radius deviates from $R = 1$, the gauge field gets a mass, providing a geometrical realization of the Higgs mechanism.
ii) In the compactified type IIA string, the gauge boson arises from wrapping a 2-brane around a collapsed 2-cycle γ_2 of X. This is a non-perturbative formulation in terms of string theory. If γ_2 does not quite vanish, the gauge field retains some non-zero mass, thereby realizing the Higgs mechanism in a different manner.
iii) In the type I string model, an $SU(2)$ gauge boson is realized by an open string stretched between two flat D-branes. This is another perturbative formulation of the Higgs mechanism. The mass of the gauge boson is proportional to the length of the open string, and thus vanishes if the two D-branes move on top of each other.

We thus see that very different mathematical geometries can represent the identical physical theory, here the $SU(2)$ Higgs model – these geometries really should be identified in string theory. This provides a special example of a more general concept, which is about getting better and better understood: "stringy geometry". In stringy geometry, certain mathematically different geometries are treated as equivalent, and just seen as different choices of "coordinates" in some abstract space of string theories. The underlying physical idea is that while in ordinary geometry point particles are used to measure properties of space-time, in stringy geometry one augments this by string and other p-brane probes. It is the wrappings and stretchings that these extra objects are able to do that can wash out the difference between topologically and geometrically distinct manifolds.

In effect, a string theory of some type "A" when compactified on some manifold X_A, can be dual to another string theory "B" on some manifold X_B – and this even if A, B and/or X_A and X_B are profoundly different.[11] Again, all what matters is that the full non-perturbative theories coincide, while there is no need for the perturbative approximations to be even remotely similar.

3 The Grand Picture

3.1 Dualities of higher dimensional string theories

We are now prepared to go back to ten dimensions and revisit the five perturbatively defined string models of section 1.2. In view of the remarks of the preceding sections concerning the irrelevance of perturbative concepts, we will now find it perhaps not too surprising to note that these five theories are really nothing but different approximations of just one theory. In complete analogy to what we said about $N = 2$ supersymmetric gauge theory in section 2.1, they simply correspond to certain choices of preferred "coordinates" that are adapted to specific parameter regions.

Although these facts can be stated in such simple terms, they are so non-trivial that it took more than a decade to discover them. It can now be explicitly shown that by compactifying any one of the five theories on a suitable manifold, and then de-compactifying it in another manner, one can reach any other of the five theories in a continuous way.

A surprise happened when the strong coupling limit of the type IIA string was investigated[12]: it turned out that in this limit, certain non-perturbative "D0-branes" form a continuous spectrum and effectively generate an extra 11th dimension. That is, at ultra strong coupling the ten-dimensional type IIA string theory miraculously gains 11-dimensional Lorentz invariance, and the low-energy theory turns into $D = 11$ supergravity. This was especially surprising because eleven dimensional supergravity is not a low energy limit of a string theory, but rather seems to be related to supermembranes. In other words, non-perturbative dualities take us beyond string theory !

So what we have are not five but six 10- or 11-dimensional approximations, or local coordinate patches on some moduli space – see Fig.6. But to what entity are these theories approximations ? Do we have here the moduli space of some underlying microscopic theory ? Indeed there is a candidate

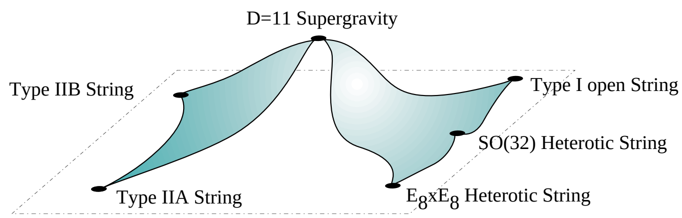


Figure 6: Moduli space in ten and eleven dimensions. Its singular asymptotic regions correspond to the five well-known supersymmetric string theories in $D = 10$, plus an eleven dimensional M-theory. The vertical direction roughly reflects the space-time dimension.

for such a theory, dubbed "M-theory"[13,14]. Its low energy limit does give $D = 11$ supergravity, and simple compactifications of it give all of the five string theories in $D = 10$. It may well be that M-theory, currently under intense investigation, holds the key for a detailed understanding of non-perturbative string theory. However, since M-theory is (presumably) a main topic of Susskind's lecture in these proceedings, we will not discuss any further details here. We will rather return to the lower dimensional theories.

3.2 Quantum moduli space of four-dimensional strings

Fig.6. shows only a small piece of a much more extended moduli space, namely only the piece that describes higher dimensional theories. These are relatively simple and there is only a small number of them. As discussed above, the lower dimensional theories obtained by compactification are much more intricate.

Among the best-investigated string theories in four dimensions are the ones with $N = 2$ supersymmetry – these are the closest analogs of the $N = 2$ gauge theory that is so well understood. They can be obtained by compactifications of type IIA/B strings on Calabi-Yau manifolds X_6, or dually, by compactifications of heterotic strings on[c] $K3 \times T_2$. Since there are roughly 10^4 of such Calabi-Yau manifolds X_6, the complete $D = 4$ string moduli space will have roughly 10^4 components, each with typical dimension 100 (keeping in mind that there can be non-trivial identifications between parts of this moduli space)– see Fig.7. We see that the moduli space is drastically more complicated as it is either for the high-dimensional theories, or for the $N = 2$ gauge theory. Each of the 10^4 components typically has several ten or eleven dimensional decompactification limits, so that one should imagine very many connections between the upper and lower parts of Fig.7 (indicated by dashed lines).

An interesting fact is that all of the known[d] 10^4 families of perturbative $D = 4$ string vacua are connected by non-perturbative extremal transitions. To understand what we mean by that, simply follow a path in the moduli space as indicated in Fig.7, starting somewhere in the interior of a blob. The massive spectrum will continuously change, and when we hit singularities, extra massless states appear and perturbation theory breaks down.

It can in particular happen that a non-perturbative massless Higgs field appears, to which we can subsequently give a vacuum expectation value to deform the theory in a direction that was not visible before. In this way, we can leave the moduli space of a single perturbative string compactification, and enter the moduli space of another one. Thus, non-perturbative extra states can glue together different perturbative families of vacua in a physically smooth way.[9] It can be proven that one can connect in this manner all of the roughly 10^4 known components that were previously attributed to

[c] Here, T_2 denotes the two-torus, and $K3$ the four dimensional version of a Calabi-Yau space.
[d] Strictly speaking, we cannot exclude further, disconnected components to exist.

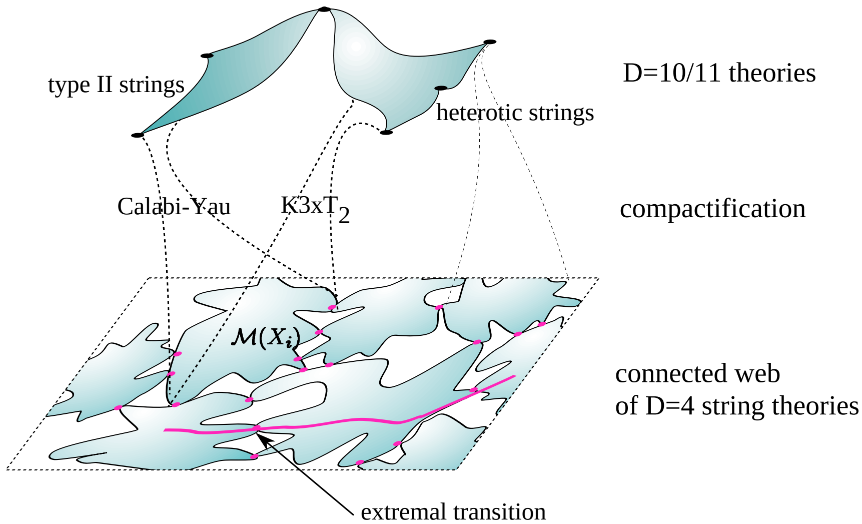


Figure 7: Rough sketch of the space of $N = 2$ supersymmetric string theories in four dimensions. In general, a given region in the 4d moduli space can be reached in several dual ways via compactification of the higher dimensional theories. Each of the perhaps 10^4 components corresponds typically to a 100 dimensional family of perturbative string vacua, and represents the moduli space of a single Calabi-Yau manifold (like the one shown in Fig.4). Non-perturbatively, all these vacua turn out to be connected by extremal transitions and thus form a continuous web.

different four dimensional string theories. In other words, the full non-perturbative quantum moduli space of $N = 2$ supersymmetric strings seems to form *one* single entity.

This is not much of a practical help for solving the vacuum degeneracy problem, but it is conceptually satisfying: instead having to choose between many four dimensional string theories, each one equipped with its own moduli space, we really have just one theory with however very many facets.

This as far as $N = 2$ supersymmetric strings in four dimensions are concerned – the situation will still be much more complicated for the phenomenologically important $N = 1$ supersymmetric string theories, whose investigation is currently under way. The main novel features that can be addressed in these theories are chirality and supersymmetry breaking. It seems that certain aspects of these theories are best described by choosing still another dual formulation, called "F-theory"[15]. This is a construction formally living in twelve dimensions, and whether it is simply a trick to describe certain features in an elegant fashion, or whether it is a honest novel theory in its own, remains to be seen.

4 "Theoretical experiments"

How can we convince ourselves that the considerations of the previous sections really make sense ? Clearly, all what we can do for the foreseeable future to test these ideas are consistency checks. Such checks can be highly non-trivial, from a formal as well as from a physical point of view. So far, numerous qualitative and quantitative tests have been performed, and not a single test on the dualities has ever failed !

To give some flavor, let us recapitulate a few characteristic checks:

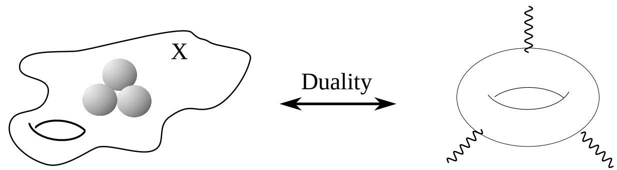


Figure 8: Counting spheres in a Calabi-Yau space X in type IIA string theory, leads ultimately to the same expressions for certain low energy couplings as a standard one-loop computation in the heterotic string.

i) *Complicated perturbative corrections* can be computed and compared for string models that are dual to each other. In all cases, the results perfectly agree. Consider for example certain 3-point couplings κ in $N = 2$ supersymmetric string compactifications. As mentioned before, such theories can be obtained either from type II strings on Calabi-Yau manifolds X_6, or dually from heterotic string compactifications on $K3\times T_2$. In the type II formulation, these couplings can be computed (via "mirror symmetry"[16]) by counting world-sheet instantons (embedded spheres) inside the Calabi-Yau space. Concretely, in a specific model the result takes the following explicit form:

$$\kappa \;=\; \frac{i}{2\pi}\frac{E_4(T)E_4(U)E_6(U)(E_4(T)^3 - E_6(T)^2)}{E_4(U)^3E_6(T)^2 - E_4(T)^3E_6(U)^2}\ ,$$

in terms of certain modular functions E_4, E_6 depending on moduli fields T, U. The very same expression can be obtained also in a completely different manner, namely by performing a standard one-loop computation in the dual heterotic string model – in precise agreement with the postulated string duality; see Fig.8. Many similar tests, also involving higher loops and gravitational couplings, have been shown to work out as well.

ii) *State count in black holes.* This is a highly non-trivial physics test. The issue is to compute the Bekenstein-Hawking entropy S_{BH} (=area of horizon) of an extremal (or near-extremal) black hole. Strominger and Vafa[17] considered the particular case of an extremal $N = 4$ supersymmetric black hole in $D = 5$, where one knows that

$$S_{BH} \;=\; 2\pi\sqrt{\frac{q_f q_h}{2}}\ ,$$

where q_f, q_h are the electric and axionic charges of the black hole. The idea is to use string duality to represent a large semi-classical black hole in terms of a type IIB string compactification on $K3 \times S^1$. This eventually boils down to a 2d sigma model on the moduli space of a gas of $D0$-branes on $K3$. Counting states in this model indeed exactly reproduces the above entropy formula for large charges.

This test (and refinements of it[18]) does not only add credibility to the string duality claims from a new perspective, but also tells that there are *no missing degrees of freedom* that we might have been overlooking – any theory of quantum gravity better comes up with the same count of relevant microscopic states.

ii) *Recovering exact non-perturbative field theory results.* One can derive the exact solution of the $N = 2$ supersymmetric gauge theory described in section 2.1 as a consequence of string duality. The point is that duality often maps classical into quantum effects and vice versa. This makes it for example possible[19] to obtain with a *classical* computation in the compactified type II string, certain non-perturbative results for the compactified heterotic string. In particular, counting world-sheet instantons similarly as in point i) above, and suitably decoupling gravity (see Fig.9), allows to exactly reproduce[20] the non-perturbative effective gauge coupling $g(\phi)$ of Fig.2.

This is very satisfying, as it gives support to both the underlying string duality and the solution of the $N = 2$ gauge theory. Indeed, while the basic heterotic-type II string duality[11] and the Seiberg-

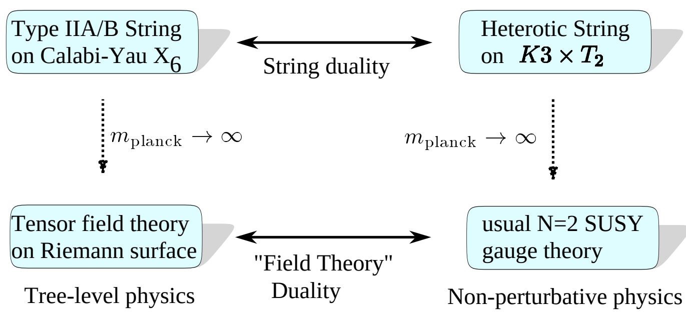


Figure 9: Recovering Seiberg-Witten theory from string duality. In the field theory limit, one sends the Planck mass to infinity in order to decouple gravity and other effects that are not important. A remnant of the string duality survives, which is a duality between the standard formulation of the gauge theory, and a novel one that was not known before. It gives a physical interpretation of the geometry underlying the non-perturbative solution of the $N = 2$ gauge theory.

Witten theory[7] have been found independently from each other around the same time, their mutual compatibility was shown only later.

It is the clearly visible, convergent evolution of a priori separate physical concepts, besides overwhelming "experimental" evidence, what gives the string theorists confidence in the validity of their ideas.

5 Spinoff: D-brane technology, and novel quantum theories without gravity.

The techniques for obtaining exact non-perturbative results for ordinary field theories from string duality are not limited to only reproducing results that one already knows – in fact, they have opened the door for deriving new results for a whole variety of field theories in various dimensions.

More specifically, there are currently two main approaches (related by duality) for obtaining standard and non-standard field theories from string theory – each one has its own virtues. One can either study the singular geometry of Calabi-Yau or other compactification manifolds, and consider the effect of wrapped branes – much like it is depicted in Fig.5 part ii).

Alternatively, one can model[21] the relevant string geometry in terms of parallel D-branes, with open strings and other branes running between them (remember that D-branes are roughly perturbative duals of p-brane solitons). This refers to part iii) of Fig.5. For example, $N = 2$ Yang-Mills theory in four dimensions can be represented by a configuration of D-branes as shown in Fig.10 a). From this simple picture one can reproduce the non-perturbative solution of the gauge theory[22]. Similar arrangements can describe $N = 1$ supersymmetric gauge theories as well, like supersymmetric QCD (Fig.10 b)). Some qualitative features, like chiral symmetry breaking or confinement, can be seen in this model.[23]

In addition, somewhat unexpectedly, these methods have also led to the discovery of completely new kinds of quantum theories that were not known to exist before. More specifically, when decoupling gravity there is no reason why one should always end up with a standard field theory that one already knows, like gauge theory coupled to some matter fields. Rather, by either looking to specific singular geometries or to appropriate D-brane configurations, one has found a number of exotic limits.[12,24] Such theories are typically strongly coupled and do not have any known description in terms of traditional quantum field theory; they comprise (non-gravitational) tensionless strings and novel non-trivial IR fixed points. The main indication that they really exist is that they arise as decoupling limits of a larger string (or M-) theory that is consistent by itself.

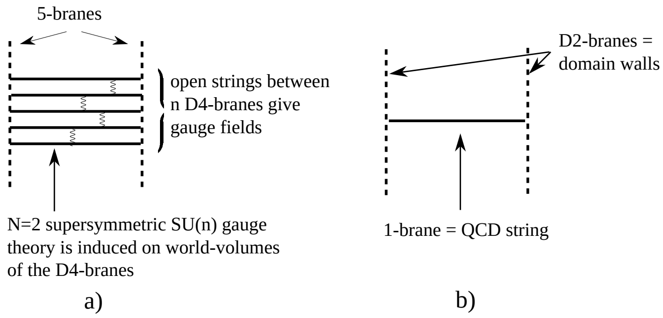


Figure 10: *D*-brane technology allows to represent gauge theories in a dual way, in which non-trivial properties are encoded in simple geometric pictures.

Especially interesting in this context are exotic theories that serve as transition points between $N = 1$ string vacua with different net numbers of chiral fermions. Such smooth chirality changing processes are not possible in conventional field theory, but they can occur string theory.[25] This opens up the possibility that $N = 1$ supersymmetric string vacua are (to some extent) unified in a manner analogous to the $N = 2$ vacua, similar to but much more complicated as hinted at in Fig.7.

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