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Theoretical aspects of W physics*

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High-precision predictions for W-production processes are complicated by the instability of the W bosons, requirements of gauge invariance, and the necessity to include radiative corrections. Salient features and recent progress concerning these issues are discussed for the process $\mathrm{ee} \to \mathrm{WW} \to 4f$.

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Abstract. High-precision predictions for W-production processes are complicated by the instability of the W bosons, requirements of gauge invariance, and the necessity to include radiative corrections. Salient features and recent progress concerning these issues are discussed for the process ee $\to$ WW $\to 4f$.

1 Introduction

The investigation of the W boson and its properties at LEP2 [1] and possible future linear e^+e^- colliders [2] is very promising. Together with the Fermi constant and the LEP1 observables, an improvement of the empirical value of the W-boson mass M_W will put better indirect constraints on the mass of the Standard-Model Higgs boson and on new-physics parameters. The W-boson mass can be obtained by inspecting the total W-pair production cross-section near threshold, where it is most sensitive to M_W, or by reconstructing the invariant masses of the W decay products. W-boson production in ee-, eγ-, and $\gamma\gamma$-collisions also yields direct information on the vector-boson self-couplings, which are governed by the gauge symmetry. For low and intermediate centre-of-mass (CM) energies, useful information can be obtained by investigating the distributions over the W-production angles. For higher energies also the total cross-sections become very sensitive to anomalous couplings.

The described experimental aims require the knowledge of the Standard-Model predictions for the mentioned observables to a high precision, e.g. for the cross-section of W-pair production at LEP2 to $\sim$0.5% [3]. The instability of the W bosons, the issue of gauge invariance, and the relevance of radiative corrections render this task highly non-trivial. In this short presentation these sources of complications and their consequences for actual calculations are discussed, and special emphasis is laid on recent developments. For definiteness, we consider the process ee $\to$ WW $\to 4f$, which is the most important one for W physics at LEP2.

2 Gauge invariance and finite-width effects

At and beyond a per-cent accuracy, gauge-boson resonances cannot be treated as on-shell states in lowest-order calculations, since the impact of a finite decay width Γ_V for a gauge boson V of mass M_V can be roughly estimated to Γ_V/M_V, which is, for instance, $\sim$3% for the W boson. Therefore, the full set of tree-level diagrams for a given fermionic final state has to be taken into account. For ee $\to$ WW $\to 4f$ this includes graphs with two resonant W-boson lines ("signal diagrams") and graphs with one or no W resonance ("background diagrams"), leading to the following structure of the amplitude [3–5]:

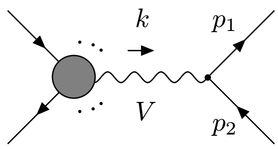
$$\mathcal{M} = \underbrace{\frac{R_{+-}(k_+^2,k_-^2)}{(k_+^2-M_\mathrm{W}^2)(k_-^2-M_\mathrm{W}^2)}}_{\text{doubly-resonant}} + \underbrace{\frac{R_{+}(k_+^2,k_-^2)}{k_+^2-M_\mathrm{W}^2} + \frac{R_{-}(k_+^2,k_-^2)}{k_-^2-M_\mathrm{W}^2}}_{\text{singly-resonant}} + \underbrace{N(k_+^2,k_-^2)}_{\text{non-resonant}} . \tag{1}$$

Gauge invariance implies that $\mathcal{M}$ is independent of the gauge fixing used for calculating Feynman graphs (gauge-parameter independence), and that gauge cancellations between different contributions to $\mathcal{M}$ take place. These gauge cancellations are ruled by Ward identities. For a physical description of the W resonances, the finite W decay width has to be introduced in the resonance poles. However, since only the sum in (1), but not the single contributions to $\mathcal{M}$, possesses the gauge-invariance properties, the simple replacement

$$\left[k_\pm^2 - M_\mathrm{W}^2\right]^{-1} \quad \to \quad \left[k_\pm^2 - M_\mathrm{W}^2 + iM_\mathrm{W}\mathit{\Gamma}_\mathrm{W}(k_\pm^2)\right]^{-1} \tag{2}$$

in general violates gauge invariance.

Although such gauge-invariance-breaking terms are formally suppressed by a factor $\mathit{\Gamma}_\mathrm{V}/M_\mathrm{V}$ [6], they can completely destroy the consistency of predictions if they disturb gauge cancellations [7–9]. Gauge cancellations can occur if a current $\bar{u}(p_1)\gamma^\mu u(p_2)$ that is associated to a pair of external fermions becomes proportional to the momentum k of the attached gauge boson:



$$\sim \quad \frac{1}{k^2 - M_\mathrm{V}^2}\, k^\mu T_\mu^V .$$

T_μ^V represents the set of subgraphs hidden in the blob. The cancellations in $k^\mu T_\mu^V$ are governed by the Ward identities

$$k^\mu T_\mu^\gamma = 0, \qquad k^\mu T_\mu^Z = iM_\mathrm{Z}T^\chi, \qquad k^\mu T_\mu^{W^\pm} = \pm M_\mathrm{W} T^{\phi^\pm}. \tag{3}$$

The first one expresses electromagnetic current conservation and is relevant, e.g., for forward scattering of $\mathrm{e}^\pm$ $(k \to 0)$. The others imply the Goldstone-boson equivalence theorem, which relates the amplitudes for high-energetic longitudinal W and Z bosons $(k^0 \gg M_\mathrm{V})$ to the ones for their respective would-be Goldstone bosons ϕ and χ.

Among the proposed methods (see Refs. [3, 9] and references therein) to introduce finite widths for W and Z bosons in tree-level amplitudes, the field-theoretically most convincing one is provided by the "fermion-loop scheme". This scheme goes beyond a pure tree-level calculation by including and consistently Dyson-summing all closed fermion loops in $\mathcal{O}(\alpha)$. This procedure introduces the running tree-level width in gauge-boson propagators via the imaginary parts of the fermion loops. The Ward identities (3) are not violated, since the fermion-loop (as well as the tree-level) contributions to vertex functions obey the simple linear (also called "naive") Ward identities that are related to the original gauge invariance rather than to the more involved BRS invariance of the quantized theory. Owing to the linearity of the crucial Ward identities for the vertex functions, the fermion-loop scheme works both with the full fermion loops and with the restriction to their imaginary parts. The full fermion-loop scheme has been worked out for ee $\to$ WW $\to 4f$ in Ref. [9], where applications are discussed as well. Simplified versions of the scheme have been introduced in Ref. [8].

The fermion-loop scheme is not applicable in the presence of resonant particles that do not exclusively decay into fermions. For such particles, parts of the decay width are contained in bosonic corrections. The Dyson summation of fermionic *and* bosonic $\mathcal{O}(\alpha)$ corrections leads to inconsistencies in the usual field-theoretical approach, i.e. the Ward identities (3) are broken in general. This is due to the fact that the bosonic $\mathcal{O}(\alpha)$ contributions to vertex functions do not obey the "naive Ward identities". The problem is circumvented by employing the background-field formalism [10], in which these naive identities are valid. This implies [11] that a consistent Dyson summation of fermionic and bosonic corrections to any order in α does not disturb the Ward identities (3). Therefore, the background-field approach provides a natural generalization of the fermion-loop scheme. We recall that any resummation formalism goes beyond a strict order-by-order calculation and necessarily involves ambiguities in relative order α^n if not all n-loop diagrams are included. This kind of scheme dependence, which in particular concerns gauge dependences, is only resolved by successively calculating the missing orders.

Note that the consistent resummation of all $\mathcal{O}(\alpha)$ loop corrections does not automatically lead to $\mathcal{O}(\alpha)$ precision in the predictions if resonances are involved. The imaginary parts of one-loop self-energies generate only tree-level decay widths so that directly on resonance one order in α is lost. To obtain also full $\mathcal{O}(\alpha)$ precision in these cases, the imaginary parts of the two-loop self-energies are required. However, how and whether this two-loop contribution can be included in a practical way without violating the Ward identities (3) is still an open problem. Taking the imaginary parts of all two-loop contributions solves the problem in principle at least for the background-field approach, but this is certainly impractical.

Fortunately, the full off-shell calculation for the process ee $\to$ WW $\to 4f$ in $\mathcal{O}(\alpha)$ is not needed for most applications. Sufficiently above the W-pair threshold a good approximation should be obtained by taking into account only the doubly-resonant part of the amplitude (1), leading to an error of the order of $\alpha\Gamma_{\mathrm{W}}/(\pi M_{\mathrm{W}}) \lesssim 0.1\%$. In such a "pole scheme" calculation [4,12] the numerator $R_{+-}(k_+^2, k_-^2)$ has to be replaced by the gauge-independent residue $R_{+-}(M_{\mathrm{W}}^2, M_{\mathrm{W}}^2)$. The structure of this approach, which is in fact non-trivial and has not been completely carried out so far, is described below.

3 Electroweak radiative corrections

Present-day Monte Carlo generators for off-shell W-pair production (see e.g. Ref. [13]) include only universal electroweak $\mathcal{O}(\alpha)$ corrections[1] such as the running of the electromagnetic coupling, $\alpha(q^2)$, leading corrections entering via the ρ-parameter, the Coulomb singularity [15], which is important near threshold, and mass-singular logarithms $\alpha\ln(m_\mathrm{e}^2/Q^2)$ from initial-state radiation. In leading order, the scale Q^2 is not determined and has to be set to a typical scale for the process; for the following we take $Q^2 = s$. Since the full $\mathcal{O}(\alpha)$ correction is not known for off-shell W pairs, the size of the neglected $\mathcal{O}(\alpha)$ contributions is estimated by inspecting on-shell W-pair production, for which the exact $\mathcal{O}(\alpha)$ correction and the leading contributions were presented in Refs. [16] and [17], respectively. These $\mathcal{O}(\alpha)$ corrections have already been implemented in an event generator for on-shell W pairs [18]. The following table shows the difference between an "improved Born approximation" δ_IBA, which is based on the above-mentioned universal corrections, and the corresponding full $\mathcal{O}(\alpha)$ correction δ to the Born cross-section integrated over the W-production angle θ for some CM energies $\sqrt{s}$.

θ range	$\sqrt{s}$/ GeV	161	175	200	500	1000	2000
$0°<\theta<180°$	$(\delta_\mathrm{IBA}-\delta)/\%$	1.5	1.3	1.5	3.7	6.0	9.3
$10°<\theta<170°$		1.5	1.3	1.5	4.7	11	22

Here the corrections δ_IBA and δ include only soft-photon emission. For more details and results we refer to Refs. [3, 19]. The quantity $\delta_\mathrm{IBA}-\delta$ corresponds to the neglected non-leading corrections and amounts to $\sim$1–2% for LEP2 energies, but to $\sim$10–20% in the TeV range. Thus, in view of the aimed 0.5% level of accuracy for LEP2 and all the more for energies of future linear colliders, the inclusion of non-leading corrections is indispensable. The large contributions in $\delta_\mathrm{IBA}-\delta$ at high energies are due to terms such as $\alpha\ln^2(s/M_\mathrm{W}^2)$, which arise from vertex and box corrections and can be read off from the high-energy expansion [20] of the virtual and soft-photonic $\mathcal{O}(\alpha)$ corrections.

As explained above, a reasonable starting point for incorporating $\mathcal{O}(\alpha)$ corrections beyond universal effects is provided by a double-pole approximation. Doubly-resonant corrections to $\mathrm{ee}\to\mathrm{WW}\to 4f$ can be classified into two types: factorizable and non-factorizable corrections [3–5]. The former are those that correspond either to W-pair production [16] or to W decay [21]. Since these corrections were extensively discussed in the literature, we focus on the non-factorizable corrections. They are furnished by diagrams in which the production subprocess and/or the decay subprocesses are not independent. Among such corrections, doubly-resonant contributions only arise if a "soft" photon of energy $E_\gamma \lesssim \Gamma_\mathrm{W}$ is exchanged between the subprocesses.

In Ref. [22] it was shown that the non-factorizable corrections vanish if the invariant masses of both W bosons are integrated over. Thus, these cor-

[1] The QCD corrections for hadronic final states are discussed in Ref. [14].

rections do not influence pure angular distributions, which are of particular importance for the analysis of gauge-boson couplings. For exclusive quantities the non-factorizable corrections are non-vanishing and have been calculated in Refs. [23–25][2]. It turns out [25] that the correction factor to the differential Born cross-section is non-universal in the sense that it depends on the parametrization of phase space. The calculations [23–25] have been carried out using the invariant masses $M_\pm$ of the $W^\pm$ bosons, which are identified with the invariant masses of the respective final-state fermion pairs, as independent variables. Since all effects from the initial e^+e^- state cancel, the resulting correction factor does not depend on the W-production angle and is also applicable to processes such as $\gamma\gamma \to WW \to 4f$. After integrating over production and decay angles, i.e. when inspecting pure invariant-mass distributions, the non-factorizable corrections become independent of the fermionic final state. Figure 1 shows that non-factorizable corrections to a single invariant-mass distribution are of the order of ~1% for LEP2 energies, shifting the maximum of the distribution by an amount of 1–2 MeV, which is small with respect to the experimental uncertainty at LEP2 [26]. For higher energies the non-factorizable correction is more and more suppressed.

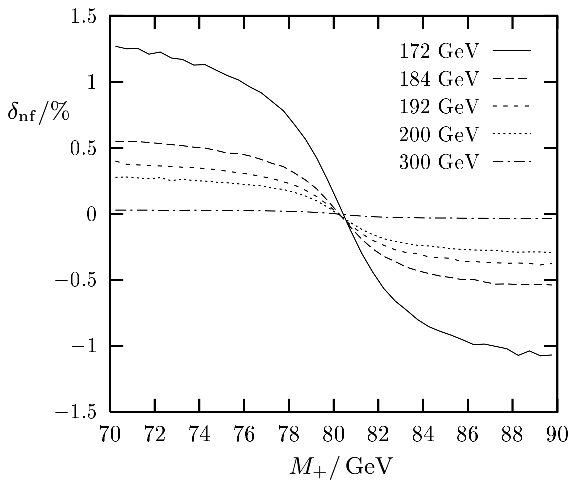


Fig. 1. Relative non-factorizable corrections to the invariant-mass distribution $d\sigma/dM_+$ for some CM energies (plot taken from Ref. [25]).

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