QCD TESTS AT e^+e^- Colliders

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A short review of the history and a 'slide-show' of QCD tests in e$^+$e$^-$ annihilation is given. The world summary of measurements of α_s is updated.

1. INTRODUCTION

Hadronic final states of e$^+$e$^-$ annihilations have proven to provide precise tests of the strong interaction and of its underlying theory, Quantum Chromodynamics (QCD). The success and increasing significance of QCD tests which was achieved in the past few years, especially with the precise data from LEP, was based on improvements and a deeper understanding of the theoretical predictions as well as of the experimental techniques, which both substantially profited from the experience of earlier studies with data from lower energy colliders.

In fact, in many respects it seems desirable to return to the low energy e$^+$e$^-$ data and reanalyse them with the knowledge of today. One reason behind this demand is that the size as well as the energy dependence of the strong coupling parameter α_s is largest at lower energies, and thus the characteristic feature of QCD, asymptotic freedom and the running of α_s, can be most significantly tested if reliable data at 'low' energies are available; see [1] for a recent review.

In the light of these remarks, a short historical review of QCD tests in e$^+$e$^-$ annihilation will be given in this report. In Section 2, selected highlights of early QCD studies, at energies below the Z^0 pole, will be reviewed, and some of the original illustrations will be reproduced. Section 3 contains a brief overview of QCD tests achieved at LEP and at the SLC. In section 4, the latest measurements of α_s from LEP at energies above the Z^0 pole will be summarised. A comparison with measurements of α_s from other processes will be given in Section 5, including an update of the world summary of α_s determinations [1].

2. QCD IN e$^+$e$^-$ ANNIHILATION – FROM SPEAR TO SLC AND LEP

In 1975, the first "Evidence for Jet Structure in Hadron Production by e$^+$e$^-$ Annihilation" was reported [2]. The data, taken with the SLAC-LBL magnetic detector at the SPEAR storage ring, showed increasing evidence for the production of two-jet like events when the center of mass energy, E_{cm}, was raised from 3 to 7.4 GeV (see Fig. 1). The jet structure manifested itself as a decrease of the mean sphericity, which is a measure of the global shape of hadronic events. The angular distribution of the jet axes from the same measurement provided evidence that the underlying partons must have spin $\frac{1}{2}$. These observations, which were further corroborated by similar measurements at $E_{cm} = 14$ to 34 GeV at the e$^+$e$^-$ storage ring PETRA [3], see Fig. 2, confirmed the basic ideas of the quark-parton model, which relates the constituents of the static quark model with partons produced in particle collisions at high energies.

In 1979, a small fraction of planar 3-jet events was observed by the PETRA experiments around $E_{cm} \approx 30$ GeV [4], which was attributed to the emission of a third parton with zero electric charge and spin 1 [5], as expected for gluon bremsstrahlung predicted by QCD [6]. First evidence for 3-jet like events came from visual scans of the energy flow within hadronic events (Fig. 3; see also Fig. 5), followed by statistical analyses of global event shapes like oblateness (Fig. 4) and of angular correlations which are sensitive to the gluon spin [5]. At the end of 1979, the MARK-J collaboration reported a first measurement of α_s, in first order perturbative QCD, from the shape
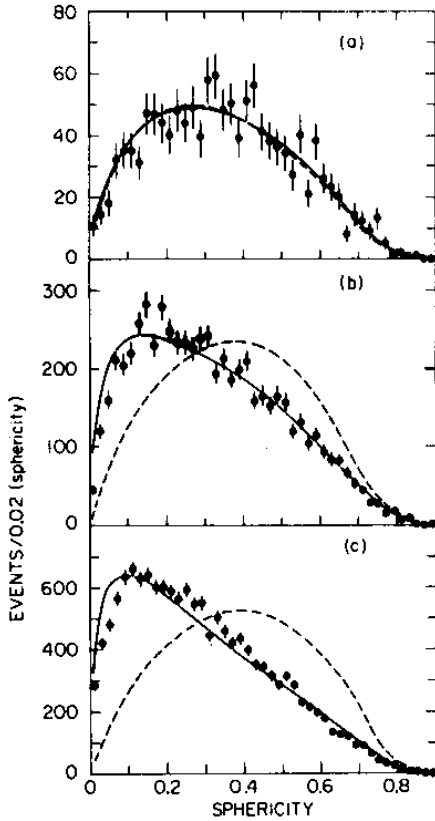
Figure 1. Sphericity distributions observed at E_{cm} = 3.0 GeV (a), 6.2 GeV (b) and at 7.4 GeV (c), compared with a jet (solid curves) and a phase space model (dashed curves). Data are from the SLAC-LBL Magnetic Detector at SPEAR [2].
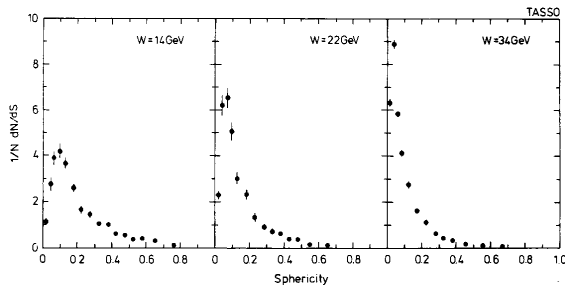

Figure 2. Sphericity distributions at E_{cm} = 14 GeV, 22 GeV and 34 GeV (from TASSO [3]).
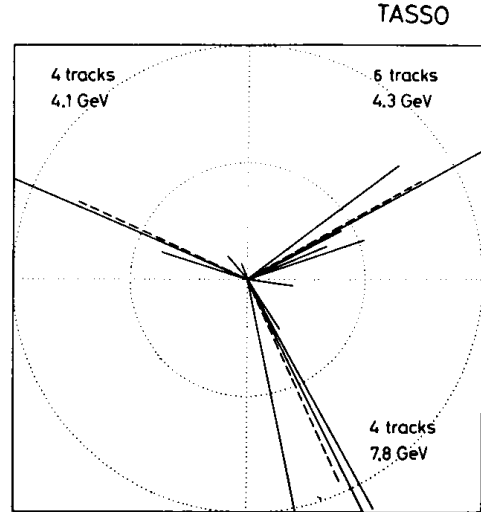
TASSO

Figure 3. The first 3-jet event observed by TASSO at PETRA. Plotted are the momentum vectors of the charged particles, projected into the principal event plane; the dotted lines indicate the reconstructed jet axes (from [7]).

of the oblateness distribution [8]. The first analysis based on the definition and reconstruction of resolvable jets was published in 1980 by the PLUTO collaboration [9], see Fig. 7.

In 1981, the JADE collaboration found first evidence [10] that the string hadronization model [11] provides a better description of the observed hadron flow in 3-jet events than does an independent jet hadronization model.

The year 1982 brought first evidence for 4-jet like events, observed by JADE at E_{cm} = 33 GeV [12], and the first determination of α_s in second order perturbation theory ($\mathcal{O}(\alpha_s^2)$) [13], again by JADE [14]. The latter analysis was already based on principles and ideas which nowadays are standard for most of the α_s analyses at SLC and LEP: An analytic QCD calculation in $\mathcal{O}(\alpha_s^2)$ perturbation theory was fitted to a measured event shape distribution which had been corrected for detector resolution and hadronization effects (Fig. 8).

In 1986 JADE published the first detailed analysis of n-jet event production rates [15], introducing a jet finding mechanism which has since then been used in many other studies. The ra-
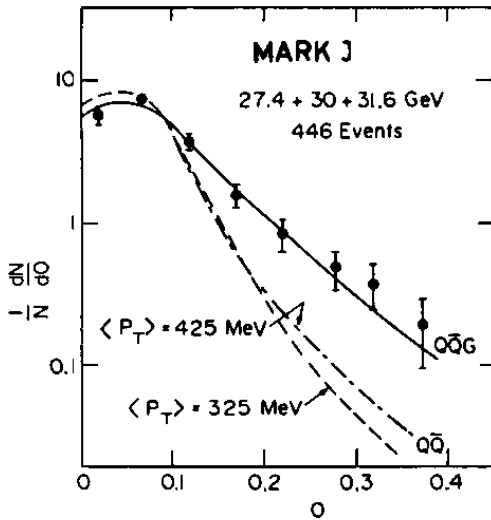
Figure 4. Oblateness distribution measured at PETRA, compared with the predictions based on a $q\bar{q}g$ model (full line) and a $q\bar{q}$ model (from [7]).
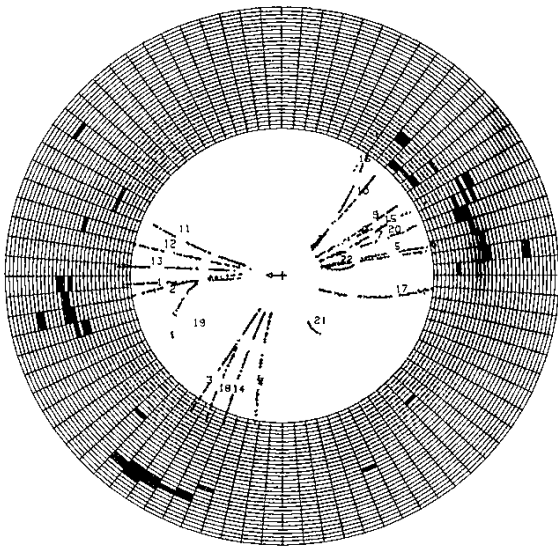

Figure 6. Distribution of the Ellis-Karliner angle $\tilde{\Theta}$ measured by TASSO [5], compared with model predictions for a vector and a scalar gluon.

Figure 5. Detector view of a 'typical' 3-jet event recorded with the JADE detector at PETRA.
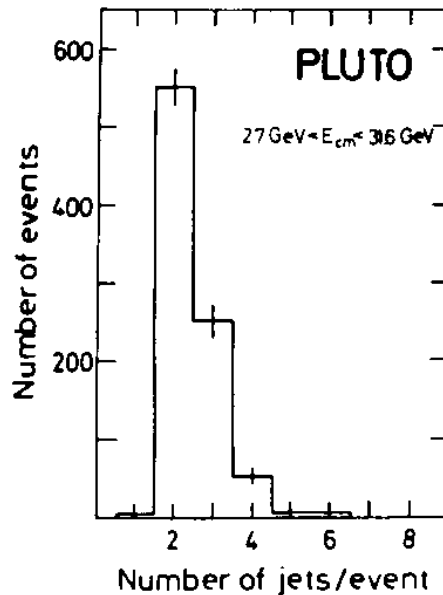
Figure 7. First jet multiplicity distribution measured at PETRA [9].

tio of 4-jet over 3-jet event production rates was found to be significantly larger than predicted by $\mathcal{O}(\alpha_s^2)$ QCD, an observation that motivated studies of the influence of the choice of renormalization scales in finite order perturbative QCD
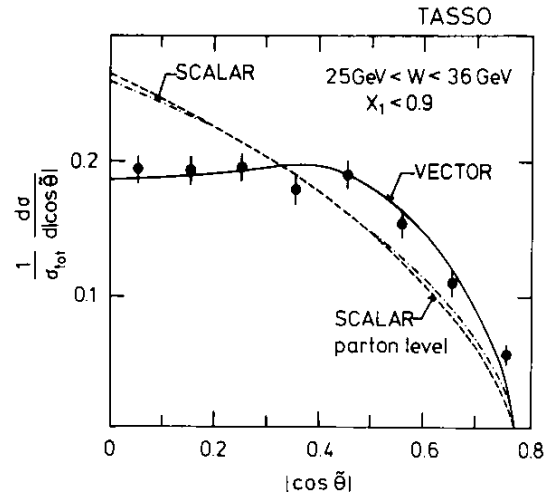
[16]. In 1988 it was demonstrated by JADE that the energy dependence of 3-jet event production rates gives evidence for the running of α_s [17], see Fig. 9. First signs of the presence of the gluon self coupling were observed in a study of 4-jet events
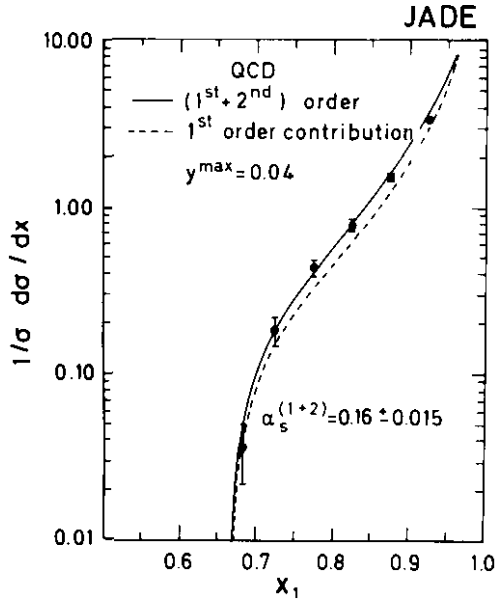
Figure 8. Corrected distribution of the scaled energy of the most energetic jet of reconstructed 3-jet events, compared with analytic QCD predictions in leading (dashed) and in next-to-leading order (full line) [14].
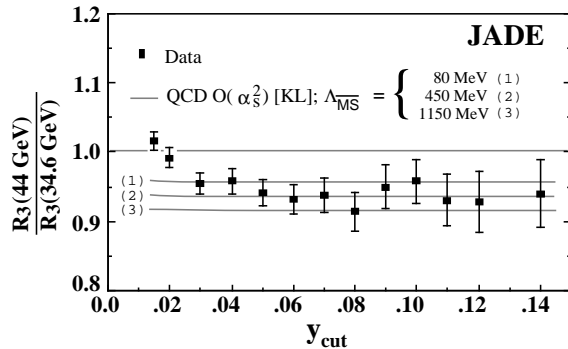

Figure 9. The ratio of 3-jet productions rates at 44 and at 34.6 GeV c.m. energy, analysed with the JADE jet finder [17]. The data are compared with QCD analytic predictions. For the hypothesis of an α_s which does *not* run with energy, a constant ratio of 1 would be expected.

by AMY [18] around $E_{cm} = 56$ GeV (Fig. 10).

More detailed reviews about jet physics in e^+e^- annihilations below the Z^0 resonance can
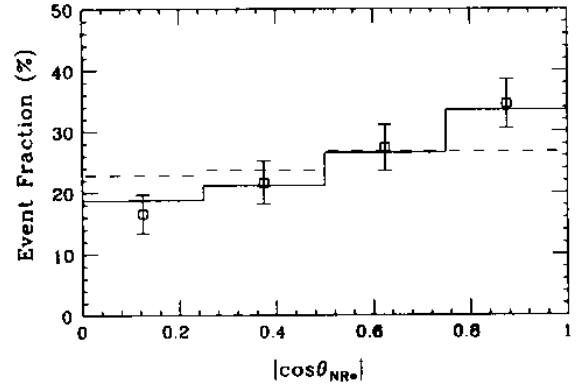

Figure 10. Distribution of the cosine of the modified Nachtmann-Reiter angle for 4-jet events measured by AMY at TRISTRAN [18], compared with the predictions of a QCD (solid line) and an Abelian (dashed line) model.

be found e.g. in [7,19,20] .

3. QCD AT AND ABOVE THE Z^0 POLE

In 1989, both the SLC and LEP started to deliver e^+e^- collisions at and around the Z^0 resonance, $E_{cm} \sim 91$ GeV. An example of an event of the type $e^+e^- \rightarrow Z^0 \rightarrow 3$ jets, observed with the Mark-II detector at the SLC, is shown in Fig. 11.

There exist several detailed reviews of QCD studies at LEP and SLC, see e.g. [21–24], and many of the most recent results were presented at this conference. Here, only an overview of some of the major achievements will be given:
New QCD calculations...

- $\mathcal{O}(\alpha_s^3)$ QCD predictions for the hadronic branching fraction of the Z^0 boson and the τ lepton [25,26],

- resummation of leading and next-to-leading logarithms and their matching with caluclations in complete $\mathcal{O}(\alpha_s^2)$, for several hadronic event shapes observables [27],

- $\mathcal{O}(\alpha_s^2)$ predictions for massive quarks [28];

new observables...

- jet finding algorithms [29] like the Durham scheme [30] or the cone algorithm [31],
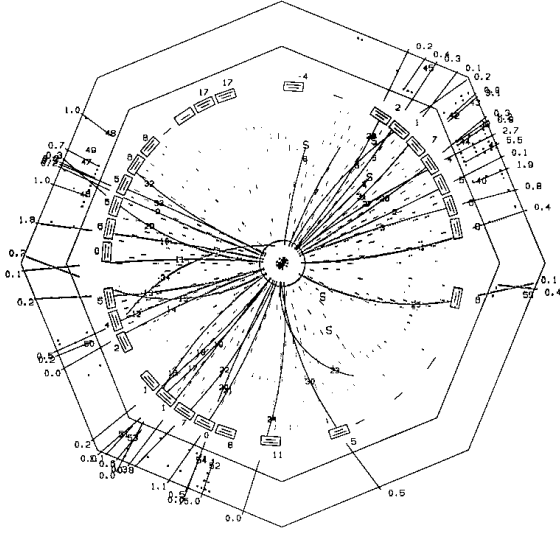
Figure 11. One of the first hadronic Z^0-decays, observed with the Mark-II detector at the SLAC Linear Collider.

- event shape variables like total and wide jet broadening, B_T and B_w [32],

- observables to study 4-jet kinematics [33];

and new experimental techniques...

- simultaneous use of many observables to study and reduce systematic and theoretical uncertainties in measurements of α_s [34],

- variations of renormalisation scale, parton virtuality, quark masses etc. to estimate theoretical uncertainties [34],

- tagging of quark - and gluon-jets [35],

- detailed studies of τ-lepton decays ([36]);

... resulted in:

- precise determinations of α_s (see Sections 4 and 5),

- tests of the QCD group structure (see [37]),

- detailed studies of differences between quark- and gluon-jets (see [38]),

- many details of the hadronisation process (see e.g. [39]),

- measurements of the heavy quark masses, $m_c(M_\tau)$ and $m_b(M_z)$ [40].

4. α_s AT LEP ABOVE THE Z^0-POLE

The most recent data from LEP around $E_{cm} = 133$ GeV, 161 GeV and 172 GeV - although with rather limited statistics of only a few hundred hadronic events for each experiment and at each energy point - were analysed in terms of hadronic event shape distributions, jet production rates, charged particle multiplicities, momentum distributions of charged particles and other observables. These analyses revealed that the new data are well described by standard QCD and hadronisation models, which were tuned to the high statistics data sets at the Z^0 pole. They also provided first measurements of α_s at these new energies which are summarised in Table 1.

It should be noted that several of these measurements are not yet officially published; those results as well as the combined values of α_s shown in Table 1 are therefore still preliminary.

5. UPDATE OF THE WORLD SUMMARY OF α_s

Significant determinations of α_s, based on perturbative QCD predictions which were, at least, complete to next-to-leading order (NLO), date back to 1979/1980 (from structure functions in deep inelastic scattering; see e.g. [45]) and to 1982 (from e^+e^- annihilation [14]). Many of the early data and determinations of α_s, see e.g. [46], have been superseded and/or replaced by more actual measurements and analyses, c.f. [47–50].

An update of a 1996 review of α_s [1] is given in Table 2. Since last year, new results from polarized structure functions [52], a correction of the results from ν-nucleon deep inelastic scattering [53], new results from lattice QCD [54] and from a study of hadronic event shapes at HERA [55], a reanalysis of event shapes from PETRA data [50], an update of α_s from the hadronic width of the Z^0 [56] and new results from LEP data above the Z^0 pole (see Section 4) became available. The

Table 1. Summary of measurements of α_s at LEP-1.5 and at LEP-2.

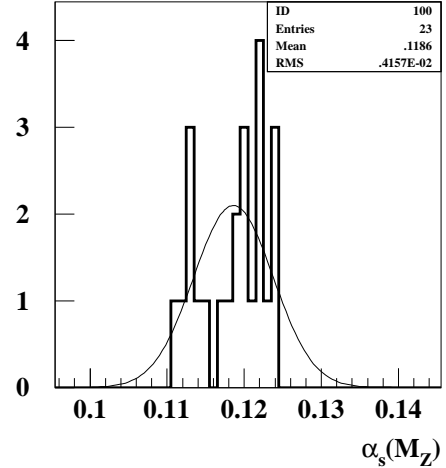| Exp.	$\alpha_s(\sqrt{s} = 133\text{ GeV})$	$\alpha_s(\sqrt{s} = 161\text{ GeV})$	$\alpha_s(\sqrt{s} = 172\text{ GeV})$	refs.
ALEPH	$0.115 \pm 0.008 \pm 0.005$	$0.111 \pm 0.009 \pm 0.005$	$0.105 \pm 0.010 \pm 0.004$	[41]
DELPHI	$0.116 \pm 0.007 \, ^{+0.005}_{-0.004}$	$0.107 \pm 0.008 \, ^{+0.005}_{-0.004}$	$0.104 \pm 0.013 \, ^{+0.005}_{-0.004}$	[42]
L3	$0.107 \pm 0.005 \pm 0.006$	$0.103 \pm 0.005 \pm 0.005$	$0.104 \pm 0.006 \pm 0.005$	[43]
OPAL	$0.110 \pm 0.009 \pm 0.009$	$0.101 \pm 0.009 \pm 0.009$	$0.093 \pm 0.009 \pm 0.009$	[44]
Average	$0.111 \pm 0.003 \pm 0.007$	$0.105 \pm 0.004 \pm 0.006$	$0.102 \pm 0.004 \pm 0.006$	

data are displayed in Figures 12 and 15.

Perhaps the most relevant change of previously reported results is the one from ν-nucleon scattering [53], which increased - due to a new energy calibration of the detector - from $\alpha_s(M_{Z^0}) = 0.111 \pm 0.006$ to 0.119 ± 0.005. This partly resolves the outstanding problem that deep inelastic scattering results preferred lower values of $\alpha_s(M_{Z^0})$ then did measurements in e^+e^- annihilation.

Figure 13. Frequency distribution of central values of $\alpha_s(M_{Z^0})$ (histogram; data from Table 2) and a gaussian with the same mean and width (curve).

Figure 12. World summary of $\alpha_s(Q)$ (status of July 1997).

The distribution of the central values of $\alpha_s(M_{Z^0})$ is displayed in Figure 13, whereby the result from lattice QCD is not included because the relevance of its given uncertainty is not en-

tirely clear. The unweighted mean of this distribution is 0.1186, the root mean squared (r.m.s.) is 0.0042. This distribution, however, does not have a gaussian shape, and so the r.m.s. is not regarded as a good measure of the error of the average value of $\alpha_s(M_{Z^0})$. Owing to the fact that the dominating uncertainties of most α_s measurements are theoretical rather than statistical or experimental, which may then be correlated to an unknown degree, two alternatives to estimate the uncertainty of the average $\alpha_s(M_{Z^0})$ are applied:

First, it is demanded that at least 90% of the central values of all measurements must be inside the error interval. This gives $\Delta\alpha_s(M_{Z^0}) = \pm 0.006$, which can be regarded as a '90% confidence interval' (68% confidence would correspond to ± 0.005).

Table 2. World summary of measurements of α_s. Entries preceded by ● are new or updated since summer 1996 [1]. Abbreviations: DIS = deep inelastic scattering; GLS-SR = Gross-Llewellyn-Smith sum rules; Bj-SR = Bjorken sum rules; (N)NLO = (next-)next-to-leading order perturbation theory; LGT = lattice gauge theory; resum. = resummed next-to-leading order.

	Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory
●	DIS [pol. strct. fctn.]	0.7 - 8		$0.120 ^{+\ 0.010}_{-\ 0.008}$	$^{+0.004}_{-0.005}$	$^{+0.009}_{-0.006}$	NLO
	DIS [Bj-SR]	1.58	$0.375 ^{+\ 0.062}_{-\ 0.081}$	$0.122 ^{+\ 0.005}_{-\ 0.009}$	–	–	NNLO
	DIS [GLS-SR]	1.73	0.32 ± 0.05	0.115 ± 0.006	0.005	0.003	NNLO
	τ-decays	1.78	0.330 ± 0.030	0.119 ± 0.004	0.001	0.004	NNLO
●	DIS [ν; F_2 and F_3]	5.0	0.215 ± 0.016	0.119 ± 0.005	0.002	0.004	NLO
	DIS [μ; F_2]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
	DIS [HERA; F_2]	2 - 10		0.120 ± 0.010	0.005	0.009	NLO
	DIS [HERA; jets]	10 - 60		0.120 ± 0.009	0.005	0.007	NLO
●	DIS [HERA; ev.shps.]	7 - 100		$0.118 ^{+\ 0.007}_{-\ 0.006}$	0.001	$^{+0.007}_{-0.006}$	NLO
	$Q\overline{Q}$ states	4.1	0.223 ± 0.009	0.117 ± 0.003	0.000	0.003	LGT
	$J/\Psi + \Upsilon$ decays	10.0	$0.167 ^{+\ 0.015}_{-\ 0.011}$	$0.113 ^{+\ 0.007}_{-\ 0.005}$	0.001	$^{+\ 0.007}_{-\ 0.005}$	NLO
●	e^+e^- [ev. shapes]	22.0	$0.161 ^{+\ 0.016}_{-\ 0.011}$	$0.124 ^{+\ 0.009}_{-\ 0.006}$	± 0.005	$^{+0.008}_{-0.003}$	resum
	e^+e^- [σ_{had}]	34.0	$0.146 ^{+\ 0.031}_{-\ 0.026}$	$0.124 ^{+\ 0.021}_{-\ 0.019}$	$^{+\ 0.021}_{-\ 0.019}$	–	NLO
●	e^+e^- [ev. shapes]	35.0	$0.143 ^{+\ 0.011}_{-\ 0.007}$	$0.122 ^{+\ 0.008}_{-\ 0.006}$	± 0.002	$^{+0.008}_{-0.005}$	resum
●	e^+e^- [ev. shapes]	44.0	$0.137 ^{+\ 0.010}_{-\ 0.007}$	$0.122 ^{+\ 0.008}_{-\ 0.006}$	± 0.003	$^{+0.007}_{-0.005}$	resum
	e^+e^- [ev. shapes]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum
	$p\bar{p} \to b\bar{b}X$	20.0	$0.145 ^{+\ 0.018}_{-\ 0.019}$	0.113 ± 0.011	$^{+\ 0.007}_{-\ 0.006}$	$^{+\ 0.008}_{-\ 0.009}$	NLO
	$p\bar{p}$, $pp \to \gamma X$	24.2	$0.137 ^{+\ 0.017}_{-\ 0.014}$	$0.112 ^{+\ 0.012}_{-\ 0.008}$	0.006	$^{+\ 0.010}_{-\ 0.005}$	NLO
	$\sigma(p\bar{p} \to$ jets)	30 - 500		0.121 ± 0.009	0.001	0.009	NLO
●	e^+e^- [$\Gamma(Z^0 \to$ had.)]	91.2	0.124 ± 0.005	0.124 ± 0.005	0.004	0.003	NNLO
	e^+e^- [ev. shapes]	91.2	0.122 ± 0.006	0.122 ± 0.006	0.001	0.006	resum.
●	e^+e^- [ev. shapes]	133.0	0.111 ± 0.008	0.117 ± 0.008	0.004	0.007	resum
●	e^+e^- [ev. shapes]	161.0	0.105 ± 0.007	0.114 ± 0.008	0.004	0.007	resum
●	e^+e^- [ev. shapes]	172.0	0.102 ± 0.007	0.111 ± 0.008	0.004	0.007	resum

Second, assuming that each result of $\alpha_s(M_{Z^0})$ has a boxed-shaped rather than a gaussian probability distribution over the interval of its quoted error, and summing all resulting weights[1] for each numerical value of $\alpha_s(M_{Z^0})$, leads to the distribution shown in Figure 14. The (weighted) mean of this distribution is almost identical to the un-weighted average (see Fig. 13), $\alpha_s(M_{Z^0}) = 0.1188$. This distribution now has a gaussian shape; the r.m.s. is 0.0061 and the width of a gaussian fit to that distribution is 0.0057. Therefore the world

average result is quoted to be

$$\alpha_s(M_{Z^0}) = 0.119 \pm 0.006,$$

where the error of 0.006 is perhaps a conservative but well defined estimate of the overall uncertainties. This average corresponds to the full line in Figure 12 and to the dashed line plus the shaded band in Figure 15. The agreement between the different measurements is excellent. Due to the many new and recent measurements, the running of α_s as predicted by QCD is significantly proven by the data.

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[1]The weight of a measurement is defined to be the inverse of the square of its total error.
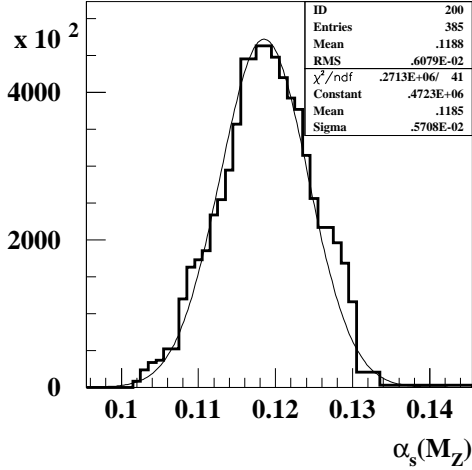
Figure 14. Sum of weights of $\alpha_s(M_{Z^0})$ (histogram; calculated from data of Table 2) and gaussian fit (curve).

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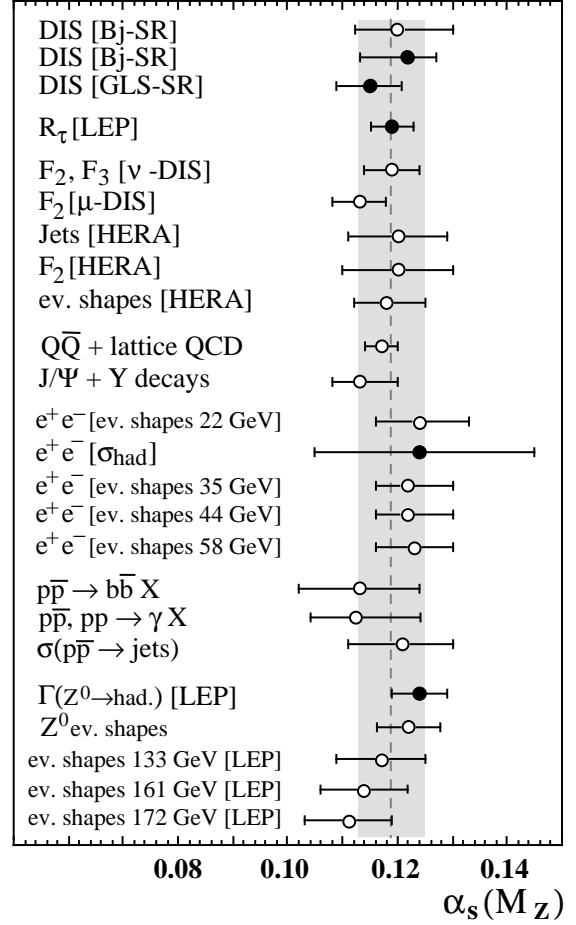
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Figure 15. World summary of $\alpha_s(M_{Z^0})$.

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