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INDIRECT CP VIOLATION IN THE B-SYSTEM

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ABSTRACT

We show that, contrary to the flavour mixing amplitude q/p, both $Re(\varepsilon)$ and $Im(\varepsilon)$ are observable quantities, where ε is the phase-convention-independent CP mixing. We consider semileptonic B_d decays from a CP tag and build appropriate time-dependent asymmetries to separate out $Re(\varepsilon)$ and $Im(\varepsilon)$. "Indirect" CP violation would have in $Im(\varepsilon)/(1+|\varepsilon|^2)$ its most prominent manifestation in the B-system, with expected values in the standard model ranging from -0.37 to -0.18. This quantity is controlled by a new observable ph ase: the relative one between the CP-violating and CP-conserving parts of the effective hamiltonian. For time-integrated rates we point out a $\Delta\Gamma \to \Sigma\Gamma$ transmutation which operates in the perturbative CP mixing.

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Since its discovery [1] in 1964, CP violation has only been seen in the $K^0 - \bar{K}^0$ system, in a few decay channels of the long-lived kaon K_L [2] and in a difference of decay rates between K^0 and $\bar{K}^0$ [3]. In the kaon system, the mechanism of CP violation due to mixing of K^0 and $\bar{K}^0$ plays the most prominent role and there is at present conflicting evidence on the existence of "direct" CP violation in the decay amplitude. Several experiments are planned to measure CP violating parameters in K-physics. CP violation can be naturally described in the standard electroweak model as long as there are, at least, three quark families [4], whereby the elements of the quark mixing matrix need not be relatively real. In the case of three families, the standard model has a great deal of predictive power, as the complexity of the quark mixing matrix is governed by a single weak phase. One of the most important goals of particle physics is to determine precisely the elements of the quark mixing matrix and test the standard model picture. The study of CP violation in B decays by means of dedicated experimental facilities in the coming years can provide such an overdetermination of the parameters of the quark mixing matrix.

In the case of the B system, the standard model prospects [5] for observation of CP violation due to flavour mixing alone are quite discouraging. The reason is that, to a good approximation, the flavour mixing amplitude q/p in the physical eigenstates of mass,

$$\begin{aligned} |B_1> &= \frac{1}{\sqrt{|p|^2+|q|^2}} \left\{ p|B^0> + q|\bar{B}^0> \right\} \\ |B_2> &= \frac{1}{\sqrt{|p|^2+|q|^2}} \left\{ p|B^0> - q|\bar{B}^0> \right\} \end{aligned} \quad (1)$$

is just a pure phase. The parameter q/p is phase-convention-dependent on the definition of the CP-transformed states and thus its phase is not, by itself, observable. The best prospects [6] then make use of the interplay between flavour mixing and decay. The non-observability of the flavour mixing phase is made apparent in the CP violating rate asymmetry, from a flavour tag, in the semileptonic decay $B^0 \to \ell\ \nu_l\ X$:

$$a_{SL} \equiv \frac{N(\ell^+\ell^+) - N(\ell^-\ell^-)}{N(\ell^+\ell^+) + N(\ell^-\ell^-)} = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} \quad (2)$$

To generate $|q/p| \neq 1$, one would need both $\Delta\Gamma_B \neq 0$ and a misalignment of the (complex) values of Γ_{12} and M_{12} in the $B^0 - \bar{B}^0$ mass matrix. All in all, the standard model a_{SL} is expected to be beyond the capabilities of the next experimental facilities, although some prospects could appear for physics beyond the standard model [7].

In this paper we show that, on the contrary, the semileptonic decay rate asymmetry based on the CP-tag has access to both $Re(\varepsilon)$ and $Im(\varepsilon)$, where ε is the phase-convention-independent parameter which governs CP-mixing in the physical states

$$\begin{aligned} |B_1> &= \frac{1}{\sqrt{1+|\varepsilon|^2}} (|B_+> + \varepsilon|B_->) \\ |B_2> &= \frac{1}{\sqrt{1+|\varepsilon|^2}} (|B_-> + \varepsilon|B_+>) \end{aligned} \quad (3)$$

and $|B_\pm>$ are the CP eigenstates. In the literature it is common to find the use of a different parameter, $\bar{\varepsilon}$, which controls the mixing between the states $1/\sqrt{2}(|B^0> \pm|\bar{B}^0>)$. These last states are not the CP eigenstates unless we fix the phase to $|\bar{B}^0> \equiv \pm CP|B^0>$. As a consequence, the parameter $\bar{\varepsilon}$ changes under a phase redefinition. For an arbitrary phase convention, $|q/p|$ is connected to $Re(\varepsilon)$ as

$$\frac{2Re(\varepsilon)}{1+|\varepsilon|^2} = \frac{1-|q/p|^2}{1+|q/p|^2} \tag{4}$$

The almost pure phase character of q/p translates into a very small value of $Re(\varepsilon)$. Experimentally [8], one has $|Re(\varepsilon)| < 0.045$. From a flavour tag, the semileptonic decay of B^0 has no access to $Im(\varepsilon)$. Both $Re(\varepsilon)$ and $Im(\varepsilon)$ are, however, observable quantities. It is of interest to illustrate by model-independent methods how to separate out these two observables.

Let us assume that, at $t = 0$, the B-meson is prepared, in the quantum mechanical sense, as a $|B_+>$. After this CP tag, the time evolution of $|B_+>$ yields the probability amplitudes for the meson to behave as $|B_+>, |B_->$

$$\begin{aligned} |B_+(t)> &= \frac{1}{1-\varepsilon^2}\Big\{[e^{-i\lambda_1 t} - \varepsilon^2 e^{-i\lambda_2 t}]|B_+> \\ &+ \varepsilon[e^{-i\lambda_1 t} - e^{-i\lambda_2 t}]|B_->\Big\} \end{aligned} \tag{5}$$

where $\lambda_j = m_j - i/2\,\gamma_j$ ($j = 1,2$). Equation (5) shows that the CP mixing amplitude is linear in ε. However, the survival probability differs from the exponential decay law only by terms of order ε^2, and the probability of becoming $|B_->$ is of order $|\varepsilon|^2$. More importantly, the observation of a final state $|\phi>$ which is accessible from the two slits $|B_\pm>$ leads to an interference pattern of order ε. The corresponding decay rate can be written as

$$\begin{aligned} |<\phi|B_+(t)>|^2 &= \frac{(1+|\varepsilon|^2)^2}{|1-\varepsilon^2|^2}\, e^{-\gamma_1 t}\, [a + b\, e^{-\Delta\Gamma t} \\ &+ c\, e^{-\frac{\Delta\Gamma}{2}t}\cos(\Delta m t) + d e^{-\frac{\Delta\Gamma}{2}t}\sin(\Delta m t)] \end{aligned} \tag{6}$$

where

$$\begin{aligned} a &= \frac{1}{1+|\varepsilon|^2}\left[\frac{|T_+|^2+|\varepsilon|^2|T_-|^2}{1+|\varepsilon|^2} + 2Re\left(\frac{\varepsilon}{1+|\varepsilon|^2}T_+^*T_-\right)\right] \\ b &= \frac{|\varepsilon|^2}{1+|\varepsilon|^2}\left[\frac{|T_-|^2+|\varepsilon|^2|T_+|^2}{1+|\varepsilon|^2} + 2Re\left(\frac{\varepsilon^*}{1+|\varepsilon|^2}T_+^*T_-\right)\right] \\ c &= -2\left[Re\left[\left(\frac{\varepsilon}{1+|\varepsilon|^2} + |\varepsilon|^2\frac{\varepsilon^*}{1+|\varepsilon|^2}\right)T_+^*T_-\right] + Re\left(\frac{\varepsilon}{1+|\varepsilon|^2}\right)^2|T_+|^2 + \left|\frac{\varepsilon}{1+|\varepsilon|^2}\right|^2|T_-|^2\right] \\ d &= -2\left[Im\left[\left(\frac{\varepsilon}{1+|\varepsilon|^2} - |\varepsilon|^2\frac{\varepsilon^*}{1+|\varepsilon|^2}\right)T_+^*T_-\right] + Im\left(\frac{\varepsilon}{1+|\varepsilon|^2}\right)^2|T_+|^2\right] \end{aligned} \tag{7}$$

$T_\pm \equiv<\phi|B_\pm>$ are the decay amplitudes and $\Delta m - i\frac{\Delta\Gamma}{2} \equiv \lambda_2 - \lambda_1$. For the semileptonic decay $|\phi>=|\ell^->$, the two amplitudes $T_\pm$ are separately phase-convention-dependent, but the product $T_+^* T_-$ is not and is real: due to the $\Delta B = \Delta Q_\ell$ rule, the charge of the lepton selects the flavour and both T_+ and T_- come from the same flavour component. For this semileptonic decay, we have

$$\begin{aligned}
a &= \frac{1}{2}|T|^2 \left[1 - 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{1}{1+|\varepsilon|^2} \\
b &= \frac{1}{2}|T|^2 \left[1 - 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{|\varepsilon|^2}{1+|\varepsilon|^2} \\
c &= |T|^2 \left[1 - 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{Re(\varepsilon)}{1+|\varepsilon|^2} \\
d &= |T|^2 \left[1 - 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{Im(\varepsilon)}{1+|\varepsilon|^2}
\end{aligned} \qquad (8)$$

where $T \equiv < \ell^- | \bar{B}^0 >$ is the decay amplitude from the the flavour state. Equations (7) or (8) include an interference term, modulated by $\sin(\Delta m t)$, proportional to $Im(\varepsilon)/(1+|\varepsilon|^2)$.

For the CP conjugate final state $|\bar{\phi}> = |\ell^+>$ from the same initial state, Eq. (6) has the same form with the coefficients of the time-dependent terms replaced by

$$\begin{aligned}
\bar{a} &= \frac{1}{2}|\bar{T}|^2 \left[1 + 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{1}{1+|\varepsilon|^2} \\
\bar{b} &= \frac{1}{2}|\bar{T}|^2 \left[1 + 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{|\varepsilon|^2}{1+|\varepsilon|^2} \\
\bar{c} &= -|\bar{T}|^2 \left[1 + 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{Re(\varepsilon)}{1+|\varepsilon|^2} \\
\bar{d} &= -|\bar{T}|^2 \left[1 + 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}\right] \frac{Im(\varepsilon)}{1+|\varepsilon|^2}
\end{aligned} \qquad (9)$$

where $\bar{T} \equiv < \ell^+ | B^0 >$. CPT invariance imposes the equality of probabilities $|\bar{T}|^2 = |T|^2$. CP invariance is clearly violated if

$$a \neq \bar{a} \quad \text{or} \quad b \neq \bar{b} \quad \text{or} \quad c \neq \bar{c} \quad \text{or} \quad d \neq \bar{d} \qquad (10)$$

To exhibit CP violation, we consider the corresponding CP asymmetry between Eqs. (8) and (9). To first order in $\varepsilon/(1+|\varepsilon|^2)$ we have

$$\begin{aligned}
A_+^{CP}(t) &\equiv \frac{\Gamma[B_+(t) \to \ell^+] - \Gamma[B_+(t) \to \ell^-]}{\Gamma[B_+(t) \to \ell^+] + \Gamma[B_+(t) \to \ell^-]} \\
&= 2\frac{Re(\varepsilon)}{1+|\varepsilon|^2}[1 - e^{\frac{\Delta\Gamma}{2}t}\cos(\Delta m t)] - 2\frac{Im(\varepsilon)}{1+|\varepsilon|^2} e^{\frac{\Delta\Gamma}{2}t}\sin(\Delta m t)
\end{aligned} \qquad (11)$$

The different time-dependence of the two terms on the right-hand side of Eq. (11) allows a separation of both parts, $Re(\varepsilon)/(1+|\varepsilon|^2)$ and $Im(\varepsilon)/(1+|\varepsilon|^2)$, of the CP mixing parameter.

For a $t = 0$ preparation of the B-meson as $|B_- >$, the replacement $B_+ \leftrightarrow B_-$ is accompanied by $\lambda_1 \leftrightarrow \lambda_2$ in Eq. (5). The decay rate can still be written as Eq. (6), but

$$a \to b, \quad b \to a, \quad c \to c, \quad d \to -d \tag{12}$$

with the simultaneous exchange $T_+ \leftrightarrow T_-$. The corresponding CP violating asymmetry in semileptonic decay is then

$$\begin{aligned} A_-^{CP}(t) &\equiv \frac{\Gamma[B_-(t) \to \ell^+] - \Gamma[B_-(t) \to \ell^-]}{\Gamma[B_-(t) \to \ell^+] + \Gamma[B_-(t) \to \ell^-]} \\ &= 2\frac{Re(\varepsilon)}{(1+|\varepsilon|^2}[1 - e^{-\frac{\Delta\Gamma}{2}t}\cos(\Delta m t)] + 2\frac{Im(\varepsilon)}{1+|\varepsilon|^2}e^{-\frac{\Delta\Gamma}{2}t}\sin(\Delta m t) \end{aligned} \tag{13}$$

The analysis of Eq. (13) offers a complementary means to that of Eq. (11) for separating out $Re(\varepsilon)/(1+|\varepsilon|^2)$ and $Im(\varepsilon)/(1+|\varepsilon|^2)$.

Let us now consider time-integrated rates to first order in $\varepsilon/(1+|\varepsilon|^2)$. From Eqs. (6) and (8) it is straightforward to get for the semileptonic decay:

$$\int_0^\infty dt| < \ell^-|B_+(t) > |^2 = \frac{|T|^2}{2\gamma_2}\left\{1 - 2Re\left[\frac{\varepsilon}{1+|\varepsilon|^2}\frac{\Delta m + i\frac{\Delta\Gamma}{2}}{\Delta m - i\frac{\Sigma\Gamma}{2}}\right]\right\} \tag{14.a}$$

$$\int_0^\infty dt| < \ell^+|B_+(t) > |^2 = \frac{|\bar{T}|^2}{2\gamma_2}\left\{1 + 2Re\left[\frac{\varepsilon}{1+|\varepsilon|^2}\frac{\Delta m + i\frac{\Delta\Gamma}{2}}{\Delta m - i\frac{\Sigma\Gamma}{2}}\right]\right\} \tag{14.b}$$

$$\int_0^\infty dt| < \ell^-|B_-(t) > |^2 = \frac{|T|^2}{2\gamma_1}\left\{1 - 2Re\left[\frac{\varepsilon}{1+|\varepsilon|^2}\frac{\Delta m + i\frac{\Delta\Gamma}{2}}{\Delta m + i\frac{\Sigma\Gamma}{2}}\right]\right\} \tag{14.c}$$

$$\int_0^\infty dt| < \ell^+|B_-(t) > |^2 = \frac{|\bar{T}|^2}{2\gamma_1}\left\{1 + 2Re\left[\frac{\varepsilon}{1+|\varepsilon|^2}\frac{\Delta m + i\frac{\Delta\Gamma}{2}}{\Delta m + i\frac{\Sigma\Gamma}{2}}\right]\right\} \tag{14.d}$$

where $\Sigma\Gamma \equiv \gamma_1 + \gamma_2$. The simplicity and interpretation of these results is amazing: in the time-integrated rates, the effective CP-mixing [compare the signs for CP-conjugate decay modes (14.a) and (14.b)] is not $\varepsilon/(1+|\varepsilon|^2)$ but that obtained by the recipe

$$\frac{\varepsilon}{1+|\varepsilon|^2} \to \frac{\varepsilon}{1+|\varepsilon|^2}\frac{\Delta m + i\frac{\Delta\Gamma}{2}}{\Delta m - i\frac{\Sigma\Gamma}{2}} \tag{15}$$

In first order perturbation theory, the energy difference $\Delta m - i\frac{\Delta\Gamma}{2}$ is expected to appear in the denominator of the CP mixing parameter $\varepsilon/(1+|\varepsilon|^2)$. Equation (15) tells us that it is rather $\Delta m - i\frac{\Sigma\Gamma}{2}$ which is the relevant denominator. Such a $\Delta\Gamma \to \Sigma\Gamma$ transmutation was noted [9] two decades ago in the context of parity violation by neutral currents in muonic atoms [10]. The appearance of the sum of the widths is connected to the fact that the transitions from the two admixed states $|B_+ >$ and $|B_- >$ are not resolved experimentally. The occurrence of the sum of the widths implies that the maximum effect occurs for $\Delta m = \frac{1}{2}\Sigma\Gamma$, a condition not far from being valid for the B_d system, where [8] $x_d = \frac{\Delta m}{\Gamma} = 0.73 \pm 0.05$.

Even if $\frac{\Delta\Gamma}{2\Delta m}$ is expected to be very small in the B-system [5], of the order of m_b^2/m_t^2, the comparable values of Δm and $\frac{\Sigma\Gamma}{2}$ help the objective of separating out $Re(\varepsilon)/(1+|\varepsilon|^2)$ and $Im(\varepsilon)/(1+|\varepsilon|^2)$ from the time-integrated rates given by Eqs. (14). As emphasized above, in the limit $\Delta\Gamma \to 0$, one expects $Re(\varepsilon) \to 0$, and the observation of CP mixing would be based on a non-vanishing value of $Im(\varepsilon)/(1+|\varepsilon|^2)$.

In order to clarify the meaning of $Im(\varepsilon)/(1+|\varepsilon|^2)$, we obtain the expression for our phase-convention-independent ε in terms of the matrix elements of the effective hamiltonian in the flavour basis, $H_{12} =< B^o|H|\bar{B}^o >$. Both the dispersive part M_{12} and the absorptive part Γ_{12} of H_{12} are phase-convention-dependent, so that only their relative phase, which determines $Re(\varepsilon)$, is physical and a manifestation of CP violation. There is, however, a third phase-convention-dependent matrix element which is involved in the connection between CP eigenstates and Flavour eigenstates: $CP_{12} =< B^o|CP|\bar{B}^o >$. The relative phase between M_{12} and CP_{12} is physical and a (new) measure of indirect CP violation. We obtain

$$\varepsilon = \frac{Im(\Gamma_{12}CP^*_{12}) + 2iIm(M_{12}CP^*_{12})}{2Re(M_{12}CP^*_{12}) - iRe(\Gamma_{12}CP^*_{12}) + \Delta m - \frac{i}{2}\Delta\Gamma} \tag{16}$$

where $(\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$. In the standard model, indirect CP violation in the B system is in fact dominated by $Im(\varepsilon)/(1+|\varepsilon|^2)$ and thus given by a CP phase in the dispersive part of the $B^o - \bar{B}^o$ mixing amplitude, relative to the phase convention CP_{12}. The main contribution to this dispersive part comes [11] from the box diagram with the top quark running in the loop. Using the matrix elements M_{12} and Γ_{12} as given in Ref. [11], compatible with $CP_{12} = -1$, we have calculate d the values of $Im(\varepsilon)/(1+|\varepsilon|^2)$ in terms of the Wolfenstein [12] parametrization of the quark mixing matrix. In the limit of dominance of the intermediate top quark, for which ε is purely imaginary, we get

$$\frac{Im(\varepsilon)}{1+|\varepsilon|^2} \simeq \frac{Im(M_{12}CP^*_{12})}{\Delta m} \simeq -\frac{\eta(1-\rho)}{(1-\rho)^2+\eta^2} \tag{17}$$

Recent estimates [13] of η and ρ, constrained from existing measurements, give an appreciable value for $Im(\varepsilon)/(1+|\varepsilon|^2)$ ranging from -0.37 to -0.18.

The reference phase CP_{12} is given by the flavour mixing amplitude in the CP-conserving limit, $(q/p)_{CP}$:

$$\frac{1-\varepsilon}{1+\varepsilon} = \frac{q}{p}CP_{12} = -\frac{(q/p)}{(q/p)_{CP}} \tag{18}$$

Therefore, CP violation in a given system will be realized either by $|q/p| \neq 1$ or by a relative phase between the CP-violating and the CP-conserving flavour mixing amplitudes, or by both of them. Contrary to the K-system, this new observable phase would have the most prominent role in B-physics, where $|q/p| \simeq 1$.

To conclude, the rate asymmetries in semileptonic B decays from a CP-tag offer an illustration of the separate observable character of $Re(\varepsilon)$ and $Im(\varepsilon)$, where ε is our phase-convention-independent CP-mixing parameter. Our study yields an additional result: for time-integrated rates, the effective CP-mixing contains an "energy difference" denominator given by $\Delta m - i\,\frac{\Sigma\Gamma}{2}$, instead of $\Delta m - i\,\frac{\Delta\Gamma}{2}$. This allows the extraction of $Im(\varepsilon)/(1+|\varepsilon|^2)$, whose values in the standard model are expected to range from -0.37 to -0.18. The new phase discussed in this paper can be understood as the relative phase between the CP-violating a nd CP-conserving parts of the effective hamiltonian.

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