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Soffer's inequality and the transversely polarized Drell-Yan process at next-to-leading order

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Abstract

We check numerically if Soffer's inequality for quark distributions is preserved by next-to-leading order QCD evolution. Assuming that the inequality is saturated at a low hadronic scale we estimate the maximal transverse double spin asymmetry for Drell-Yan muon pair production to next-to-leading order accuracy.

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1 Introduction

The transversity distribution $\delta q(x, Q^2)$ is the only completely unknown twist-2 parton distribution function of the nucleon. In a transversely polarized nucleon it counts the number of quarks with spin parallel to the nucleon spin minus the number of quarks with antialigned spin [1]. In field theory the transversity distribution is defined by the expectation value of a chiral-odd operator between nucleon states which is the reason why it is not experimentally accessible in inclusive deep inelastic lepton-nucleon scattering (DIS) [2, 3]. The most promising hard process allowed by this chirality selection rule seems to be Drell-Yan dimuon production, and exactly this reaction will be utilized for attempting a first measurement of $\delta q(x, Q^2)$ at RHIC [4]. What actually will be measured is not the transversity distribution itself, but the transverse double spin asymmetry $A_{TT} = d\delta\sigma/d\sigma$ where the polarized and unpolarized hadronic cross sections are defined as

$$d\delta\sigma \equiv \frac{1}{2}\left(d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}\right) \quad , \qquad d\sigma \equiv \frac{1}{2}\left(d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}\right) \quad . \tag{1}$$

In perturbative QCD (pQCD) A_{TT} can be expressed in terms of unpolarized parton distributions, the yet unknown transversity distributions and the relevant partonic cross sections. Although the latter have been known to next-to-leading order (NLO) accuracy in the strong coupling for several years by now [5, 6, 7], consistent NLO calculations were not possible because of the lack of the two-loop transversity splitting functions. This situation changed only very recently [8, 9, 10], allowing for the first time for a consistent calculation of pQCD corrections to the transverse double spin asymmetry for the Drell-Yan process.

The unpolarized, longitudinally and transversely polarized quark distributions (q, Δq, δq) of the nucleon are expected to obey the rather interesting relation

$$2|\delta q(x)| \leq q(x) + \Delta q(x) \tag{2}$$

derived by Soffer [11]. It has been recently clarified that Soffer's inequation is preserved by leading order (LO) QCD evolution, i.e. if (2) is valid at some scale

Q_0 it will also be valid at $Q > Q_0$ [12]. To NLO the situation is not as simple. The parton distributions are now subject to the choice of the factorization scheme which one may fix independently for q, Δq and δq. One can therefore always find "sufficiently incompatible" schemes in which a violation of (2) occurs. However, in [8] it was shown with analytical methods that the inequation for valence densities is preserved by NLO QCD evolution in a certain "Drell-Yan scheme" in which the NLO cross sections for dimuon production maintain their LO forms, and also in the $\overline{\mathrm{MS}}$ scheme. An analytical check of the sea part is difficult since the singlet mixing between quarks and gluons has to be taken into account for the unpolarized and longitudinally polarized quantities on the right-hand-side (r.h.s.) of (2). In Section 2 of this article we shall show numerically that Soffer's inequation for sea quarks is also preserved under NLO evolution.

Estimates of A_{TT} suffer of course from the fact that no experimental information on the transversity distribution is available at the moment. Therefore one has to rely on ansätze or model calculations of $\delta q(x, Q^2)$ at some reference scale Q_0 [13]. For example, a popular assumption is $\delta q(x, Q_0^2) = \Delta q(x, Q_0^2)$ which, however, is in general incompatible with Soffer's inequality (2), in particular in a situation in which $\Delta q(x, Q_0^2) \approx -q(x, Q_0^2)$. Our aim in Sections 3 and 4 will be to estimate within LO and NLO an upper bound on the transverse double spin asymmetry for the Drell-Yan process. To do so, we will first of all assume validity of Soffer's inequality which seems reasonable and is corroborated by our finding of Section 2 that the NLO evolution to $Q^2 > Q_0^2$ preserves the inequation once it is satisfied at the input scale. The maximal asymmetry A_{TT} can then be estimated by further assuming *saturation* of the Soffer bound (2). The result obtained for A_{TT} under this assumption obviously strongly depends on the value chosen for Q_0. If Q_0 is taken to be large, i.e. of the order of the invariant mass M of the lepton pair which sets the typical hard scale for the Drell-Yan process, the largest possible values for A_{TT} will be reached. However, for several reasons it does not seem convincing to assume saturation of (2) by the input distributions employing such a high $Q_0^2 \sim M^2$: Firstly, evolving backwards to $Q^2 < Q_0^2$ – which should be a completely legitimate procedure if Q_0 is not small – will under such circumstances immediately yield a violation of Soffer's inequality. Secondly, the r.h.s. of (2) will almost certainly lead to an overestimation for the δq if saturation is assumed at a large Q_0. For instance, for sea quarks the first moment (x-integral) of the r.h.s. in (2) will diverge, which is not expected for the integral over $\delta\bar{q}$ at any Q^2. Therefore, to obtain a realistic estimate for an upper bound on A_{TT} by assuming saturation of Soffer's inequation, two requirements have to be met: (i) The saturation should be adopted only at a rather low "hadronic" input scale where (ii) the integral over the r.h.s. of (2) is finite. Both demands are automatically fulfilled if we choose the unpolarized and longitudinally polarized input parton distributions of the radiative parton model analyses [14, 15, 16] and set[1]

$$2\delta q(x, Q_0^2) = q(x, Q_0^2) + \Delta q(x, Q_0^2) \ , \tag{3}$$

where Q_0 now is identified with the input scale $\mu \sim \mathcal{O}(0.6 \text{ GeV})$ of the radiative parton model [14] and is considered the smallest scale from which perturbative evolution can be performed, such that no backward evolution from μ makes sense. While we are aware that our approach with its rather small Q_0 may lead to an underestimation of the maximally possible A_{TT}, we still believe our results to be built on a firm basis, given the large phenomenological success [15, 16] of the radiative parton model for the q, Δq. In any case, our results for A_{TT} under the assumption of (3) are the largest the radiative parton model can predict and will provide a useful target for future experiments. We emphasize that our NLO results presented in Section 4 are the first ones to be obtained to true and consistent NLO accuracy. Section 4 will also provide a discussion of other possible uncertainties of our results.

In Section 5 we will present our conclusions.

[1]The possibility of choosing a different sign in front of the r.h.s. of (3) will be discussed later.

2 Preservation of Soffer's inequation by NLO evolution

Unlike the case of the more familiar unpolarized and longitudinally polarized densities, all transversity distributions obey simple non-singlet type evolution equations because there is no corresponding gluonic quantity due to angular momentum conservation [17, 3]. Introducing

$$\delta q_{\pm}(x, Q^2) \equiv \delta q(x, Q^2) \pm \delta \bar{q}(x, Q^2) \tag{4}$$

and Mellin moments $\delta q_{\pm}^n(Q^2) \equiv \int_0^1 dx x^{n-1} \delta q_{\pm}(x, Q^2)$ the evolution equations are given by [5]

$$Q^2 \frac{d}{dQ^2} \delta q_{\pm}^n(Q^2) = \delta P_{qq,\pm}^n(\alpha_s(Q^2)) \delta q_{\pm}^n(Q^2) \quad . \tag{5}$$

The Mellin moments of the transverse splitting functions $\delta P_{qq,\pm}^n$ are taken to have the following perturbative expansion

$$\delta P_{qq,\pm}^n(\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right) \delta P_{qq}^{(0),n} + \left(\frac{\alpha_s}{2\pi}\right)^2 \delta P_{qq,\pm}^{(1),n} + \dots \quad , \tag{6}$$

i.e. both are equal to LO. We use the following NLO expression for the strong coupling constant,

$$\frac{\alpha_s(Q^2)}{2\pi} = \frac{2}{\beta_0 \ln Q^2/\Lambda^2} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\ln \ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2}\right) \; , \tag{7}$$

where Λ is the QCD scale parameter and $\beta_0 = 11 - 2n_f/3$, $\beta_1 = 102 - 38n_f/3$ with n_f being the number of active flavors. The solution of (5) is then simply given by [5]

$$\begin{aligned} \delta q_{\pm}^n(Q^2) = & \left[1 + \frac{\alpha_s(Q_0^2) - \alpha_s(Q^2)}{\pi \beta_0} \left(\delta P_{qq,\pm}^{(1),n} - \frac{\beta_1}{2\beta_0} \delta P_{qq}^{(0),n}\right)\right] \\ & \times \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{-2\delta P_{qq}^{(0),n}/\beta_0} \delta q_{\pm}^n(Q_0^2) \quad . \end{aligned} \tag{8}$$

Needless to say that the LO evolutions are entailed in the above equations when we put the NLO quantities, $\delta P_{qq,\pm}^{(1),n}$, β_1, to zero.

Eq. (8) can be very conveniently employed for a numerical calculation of the NLO evolution of the transversity distributions. As discussed in the introduction, we will assume saturation of Soffer's inequality at the input scale, see Eq. (3). Our choice for the r.h.s. of (3) will then be the NLO $\overline{\mathrm{MS}}$ radiative parton model inputs for $q(x, Q_0^2)$ of [15] and for the longitudinally polarized $\Delta q(x, Q_0^2)$ of the "standard" scenario of [16] at[2] $Q_0^2 = \mu_{NLO}^2 = 0.34$ GeV^2. For simplicity we will slightly deviate from the actual $q(x, Q_0^2)$ of [15] in so far as we will neglect the breaking of SU(2) in the input sea quark distributions originally present in this set. This seems reasonable as SU(2)-symmetry was also assumed for the $\Delta\bar{q}(x, Q_0^2)$ of [16], which in that case was due to the fact that in the longitudinally polarized case there are no data yet that could discriminate between $\Delta\bar{u}$ and $\Delta\bar{d}$. We therefore prefer to assume $\delta\bar{u}(x, Q_0^2) = \delta\bar{d}(x, Q_0^2)$ also for the transversity input. We will examine the possible effects of SU(2)-breaking later. The moments of the resulting input distributions $\delta q(x, Q_0^2)$ are easily taken, and the $\delta q_\pm^n(Q_0^2)$ are then evolved to higher scales $Q^2 > Q_0^2$ with the help of (8). A standard inverse Mellin transformation finally gives the desired transversity distribution in x-space. In order to perform this inverse Mellin transformation, Eq. (8) has to be analytically continued to complex n [14]. The evolutions of the $q(x, Q_0^2)$ (neglecting the SU(2)-breaking) and the $\Delta q(x, Q_0^2)$, which both involve the singlet mixing between quarks and gluons, proceed as explained in [14, 15, 16].

In order to numerically check the preservation of (2), Fig. 1 shows the ratio

$$R_q(x, Q^2) = \frac{2|\delta q(x, Q^2)|}{q(x, Q^2) + \Delta q(x, Q^2)} \tag{9}$$

as a function of x for several different Q^2 values for $q = u_v = u_-$, $\bar{u} = (u_+ - u_-)/2$, $d_v = d_-$, $\bar{d} = (d_+ - d_-)/2$ (cf. Eq. (4)). If NLO evolution preserves Soffer's inequality then $R_q(x, Q^2)$ should not become larger than 1 for any $Q^2 \geq Q_0^2$. As we already know from [8] this is the case for the two valence distributions. Fig. 1 confirms that Soffer's inequality is indeed also preserved for sea distributions. Furthermore, in Fig. 1 we see that evolution leads to a strong suppression of $R_q(x, Q^2)$ at small values of x, in particular for the sea quarks. This can be understood by the fact that $\delta P_{qq,\pm}(x)$ has a very mild behaviour for $x \to 0$ [8],

[2]Note that for the purpose of checking the preservation of Soffer's inequality by evolution the choice of the initial scale Q_0^2 is actually irrelevant.

and by the well-known sharp small-x rise of the unpolarized sea distributions in the denominator of R_q due to Q^2-evolution. We note that our numerical results for the sea quarks became somewhat unstable at large x, probably caused by the fact that the sea distributions are obtained here as differences of two much larger quantities.

As is obvious from (2), Soffer's inequality only restricts the absolute value of the transversity distribution. Therefore, we are free to choose a different sign in front of the r.h.s. of (3) and have to check the results for the two distinct possibilities $\delta q_v(x, Q_0^2) > 0$, $\delta\bar{q}(x, Q_0^2) > 0$ and $\delta q_v(x, Q_0^2) > 0$, $\delta\bar{q}(x, Q_0^2) < 0$. Our results do not noticeably depend on the actual choice.

As we have neglected any possible SU(2)-breaking in all the sea input distributions $\bar{q}(x, Q_0^2)$, $\Delta\bar{q}(x, Q_0^2)$, $\delta\bar{q}(x, Q_0^2)$, any difference between the curves for $R_{\bar{u}}$, $R_{\bar{d}}$ can necessarily only result from the dynamical breaking of SU(2) first induced by NLO evolution. The occurrence of a small breaking from this source is well-known from the unpolarized [18] and longitudinally polarized [19] cases. For the transversity densities it is given by

$$\begin{aligned} 2\left(\delta\bar{u} - \delta\bar{d}\right)^n (Q^2) \;\; &= \;\; \frac{(\alpha_s(Q^2) - \alpha_s(Q_0^2))}{\pi\beta_0} \left(\delta P_{qq,-}^{(1),n} - \delta P_{qq,+}^{(1),n}\right) \\ &\times \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{-2\delta P_{qq}^{(0),n}/\beta_0} (\delta u_v - \delta d_v)^n (Q_0^2) \quad . \end{aligned} \tag{10}$$

Fig. 2 displays the resulting effect via the ratio

$$\delta D(x, Q^2) = \frac{\delta\bar{u}(x, Q^2) - \delta\bar{d}(x, Q^2)}{\delta\bar{u}(x, Q^2) + \delta\bar{d}(x, Q^2)} \tag{11}$$

for various Q^2. One can see that – apart from the region of very large x – the dynamical breaking of SU(2) is rather small and could in reality well be entirely masked by the explicit breaking in the non-perturbative sea input.

3 Upper bounds on A_{TT}: Framework

Now that we have shown that NLO evolution preserves Soffer's inequation, we want to utilize it to derive upper bounds on the transverse double spin asymmetry to be measured in polarized Drell-Yan muon pair production. For this purpose we choose again the maximally allowed value (3) for the transversity distributions, which should yield the maximal double spin asymmetry. We employ the same unpolarized and longitudinally polarized input distributions as in the previous section, along with the same value for the initial scale Q_0.

The scaling variable for the Drell-Yan process is $\tau = M^2/S$, where M is the invariant mass of the produced muon pair and $\sqrt{S}$ is the center-of-mass energy of the hadronic collision. Since in unpolarized reactions only the collision axis is specified, the distribution of the produced muon pairs cannot depend on the azimuth ϕ. If the colliding nucleons are transversely polarized then the collision and spin axes specify a plane in space and consequently the polarized cross section will depend on ϕ. Instead of working with τ-dependent cross sections we again prefer Mellin moments defined by

$$\frac{d(\delta)\sigma^n}{d\phi} \equiv \int_0^1 d\tau \tau^{n-1} \frac{\tau d(\delta)\sigma}{d\tau d\phi} \quad . \tag{12}$$

Including NLO corrections to these cross sections one obtains the generic expression [6, 7]

$$\begin{aligned} \frac{d(\delta)\sigma^n}{d\phi} &= \frac{\alpha_{em}^2}{9S}\,(\delta)\Phi(\phi)\left[(\delta)H_q^n(Q_F^2)\left(1+\frac{\alpha_s(Q_R^2)}{2\pi}\,(\delta)C_q^{DY,n}(Q_F^2)\right)\right. \\ &\left. \qquad + H_g^n(Q_F^2)\frac{\alpha_s(Q_R^2)}{2\pi}\,(\delta)C_g^{DY,n}(Q_F^2)\right] \quad , \end{aligned} \tag{13}$$

where

$$(\delta)H_q^n(Q_F^2) \equiv \sum_q e_q^2\left[(\delta)q_A^n(Q_F^2)\,(\delta)\bar{q}_B^n(Q_F^2) + (A \leftrightarrow B)\right] \ , \tag{14}$$

$$H_g^n(Q_F^2) \equiv \sum_q e_q^2\left[g_A^n(Q_F^2)\left(q_B^n(Q_F^2)+\bar{q}_B^n(Q_F^2)\right) + (A \leftrightarrow B)\right] \quad . \tag{15}$$

The dependence on the azimuth is given by $\Phi(\phi) = 1$ and $\delta\Phi(\phi) = \cos 2\phi$. Integration over ϕ thus isolates the unpolarized part and $\Phi(\phi)$ is then replaced by 2π. On the other hand the integration [20] $\left(\int_{-\pi/4}^{\pi/4} - \int_{\pi/4}^{3\pi/4} + \int_{3\pi/4}^{5\pi/4} - \int_{5\pi/4}^{7\pi/4}\right) d\phi$ extracts the polarized cross section and $\delta\Phi(\phi)$ can then be simply substituted by 4. In the following we will always assume appropriate integration over the azimuth.

The unpolarized NLO $\overline{\mathrm{MS}}$ QCD coefficients in τ-space can be found, e.g., in [5]. Their Mellin moments are

$$
\begin{aligned}
C_q^{DY,n}(Q_F^2) &= C_F\left(4S_1^2(n) - \frac{4}{n(n+1)}S_1(n) + \frac{2}{n^2} + \frac{2}{(n+1)^2} - 8 + \frac{4\pi^2}{3}\right) + \\
&\quad C_F\left[\frac{2}{n(n+1)} + 3 - 4S_1(n)\right] \ln\left(\frac{M^2}{Q_F^2}\right) \quad , \qquad (16)\\
C_g^{DY,n}(Q_F^2) &= T_R\left(-2\frac{n^2+n+2}{n(n+1)(n+2)}S_1(n) + \frac{n^4+11n^3+22n^2+14n+4}{n^2(n+1)^2(n+2)^2}\right) + \\
&\quad T_R\,\frac{n^2+n+2}{n(n+1)(n+2)}\ \ln\left(\frac{M^2}{Q_F^2}\right) \quad , \qquad (17)
\end{aligned}
$$

where $C_F = 4/3$, $T_R = 1/2$. The polarized ones can be found in [8] and read

$$
\begin{aligned}
\delta C_q^{DY,n}(Q_F^2) &= C_F\left[4S_1^2(n) + 12\left(S_3(n) - \zeta(3)\right) + \frac{4}{n(n+1)} - 8 + \frac{4\pi^2}{3}\right] + \\
&\quad C_F\left[3 - 4S_1(n)\right]\ \ln\left(\frac{M^2}{Q_F^2}\right) \quad , \qquad (18)\\
\delta C_g^{DY,n}(Q_F^2) &= 0 \quad . \qquad (19)
\end{aligned}
$$

In the above formulas we used the abbreviation $S_k(n) = \sum_{j=1}^{n} j^{-k}$. Since there is no gluon transversity distribution for the nucleon, the gluonic part of (13) drops out for the polarized case. The indices A and B in (14) and (15) take into account the possibility of having two different scattering hadrons, although only *pp* collisions are planned at the moment. Finally, Q_F and Q_R in Eqs. (13)-(19) are the factorization and renormalization scales, respectively, for which we will choose $Q_F = Q_R = M$ unless stated otherwise.

Z^0 production and γZ^0-interference can be easily included by the substitution (see also [7])

$$
\begin{aligned}
e_q^2 \quad \rightarrow \quad & e_q^2 - 8e_qV_lV_q\kappa\frac{M^2(M^2-M_Z^2)}{(M^2-M_Z^2)^2+\Gamma_Z^2M_Z^2} \\
& +16(V_l^2+A_l^2)(V_q^2 \pm A_q^2)\kappa^2\frac{M^4}{(M^2-M_Z^2)^2+\Gamma_Z^2M_Z^2} \quad , \qquad (20)
\end{aligned}
$$

where

$$\kappa \equiv \frac{\sqrt{2} G_F M_Z^2}{16\pi\alpha_{em}} \quad , \quad V_f \equiv T_f^3 - 2e_f \sin^2\Theta_W \quad , \quad A_f \equiv T_f^3 \quad . \tag{21}$$

The positive sign in front of the A_q^2 term is appropriate for the unpolarized cross section, the negative sign for the polarized one. As usual, G_F denotes the Fermi constant, Θ_W the Weinberg angle ($\sin^2\Theta_W = 0.224$) and T_f^3 the third component of the weak isospin.

For examining the perturbative stability of our results we will also calculate the cross section at LO. In this case one simply needs to set the QCD coefficients to zero in the above formulas and to replace the NLO parton distributions by ones evolved in LO. As LO input distributions for (3) we will use the unpolarized LO parametrizations of [15] (neglecting again the SU(2) breaking in the quark sea) and the polarized ones of [16] at the LO input scale $Q_0^2 = 0.23$ GeV^2.

4 Results

Fig. 3 shows the transversely polarized *pp* cross section and the "maximal" double spin asymmetry A_{TT} for $\sqrt{S} = 40$ GeV, corresponding to the proposed [21] HERA-$\vec{\mathrm{N}}$ fixed target experiment which would utilize the possibly forthcoming polarized 820 GeV proton beam at HERA on a transversely polarized target. We show results at both LO and NLO. For illustration we have also included the expected statistical errors for a measurement of A_{TT} by HERA-$\vec{\mathrm{N}}$ which can be estimated from

$$\delta A_{TT} = \frac{1}{P_B P_T \sqrt{\mathcal{L}\sigma\epsilon}} \, , \tag{22}$$

where P_B and P_T are the beam and target polarizations for which we will use $P_B = P_T = 0.7$. $\mathcal{L}$ is the anticipated integrated luminosity of $\mathcal{L} = 240$ pb^{-1}, σ the unpolarized cross section integrated over bins of M, and ϵ the detection efficiency for which we will take for simplicity $\epsilon = 100\%$. Note that full 4π coverage of the detector is assumed. Fig. 3 shows that the maximal asymmetry for HERA- $\vec{\mathrm{N}}$ is actually fairly large and would be accessible in that experiment.

In Fig. 4 we present results similar to Fig. 3, but now for $\sqrt{S} = 150$ GeV, corresponding to the RHIC collider. We note that the region 9 GeV $\lesssim M \lesssim 11$ GeV will presumably not be accessible experimentally since it will be dominated by muon pairs from bottomonium decays. Again the predicted maximal asymmetry is of the order of a few per cents. From the expected error bars calculated again for 70% beam polarization, $\mathcal{L} = 240$ pb^{-1} and $\epsilon = 100\%$ one concludes that asymmetries of this size should be also well measurable at RHIC.

Fig. 5 shows similar results for the high-energy end of RHIC, $\sqrt{S} = 500$ GeV, where the integrated luminosity is expected to be $\mathcal{L} = 800$ pb^{-1}. It turns out that the asymmetries become smaller as compared to the lower energies, but thanks to the higher luminosity the error bars become relatively smaller as well, at least in the region 5 GeV $\lesssim M \lesssim 25$ GeV where the errors are approximately 1/10 of the maximal asymmetry. One can also clearly see in Fig. 5 the effect of Z-exchange and the Z resonance.

We have already mentioned before that Soffer's inequation does not determine the sign of $\delta q(x, Q^2)$, so that in principle we have to check all different combinations in order to find the "true" maximal value for A_{TT}. It turns out, e.g., that keeping a positive sign only for $\delta u_v(x, Q^2)$ leads to a reduction of $|A_{TT}|$ at small M but an enhancement at the experimentally not accessible region of large M. We have checked that for small M the asymmetry takes its largest values if all signs are chosen to be positive, as was done in Eq. (3) and in the above plots.

A comparison of the LO and NLO results in Figs. 3-5 answers one key question concerning the transversely polarized Drell-Yan process: Our predictions for the maximal A_{TT} show good perturbative stability, i.e. the NLO corrections to the cross sections and A_{TT} are of moderate size, albeit not negligible. There seems to be a general tendency towards smaller corrections when the energy is increasing, which should be mainly due to the larger invariant masses probed and to a resulting smaller $\alpha_s(M^2)$.

Let us finally address some of the the main uncertainties in our predictions for the maximal asymmetry A_{TT}. The first issue is the scale dependence of the results. This is examined in Fig. 6 for the case $\sqrt{S} = 150$ GeV. We plot here the maximal asymmetry in LO and NLO, varying the renormalization and factorization scales in the range $M/2 \leq Q_F = Q_R \leq 2M$. One can see that already the LO asymmetry is fairly stable with respect to scale changes, which is in accordance with the finding of generally moderate NLO corrections. The NLO asymmetry even shows a significant improvement, so that A_{TT} becomes largely insensitive to the choice of scale.

In order to get a rough idea about the uncertainty caused by our imperfect knowledge of the longitudinally polarized parton densities $\Delta q(x, Q^2)$, $\Delta g(x, Q^2)$, we have also calculated the asymmetries using the NLO "valence" scenario input distributions of [16] instead of the "standard" ones in (3). As can be seen in Fig. 7 for the case $\sqrt{S} = 150$ GeV, the difference for experimentally significant M turns out to be quite small, with the predictions based on the "valence" scenario distributions having slightly smaller asymmetries.

As we already mentioned in Sec. 2, neither the "standard" nor the "valence" scenario parametrizations take into account a possible SU(2) breaking in the polarized sea because only neutral current polarized DIS data are available at the moment. This led us to neglecting also any SU(2) breaking in the transversity input densities for our calculations, just keeping the dynamical SU(2) breaking produced by NLO evolution (cf. Fig. 2). On the other hand, it seems rather likely that a certain amount of SU(2) breaking – possibly much more than the one generated by evolution – could be realized in nature. One possible way of estimating the uncertainty entering our predictions for A_{TT} through this source, is to reintroduce the hitherto neglected amount of SU(2) breaking in the unpolarized input densities as fixed in the original input distributions of [15]. The SU(2) breaking will also influence the transversity input via Eq. (3). The resulting asymmetry is also depicted in Fig. 7. As can be seen, the effect is sizeable only at rather large M.

5 Conclusions

We have shown numerically that Soffer's inequation is preserved by NLO QCD evolution provided it is satisfied by the input distributions.

For the first time, we have presented a complete and consistent NLO calculation of the transverse double spin asymmetry A_{TT} for the Drell-Yan process, employing the NLO corrections to the hard subprocess cross sections as well as performing the Q^2-evolutions in NLO. Here we have estimated the maximally possible A_{TT} in the framework of the radiative parton model by assuming that Soffer's inequality is saturated at a low hadronic scale. For $\sqrt{S} = 40$ GeV the maximal value of A_{TT} for pp collisions was found to be approximately 4% with an expected statistical error for HERA-$\vec{\text{N}}$ of about 1% at an invariant mass of $M = 4$ GeV. The situation for RHIC with $\sqrt{S} = 150$ GeV turns out to be rather similar. The prospects of measuring A_{TT} somewhat improve when going to $\sqrt{S} = 500$ GeV where the maximal asymmetry is of the order of 1% for small M with an expected relative statistical error of approximately 1/10. We emphasize again, however, that our results only represent an *upper bound* on A_{TT}, so that the "true" asymmetry may well be much smaller and even experimentally not measurable.

Comparing to corresponding LO calculations, we find that the QCD corrections turn out to be moderate but non-negligible, putting our predictions on a firm basis. We have also examined the main uncertainties of our predictions, such as the scale dependence of the asymmetry and our imperfect knowledge of the longitudinally polarized parton densities to be utilized for the saturation of Soffer's inequality at the input scale. We found that these uncertainties seem to have rather little impact on our results in the regions hopefully accessible in future experiments with transversely polarized protons.

Note added: After completing this work, we received the paper [22] in which a mathematical proof of the preservation of Soffer's inequality under NLO Q^2-evolution is given.

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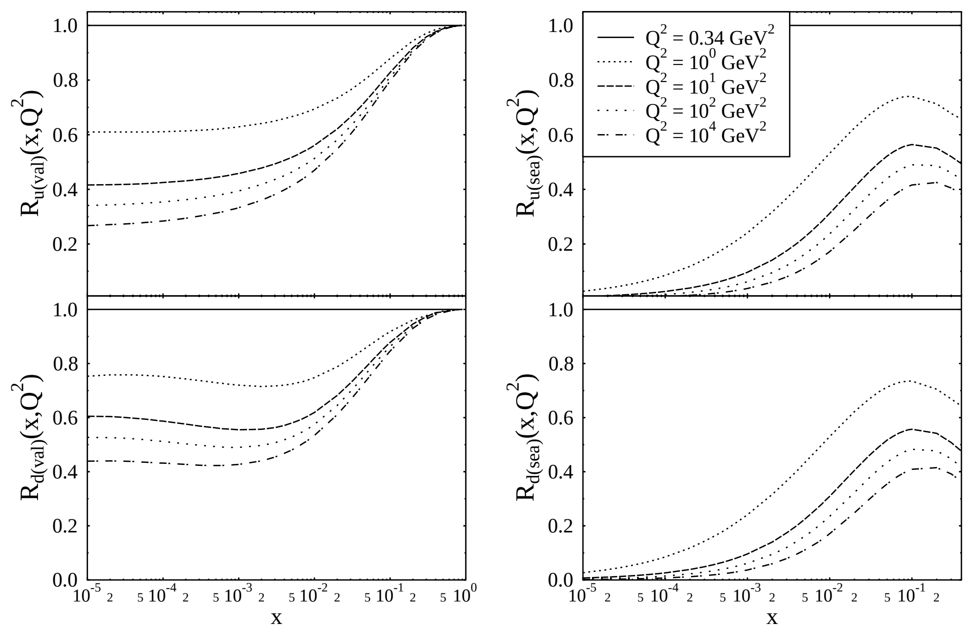

Figure 1: The ratio $R_q(x, Q^2)$ as defined in (9) for $q = u_v, \bar{u}, d_v, \bar{d}$ and several fixed values of Q^2.

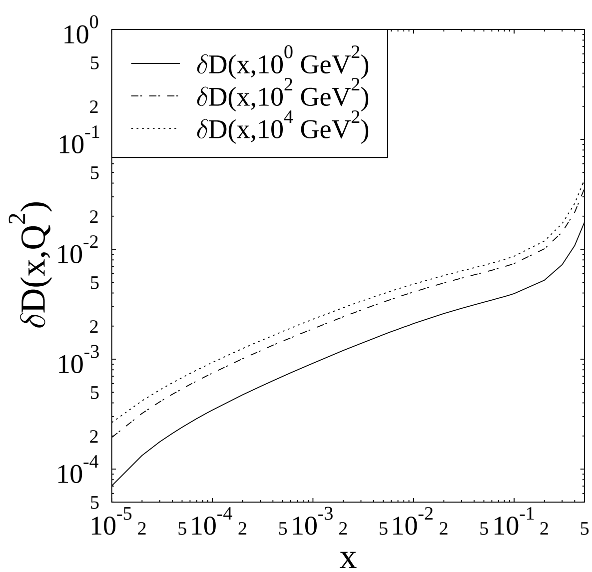

Figure 2: The dynamical SU(2)-breaking in the NLO transversity densities expressed by the ratio $\delta D(x, Q^2)$ as defined in (11) for several fixed values of Q^2.

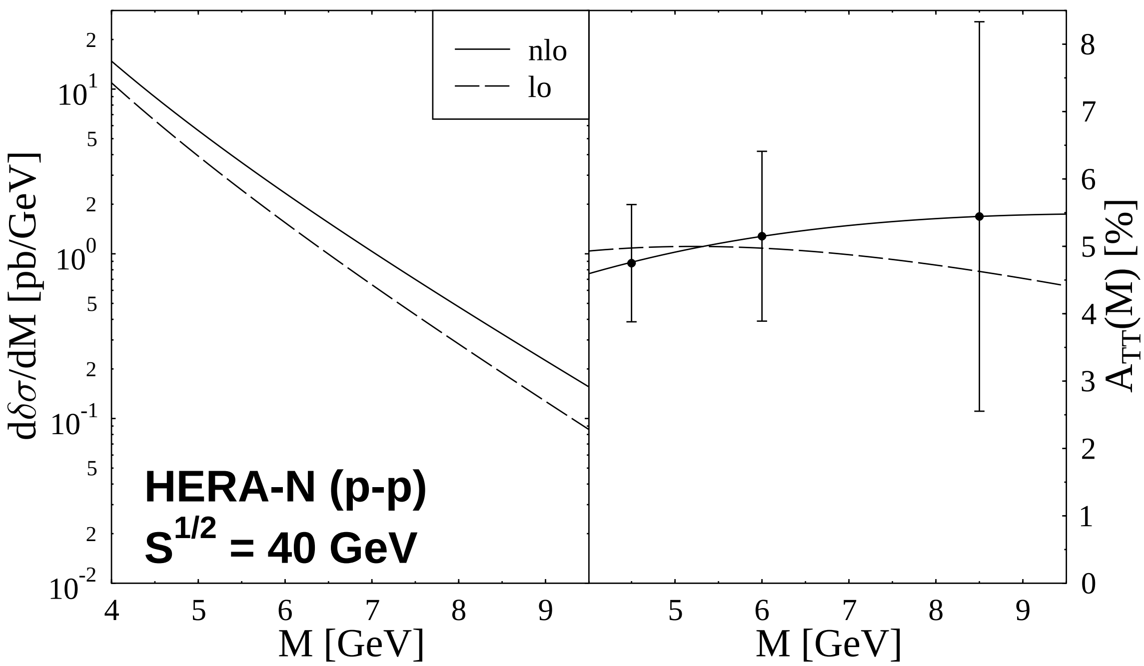

Figure 3: NLO and LO maximal polarized Drell-Yan cross sections and asymmetries for HERA-$\vec{\mathrm{N}}$. The error bars have been calculated according to Eq. (22) and are based on $\mathcal{L} = 240\,\mathrm{pb}^{-1}$, 70% polarisation of beam and target and 100% detection efficiency.

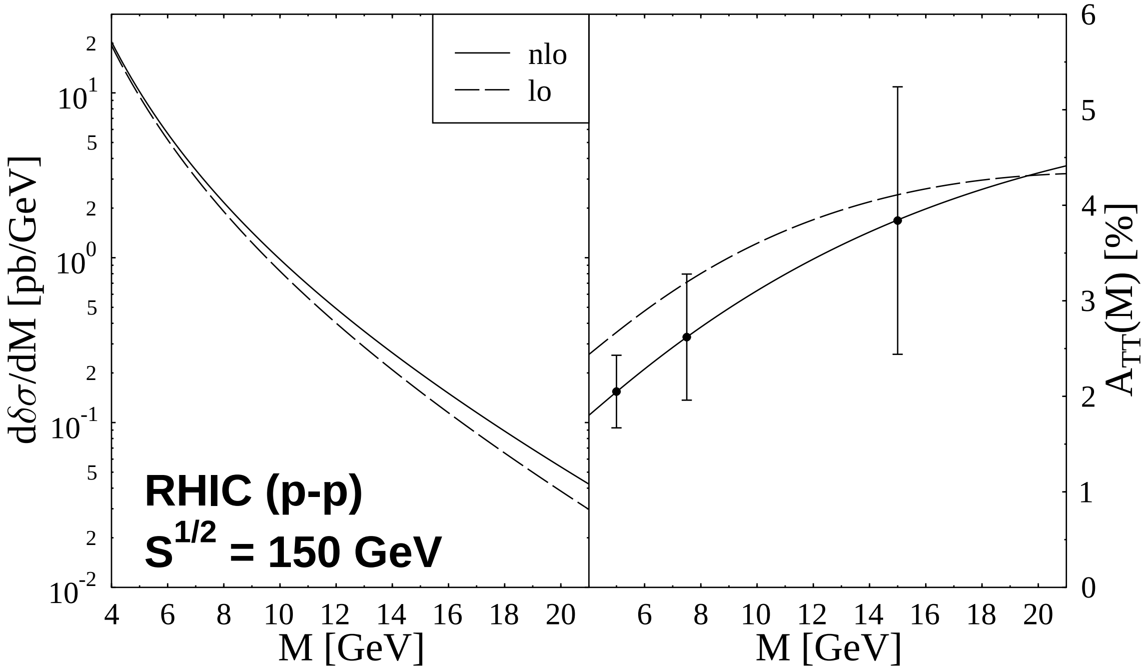

Figure 4: As in Fig. 3, but for RHIC at $\sqrt{S} = 150\,\mathrm{GeV}$ assuming $\mathcal{L} = 240\,\mathrm{pb}^{-1}$, 70% polarisation of each beam and 100% detection efficiency.

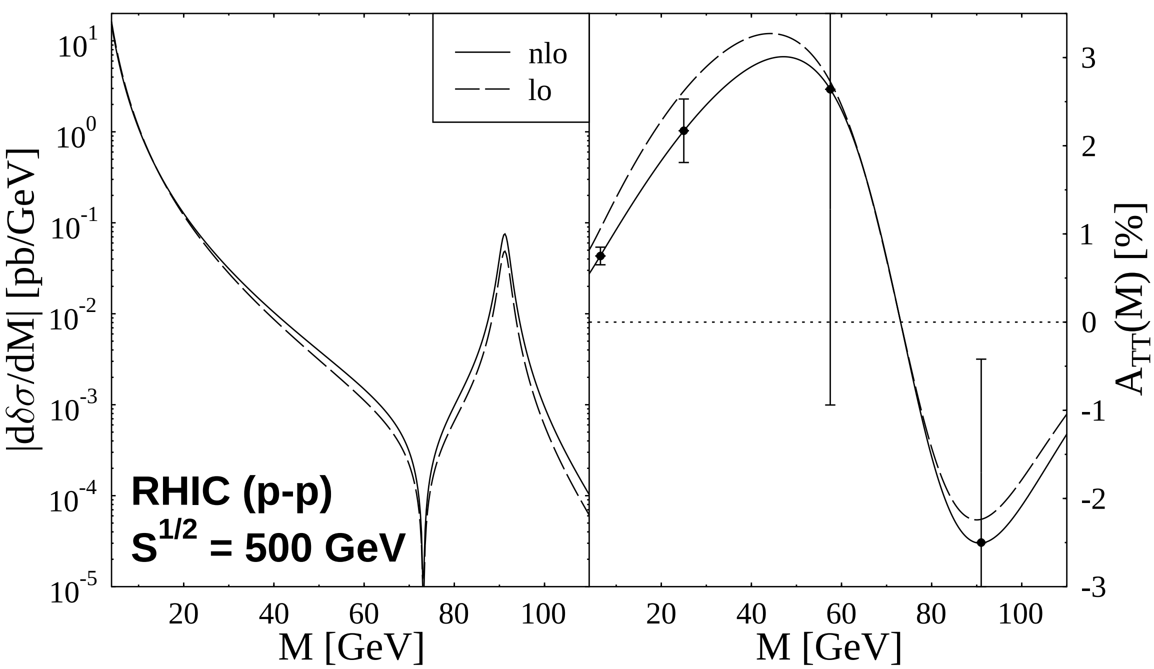

Figure 5: As in Fig. 4, but for $\sqrt{S} = 500\,\mathrm{GeV}$ and $\mathcal{L} = 800\,\mathrm{pb}^{-1}$.

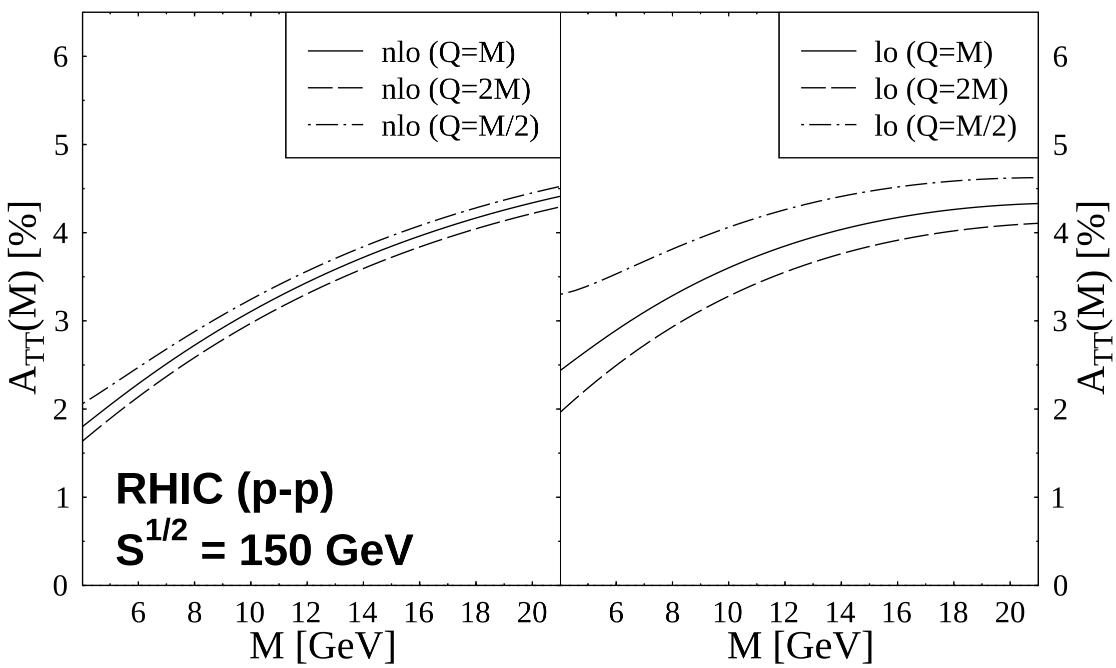

Figure 6: Scale dependence of the LO and NLO asymmetries at $\sqrt{S} = 150\,\mathrm{GeV}$. The renormalization and factorization scales in (13)-(15) were chosen to be $Q_R = Q_F = M/2, M, 2M$. The solid line is as in Fig. 4.

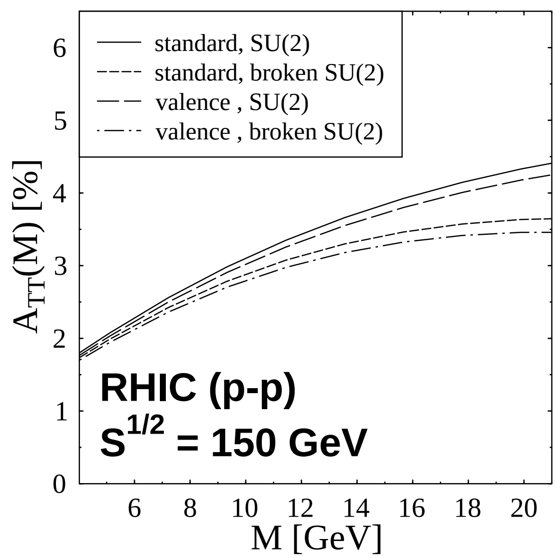

Figure 7: The NLO asymmetry at $\sqrt{S} = 150\,\mathrm{GeV}$, using the "standard" and the "valence" sets of [16] for the $\Delta q(x, Q_0^2)$ in (3). Also shown is the change caused by not neglecting the SU(2) breaking in the $q(x, Q_0^2)$ of [15] (see text). The solid line is as in Fig. 4.