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On the Exact Quartic Effective Action for the Type IIB Superstring*

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Abstract

We propose a four-point effective action for the graviton, antisymmetric two-forms, dilaton and axion of type IIB superstring in ten dimensions. It is explicitly $SL(2,\mathbf{Z})$-invariant and reproduces the known tree-level results. Perturbatively, it has only one-loop corrections for the NS-NS sector, generalizing the non-renormalization theorem of the R^4 term. Finally, the non-perturbative corrections are of the expected form, namely, they can be interpreted as arising from single D-instantons of multiple charge.

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1. Introduction

There is a lot of activity nowadays towards understanding the non-perturbative structure in string theory. In type IIB theory, in particular, the non-perturbative physics is intimately related with the existence of the $SL(2,\mathbf{Z})$ symmetry [1, 2]. The spectrum of the type IIB theory contains an $SL(2,\mathbf{Z})$ multiplet of strings and five-branes, the self-dual three-brane, the seven-brane, as well as D-instanton solutions. The latter are the only ones that give non-perturbative corrections in ten dimensions. This can be seen by compactifying the theory. In this case, the various Euclidean $(p+1)$-word-volumes of p-branes have an infinite action in the decompactification limit except when $p=-1$, which is just the type IIB D-instanton.

In σ-model perturbation theory, there exists a four-loop divergence that contributes to the β-functions [3] and gives α'^3 corrections to the effective action. This can also be confirmed by string four-point amplitude calculations [4, 6]. For four gravitons, in particular, there exists also a one-loop result [7] for the R^4 corrections and non-renormalization theorems have been conjectured for their structure [8, 9]. One expects that all contributions higher than one loop to vanish, since for higher genus surfaces there are more than eight fermionic zero modes; this is exactly the number needed to saturate the external particles in a four-point amplitude [9]. This heuristic argument has been proved by using superspace techniques [10].

Besides the perturbative corrections to the R^4 term, there also exist non-perturbative ones. Their form has recently been conjectured by Green and Gutperle on the basis of $SL(2,\mathbf{Z})$ invariance [9]. In particular, the modular invariance of the effective action is achieved by employing a certain non-analytic modular form. The structure of the latter is such that it gives only tree- and one-loop corrections to the R^4 term besides the instanton ones. An ansatz for the form of the corresponding four-graviton amplitude has been given in [11]. Moreover, the R^4 term gives rise to a similar term in M-theory [12, 13, 14] and the compactification of the latter gives results consistent with string theory expectations [15, 16].

One may now proceed further by including the other massless modes of the type IIB theory. In this case, the tree-level result for the four-point amplitudes of the dilaton and the antisymmetric tensor has been given in [6], while a one-loop calculation is lacking. Now, arguments similar to those above seem to suggest that the non-renormalization theorem for the R^4 term may also be extended to the full effective theory when all modes are included. Namely, the perturbative expansion for the NS-NS sector stops at one loop and all other corrections are non-perturbative. This can also be justified by consistency conditions related to M-theory [17]. However, the inclusion of the other modes at the tree level has a serious drawback. It breaks the manifest $SL(2,\mathbf{Z})$ invariance of the theory. Here, we propose an effective action for all bosonic massless modes of type IIB, except for the self-dual four-form. We do not consider the latter because of the lack of any perturbative information at the eight-derivative level. The action we propose respects the $SL(2,\mathbf{Z})$ symmetry and reproduces the effective action of [6] when all R-R fields are switched off. In particular, the NS-NS sector has only tree- and one-loop corrections besides the non-perturbative ones.

In the following section, we recall perturbative results in the type IIB effective theory. In section 3, we summarize the analysis of [9] concerning the non-perturbative corrections to the R^4 term. In section 4, we propose an $SL(2,\mathbf{Z})$-invariant effective action and discuss its compatibility with a recent calculation [21] of the $R^2(\partial\partial\phi)^2$ term in type IIB on $K3$.

2. Perturbative Effective Type IIB Theory

The massless bosonic spectrum of type IIB superstring theory consists in the graviton g_{MN}, the dilaton ϕ and the antisymmetric tensor B^1_{MN} in the NS-NS sector and the axion χ, the two-form B^2_{MN} and the self-dual four-form field A_{MNPQ} in the R-R sector. The two scalars of the theory can be combined into a complex one, $\tau = \tau_1 + i\tau_2$, defined by

$$\tau = \chi + ie^{-\phi} \,. \tag{2.1}$$

The theory has two supersymmetries generated by two supercharges of the same chirality. It has in addition a conserved $U(1)$ charge which generates rotations of the two supersymmetries and under which some of the fields are charged [1]. The graviton and the four-form field are neutral, the antisymmetric tensors have charge $q = 1$, whereas the complex scalar τ has $q = 2$. The fermionic superpartners of the above fields are a complex Weyl gravitino and a complex Weyl dilatino.

The bosonic effective Lagrangian of the theory in lowest order in α' takes the form[1]

$$\mathcal{L}_0 = R - \frac{1}{2\tau_2^2}\partial_M\tau\partial^M\bar{\tau} - \frac{1}{12\tau_2}(\tau H^1 + H^2)_{KMN}(\bar{\tau}H^1 + H^2)^{KMN} \,, \tag{2.2}$$

where $H^\alpha_{KMN} = \partial_K B^\alpha_{MN} +$ cyclic for $\alpha = 1, 2$ and we have set the four-form to zero. The theory has an $SL(2,\mathbf{R})$ symmetry that acts as

$$\tau \to \frac{a\tau + b}{c\tau + d}\,, \qquad B^\alpha_{MN} \to (\Lambda^T)^{-1\alpha}{}_\beta B^\beta_{MN}\,, \quad \Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbf{R})\,, \tag{2.3}$$

and leaves the Lagrangian (2.2) invariant. The complex scalar τ parametrizes an $SL(2,\mathbf{R})/U(1)$ coset space. In general, the group $SL(2,\mathbf{R})$ can be represented by a matrix $V^\alpha_\pm$ [1, 9]

$$V = \begin{pmatrix} V^1_- & V^1_+ \\ V^2_- & V^2_+ \end{pmatrix} = \frac{1}{\sqrt{-2i\tau_2}} \begin{pmatrix} \bar{\tau}e^{-i\theta} & \tau e^{i\theta} \\ e^{-i\theta} & e^{i\theta} \end{pmatrix} \,. \tag{2.4}$$

The local $U(1)$ is realized by the shift $\theta \to \theta + \Delta\theta$ and the global $SL(2,\mathbf{R})$ acts from the left. One may define the quantities

$$P_M = -\epsilon_{\alpha\beta}V^\alpha_+\partial_M V^\beta_+ = ie^{2i\theta}\frac{\partial_M\tau}{2\tau_2}\,, \quad Q_M = -i\epsilon_{\alpha\beta}V^\alpha_+\partial_M V^\beta_- = \partial_M\theta - \frac{\partial_M\tau_1}{2\tau_2}\,, \tag{2.5}$$

where Q_M is a composite $U(1)$ gauge connection and P_M has charge $q = 2$. We also define the complex three-form

$$G_{KMN} = -\sqrt{2i}\delta_{\alpha\beta}V^\alpha_+ H^\beta_{KMN} = -i\frac{e^{i\theta}}{\sqrt{\tau_2}}(\tau H^1_{KMN} + H^2_{KMN})\,, \tag{2.6}$$

[1]We set $\alpha' = 1$ from now on.

with charge $q = 1$. We fix the gauge by choosing $\theta \equiv 0$ from now on. In this case, the global $SL(2,\mathbf{R})$ transformation is non-linearly realized and the various quantities in eqs.(2.5) and (2.6) transform as

$$P_M \to \frac{c\bar\tau + d}{c\tau + d} P_M\,, \quad Q_M \to Q_M + \frac{1}{2i}\partial_M \ln\left(\frac{c\bar\tau + d}{c\tau + d}\right)\,, \quad G_{KMN} \to \left(\frac{c\bar\tau + d}{c\tau + d}\right)^{1/2} G_{KMN}\,. \tag{2.7}$$

We may also define the covariant derivative $D_M = \nabla_M - iqQ_M$, which transforms under $SL(2,\mathbf{R})$ as

$$D_M \to \left(\frac{c\bar\tau + d}{c\tau + d}\right)^{q/2} D_M\,. \tag{2.8}$$

There exists α'^3 corrections to the effective Lagrangian (2.2) above, which have been evaluated in [6] and are written as

$$\begin{aligned}\mathcal{L}_{4pt} &= \frac{\zeta(3)}{3\cdot 2^6}\tau_2^{3/2}\Big(t_8^{ABCDEFGH}t_8^{MNPQRSTU} + \frac{1}{8}\varepsilon_{10}^{ABCDEFGHIJ}\varepsilon_{10}^{MNPQRSTU}{}_{IJ}\Big) \\ &\quad\times \hat R_{ABMN}\hat R_{CDPQ}\hat R_{EFRS}\hat R_{GHTU}\,,\end{aligned} \tag{2.9}$$

where

$$\hat R_{MN}{}^{PQ} = R_{MN}{}^{PQ} + \frac{1}{2}e^{-\phi/2}\nabla_{[M}H^1_{N]}{}^{PQ} - \frac{1}{4}g_{[M}{}^{[P}\nabla_{N]}\nabla^{Q]}\phi\ . \tag{2.10}$$

The tensor t_8 is defined in [4], ε_{10} is the totally antisymmetric symbol in ten dimensions and the square brackets are defined without the combinatorial factor 1/2 in front. The Lagrangian (2.9) reproduces the four-point amplitude calculated in string theory and it is in agreement with σ-model perturbative calculations. In addition, at the same order in α' one expects contributions coming from five- to eight-point amplitudes. These contributions can be implemented by adding the terms

$$\begin{aligned}&-\frac{1}{4}e^{-\phi}H^1_{[M}{}^{C[P}H^1_{N]C}{}^{Q]} + \frac{1}{16}g_{[M}{}^{[P}\partial_{N]}\phi\partial^{Q]}\phi - \frac{1}{8}g_{[M}{}^{[P}g_{N]}{}^{Q]}\partial_K\phi\partial^K\phi \\ &-\frac{1}{4}e^{-\phi/2}\partial^{[P}\phi H^{1Q]}{}_{MN} - \frac{1}{4}e^{-\phi/2}\partial_{[M}\phi H^1_{N]}{}^{PQ} - \frac{1}{8}e^{-\phi/2}g_{[M}{}^{[P}H^1_{N]}{}^{Q]C}\partial_C\phi\end{aligned} \tag{2.11}$$

to the r.h.s. of eq.(2.10) and then $\hat R_{PQMN}$ turns out to be the Riemann tensor associated to the generalized connection, which includes the torsion and the Weyl connection [6]

$$T_{MNP} = \frac{1}{2} e^{-\phi/2} H^1_{MNP} \,, \quad \tilde{\omega}_{MNP} = \frac{1}{4} g_{P[N} \partial_{M]} \phi \,. \tag{2.12}$$

Expanding the $\hat{R}$ terms in eq.(2.9) we find

$$\begin{aligned} \mathcal{L}_{4pt} \; = \; & \frac{\zeta(3)}{3 \cdot 2^6} \tau_2^{3/2} \left(t_8 t_8 + \frac{1}{8} \varepsilon_{10} \varepsilon_{10} \right) \Big(R^4 + e^{-2\phi} (\nabla H^1)^4 + (\partial\partial\phi)^4 \\ & -12 e^{-\phi} R (\nabla H^1)^2 \partial\partial\phi - 4 R^3 \partial\partial\phi + 6 e^{-\phi} R^2 (\nabla H^1)^2 \\ & +6 e^{-\phi} (\nabla H^1)^2 (\partial\partial\phi)^2 + 6 R^2 (\partial\partial\phi)^2 - 4 R (\partial\partial\phi)^3 \Big) \,, \end{aligned} \tag{2.13}$$

where $\partial\partial\phi$ stands for $(\partial\partial\phi)_{MNPQ} \equiv g_{MP} \nabla_N \partial_Q \phi$ and ∇H^1 for $(\nabla H^1)_{MNPQ} \equiv \nabla_M H^1_{NPQ}$. It should be noted that there are no terms with odd powers of ∇H^1, because such terms vanish, owing to the Bianchi identity. One can check for example that the amplitude for three gravitons and one antisymmetric field

$$t_8^{ABCDEFGH} t_8^{MNPQRSTU} R_{ABMN} R_{CDPQ} R_{EFRS} \nabla_{[G} H^1_{H]TU} \tag{2.14}$$

is zero. This can also be argued on the basis of the invariance of the type IIB superstring under the world-sheet parity that acts on B^1 as $B^1 \to -B^1$.

At the perturbative level, there exist string one-loop corrections to the four-point functions. For four gravitons these corrections have been calculated [7] and amount to the exchange

$$\zeta(3) \to \zeta(3) + \frac{\pi^2}{3} \tau_2^{-2} \,, \tag{2.15}$$

in eq.(2.9).

3. Non-perturbative R^4 couplings

In the type IIB theory, there exist D-instantons that contribute to the four-point amplitudes. Instantons are solutions of the tree-level Lagrangian (2.2) in Euclidean space with vanishing antisymmetric fields and non-trivial profile for the complex scalar τ [18]. In general, they break half of the supersymmetries and the broken ones generate fermionic zero modes. Since the supersymmetry in type IIB theory is generated by a complex Weyl spinor with 16 components, we expect eight fermionic zero modes, which can give a non-zero contribution to four-point amplitudes. In such a background, the action is finite and its value is $S^{(Q)} = -2\pi|Q|i\tau_0$, where Q is the instanton charge and τ_0 is the value of τ at infinity. Thus, we expect the contribution from a single instanton of charge 1 to be proportional to $e^{2\pi i\tau_0}$. The multi-instanton contributions may be determined by T-duality arguments as follows [9]. Under compactification on S^1, the type IIB D-instanton is mapped to the type IIA D-particle. There are arguments to support the fact that n such single charged D-particles combine to a single bound state of charge n [19]. The world line of this bound state can wrap m times around the compact S^1 so that its topological charge is mn. Then, its T-dual counterpart in type IIB should be a D-instanton of charge $Q = mn$ whose contribution is proportional to $e^{2i\pi|mn|\tau_0}$. Notice that separated instantons are accompanied by additional fermionic zero modes which can only be soaked up with higher than four-derivative interactions.

For four gravitons the full instanton corrections have been conjectured to take the form [9]

$$\mathcal{L}_{R^4} = \frac{1}{3\cdot 2^7} f_0(\tau,\bar\tau)\left(t_8t_8 + \frac{1}{8}\varepsilon_{10}\varepsilon_{10}\right) R^4\,, \tag{3.1}$$

where $f_0(\tau,\bar\tau)$ is the non-holomorphic modular form [20]

$$f_0(\tau,\bar\tau) = {\sum_{m,n}}' \frac{\tau_2^{3/2}}{|m+n\tau|^3}\,, \tag{3.2}$$

and the sum extends over integers $(m,n) \neq (0,0)$. In addition, $f_0(\tau,\bar\tau)$ has the small τ_2^{-1}

expansion

$$
\begin{aligned}
f_0(\tau,\bar\tau) &= 2\zeta(3)\tau_2^{3/2}+\frac{2\pi}{3}\tau_2^{-1/2}+8\pi\tau_2^{1/2}\sum_{m\neq 0,n\geq 1}\left|\frac{m}{n}\right|e^{2i\pi mn\tau_1}K_1(2\pi|mn|\tau_2)\\
&= 2\zeta(3)\tau_2^{3/2}+\frac{2\pi}{3}\tau_2^{-1/2} \qquad (3.3)\\
&\quad +4\pi^{3/2}\sum_{m,n\geq 1}\left(\frac{m}{n^3}\right)^{1/2}\left(e^{2i\pi mn\tau}+e^{-2i\pi mn\bar\tau}\right)\left(1+\sum_{k=1}^{\infty}(4\pi mn\tau_2)^{-k}\frac{\Gamma(k-1/2)}{\Gamma(-k-1/2)k!}\right),
\end{aligned}
$$

where K_1 is a Bessel function. This proposal for the exact R^4 corrections satisfies several consistency requirements:

i) $SL(2,\mathbf{Z})$ invariance. The function $f_0(\tau,\bar\tau)$ is modular-invariant, so that $\mathcal{L}_{R^4}$ is invariant as well.

ii) It reproduces the correct perturbative expansion.

iii) The non-perturbative corrections are of the expected form: there are only multiply-charged single D-instanton contributions.

It should be noted that although the proposed form of the R^4 corrections to the effective action satisfy the above constraints, there is no proof for its validity. However, from the type IIA side [13, 12], as well as from lower dimensional compactifications of type IIB [22], there exist strong arguments supporting this form of the R^4 terms. Nevertheless, it does not provide the full four-point Lagrangian since the analogous corrections to the other modes (antisymmetric fields and scalars) are lacking. One should expect that the substitution of the Riemann tensor R_{MNPQ} with the modified one in eq.(2.10), as suggested by the tree-level result, is the full answer. However, in this case, the complex scalar and the antisymmetric fields are included in an non-modular-invariant way, which explicitly breaks the conjectured $SL(2,\mathbf{Z})$ symmetry of the type IIB theory. We will construct below a full $SL(2,\mathbf{Z})$-invariant effective action.

4. The $SL(2,\mathbf{Z})$-invariant type IIB effective action

The proposed $SL(2,\mathbf{Z})$-invariant four-point effective action compatible with the tree-level NS-NS sector, which includes the complex scalar and the antisymmetric two-form fields, is

$$
\begin{aligned}
\mathcal{S} \;=\; & \frac{1}{2}\int d^{10}x\sqrt{-g}\,\Big\{R-\frac{1}{2\tau_2^2}\partial_M\tau\partial^M\bar\tau-\frac{1}{6}G_{KMN}\bar G^{KMN}+\frac{1}{3\cdot 2^7}(t_8t_8+\frac{1}{8}\varepsilon_{10}\varepsilon_{10})\times\\
&\Big[\frac{1}{2}f_0(\tau,\bar\tau)\Big(R^4+12R^2DP\,D\bar P-6RDP\,D\bar G^2+3R^2DG\,D\bar G\\
&+6DP^2\,D\bar P^2+\frac{3}{8}DG^2\,D\bar G^2+6DP\,D\bar P\,DG\,D\bar G\Big)\\
&+f_1(\tau,\bar\tau)\Big(-4R^3DP+\frac{3}{2}R^2DG^2-12RDP^2\,D\bar P-6RDP\,DG\,D\bar G\\
&+3DP\,D\bar P\,DG^2+\frac{3}{2}DP^2\,D\bar G^2+\frac{1}{4}DG^3\,D\bar G\Big)\\
&+f_2(\tau,\bar\tau)\Big(6R^2DP^2-3RDPDG^2+4DP^3\,D\bar P+3DP^2\,DG\,D\bar G+\frac{1}{16}DG^4\Big)\\
&+f_3(\tau,\bar\tau)\Big(-4RDP^3+\frac{3}{2}DP^2DG^2\Big)+f_4(\tau,\bar\tau)DP^4+c.c.\Big]\Big\}\;,
\end{aligned}
\tag{4.1}
$$

where DG stands for $(DG)_{MNPQ}\equiv D_MG_{NPQ}$, DP for $(DP)_{MNPQ}\equiv g_{MP}D_NP_Q$ and similarly for $D\bar G$ and $D\bar P$ of $U(1)$ charge $q=-1,-2$, respectively. The functions $f_k(\tau,\bar\tau)$ are defined as

$$
f_k(\tau,\bar\tau)=\sum_{m,n}{}'\frac{\tau_2^{3/2}}{(m+n\tau)^{3/2+k}(m+n\bar\tau)^{3/2-k}}\;,
\tag{4.2}
$$

for $k\in\mathbf{Z}$. They transform under $SL(2,\mathbf{Z})$ as

$$
f_k(\tau,\bar\tau)\to\left(\frac{c\tau+d}{c\bar\tau+d}\right)^k f_k(\tau,\bar\tau)\,,\qquad \begin{pmatrix}a&b\\c&d\end{pmatrix}\in SL(2,\mathbf{Z})\,.
\tag{4.3}
$$

These functions have the correct modular transformation properties to render the action $SL(2,\mathbf{Z})$-invariant. Moreover, the effective action we propose satisfies all the criteria listed in the previous section. Namely, it has only tree-level and one-loop perturbative corrections in the NS-NS sector, which is an extension of the non-renormalization theorem of the R^4 term. This can be seen by examining the small τ_2^{-1} expansion of f_k, which follows from

$$\left(k + 2i\tau_2 \frac{\partial}{\partial\tau}\right) f_k \;=\; \left(\frac{3}{2} + k\right) f_{k+1} \,. \tag{4.4}$$

As a result we obtain

$$f_k(\tau,\bar\tau) = 2\zeta(3)\tau_2^{3/2} + c_k \tau_2^{-1/2} + \cdots \,, \tag{4.5}$$

where

$$c_0 = \frac{2\pi^2}{3}\,, \quad \text{and} \quad c_{k+1} = \frac{2k-1}{2k+3} c_k\,, \quad k = 0, 1, \ldots\,, \tag{4.6}$$

and the dots in eq.(4.5) stand for instanton corrections.

The action (4.1) we propose has been constructed from the tree-level one (2.13) by replacing

$$\nabla_M \partial_N \phi \to D_M P_N + D_M \bar{P}_N \tag{4.7}$$

for the dilaton and

$$e^{-\phi/2} \nabla_M H^1_{NPQ} \to -\frac{i}{\sqrt{2}} D_M G_{NPQ} + \frac{i}{\sqrt{2}} D_M \bar{G}_{NPQ} \tag{4.8}$$

for the NS-NS antisymmetric tensor. Notice that when $\chi = 0\,, H^2 = 0$, the r.h.s. of eq.(4.8) gives $e^{-\phi/2}\nabla_M H^1_{NPQ} + e^{-\phi/2}\partial_M \phi H^1_{NPQ}$, where the second term contributes to at least six-point amplitudes, which is not relevant to our discussion. Under modular transformations, each term in the resulting expression is multiplied by a factor $\left(\frac{c\tau+d}{c\bar\tau+d}\right)^{q/2}$, which we finally compensate by replacing $2\zeta(3)\tau_2^{3/2}$ by $f_{q/2}(\tau,\bar\tau)$. Notice that the terms linear or cubic in ∇H^1, which vanish anyway at tree level in the expansion (2.13) because of the Bianchi identity would have implied additional contributions in (4.1) involving $f_{k/2}$'s with k odd. However, it is easily seen from their definition in eq.(4.2) that these functions vanish identically.

As a non-trivial check of our proposal, there exists a non-perturbative calculation of type IIB on $K3$ for the four-point amplitude involving two gravitons and two dilatons [21].

This term corresponds to

$$A_{hh\phi\phi} \propto R^{ijkl} R_{ijkl} (\partial_\mu \partial_\nu \phi)(\partial^\mu \partial^\nu \phi) \,, \tag{4.9}$$

where the Latin and Greek indices refer to the internal $K3$ and to the six-dimensional spacetime, respectively. By taking the large $K3$ volume limit, one finds that this amplitude is multiplied by the function $f_0(\tau, \bar{\tau})$ in ten dimensions. However, in our case from the action (4.1) this amplitude is proportional to

$$\left(t_8^{ijkl\mu\nu\rho\sigma} t_8^{mnpq\kappa\lambda\varphi\omega} + \frac{1}{8} \varepsilon_{10}^{ijkl\mu\nu\rho\sigma\alpha\beta} \varepsilon_{10}^{mnpq\kappa\lambda\varphi\omega}{}_{\alpha\beta} \right) R_{ijmn} R_{klpq} g_{\nu\lambda} g_{\sigma\omega} \nabla_\mu \partial_\kappa \phi \nabla_\rho \partial_\varphi \phi \,, \tag{4.10}$$

and can be seen to vanish by a straightforward calculation. This apparent paradox can be avoided[2] by recalling that there exists an additional contribution to the six-dimensional amplitude. This arises from the the second and third terms in eq.(2.11) in the definition of $\hat{R}$ in eq.(2.10). In that case one obtains the additional contribution

$$\left(t_8^{ijkl\mu\nu\rho\sigma} t_8^{mnpq\kappa\lambda\varphi\omega} + \frac{1}{8} \varepsilon_{10}^{ijkl\mu\nu\rho\sigma\alpha\beta} \varepsilon_{10}^{mnpq\kappa\lambda\varphi\omega}{}_{\alpha\beta} \right) R_{ijmn} R_{klpq} R_{\mu\nu\kappa\lambda} \times$$
$$\left(\frac{1}{2} g_{\sigma\omega} \partial_\rho \phi \partial_\varphi \phi - g_{\rho\varphi} g_{\sigma\omega} \partial_\gamma \phi \partial^\gamma \phi \right) . \tag{4.11}$$

By expanding around the background $g_{MN} = (\eta_{\mu\nu}, g_{mn}^{K3})$ and by partially integrating the two derivatives in $R_{\mu\nu\kappa\lambda}$, we get a term proportional to the r.h.s. of eq.(4.9). Finally, this term should be promoted in the exact $SL(2,\mathbf{Z})$-invariant Lagrangian to

$$f_0(\tau, \bar{\tau}) R^{ijkl} R_{ijkl} D_\mu P_\nu D^\mu \bar{P}^\nu \,, \tag{4.12}$$

which indeed reproduces the result of [21].

5. Conclusions

We have conjectured here an S-duality invariant effective four-point action of type IIB theory. The guiding principles we used were basically the $SL(2,\mathbf{Z})$ invariance and the

[2]We would like to thank I. Antoniadis and B. Pioline for their contribution on this point.

expectation that the perturbative corrections stop at one loop for the NS-NS sector. We found that these principles can be satisfied by using the functions $f_k(\tau,\bar{\tau})$ we introduced. They give the correct perturbative corrections and the non-perturbative ones are of the expected form. Moreover, the vanishing of four-point amplitudes that involve one or three antisymmetric fields is consistent, as we have discussed, with the vanishing of the f_k forms for half-integer k. Finally, the result of [21] is consistent with our proposal since, for the $K3$ compactification they consider, it arises from a five-point term in ten dimensions. Finally, a simple test of our proposal would be an explicit determination of the one-loop coefficients c_k defined in eq.(4.5).

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References

[1] J.H. Schwarz and P. West, *Symmetries and Transformations of Chiral N=2 D=10 Supergravity*, Phys. Lett. **B126** (1983) 301;
P.S. Howe and P. West, *The Complete N=2, D=10 Supergravity*, Nucl. Phys. **B238** (1984) 181;
J.H. Schwarz, *Covariant Field Equations of Chiral* $N = 2$ $D = 10$ *Supergravity*, Nucl. Phys. **B226** (1983) 269.

[2] C.M. Hull and P.K. Townsend, *Unity of Superstring Dualities*, Nucl. Phys. **B438** (1995) 109, hep-th/9410167.

[3] M.T. Grisaru, A.E.M. van de Ven and D. Zanon, *Two-Dimensional Supersymmetric Sigma-Models on Ricci Flat Kähler Manifolds are not Finite*, Nucl. Phys **B277** (1986) 388; *Four Loop Divergencies for the $N = 1$ Supersymmetric Nonlinear Sigma Model in Two-Dimensions*, Nucl. Phys. **B277** (1986) 409.

[4] J.H. Schwarz, *Superstring Theory*, Phys. Rep. **89** (1982) 223.

[5] D.J. Gross and E. Witten, *Superstring Modifications of Einstein's Equations*, Nucl. Phys. **B277** (1986) 1.

[6] D.J. Gross and J.H. Sloan, *The Quartic Effective Action for the Heterotic String*, Nucl. Phys. **B291** (1987) 41.

[7] N. Sakai and Y. Tanii, *One-Loop Amplitudes and Effective Action in Supertring Theories*, Nucl. Phys. **B287** (1987) 457.
M. Abe, H. Kubota and N. Sakai, *Loop Corrections to the Heterotic String Effective Lagrangian*, Phys. Lett. **B200** (1988) 461; **B203** (1988) 474.

[8] A.A. Tseytlin, *Heterotic Type I Superstring Duality and Low Energy Effective Action*, Nucl. Phys. **B467** (1996) 383, hep-th/9512081.

[9] M.B. Green and M. Gutperle, *Effects of D-Instantons*, Nucl. Phys. **B498** (1997) 195, hep-th/9701093.

[10] N. Berkovits, *Construction of R^4 Terms in $N = 2$ $D = 8$ Superspace*, hep-th/9709116.

[11] J.G. Russo, *An Ansatz for a Non-Perturbative Four-Graviton Amplitude in Type IIB Superstring Theory*, hep-th/9707241.

[12] M.B. Green, P. Vanhove and M. Gutperle, *One Loop in Eleven Dimensions*, hep-th/9706175.

[13] M.B. Green and P. Vanhove, *D-Instantons, Strings and M-Theory*, hep-th/9704145.

[14] J.G. Russo and A.A. Tseytlin, *One-Loop Four Graviton Amplitude in Eleven-Dimensional Supergravity*, hep-th/9707134.

[15] A. Strominger, *Loop Corrections to the Universal Hypermultiplet*, hep-th/9706195.

[16] I. Antoniadis, S. Ferrara, R. Minasian and K.S. Narain, R^4 *Couplings in M and Type II Theories on Calabi-Yau Spaces*, hep-th/9707013.

[17] A. Kehagias and H. Partouche, *in preparation.*

[18] G.W. Gibbons, M.B. Green and M.J. Perry, *Instantons and Seven-Branes in Type IIB Superstring Theory*, Phys. Lett. **B370** (1996) 37, hep-th/9511080.

[19] E. Witten, *Bound States of Strings and p-Branes*, Nucl. Phys **B460** (1996) 335, hep-th/9510135.

[20] H. Maass, *Lectures on Modular Functions of One Complex Variable*, TATA Instit. of Fund. Research, Springer Verlag (1983).

[21] I. Antoniadis, B. Pioline and T.R. Taylor, *Calculable* $e^{-1/\lambda}$ *Effects*, hep-th/9707222.

[22] E. Kiritsis and B. Pioline, *On* R^4 *Threshold Corrections in IIB String Theory and (p,q)-String Instantons*, hep-th/9707018.