

CERN-TH/97-262

hep-ph/9709502

## A Next-to-Minimal Supersymmetric Model of Hybrid Inflation

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### Abstract

We propose a model of inflation based on a simple variant of the NMSSM, called  $\phi$ NMSSM, where the additional singlet  $\phi$  plays the role of the inflaton in hybrid (or inverted hybrid) type models. As in the original NMSSM, the  $\phi$ NMSSM solves the  $\mu$  problem of the MSSM via the VEV of a gauge singlet  $N$ , but unlike the NMSSM does not suffer from domain wall problems since the offending  $Z_3$  symmetry is replaced by an approximate Peccei-Quinn symmetry which also solves the strong CP problem, and leads to an invisible axion with interesting cosmological consequences. The PQ symmetry may arise from a superstring model with an exact discrete  $Z_3 \times Z_5$  symmetry after compactification. The model predicts a spectral index  $n = 1$  to one part in  $10^{12}$ .

CERN-TH/97-262

February 20, 1998

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<sup>1</sup>On leave of absence from \*.

There is to date no standard model of inflation, and although there has been a good deal of progress in recent years in this area much of the current activity has been concerned with conceptualised field theoretic models rather than well motivated particle physics based models [1]. Possibly the best motivated particle physics model beyond the standard model is the minimal supersymmetric standard model (MSSM). However the only Higgs fields in the MSSM are the two doublets  $H_1, H_2$ , which develop vacuum expectation values (VEVs) of order the weak scale, and it is very difficult if not impossible to develop a model of inflation using only these fields for several reasons. The primary reasons are that the electroweak scale turns out to be too small and the Higgs potential is not sufficiently flat. The so called next-to-minimal supersymmetric standard model (NMSSM) is more promising from the point of view of inflation since it contains, in addition to the two Higgs doublets, a Higgs singlet  $N$  which may develop a large VEV.

The usual NMSSM does not require the  $\mu H_1 H_2$  term of the MSSM, replacing it with a  $\lambda N H_1 H_2$  term, and thereby solving the  $\mu$  problem<sup>2</sup>. The NMSSM also involves a term  $kN^3$  in the superpotential so that the model has an exact  $Z_3$  symmetry [3, 4]. However this is broken at the weak scale leading to a serious domain wall problem [5, 6]. Originally it was thought that the  $Z_3$  may be slightly violated by Planck scale operators, leading to a pressure term that removes the walls. However without an exact  $Z_3$  symmetry supergravity tadpole diagrams will lead to a large singlet mass in the low energy theory, and the amount of  $Z_3$  breaking required to solve the domain wall problem is in conflict with requirement that tadpoles do not make the singlet too heavy [6, 7].

It transpires that, without fine-tuning, the NMSSM does not lead to a sufficiently flat potential along which the inflaton may roll. In order to overcome this we introduce

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<sup>2</sup>Note that the Giudice-Masiero mechanism [2] presents a solution to the  $\mu$  problem within the MSSM by generating the  $\mu$  term via a non-minimal Kahler potential.

a second singlet  $\phi$ , and replace the term  $N^3$  in the NMSSM by  $\phi N^2$ . Thus our model is based on the superpotential:

$$W_{\phi NMSSM} = \lambda N H_1 H_2 - k \phi N^2 \quad (1)$$

Note that our model has the same number of dimensionless couplings as the original NMSSM, and we have used the same notation  $\lambda, k$  to emphasise this. With this modification the field  $\phi$  appears only linearly in the superpotential and so will have a very flat potential, lifted only by a tiny mass  $m_\phi$  of order electronvolts, and will play the role of the inflaton field of hybrid inflation [8, 9, 10] if  $m_\phi^2 > 0$  or inverted hybrid inflation [11] if  $m_\phi^2 < 0$ . In the case of inverted hybrid inflation the present model provides an interesting counter example to the problems raised in Ref. [12]. Inflation ends when  $\phi$  reaches a critical value  $\phi_c \sim 10^{13}$  GeV after which the  $N$  field, which has a zero value during inflation, develops a VEV  $\langle N \rangle \sim \phi_c$ . Interestingly the inflaton also develops an eventual VEV  $\langle \phi \rangle \sim \phi_c$  via a tadpole coupling, which is typical of inverted hybrid inflation but quite extraordinary for hybrid inflation. The resulting dimensionless couplings are  $\lambda, k \sim 10^{-10}$ , whose smallness will be explained by embedding the model into a string inspired model where the couplings result from higher dimension operators, controlled by discrete symmetries. Note that radiative corrections to the inflaton mass are controlled by  $\lambda, k$  and are of order the inflaton mass itself.

Having replaced the NMSSM superpotential by Eq. (1), the troublesome  $Z_3$  symmetry is replaced by a global  $U(1)_{PQ}$  Peccei-Quinn symmetry where the global charges of the fields satisfy:

$$Q_N + Q_{H_1} + Q_{H_2} = 0, \quad Q_\phi + 2Q_N = 0. \quad (2)$$

with the quark fields having the usual axial PQ charges. The global symmetry forbids additional couplings such as  $N^3$ ,  $\phi H_1 H_2$  and so on, but is broken at the scale of the VEVs releasing a very light axion. The axion scale  $f_a$  is therefore of order  $\phi_c$  in this

model. The axion will be an invisible Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) [13] type axion, which couples to ordinary matter through its mixing with the standard Higgses after the electroweak phase transition. Once we embed our model into a string motivated model, the global PQ symmetry will emerge as an approximate accidental symmetry of an underlying discrete symmetry, and we need to discuss such questions as the solution to the strong CP problem in this wider context. Note that if we had simply removed the  $N^3$  term from the NMSSM superpotential and not replaced it with anything then the theory would also have a PQ symmetry, and the potential would also be flat in the  $N$  direction, and then one might be tempted to identify  $N$  with the inflaton of hybrid inflation. However in such a scenario the height of the potential during inflation would be of order 1 TeV, leading to an inflaton mass very much smaller than the radiative corrections to its mass of order 1 eV, which would require unnatural fine-tuning. By contrast, with the  $\phi N^2$  term present, the height of the potential during inflation is about  $10^8$  GeV and the COBE constraint may be satisfied by an inflaton mass of about 1 eV which is the same order as the radiative corrections to its mass, leading to a natural scenario with no fine-tuning required.

The tree-level potential which follows from the superpotential in Eq. (1) can be written, if we ignore  $H_1, H_2$  which have smaller VEVs,

$$\begin{aligned} V_0 &= V(0) + V(\phi, N) \\ V(\phi, N) &= k^2 N^4 + m^2(\phi) N^2 + m_\phi^2 \phi^2, \end{aligned} \quad (3)$$

with the field dependent  $N$  mass given by,

$$m^2(\phi) = m_N^2 - 2kA_k\phi + 4k^2\phi^2. \quad (4)$$

We have taken  $\phi$  and  $N$  to be the real components of the complex singlets, and included the soft breaking parameters from the soft supersymmetry breaking potential terms  $m_N N^2$ ,  $m_\phi \phi^2$  and  $A_k k \phi N^2$ . We have also added by hand a constant vacuum

energy  $V(0)$  to the potential, about which we shall say more later. Note that  $m^2(\phi) = 0$  for  $\phi$  equal to a critical value<sup>3</sup>:

$$\phi_c^\pm = \frac{A_k}{4k} \left( 1 \pm \sqrt{1 - 4 \frac{m_N^2}{A_k^2}} \right). \quad (5)$$

In order to discuss inflation we need to specify the sign of the inflaton mass squared  $m_\phi^2$ . If  $m_\phi^2 > 0$  (as in hybrid inflation) then, for  $\phi > \phi_c^+$ ,  $N$  will be driven to a local minimum (false vacuum) with  $N=0$ . Having a positive mass squared,  $\phi$  will roll towards the origin and  $m^2(\phi)$  will become negative once the field  $\phi$  reaches  $\phi_c^+$ . After that, the potential develops an instability in the  $N=0$  direction, and both singlets roll down towards the global minimum,

$$\langle \phi \rangle = \frac{A_k}{4k}, \quad (6)$$

$$\langle N \rangle = \frac{A_k}{2\sqrt{2}k} \sqrt{1 - 4 \frac{m_N^2}{A_k^2}} = \sqrt{2} |\phi_c^\pm - \langle \phi \rangle|, \quad (7)$$

signaling the end of the inflation. On the other hand if  $m_\phi^2 < 0$  (corresponding to inverted hybrid inflation) then we shall suppose that during inflation  $\phi < \phi_c^-$ , with the inflaton rolling away from the origin, eventually reaching  $\phi_c^-$  and ending inflation with the same global minimum as before. Note that the global minimum VEV  $\langle \phi \rangle$  is sandwiched in between  $\phi_c^-$  and  $\phi_c^+$  so either hybrid or inverted hybrid inflation is possible in this model depending on the sign of  $m_\phi^2$ .

Since  $A_k$  is a soft SUSY breaking parameter of order 1 TeV we have the order of magnitude results:

$$k\phi_c^\pm \sim k \langle N \rangle \sim k \langle \phi \rangle \sim 1 \text{ TeV}. \quad (8)$$

Since the VEVs are associated with the large axion scale, we see that the parameter  $k \sim O(10^{-10})$ . Similarly since  $\lambda \langle N \rangle$  plays the role of the  $\mu$  parameter of the MSSM we require  $\lambda$  to have a similarly small value. We shall discuss the origin of

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<sup>3</sup>We require that the condition  $A_k^2 > 4m_N^2$  is fulfilled.

such a small values of  $\lambda, k$  later in the context of the string motivated model, but for now we simply note their smallness and continue.

The negative value of  $V(\phi, N)$  at the global minimum, is compensated by  $V(0)$  which is assumed to take an equal and opposite value, in accordance with the observed small cosmological constant. Thus we assume:

$$V(0) = -V(\langle \phi \rangle, \langle N \rangle) = k^2 \langle N \rangle^4 = 4k^2(\phi_c^\pm - \langle \phi \rangle)^4. \quad (9)$$

During inflation we may set the field  $N = 0$  so that the potential simplifies to:

$$V = V(0) + m_\phi^2 \phi^2 \quad (10)$$

The slow roll conditions are given by:

$$\epsilon_N = \frac{1}{16\pi} \frac{M_P^2 m_\phi^4 \phi_N^2}{V(0)^2} \ll 1, \quad (11)$$

$$|\eta_N| = \frac{M_P^2 |m_\phi^2|}{8\pi V(0)} \ll 1. \quad (12)$$

The subscripts “N” means that  $\phi$  and  $\epsilon$  have to be evaluated N e-folds before the end of inflation, when the largest scale of cosmological interest crosses the horizon<sup>4</sup>, that is,  $N \simeq 60$ . The height of the potential during inflation is approximately constant and given by  $V(0)^{\frac{1}{4}} = k^{\frac{1}{2}} \langle N \rangle \sim 10^8$  GeV.

Assuming that  $V(0)$  dominates the potential during inflation,  $\phi_N = \phi_c^\pm e^{\eta N} \approx \phi_c^\pm$ , where the last approximation follows since in our model  $|\eta| \ll 1/N$ . We need further to check that our inflationary model is able to produce the correct level of density perturbation, responsible for the large scale structure in the Universe, accordingly to the COBE anisotropy measurements. The spectrum of the density perturbations is

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<sup>4</sup>The required number of e-folds is roughly 60 for a potential barrier  $V(0)^{1/4} \simeq 10^{16}$  GeV and very efficient reheating in the post-inflationary period. It diminishes when the value of  $V(0)$ , and/or the reheating temperature decrease. This will be the case for this model, where we will see that the needed value of  $V(0)$  is lower than  $10^{11}$  GeV, and the reheating temperature is quite low. Nevertheless, because  $|\eta|$  is extremely small, the particular value of  $N$  is not relevant to this stage.

given by the quantity [14],

$$\delta_H^2 = \frac{32 V(0)}{75 M_P^4} \frac{1}{\epsilon_N}, \quad (13)$$

with the COBE value,  $\delta_H = 1.95 \times 10^{-5}$  [15]. Writing  $\phi_c^\pm \sim \phi_c$ , COBE gives the order of magnitude constraint:

$$|km_\phi| \simeq 8 \left(\frac{8\pi}{75}\right)^{1/4} \delta_H^{-1/2} \frac{(k\phi_c)^{5/2}}{M_P^{3/2}} \simeq 10^{-18} \text{ GeV} \left(\frac{k\phi_c}{1 \text{ TeV}}\right)^{5/2}. \quad (14)$$

This, in turn, is more than enough to broadly satisfy the slow-roll conditions. In particular,

$$|\eta_N| \simeq \frac{M_P^2}{8\pi} \frac{|km_\phi|^2}{(\sqrt{2}k\phi_c)^4} \sim 10^{-12}, \quad (15)$$

$$\epsilon_N \sim \frac{M_P^2}{16\pi} \frac{|km_\phi|^4}{(\sqrt{2}k\phi_c)^8} \phi_N^2 \sim 4\pi \frac{\phi_N^2}{M_P^2} \eta_N^2 \quad (16)$$

The model predicts a very flat spectrum of density perturbations, as usual in this type of hybrid model, with no appreciable deviation of the spectral index,  $n = 1 + 2\eta - 6\epsilon$ , from unity. Only models where the curvature (of either sign) of the inflaton potential is not very suppressed with respect to  $H$  can give rise to a blue [16] (red [17]) tilted spectrum.

Note that COBE requires the product  $|km_\phi|$  to be extremely small. If we take  $k \sim 10^{-10}$ , motivated by axion physics as discussed above, then this implies  $m_\phi$  in the electronvolt range. The requirement of such a small mass leads to several interesting requirements on the model. We envisage that at the Planck scale the  $\phi$  mass is equal to zero. This can be naturally accomplished within the framework of supergravity no-scale models [18], where *some* (not necessarily all) of the SUSY soft masses are predicted to vanish, but with non-zero and universal trilinear coupling parameters. However the high energy value of  $m_\phi$  will be subject to radiative corrections which are very small, being controlled by the small coupling  $k$ . In our model the radiative corrections at  $\phi \gg \phi_c$  arise from loops of the scalar and pseudoscalar components of the complex  $N$  field, which are split by soft SUSY breaking terms, and by their

fermionic partners. The result may be easily obtained to be [19, 20]:

$$\Delta V = \frac{k^4 \phi_c^2 \phi^2}{2\pi^2} \ln\left(\frac{4k^2 \phi^2}{\mu^2}\right) \quad (17)$$

where  $\mu$  is the modified minimal subtraction scale and we have assumed  $\phi \gg \phi_c$ . If we take  $\mu^2 = 4k^2 \phi^2 \sim 1TeV^2$  to remove the logarithm then use the renormalisation group (RG) to run the  $\phi$  mass from the Planck scale (where it is zero) down to this scale then the radiative corrections result in a correction to the mass of:

$$\delta m_\phi^2 \simeq -\frac{|c|}{\pi^2} k^2 (k\phi_c)^2, \quad (18)$$

with  $c$  a constant of order 1. The COBE constraint,  $km_\phi \sim 10^{-18}$ , together with the naturalness requirement that the radiative correction is of order the mass itself, will now translate into,

$$k \approx 10^{-10} \left( \frac{k\phi_c}{1TeV} \right)^{3/4}, \quad (19)$$

leading to a mass  $m_\phi$  in the eV range. Notice that RG equations result in a negative mass squared which would, in the absence of any other correction, appear to favour the inverted hybrid inflation scenario.

However there is another potentially large contribution to the  $\phi$  mass coming from the vacuum energy  $V(0)$  which breaks supersymmetry and which drives inflation. On general grounds, whenever local SUSY is broken by non-zero F-terms in the visible or hidden sector, the Kahler potential will generate soft SUSY breaking parameters in the observable sector which leads to the so-called  $\eta$ -problem. All the scalar fields in the observable sector will pick up masses of order of the Hubble constant [9, 21, 22], where  $H^2 \approx V_0/M_P^2$ , due to the presence of an exponential factor for the Kahler potential in front of the potential for the observable sector. The inflaton, unless fine-tuning or specific forms of the Kahler potential are assumed [23], will also get this type of mass, making it difficult to satisfy the slow-rolling constraint  $\eta \ll 1$ . A linear superpotential for the inflaton is a special case that can avoid the problem



[9] even for minimal Kahler potential, but requires an input mass parameter. One simple way out of the problem is to consider  $V(0)$  not having an F-term origin, but instead originating from some D-term [24]. Most F-type breaking supergravity models assume that the D-term contribution to the potential vanishes when the fields in the hidden sector get their VEVs. This may not be necessarily the case if these fields are charged under some  $G_H$  hidden or visible gauge symmetry, such as a gauged (possibly anomalous)  $U(1)$  symmetry. However the identification of  $V(0)$  with some D-term seems problematic, at least within the framework of string theories, since the mass scale of the parameter  $\xi$  predicted by string theories is too large compared to that required by hybrid inflation [25], and in the present model this problem is made worse due to the smallness of  $V(0)$ .

The problem of the origin of  $V(0)$  in our model is in fact the cosmological constant problem and the same problem besets the MSSM where the explicit potential at the global minimum does not vanish,  $V_{MSSM}(\langle H_1 \rangle, \langle H_2 \rangle) \neq 0$ . In order to obtain a small cosmological constant in the MSSM one is forced to add by hand a vacuum energy  $V(0)$  to accurately cancel it as in Eq.9. Here, as in the MSSM, we do not specify the origin of  $V(0)$ , but simply add  $V(0)$  by hand. The solution to the cosmological constant problem almost certainly lies beyond supergravity, and probably beyond string theory. What is crucial for the success of our model is that during inflation all the F-terms vanish, and this condition is in fact satisfied by all the explicit terms in our model. All that we require of  $V(0)$  is that it does not originate from the F-term of a supergravity model. Such a vacuum energy which has the characteristics assumed here, namely that it does not originate from the F-term of a supergravity model, and has at least the possibility of solving the cosmological constant problem has been proposed within the context of quantum cosmology [26].

Having fixed the value of the Yukawa coupling, the vacuum expectation values are

then around the scale  $\phi \sim N \sim 10^{13}$  GeV. More specifically, from Eq. (19) we have,

$$\phi_c \approx 10^{13} (10^{10} k)^{1/3} \text{ GeV}. \quad (20)$$

The value of  $\phi_c$  slightly exceeds the usually quoted upper bound for the axion VEV, derived requiring the cosmic density of axions not to exceed the critical density of the universe [27]. However a value  $f_a \sim 10^{13}$  GeV is not strictly excluded when several uncertainties entering the derivation of this bound are allowed [28]. Recently, it has been also argued [29] that a value of  $f_a$  bigger than  $10^{12}$  GeV can be allowed in models where the reheating temperature goes below 1 GeV, that is, below the temperature at which the axion field begins to oscillate. The point is that during inflation the PQ symmetry is broken and the axion field is displaced at some arbitrary angle, and it relaxes to zero only after reheating and only below the QCD phase transition when its potential is tilted. At this point the dangerous energy stored in the axion field is released, but if the reheat temperature is of order 1 GeV then the resulting axion density from the displaced axion field will be diluted by the entropy release [30] produced by the inflaton decay. On the other hand the inflaton itself may decay directly into axions, and this branching fraction must be sufficiently small so that the resulting number density of axions at the time of nucleosynthesis is only a small fraction of a neutrino species, as we now discuss.

Immediately after inflation ends the masses of the singlet scalars which correspond to the oscillating mode of the fields  $N$  and  $\phi$  just after they receive their global VEVs will be  $M_\phi \sim O(1 \text{ TeV})$ . The resulting “inflaton” mode can decay into lighter particles, with a width proportional to the coupling  $k^2/(4\pi) \sim (1 \text{ TeV})^2/f_a^2$ . As discussed in Ref. [29] in order to avoid conflict with nucleosynthesis in models where the reheat temperature is of order 1 GeV one requires the branching fraction of the decaying inflaton into axions not to exceed about 10%. As pointed out [29] the inflaton coupling to stops  $m_t^2/f_a$  may dominate over the coupling to axions  $M_\phi^2/f_a$  if

$m_{\tilde{t}}^2 > M_\phi^2$ , providing that the stop mixing results in a kinematically accessible light stop mass eigenstate  $m_{\tilde{t}1}^2 < M_\phi^2$ . In this case the inflaton may decay predominantly into stops. The resulting decay rate may be estimated as:

$$\Gamma_\phi \approx \frac{k^2}{4\pi} M_\phi \sim 10^{-8} eV, \quad (21)$$

which is quite suppressed with respect to,

$$H \simeq \frac{V_0^{1/2}}{3M_P} \approx \frac{k\phi_c^2}{3M_P} \sim 1 MeV, \quad (22)$$

The reheating temperature is given by [1]:

$$T_{RH} \simeq 0.55 g_*^{-1/4} \sqrt{\Gamma_\phi M_P}, \quad (23)$$

where  $g_*$  is the number of effective degrees of freedom at reheating, and  $\Gamma_\phi$  is the width of the inflaton decay. Conversion of the vacuum energy to thermal radiation through the decay of the inflaton  $\phi$  into light particles will be quite inefficient, because  $\Gamma_\phi \ll H$ . This gives a reheating temperature  $T_{RH} \approx O(1 - 10)$  GeV.

Despite its low value, the reheat temperature is high enough to preserve the standard scenario for nucleosynthesis,  $T_{RH} > 6$  MeV, although quite far to allow electroweak baryogenesis. Moreover, any pre-existing baryon asymmetry is likely to be diluted during inflation. Nevertheless, as has been pointed out [29, 17], the amount of baryon asymmetry needed might be produced directly by the decays of the inflaton. For this mechanism to work we require the presence of baryon-number violating operator in the superpotential, type  $\lambda_{ijk}'' U_i^c D_j^c D_k^c$ . As discussed the inflaton can decay predominantly into light stop squarks, and the subsequent decay of the stops into two down-type quarks from this R-parity baryon number violating operator will generate baryon-antibaryon asymmetry. Other mechanisms, like Affleck-Dine type baryogenesis [31], might also work.

We now turn to the question of origin of the extremely small values of the couplings  $\lambda$  and  $k$  which are required in this model, and to whether this might be understood

in terms of a deeper theory such as string theory. In fact it has been argued [32, 33] that an approximate PQ symmetry that solves the strong CP problem can arise from superstring models with exact discrete symmetries after compactification. Let us assume a  $Z_3 \times Z_5$  discrete symmetry, resulting from some string compactification and introduce singlets  $M, \bar{M}$ , with the fields transforming as in Table 1. All the fields are supposed to originate from some 27 and  $\bar{27}$  representations of  $E_6$  apart from  $\phi$  which is taken to be a singlet of  $E_6$ . To be precise one can assume that  $H_1, H_2, N, M$  originate from 27's while  $\bar{M}$  originates from a  $\bar{27}$ . The superpotential is given by,

$$W_{NR} = \lambda' H_1 H_2 N \frac{M \bar{M}}{M_P^2} - k' \phi N^2 \frac{\bar{M}^2}{M_P^2} + c \frac{(M \bar{M})^3}{M_P^3} + d \frac{(N \bar{M})^5 (M \bar{M})^2}{M_P^{11}} + \dots \quad (24)$$

where all the Yukawa couplings can be assumed to be of order unity, and we have included only the leading physically relevant terms. Note that terms such as  $N^3$  are forbidden by the  $E_6$  gauge symmetry since the product of three 27's does not contain three singlets. Here the  $E_6$  gauge symmetry is assumed to be broken at the string level by for example Wilson line breaking [34], so questions such as doublet-triplet splitting are addressed at the string level. We assume that the  $M, \bar{M}$  fields radiatively generate VEVs, as a result of a radiative mechanism due to their soft masses  $m^2$  becoming negative, stabilised by F-terms arising from the above operators with resulting VEVs

$$\langle M \rangle = \langle \bar{M} \rangle \sim \left( \frac{m M_P^3}{c} \right)^{\frac{1}{4}} \sim 10^{14} GeV \quad (25)$$

As a result of these VEVs we recover the two terms of the superpotential given in Eq. (1), with

$$\lambda \sim \lambda' \frac{M \bar{M}}{M_P^2} \sim \lambda' 10^{-10}, \quad (26)$$

$$k \sim k' \frac{\bar{M}^2}{M_P^2} \sim k' 10^{-10}, \quad (27)$$

The  $U(1)_{PQ}$  symmetry is explicitly broken by the higher order term proportional to  $d$ . As discussed [32] such higher order terms contribute an explicit axion mass  $\Delta m_a^2$

	$H_1$	$H_2$	$N$	$\phi$	$M$	$\overline{M}$
$Z_3$	$\alpha^2$	$\alpha^2$	1	$\alpha$	$\alpha$	$\alpha$
$Z_5$	1	1	1	$\beta$	$\beta^3$	$\beta^2$

Table 1:  $Z_3 \times Z_5$  charges for the chiral supermultiplets.

which tilts the axion potential slightly, and perturbs the  $\theta$  angle by an amount

$$\Delta\theta \sim \frac{\Delta m_a^2 f_a^2}{m_\pi^2 f_\pi^2}. \quad (28)$$

In order to preserve the PQ solution to the strong CP problem we require  $\Delta\theta < 10^{-8}$ . Setting  $d = 1$  and including a trilinear coupling  $A = 10^3$  GeV, the above operator leads to  $\Delta\theta \sim 10^{-11}$  thereby preserving the PQ solution to the strong CP problem in this model. Note that the question of domain walls, both coming from the breaking of the discrete string symmetries, and those associated with axion domain walls, which was discussed in detail in the second reference in [32], does not arise in our model since they are both inflated away.

To summarise, we have seen that a simple variant of the NMSSM, called  $\phi$ NMSSM, involving two singlets  $N, \phi$  but the same number of parameters as in the original NMSSM, opens up the possibility of solving the strong CP problem and the  $\mu$  problem, as well as providing also a mechanism for hybrid or inverted hybrid inflation in the early Universe, neatly side-stepping all domain wall problems. The smallness of the couplings  $\lambda$  and  $k$  can be understood in terms of a string motivated discrete symmetry, while the smallness of the  $\phi$  mass implies a no-scale supergravity origin for this parameter. We do not specify the origin of the vacuum energy  $V(0)$  which is necessary to drive inflation, and lead to an acceptably small cosmological constant. However we do require that the source of the vacuum energy not be the F-term of a supergravity model which would lead to an unacceptably large  $\phi$  mass. The magnitude of the VEVs are of the correct order of magnitude for axion cosmology, albeit on the upper edge of the allowed range. However the model may provide its own cure since it has a low reheat temperature of around 1 GeV, and the entropy produced by

the inflaton decay partially dilutes the cosmic axion density. Nevertheless, with  $f_a$  just on the upper corner, one expects axion dark matter in this model. In conclusion the  $\phi$ NMSSM has many interesting features and solves several outstanding problems of particle physics and cosmology. Like  $N$ , the inflaton  $\phi$  resides in the visible sector of the theory, and will mix with the two Higgs doublets leading to the exciting possibility of experimentally observable effects [35].

### Acknowledgements

We would like to thank S. Abel, M. Hindmarsh, D. Lyth, A. Riotto and J. Sanderson for useful discussions.

### Note added

Since our manuscript was resubmitted two further points have been brought to our attention. The first point raised by D.Lyth (hep-ph/9710347) is that M.Dine (hep-th/9207045) has showed that the model of Casas and Ross is not viable due to the presence of additional soft operators which are allowed by the discrete symmetry and which violate the PQ symmetry at an unacceptable level. In our model the dangerous Dine operators such as  $NM^*\bar{M}\bar{N}^*$ , and similar higher order operators, are not present due to the absence of the  $\bar{N}$  field, and so our model is exempt from this criticism. The second point emphasised by A.Riotto (private communication) is that in a certain class of no-scale supergravity models [18] (those in which a Heisenberg symmetry is present) the inflaton receives no mass of order the Hubble constant thereby solving the  $\eta$  problem [23]. Since we already invoke no-scale supergravity to explain the masslessness of the inflaton, it is natural to appeal to this mechanism in our model. Radiative corrections to the inflaton mass during inflation would be negligible due to the smallness of the couplings  $k$  and  $\lambda$ . This of course opens up the possibility that the vacuum energy during inflation originates from F-terms after all.

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