hep-ph/9709407

THE MOST PROMISING WAYS TO MEASURE V_{ub}

Patricia Ball

Fermilab, P.O. Box 500, Batavia IL 60510, USA

Abstract:

I review currently discussed methods to determine the CKM mixing matrix element $|V_{ub}|$ from experimental data. Although the theory of inclusive decays and their spectra has entered a model-independent stage, its predictions are still sensitive to various input parameters, in particular the poorly known b quark mass. At present, determinations from exclusive channels, notably $B \to \pi e\nu$, seem slightly more accurate and allow determination of $|V_{ub}|$ from the spectrum in the momentum transfer to the leptons with a theoretical uncertainty of $\sim 10\%$.

To Appear in the Proceedings of the
7th International Symposium on Heavy Flavor Physics
Santa Barbara (CA), USA, 7–11 July 1997

THE MOST PROMISING WAYS TO MEASURE V_{ub}

PATRICIA BALL

Fermilab, P.O. Box 500, Batavia IL 60510, USA

I review currently discussed methods to determine the CKM mixing matrix element $|V_{ub}|$ from experimental data. Although the theory of inclusive decays and their spectra has entered a model-independent stage, its predictions are still sensitive to various input parameters, in particular the poorly known b quark mass. At present, determinations from exclusive channels, notably $B \to \pi e \nu$, seem slightly more accurate and allow determination of $|V_{ub}|$ from the spectrum in the momentum transfer to the leptons with a theoretical uncertainty of $\sim 10\%$.

1 Introduction

The study of $b \to u$ transitions will enter a new stage with the ever increasing data available from CLEO and the new dedicated B factories BABAR and BELLE and will eventually allow us to measure the CKM mixing angle $|V_{ub}|$ accurately. Although most $b \to u$ are purely hadronic, reliable theoretical predictions exist only for semileptonic decays. Due to the progress of recent years in the theoretical description of heavy quark decays, as summarized e.g. in [1,2], it is fair to say that heavy quark physics has now reached the "model-independent" stage. I would like to stress, however, that "model-independent" is not equivalent to "arbitrarily precise": there is always a sensitivity to input parameters like e.g. the b quark mass, whose values will always be afflicted with some theoretical and/or experimental uncertainty. "Model-independence" can rather be viewed as the fact that attributing theoretical errors to predictions is now based on more than simple guessing or averaging over several available models.

In the following I review the various suggested observables from which $|V_{ub}|$ is likely to be measured reliably and where one can hope to reach a minimum of the combined experimental and theoretical error.

2 $|V_{ub}|$ from inclusive semileptonic decays

The main tool in the theoretical description of inclusive b decays is heavy quark expansion. As this topic is also discussed in Refs. [1,2], I will touch on it only shortly. It was realized in the pioneering papers [3] that inclusive heavy hadron decays can be described in terms of an expansion of the relevant hadronic

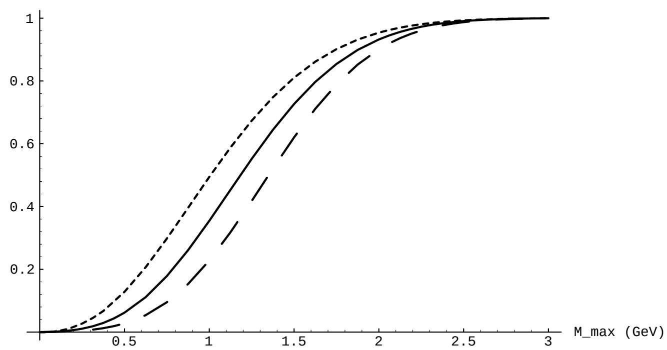

Figure 1: The integrated fraction of the events $\Phi(m_x^{\rm cut})$, Eq. (1), for different values of the b quark mass, $m_b = \{4.72, 4.82, 4.92\}$ GeV. Figure taken from Ref. [4].

matrix element in inverse powers of the heavy quark mass. The first term in this expansion is the free quark decay contribution and the first correction term is suppressed by two powers of the b quark mass (at least in the total rate). Thus the first nonperturbative corrections are small, of the order of 5%, and the decay rate of the B meson nearly equals that of the b quark. Heavy quark expansion works best in regions of phase-space where the final-state hadrons carry large energy and it breaks down if only a few resonances contribute.

Despite the smallness of nonperturbative power-suppressed corrections, nonperturbative quantities also enter from a different source, namely in form of the b quark mass, which is related to the B meson mass by heavy quark expansion. Despite much effort to determine the b quark mass, e.g. [5], it is fair to say that this quantity constitutes today one of the main sources of theoretical uncertainty in inclusive decays.

Due to the overwhelming dominance of $b \to c$ transitions, the total rate $\Gamma(B \to X_u e\nu)$ is not accessible in experiment. One thus aims to measure spectra in one (or more) variables. Two "natural" variables in that game are the charged lepton energy and the hadronic mass in the final state. Experimental data [6] exist so far only for $d\Gamma/dE_e$ with lepton energies E_e above about 2.3 GeV. The cut removes up to about 90% of all events. The theoretical description of the spectrum is very difficult in this region where fixed order heavy quark expansion breaks down. A solution is to resum terms of all orders in $1/m_b$ into a so-called shape function [7,8], which is not known from first principles, but in some rather remote future may be measurable from the photon-energy spectrum in $B \to X_s\gamma$.

Recently, two groups took up an older suggestion [9] and studied the spectrum in the invariant hadronic mass m_X [10,4]. The region of small hadronic multiplicity corresponds here to small m_X, the threshold for $b \to c$ transitions is at $m_X = m_D$. A cut at $m_X = m_D$ removes only a moderate fraction of $b \to u$ transitions. There is an experimental problem to be expected, though, the "charm-leaking" of misidentified charmed particles below the kinematical $b \to c$ threshold. In order to remove them effectively, it may be necessary to cut off the hadron mass spectrum at smaller values of m_X, say $m_X \approx 1.5\,\mathrm{GeV}$. As, on the other hand, in a fixed order heavy quark expansion one also has to cut off the contributions of small m_X, say $m_X \leq 1\,\mathrm{GeV}$, the resulting bin in hadronic mass may be too small for heavy quark expansion to be applicable [10]. Like for the endpoint spectrum in the electron energy, it may be necessary to invoke the shape-function, which is known only via its first few moments. Definitive predictions thus almost necessarily involve a certain degree of model-dependence [10,4].

In [4] the integrated fraction of events was introduced,

$$\Phi(m_x^{\mathrm{cut}}) = \frac{1}{\Gamma(B \to X_u e\nu)} \int_0^{m_X^{\mathrm{cut}}} dm_X \, \frac{d\Gamma}{dm_X}, \tag{1}$$

and studied in its sensitivity to its input parameters, in particular the b quark mass. In Fig. 1 I show $\Phi(m_X^{\mathrm{cut}})$ as function of m_X for various values of m_b. It is clear that the strong sensitivity to m_b around $m_X^{\mathrm{cut}} = 1.5\,\mathrm{GeV}$ is not favourable to extracting $|V_{ub}|$. There are also other theoretical uncertainties not shown in the plot. The authors of [4] quote a theoretical uncertainty of $|V_{ub}|$ of about (10–20)% for the pessimistic scenario $m_X^{\mathrm{cut}} = 1.5\,\mathrm{GeV}$. Both papers [10,4] agree that the main source of uncertainty is the value of m_b or $\bar{\Lambda} = m_B - m_b + O(1/m_b)$, respectively, which needs to be fixed to an accuracy of 100 MeV or better in order to reduce the theoretical error on $|V_{ub}|$ arising from m_b to 10%. The total theoretical error should also account for the model-dependence introduced by the specific choice of the shape-function and for subleading "higher-twist" effects and may be closer to the 20% mark. Heavy quark expansion to fixed order without introducing the shape-function is only sensible if m_X^{cut} can be pushed to larger values. But also in this case there is a strong dependence of the result on $\bar{\Lambda}$ [10].

3 $|V_{ub}|$ from exclusive semileptonic decays

Possible candidates for the extraction of $|V_{ub}|$ from exclusive decays are $B \to \pi e\nu$, $B \to \rho e\nu$ and $B \to \omega e\nu$. CLEO has already measured the corresponding rates [11], but the results are still slightly model-dependent. Theory has to

	$f_+(0)$	$A_1(0)$	$A_2(0)$	$V(0)$
UKQCD [12]	0.27 ± 0.11	$0.27^{+0.05}_{-0.04}$	$0.26^{+0.05}_{-0.03}$	$0.35^{+0.06}_{-0.05}$
LCSR [13]		0.27 ± 0.05	0.28 ± 0.05	0.35 ± 0.07
LCSR [14]	0.28 ± 0.05			

	$\Gamma(B^0 \to \pi^+ l^- \nu_l)$	$\Gamma(B^0 \to \rho^+ l^- \nu_l)$	$\Gamma(\rho)/\Gamma(\pi)$	Γ_L/Γ_T
UKQCD [12]	$8.5^{+3.3}_{-1.4}$	$16.5^{+3.5}_{-2.3}$	$1.9^{+0.9}_{-0.7}$	$0.80^{+0.04}_{-0.03}$
LCSR [13]		13.5 ± 4.0	1.7 ± 0.5	0.52 ± 0.08
LCSR [15]	8.7 ± 2.6			

Table 1: Form factor values at $q^2 = 0$ and decay rates and ratios for $b \to u$ transitions from lattice-constrained parametrizations and from light cone sum rules (LCSR). Decay rates are given in units of $|V_{ub}|^2\,\mathrm{ps}^{-1}$.

provide the form factors that describe the relevant hadronic matrix elements, which can be parameterized in the following form:

$$\begin{aligned} \langle \pi | V_\mu | B \rangle &= f_+(q^2)(p_B + p_\pi)_\mu + \ldots \\ \langle \rho | (V-A)_\mu | B \rangle &= -i(m_B + m_\rho) A_1(q^2) \epsilon^*_\mu + \frac{i A_2(q^2)}{m_B + m_\rho} (\epsilon^* \cdot p_B)(p_B + p_\rho)_\mu \\ &\quad + \frac{2V(q^2)}{m_B + m_\rho} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_B^\alpha p_\rho^\beta + \ldots \end{aligned} \tag{2}$$

The form factors denoted by dots do not contribute to semileptonic decays with massless leptons. The form factors are functions of q^2, the squared momentum transfer to the leptons. Reliable predictions for form factors come from both lattice calculations and light-cone sum rules; there exist also a number of useful parametrizations and constraints.

Let me first shortly review lattice results, which are discussed in detail in [16]. At present, direct calculation of form factors from lattice [17] is possible only for large $q^2 \geq 14\,\mathrm{GeV}^2$. Recently, however, the UKQCD collaboration has designed a simple parametrization for form factors that describe the decay of a B meson into a light meson [12]. This parametrization is inspired by the work of Stech [18] and consistent with heavy quark symmetry and kinematical constraints, but requires an ansatz for the q^2 dependence of one of the form factors. The parameters of this ansatz are determined by fitting to lattice results around $q^2_{\max}$. As a result, $B \to \rho e \nu$ and $B \to K^* \gamma$ decays are described with only two parameters and $B \to \pi e \nu$ decays with a further two. The form factors and decays rates are given in Table 1. The resulting spectra are shown in [16,12]. The uncertainties for f_+ and the $B \to \pi$ spectra are still rather large, whereas the uncertainties in the spectrum of $B \to \rho e\nu$ are less than 20% for large $q^2 \geq 15\,\mathrm{GeV}^2$ and thus could allow a measurement of $|V_{ub}|$ with a theoretical error of about 10%.

Progress in describing the shape of form factors has recently been made in the form of model-independent parameterizations [19,20] based on QCD dispersion relations and analyticity. These dispersion relations lead to an infinite tower of upper and lower bounds that can be derived by using the normalizations of the form factor $f_+(q_i^2)$ at a fixed number of kinematic points q_i^2 as input [21,22]. In Ref. [19,20] the most general parametrization of a form factor consistent with the constraints from QCD was derived. For a generic form factor $F(q^2)$ describing the exclusive semileptonic decay of a B meson to a final state meson H as a function of q^2, the parameterization takes the form

$$F(q^2) = \frac{1}{P(q^2)\phi(q^2)} \sum_{k=0}^{\infty} a_k \; z(q^2; q_0^2)^k \quad , \tag{3}$$

where $\phi(q^2)$ is a computable function arising from perturbative QCD. The function $P(q^2)$ depends only on the masses of mesons below the BH pair-production threshold that contribute to BH pair-production as virtual intermediate states. The variable $z(q^2; q_0^2)$ is a kinematic function of q^2 defined by

$$\frac{1 + z(q^2; q_0^2)}{1 - z(q^2; q_0^2)} = \sqrt{\frac{q_+^2 - q^2}{q_+^2 - q_0^2}} \quad , \tag{4}$$

where $q_+^2 = (M_B + M_H)^2$ is the pair-production threshold and q_0^2 is a free parameter that is often taken to be $q_-^2 = (M_B - M_H)^2$, the maximum momentum-transfer squared allowed in the semileptonic decay $B \to Hl\nu$. The coefficients a_k are unknown constants constrained to obey

$$\sum_{k=0}^{\infty} (a_k)^2 \leq 1 \quad . \tag{5}$$

The kinematic function $z(q^2; q_0^2)$ takes its minimal physical value z_{min} at $q^2 = q_-^2$, vanishes at $q^2 = q_0^2$, and reaches its maximum z_{max} at $q^2 = 0$. Thus the sum $\sum a_k \, z^k$ is a series expansion about the kinematic point $q^2 = q_0^2$. The value z_{max} can be made even smaller by choosing an optimized value $0 \leq q_0^2 \leq q_-^2$. In that case, most form factors describing $B \to Dl\nu$ and $B \to D^*l\nu$ can be

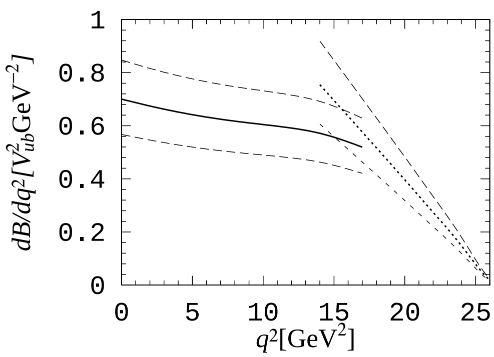

Figure 2: The spectrum $dB(B \to \pi e\nu)/dq^2$ as a function of q^2. Solid line: result from light-cone sum rules. Dotted line: B^* contribution in pole dominance approximation, which is expected to dominate for large q^2. Dashed lines: theoretical uncertainties. Figure taken from [14].

parameterized with only one unknown constant to an accuracy of a few percent (assuming the normalization at zero recoil given by heavy quark symmetry). Thus the continuous function $F(q^2)$ has been reduced to a single constant, for example the value of the form factor $F(q^2=0)$ at maximum recoil. For $\bar{B} \to \pi l \overline{\nu}$, the maximum value of z is $z_{max} = 0.52$, but even in this case Eqs. (3) and (5) severely constrain the relevant form factor [22]. The main source of uncertainty in this approach is the normalization of the form factor which has to be taken from an external source, such that the uncertainty of the predicted form factor is at least as large as the error on the input normalization. In Ref. [20] 30% are quoted.

The last method I would like to discuss in these proceedings are the so-called light-cone sum rules [23], which were applied to $B \to \pi$ and $B \to \rho$ transitions in [15,14,13,24]. The starting point in this approach is the observation that at large recoil the light quark originating from the weak decay carries large energy of order $m_b/2$ and has to transfer it to the soft cloud to recombine to the final state hadron. The probability of such a recombination depends on the parton content of both the B meson and the light meson, the valence configuration with the minimum number of Fock constituents being dominant. The valence quark configuration is characterized by the wave function $\phi(x, k_\perp)$ depending on the momentum fraction x carried by the quark and on its transverse momentum $k_\perp$. There exist two different mechanisms for the valence quark contribution to the transition form factor. The first one is the hard rescattering mechanism which requires that the recoiling and spectator

FF	$F(0)$	a_F	b_F
$f_+^{B\to\pi}$	0.30 ± 0.03	-1.32	0.21
$f_0^{B\to\pi}$	0.30 ± 0.05	-0.84	0.03
$A_1^{B\to\rho}$	0.27 ± 0.05	-0.42	-0.29
$A_2^{B\to\rho}$	0.28 ± 0.05	-1.34	0.38
$V^{B\to\rho}$	0.35 ± 0.07	-1.51	0.47

Table 2: Form factors from light-cone sum rules with functional q^2 dependence fitted to Eq. (7).

quarks are at small transverse separations. In this case the large momentum is transferred by exchange of a hard gluon with virtuality $k^2 \sim O(m_b)$. This contribution is perturbatively calculable in terms of the Bethe-Salpeter wave functions at small ($\sim 1/m_b$) transverse separations, or *distribution amplitudes* (DA):

$$\phi(x) = \int^{k_\perp^2 \sim m_b} dk_\perp^2 \phi(x, k_\perp). \tag{6}$$

The second mechanism is the soft contribution. The idea is that hard gluon exchange is not necessary, provided one picks up an "end-point" configuration with almost all momentum $1 - x \sim O(1/m_b)$ carried by one constituent. The transverse quark-antiquark separation is not constrained in this case, which implies that the soft contribution is sensitive to long-distance dynamics. To calculate the soft contribution one needs to know the wave function as a function of the transverse separation; the simpler distribution amplitude is not enough. QCD sum rules offer a nonperturbative technique to estimate the necessary convolution integral without explicit knowledge of the wave functions.

The essential nonperturbative input in this method is encoded in hadronic distribution amplitudes ordered by increasing twist. The lowest order twist 2 distributions can be experimentally accessed, e.g. in the $\pi\gamma\gamma^*$ form factor at large momentum transfer. Lacking this measurement, the most important nonperturbative terms in the DAs have been estimated from QCD sum rules [25]. Light-cone sum rules are expected to be valid for not too large q^2, $m_b^2 - q^2 \sim O(m_b)$, e.g. $q^2 \leq 17\,\mathrm{GeV}^2$, and in that respect are largely complementary to presently feasible lattice simulations. At present, the calculations for $f_+(q^2)$ include twist 3 and 4 distributions and lowest order radiative corrections to the leading twist contribution [14]. Both the twist expansion and the radiative corrections are well under control. The spectrum $d\Gamma(B \to \pi e\nu)/dq^2$ is shown

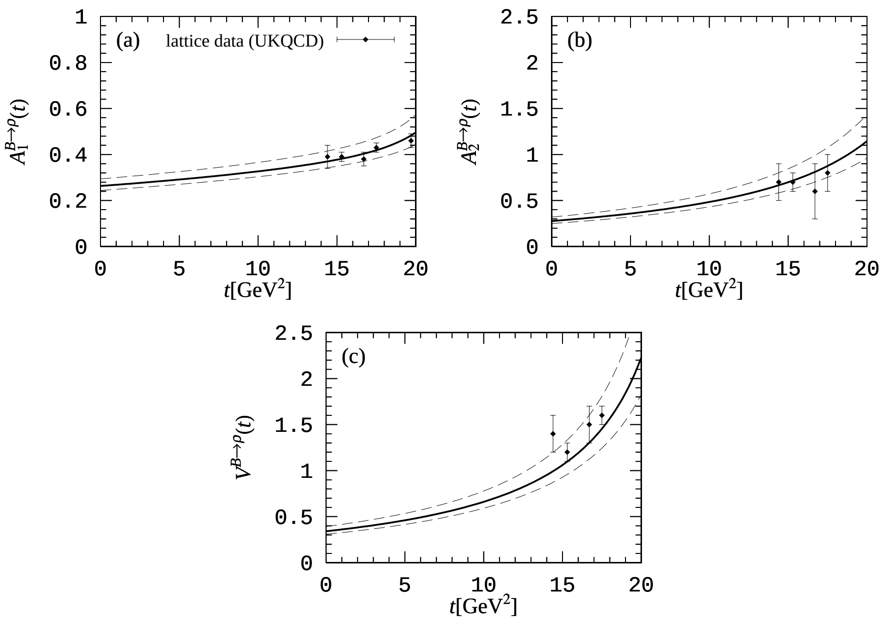

Figure 3: Form factors of the $B \to \rho$ transition from light-cone sum rules [13] (solid lines). Dashed lines: estimate of theoretical errors. For comparison I also plot the results from lattice simulations [17]. Figure taken from [13].

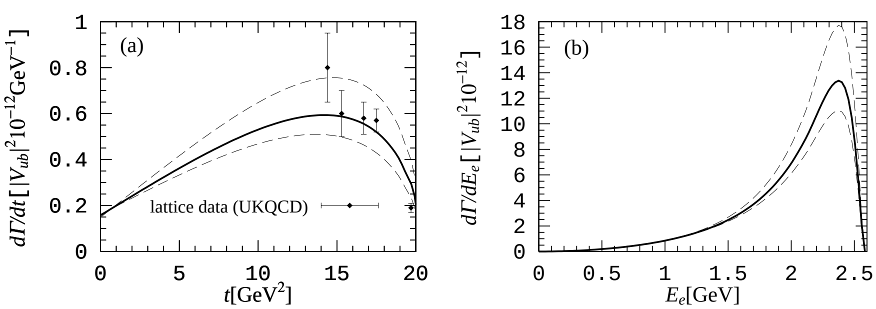

Figure 4: Spectra of the $B \to \rho$ decay in the momentum transfer (a) and the electron energy (b). Same notations as in previous figure.

in Fig. 2. The method of light-cone sum rules cannot yield arbitrarily accurate results, but always involves a certain systematic error, which cannot be reduced to below $\sim 10\,\%$ in the form factor. In this regard the results of [14] are not expected to be much improved by future calculations.

The situation is different for the $B \to \rho$ transition, where so far only the tree-level twist 2 contributions have been taken into account [13]. Results for the form factor are shown in Fig. 3, for the spectra in Fig. 4. Fig. 3 strikes by the excellent agreement between LCSR and lattice results, cf. Table 1. However, it may be premature to conclude that the question of $B \to \rho$ form factors is settled, as the LCSR results still need to be improved by including radiative and higher twist corrections. Results for form factors and branching ratios for both $B \to \pi$ and $B \to \rho$ transitions are given in Table 1. In Table 2 I also give a simple parametrization of LCSR form factors of the form

$$F(q^2) = \frac{F(0)}{1 + a_F\,\frac{q^2}{m_B^2} + b_F\,\frac{q^4}{m_B^4}}. \tag{7}$$

4 Conclusions

In my opinion $|V_{ub}|$ will finally be determined from the spectrum $d\Gamma(B \to \pi e\nu)/dq^2$. As I have discussed in Sec. 2, the connection between theory and experiment in inclusive decays may be difficult to establish, in particular if the experimental cut-off in the hadronic invariant mass has to be pessimistically low. In any case one has to await runing and data-taking at BABAR or BELLE. Exclusive decays, on the other hand, are already measured at CLEO with increasing statistics. From the experimental point of view, the $B \to \rho e\nu$ decay is slightly disfavoured, as distinguishing nonresonant $\pi\pi$ states from the broad ρ resonance poses an additional experimental challenge that is absent in $B \to \pi e\nu$. On the other hand, the branching ratio of the latter one is smaller by roughly a factor of two. Theory provides a number of largely different and complementary theoretical tools which become continually finer shaped. Also for theory $B \to \rho$ is more challenging, as three form factors need to be predicted, and to date the predicted q^2 dependence can not yet be checked for internal consistency from unitarity constraints as it is the case with $B \to \pi$.[a]In principle it is of course possible to constrain the q^2 dependence experimentally by measuring the polarization of the ρ, but in view of the lack of conclusive results for the Cabibbo-favoured decays $D \to K^* e\nu$ and even $B \to D^* e\nu$, this possibility appears remote. As none of the discussed methods can predict the form factor in the complete physical range in q^2, a determination of $|V_{ub}|$ from

[a] The parametrizations suggested in [12] are not as strict as the ones from unitarity.

the broad spectrum of $B \to \pi e \nu$ in q^2 seems most promising and also allows naturally the inclusion of experimental cuts.

References

1. T. Mannel, *these proceedings.*
2. A. Vainshtein, *these proceedings.*
3. I. Bigi et al., Phys. Lett. B **293** (1992) 430; Err. ibid. B **297** (1993) 477; Phys. Rev. Lett. **71** (1993) 496.
4. R.D. Dikeman and N.G. Uraltsev, hep–ph/9703437;
   I. Bigi, R.D. Dikeman and N.G. Uraltsev, hep–ph/9706520.
5. P. Ball and V.M. Braun, Phys. Rev. D **49** (1994) 2472;
   M. Jamin and A. Pich, hep–ph/9702276.
6. CLEO coll., Phys. Rev. Lett. **71** (1993) 4111.
7. I. Bigi et al., Int. J. Mod. Phys. A **9** (1994) 2467;
   R.D. Dikeman, M. Shifman and N.G. Uraltsev, Int. J. Mod. Phys. A **11** (1996) 571.
8. M. Neubert, Phys. Rev. D **49** (1994) 3392; Phys. Rev. D **49** (1994) 4623.
9. V. Barger, C.S. Kim and G.J.N. Phillips, Phys. Lett. B **251** (1990) 629.
10. A.F. Falk, Z. Ligeti and M.B. Wise, Phys. Lett. B **406** (1997) 225.
11. L. Gibbons, *these proceedings*
12. UKQCD coll., hep–lat/9708008.
13. P. Ball and V.M. Braun, Phys. Rev. D **55** (1997) 5561.
14. A. Khodjamirian *et al.*, hep-ph/9706303;
    E. Bagan *et al.*, hep-ph/9709243.
15. V.M. Belyaev, A. Khodjamirian and R. Rückl, Z. Phys. C **60** (1993) 349;
    A. Khodjamirian and R. Rückl, Talk at ICHEP 96, Warsaw, Poland, July 1996; published in ICHEP 96, 902 (hep–ph/9610367).
16. J. Flynn, *these proceedings.*
17. UKQCD coll., Nucl. Phys. **B461** (1996) 327; Nucl. Phys. **B476** (1996) 313.
18. B. Stech, Phys. Lett. B **354** (1995) 447.
19. G. Boyd, B. Grinstein and R. Lebed, Phys. Lett. B **353** (1995) 306; Nucl. Phys. **B461** (1996) 493.
20. C. Boyd and M. Savage, Phys. Rev. D **56** (1997) 303.
21. C. Bourrely, B. Machet and E. de Rafael, Nucl. Phys. **B189** (1981) 157.
22. C. Boyd, B. Grinstein and R. Lebed, Phys. Rev. Lett. **74** (1995) 4603;
    L. Lellouch, Nucl. Phys. **B479** (1996) 353.
23. P. Ball, V.M. Braun and H.G. Dosch, Phys. Rev. D **44** (1991) 3567;
    A. Ali, V.M. Braun and H. Simma, Z. Phys. C **63** (1994) 437.

24. P. Ball, Phys. Rev. D **48** (1993) 3190 and references therein.
25. V.M. Braun and I.E. Filyanov, Z. Phys. C **44** (1989) 157;
P. Ball and V.M. Braun, Phys. Rev. D **54** (1996) 2182.