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M-Theory Model-Building and Proton Stability

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Abstract

We study the problem of baryon stability in M theory, starting from realistic four-dimensional string models constructed using the free-fermion formulation of the weakly-coupled heterotic string. Suitable variants of these models manifest an enhanced custodial gauge symmetry that forbids to all orders the appearance of dangerous dimension-five baryon-decay operators. We exhibit the underlying geometric (bosonic) interpretation of these models, which have a $Z_2 \times Z_2$ orbifold structure similar, but not identical, to the class of Calabi-Yau threefold compactifications of M and F theory investigated by Voisin and Borcea. A related generalization of their work may provide a solution to the problem of proton stability in M theory.

There has recently been important progress towards a better understanding of the underlying non–perturbative formulation of superstring theory. The picture which emerges is that all the superstring theories in ten dimensions, as well as 11-dimensional supergravity, which were previously thought to be distinct, are in fact different limits of a single fundamental theory, often referred to as M (or F) theory [1]. To elevate this new mathematical understanding into contact with experimentally-oriented physics is a rewarding challenge. Different directions may be followed in pursuing this endeavour. On the one hand, one may look *ab initio* for generic phenomenological properties which may characterize the fundamental M (or F) theory. Or, on the other hand, one may adapt the technologies that have been developed for the analysis of realistic classes of heterotic string solutions in four dimensions [2], and explore the extent to which they may apply in the context of M and F theory.

Following the first line of thought, one of the issues in superstring phenomenology that received a great deal of attention is the problem of superstring gauge coupling unification. As is well known, the discrepancy between the Grand Unification scale of around 2×10^{16} GeV estimated by extrapolating naively from the measurements at LEP and elsewhere [3] and the estimate of around 4×10^{17} GeV found in weakly-coupled heterotic string theory [4] may be removed if the Theory of Everything is M theory in a strong-coupling limit, corresponding to an eleventh dimension that is considerably larger than the naive Planck length [5]. This scenario would explain naturally why the value of $\sin^2\theta_W$ measured at accelerators is in good agreement with minimal supersymmetric GUT predictions, a feature not shared by generic string models with extra particles at intermediate scales, large Planck-scale threshold corrections, or different Kac-Moody levels for different gauge group factors [6].

This economic strong-coupling solution to the reconciliation of the minimal supersymmetric GUT and string unification scales may be desirable for resolving other phenomenological issues, such as stabilizing the dilaton vacuum expectation value and selecting the appropriate vacuum point in moduli space [7]. However, as is only too often the case, closing the door for one genie may open a door for another. In this case, we fear that the problems associated with proton stability will resurface and in fact worsen.

This has been a prospective problem for a quantum theory of gravity ever since the no-hair theorems were discovered and it was realized that non-perturbative vacuum fluctuations could engender baryon decay [8], in the absence of any custodial exact (gauge) symmetry. This problem became particularly acute with the advent of supersymmetric GUTs, when it was realized that effective dimension-five operators of the form

$$QQQL \tag{1}$$

could induce rapid baryon decay. Operators of this form could be generated either by the exchange of GUT particles [9] or by quantum gravity effects [10]. Specifically, in the context of string theory, such an operator could in general be induced by the exchange of heavy string modes. In this case, the coefficient of the operator

(1) would be suppressed by one inverse power of the effective string scale M. In the perturbative heterotic string solutions studied heretofore, this scale is of the order $M \sim 10^{18}$ GeV, whilst in the proposed non–perturbative M-theory solution to the string-scale gauge-coupling unification problem this scale would be of the order $M \sim 10^{16}$ GeV. Thus, the magnitude of the effective dimension-five operator (1) may increase by $\sim$ two orders of magnitude. As proton stability considerations severely restrict the magnitude of such operators, and as the general expectation is that this kind of operator is abundant in a generic superstring vacuum, it would seem that the M-theory resolution of the problem of string-scale gauge-coupling unification, we have re–introduced a far more serious problem, namely that of baryon decay.

A full M- (or F-)theory solution to this problem lies beyond technical reach at this time. However, we believe that useful insight into this problem may be obtained by examining perturbative heterotic string models in four dimensions that possess some realistic properties, identifying symmetries that guarantee the absence of dangerous dimension-five operators to all orders in string perturbation theory, and then investigating the possibility of elevating such models to a full non-perturbative M- (or F-)theory formulation.

For this purpose, we choose to investigate the three-generation superstring models [11, 12, 13, 14, 15, 16] derived in the free-fermion formulation [17]. This construction produces a large number of three-generation models with different phenomenological characteristics, some of which are especially appealing. This class of models corresponds to $Z_2 \times Z_2$ orbifold compactification at the maximally-symmetric point in the Narain moduli space [18]. The emergence of three generations is correlated with the underlying $Z_2 \times Z_2$ orbifold structure. Detailed studies of specific models have revealed that these models may explain the qualitative structure of the fermion mass spectrum [19] and could form the basis of a realistic superstring model. We refer the interested reader to several review articles which summarize the phenomenological studies of this class of models [2].

For our purposes here, let us recall the main structures underlying this class of models. In the free-fermion formulation [17], a model is defined by a set of boundary condition basis vectors, together with the related one–loop GSO projection coefficients, that are constrained by the string consistency constraints. The basis vectors, b_k, span a finite additive group, $\Xi = \sum_k n_k b_k$ where $n_k = 0, \cdots, N_{z_k} - 1$. The physical states in the Hilbert space of a given sector $\alpha \in \Xi$, are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections. The $U(1)$ charges $Q(f)$ corresponding to the unbroken Cartan generators of the four-dimensional gauge group, which are in one-to-one correspondence with the $U(1)$ currents $f^* f$ for each complex fermion f, are given by:

$$Q(f) = \frac{1}{2}\alpha(f) + F(f) \tag{2}$$

where $\alpha(f)$ is the boundary condition of the world–sheet fermion f in the sector α, and $F_\alpha(f)$ is a fermion-number operator that takes the value $+1$ for each mode of f, and the value -1 for each mode of f^*, if f is complex. For periodic fermions, which have $\alpha(f) = 1$, the vacuum must be a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion f, there are two degenerate vacua $|+\rangle, |-\rangle$, annihilated by the zero modes f_0 and ${f_0}^*$, respectively, and with fermion numbers $F(f) = \pm 1$.

Realistic models in this free-fermionic formulation are generated by a suitable choice of boundary-condition basis vectors for all world–sheet fermions, which may be constructed in two stages. The first stage consists of the NAHE set [11, 20] of five boundary condition basis vectors, $\{\mathbf{1}, S, b_1, b_2, b_3\}$. After generalized GSO projections over the NAHE set, the residual gauge group is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ space–time supersymmetry *. The space–time vector bosons that generate the gauge group arise from the Neveu–Schwarz sector and from the sector $1 + b_1 + b_2 + b_3$. The Neveu–Schwarz sector produces the generators of $SO(10) \times SO(6)^3 \times SO(16)$. The sector $1 + b_1 + b_2 + b_3$ produces the spinorial 128 of $SO(16)$ and completes the hidden-sector gauge group to E_8. The vectors b_1, b_2 and b_3 correspond to the three twisted sectors in the corresponding orbifold formulation, and produce 48 spinorial 16-dimensional representations of $SO(10)$, sixteen each from the sectors b_1, b_2 and b_3.

The second stage of the basis construction consists of adding three more basis vectors to the NAHE set, corresponding to Wilson lines in the orbifold formulation, whose general forms are constrained by string consistency conditions such as modular invariance, as well as by space-time supersymmetry. These three additional vectors are needed to reduce the number of generations to three, one from each of the sectors b_1, b_2 and b_3. The details of the additional basis vectors distinguish between different models and determine their phenomenological properties.

The residual three generations constitute representations of the final observable gauge group, which can be $SU(5) \times U(1)$ [11], $SO(6) \times SO(4)$ [13] or $SU(3) \times SU(2) \times U(1)^2$ [12, 14, 16]. In the former two cases, an additional pair of 16 and $\overline{16}$ representations of $SO(10)$ is obtained from the two basis vectors that extend the NAHE set. The electroweak Higgs multiplets are obtained from the Neveu–Schwarz sector, and from a sector that is a combination of the two basis vectors which extend the NAHE set. This combination has the property that $X_R \cdot X_R = 4$, and produces states that transform solely under the observable-sector symmetries. Massless states from this sector are then obtained by acting on the vacuum with fermionic oscillators with frequency 1/2. Details of the flavor symmetries differ between models, but consist of at least three $U(1)$ symmetries coming from the observable-sector E_8. Additional flavor $U(1)$ factors arise from the complexification of real world–sheet fermions, corresponding to the internal manifold in a bosonic formulation. The models typically contain a hidden sector in which the final gauge group is a subgroup of the hidden E_8, and three matter representations in the vectorial 16 of $SO(16)$, which arise from

*The vector S in this NAHE set is the supersymmetry generator, and the superpartners of the states from a given sector α are obtained from the sector $S + \alpha$.

the breaking of the hidden E_8 to $SO(16)$ †.

The cubic and higher-order terms in the superpotential are obtained by evaluating the correlators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N \rangle, \tag{3}$$

where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators, using the rules given in [21]. Generically, correlators of the form (3) are of order $\mathcal{O}(g^{N-2})$, and hence of progressively higher orders in the weak-coupling limit. One of the $U(1)$ factors in the free-fermion models is anomalous, and generates a Fayet–Ilioupolos term which breaks supersymmetry at the Planck scale. The anomalous $U(1)$ is broken, and supersymmetry is restored, by a non–trivial VEV for some scalar field that is charged under the anomalous $U(1)$. Since this field is in general also charged with respect to the other anomaly-free $U(1)$ factors, some non-trivial set of other fields must also get non–vanishing VEVs $\mathcal{V}$, in order to ensure that the vacuum is supersymmetric. Some of these fields will appear in the nonrenormalizable terms (3), leading to effective operators of lower dimension. Their coefficients contain factors of order $\mathcal{V}/M$, which may not be very small, particularly in the context of the M-theory resolution of the unification-scale problem.

This technology has previously been used to study the issue of proton decay in the context of $SU(5)\times U(1)$ and $SU(3)\times SU(2)\times U(1)^2$ type models [22]. Here we study this issue using two specific free-fermion models [14, 15] as case studies. In both models, the color-triplet Higgs multiplets from the Neveu–Schwarz sectors are projected out by a superstring doublet–triplet splitting mechanism, so that conventional GUT-scale dimension-five operators are absent. Whilst the model of [14] contains one pair of color-triplet Higgs fields from the sector $b_1+b_2+\alpha+\beta$, in the model of [15] the Higgs color triplets from this sector are projected out by the generalized GSO projections. The two models also contain exotic color triplets from sectors that arise from the $SO(10)$ Wilson-line breaking, and carry lepton numbers $\pm 1/2$.

Examining the superpotential terms in the first model [14], we find the following non-renormalizable terms at order $N=6$

$$Q_3Q_2Q_2L_3\Phi_{45}\bar{\Phi}_2^- \quad \text{and} \quad Q_3Q_1Q_1L_3\Phi_{45}\Phi_1^+ \tag{4}$$

Thus, dangerous dimension-five operators arise in this model if either of the sets of fields $\{\Phi_{45}\ ;\ \bar{\Phi}_2^-\}$ or $\{\Phi_{45}\ ;\ \Phi_1^+\}$ gets a VEV in the cancellation of the anomalous $U(1)$ D–term equations. Even if we can choose flat directions such that these order $N=6$ terms are suppressed, higher-order terms can still generate the dangerous operators. If the suppression factor $\langle\phi\rangle/M \sim 1/10$, the dimension-five terms are suppressed by $\sim 10^{N-2}$ in each successive order. The order $N=6$ terms would certainly lead to proton decay at a rate that contradicts experiment, and the same would be true of many higher-order operators in the proposed M-theory context.

†In general, the models also contain exotic massless states that arise from the breaking of the non–Abelian $SO(10)$ symmetry at the string level.

Next we turn to the model of [15], which introduces a new feature. In this model, the observable-sector gauge group formed by the gauge bosons from the Neveu–Schwarz sector alone is

$$SU(3) \times SU(2) \times U(1)_C \times U(1)_L \times U(1)_{1,2,3,4,5,6} \ . \tag{5}$$

However, in this model there are two additional gauge bosons from the vector combination $\mathbf{1}+\alpha+2\gamma$ [15], where the vector 2γ has periodic boundary conditions for the internal fermions $\{\bar\psi^{1,\cdots,5},\bar\eta^1,\bar\eta^2,\bar\eta^3,\bar\phi^{1,\cdots,4}\}$ and anti–periodic boundary conditions for all the remaining world–sheet fermions:

$$2\gamma = (0,\cdots,0| \underbrace{1,\cdots,1}_{\bar\psi^{1,\cdots,5},\bar\eta^{1,2,3},\bar\phi^{1,\cdots,4}} ,0,\cdots,0) \ . \tag{6}$$

This model also has two new gauge generators, whose gauge bosons are singlets of the non–Abelian group, but carry $U(1)$ charges. Referring to these two generators as $T^{\pm}$, we can define the linear combination

$$T^3 \equiv \frac{1}{4}\left[U(1)_C + U(1)_L + U(1)_4 + U(1)_5 + U(1)_6 + U(1)_7 - U(1)_9\right] \tag{7}$$

such that the three generators $\{T^{\pm},T_3\}$ together form the enhanced symmetry group $SU(2)$. Thus, the original observable symmetry group (5) has been enhanced to

$$SU(3)_C \times SU(2)_L \times SU(2)_{cust} \times U(1)_{C'} \times U(1)_L \times U(1)_{1,2,3} \times U(1)_{4',5',7''} \tag{8}$$

and the remaining $U(1)$ combinations which are orthogonal to T^3 are given by

$$\begin{aligned}
U(1)_{C'} &\equiv \frac{1}{3}U(1)_C - \frac{1}{2}U(1)_7 + \frac{1}{2}U(1)_9 \\
U(1)_{4'} &\equiv U(1)_4 - U(1)_5 \\
U(1)_{5'} &\equiv U(1)_4 + U(1)_5 - 2\,U(1)_6 \\
U(1)_{7''} &\equiv U(1)_C - \frac{5}{3}\Big[U(1)_4 + U(1)_5 + U(1)_6\Big] + U(1)_7 - U(1)_9 \ .
\end{aligned} \tag{9}$$

The weak hypercharge can still be defined as the linear combination $1/3U(1)_C + 1/2U(1)_L$. However, as the $U(1)_C$ symmetry is now part of the extended $SU(2)$ gauge group, $U(1)_C$ is given as a linear combination of the generators above

$$\frac{1}{3}U(1)_C \;=\; \frac{2}{5}\Big\{U(1)_{C'} + \frac{5}{16}\left[T^3 + \frac{3}{5}U_{7''}\right]\Big\} \ . \tag{10}$$

Since the weak hypercharge is not orthogonal to the enhanced $SU(2)$ symmetry, it is convenient to define a new linear combination of the $U(1)$ factors:

$$\begin{aligned}
U(1)_{Y'} &\equiv U(1)_Y - \frac{1}{8}T^3 \\
&= \frac{1}{2}U(1)_L + \frac{5}{24}U(1)_C \\
&\qquad - \frac{1}{8}\left[U(1)_4 + U(1)_5 + U(1)_6 + U(1)_7 - U(1)_9\right] ,
\end{aligned} \tag{11}$$

so that the weak hypercharge is expressed in terms of $U(1)_{Y'}$ as

$$U(1)_Y = U(1)_{Y'} + \frac{1}{2} T^3 \quad \Longrightarrow \quad Q_{\rm e.m.} = T^3_L + Y = T^3_L + Y' + \frac{1}{2} T^3_{cust} \, . \tag{12}$$

The final observable-sector gauge group is therefore

$$SU(3)_C \times SU(2)_L \times SU(2)_{cust} \times U(1)_{Y'} \ \times \ \Big\{ \text{ seven other } U(1) \text{ factors } \Big\} \, . \tag{13}$$

The remaining seven $U(1)$ factors must be chosen as linear combinations of the previous $U(1)$ factors so as to be orthogonal to the each of the other factors in (13).

Up to $U(1)$ charges, the massless spectrum from the Neveu–Schwarz sector and the sector $b_1 + b_2 + \alpha + \beta$ is the same as in the previous model. However, because of the enhanced symmetry, the spectrum from the sectors b_j is modified. The sectors $b_j \oplus \mathbf{1} + \alpha + 2\gamma$. produce the three light generations, one from each of the sectors b_j $(j = 1, 2, 3)$, as before. However, the gauge enhancement noted above has the important corollary that only the leptons, $\{L, e^c_L, N^c_L\}$, transform as doublets of the enhanced $SU(2)_{cust}$ gauge group, whilst the quarks, $\{Q, u^c_L, d^c_L\}$, are $SU(2)_{cust}$ singlets. Therefore, terms of the form $QQQL$ are not invariant under the enhanced $SU(2)_{cust}$ gauge group. Furthermore, such terms are forbidden to all orders of non-renormalizable terms, as can be verified by an explicit computerized search. Even if we break the custodial $SU(2)_{cust}$ by, *e.g.*, the VEV of the right–handed neutrino and its complex conjugate, the higher-order terms will not be invariant under the combined symmetries $SU(2)_{cust}$ and $U(1)_L$. A computerized search for all possible operators that might lead to proton decay confirms that such terms do not arise at any order in this model.

This model therefore provides an example how the proton decay problem may be resolved in a robust way: even if the string unification scale is lowered to the minimal supersymmetric GUT scale, as proposed in the M–theory strong-coupling solution to the string gauge-coupling unification problem, such a model can evade the proton decay constraints to all orders in perturbation theory. However, we recognize that this approach does not encompass strictly non-perturbative string effects which may appear in a direct M- or F-theory construction. On the other hand, we also note that enhanced gauge symmetries appear in many M- and F-theory constructions [23], and may play a role analogous to that played by the enhancement in the above model.

Such enhanced symmetries arise frequently in the free-fermion models, whenever there is a combination of the basis vectors which extends the NAHE set:

$$X = n_\alpha \alpha + n_\beta \beta + n_\gamma \gamma \tag{14}$$

for which $X_L \cdot X_L = 0$ and $X_R \cdot X_R \neq 0$. Such a combination may produce additional space–time vector bosons, depending on the choice of generalized GSO phases. For example, in the flipped $SU(5)$ model of [24], in addition to the gauge bosons from the

Neveu–Schwarz sector and the sector $I = \mathbf{1}+b_1+b_2+b_3$, additional space–time vector bosons are obtained from the sectors $b_1 + b_4 \pm \alpha \oplus I$. In this case, the hidden-sector $SU(4)$ gauge group, arising from the gauge bosons of the NS$\oplus I$ sectors, is enhanced to $SU(5)$. This particular enhancement does not modify the observable gauge sector, and does nothing to forbid dangerous higher-order operators. However, other, more interesting, cases may exist.

The type of enhancement depends not only on the boundary-condition basis vectors, but also on the discrete choices of GSO phases. For example, in the model of [14], the combination of basis vectors $X = b_1 + b_2 + b_3 + \alpha + \beta + \gamma + (I)$ has $X_L \cdot X_L = 0$, and thus may give rise to additional space–time vector bosons. All the extra space–time vector bosons are projected out by the choice of generalized GSO projection coefficients. However, with the modified GSO phases

$$c\begin{pmatrix}1\\ \gamma\end{pmatrix} \to -c\begin{pmatrix}1\\ \gamma\end{pmatrix}, c\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \to -c\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \text{ and } c\begin{pmatrix}\gamma\\ \beta\end{pmatrix} \to -c\begin{pmatrix}\gamma\\ \beta\end{pmatrix}, \tag{15}$$

additional space–time vector bosons are obtained from the sector $b_1 + b_2 + b_3 + \alpha + \beta + \gamma + (I)$. In addition, the sector $b_1 + b_2 + b_3 + \alpha + \beta + \gamma + (I)$ produces the representations $3_1 + 3_{-1}$ of $SU(3)_H$, where one of the $U(1)$ combinations is the $U(1)$ in the decomposition of $SU(4)$ under $SU(3) \times U(1)$. In this case, the hidden-sector $SU(3)_H$ gauge group is extended to $SU(4)_H$. If instead we take the modified phases

$$c\begin{pmatrix}1\\ \gamma\end{pmatrix} \to -c\begin{pmatrix}1\\ \gamma\end{pmatrix}, c\begin{pmatrix}\gamma\\ \alpha\end{pmatrix} \to -c\begin{pmatrix}\gamma\\ \alpha\end{pmatrix}, \tag{16}$$

then the sector $\mathbf{1} + \alpha + \beta + \gamma$ produces two additional space–time vector bosons which enhances one of the $U(1)$ factors to $SU(2)$. These examples further illustrate the point that this type of enhancement is common in realistic free-fermion models, which makes it interesting to explore further in connection with the proton stability problem.

We now explore the possibility of a connection between the type of models discussed above and the vacua of M (and F) theory. At present, a direct connection between known features of the non-perturbative formulation of M theory and the above realistic free-fermion models, with a full basis consisting of eight vectors, is not yet possible. However, it is nevertheless possible to make observations suggesting a possible connection of these models to the type of M– (and F–)theory compactifications which have been discussed in the literature [25].

We start by studying in more detail the geometric interpretation of the five-basis-vector NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ that underlies the realistic free fermionic models. This set corresponds to a $Z_2 \times Z_2$ orbifold compactification of the weakly-coupled ten-dimensional heterotic string, and the basis vectors b_1, b_2 and b_3 correspond to the three twisted sectors of these orbifold models. To see this correspondence, we add to the NAHE set the basis vector

$$X = (0, \cdots, 0| \underbrace{1, \cdots, 1}_{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}}, 0, \cdots, 0) \ . \tag{17}$$

with the following choice of generalized GSO projection coefficients:

$$C\binom{X}{\mathbf{b}_j} \;=\; -C\binom{X}{S} \;=\; C\binom{X}{\mathbf{1}} \;=\; +1\,. \tag{18}$$

This set of basis vectors produces models with an $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ gauge group and $N = 1$ space–time supersymmetry. The matter fields include 24 generations in 27 representations of E_6, eight from each of the sectors $b_1 \oplus b_1 + X$, $b_2 \oplus b_2 + X$ and $b_3 \oplus b_3 + X$. Three additional 27 and $\overline{27}$ pairs are obtained from the Neveu–Schwarz $\oplus$ X sector.

The subset of basis vectors

$$\{\mathbf{1}, S, X, I = \mathbf{1} + b_1 + b_2 + b_3\} \tag{19}$$

generates a toroidally-compactified model with $N = 4$ space–time supersymmetry and $SO(12) \times E_8 \times E_8$ gauge group. The same model is obtained in the geometric (bosonic) language by constructing the background fields which produce the $SO(12)$ Narain lattice [18, 26], taking the metric of the six-dimensional compactified manifold to be the Cartan matrix of $SO(12)$:

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \tag{20}$$

and the antisymmetric tensor

$$b_{ij} = \begin{cases} g_{ij} & ; i > j, \\ 0 & ; i = j, \\ -g_{ij} & ; i < j. \end{cases} \tag{21}$$

When all the radii of the six-dimensional compactified manifold are fixed at $R_I = \sqrt{2}$, it is easily seen that the left– and right–moving momenta

$$P^I_{R,L} = [m_i - \frac{1}{2}(B_{ij} {\pm} G_{ij})n_j]e_i^{I*} \tag{22}$$

reproduce all the massless root vectors in the lattice of $SO(12)$, where in (22) the $e^i = \{e_i^I\}$ are six linearly-independent vectors normalized: $(e_i)^2 = 2$. The e_i^{I*} are dual to the e_i, and $e_i^* \cdot e_j = \delta_{ij}$.

Adding the two basis vectors b_1 and b_2 to the set (19) corresponds to the $Z_2 \times Z_2$ orbifold model with standard embedding. The fermionic boundary conditions are translated in the bosonic language to twists on the internal dimensions and shifts on the gauge degrees of freedom. Starting from the Narain model with $SO(12)\times E_8\times E_8$ symmetry [18], and applying the $Z_2\times Z_2$ twisting on the internal coordinates, we then obtain the orbifold model with $SO(4)^3\times E_6\times U(1)^2\times E_8$ gauge symmetry. There are sixteen fixed points in each twisted sector, yielding the 24 generations from the three twisted sectors mentioned above. The three additional pairs of 27 and $\overline{27}$ are obtained from the untwisted sector. This orbifold model exactly corresponds to the free-fermion model with the six-dimensional basis set $\{\mathbf{1},S,X,I=\mathbf{1}+b_1+b_2+b_3,b_1,b_2\}$. The Euler characteristic of this model is 48 with $h_{11}=27$ and $h_{21}=3$.

This $Z_2\times Z_2$ orbifold, corresponding to the extended NAHE set at the core of the realistic free fermionic models, differs from the one which has usually been examined in the literature [25]. In that orbifold model, the Narain lattice is $SO(4)^3$, yielding a $Z_2\times Z_2$ orbifold model with Euler characteristic equal to 96, or 48 generations, and $h_{11}=51$, $h_{21}=3$.

In more realistic free-fermion models, the vector X is replaced by the vector 2γ (6). This modification has the consequence of producing a toroidally-compactified model with $N=4$ space–time supersymmetry and gauge group $SO(12)\times SO(16)\times SO(16)$. The $Z_2\times Z_2$ twisting breaks the gauge symmetry to $SO(4)^3\times SO(10)\times U(1)^3\times SO(16)$. The orbifold twisting still yields a model with 24 generations, eight from each twisted sector, but now the generations are in the chiral 16 representation of $SO(10)$, rather than in the 27 of E_6. The same model can be realized with the set $\{\mathbf{1},S,X,I=\mathbf{1}+b_1+b_2+b_3,b_1,b_2\}$, projecting out the $16\oplus\overline{16}$ from the sector X by taking

$$c\binom{X}{I}\rightarrow -c\binom{X}{I}. \tag{23}$$

This choice also projects out the massless vector bosons in the 128 of $SO(16)$ in the hidden-sector E_8 gauge group, thereby breaking the $E_6\times E_8$ symmetry to $SO(10)\times U(1)\times SO(16)$. This analysis confirms that the $Z_2\times Z_2$ orbifold on the $SO(12)$ Narain lattice is indeed at the core of the realistic free fermionic models.

We can now examine whether some connection with M- (and F-)theory compactifications can be contemplated, in view of the extensive literature on $Z_2\times Z_2$ orientifolds of M and F theory [25]. In particular, these interesting papers have examined in detail the $Z_2\times Z_2$ orbifold model with $h_{11}=51$ and $h_{21}=3$. This model is precisely the $Z_2\times Z_2$ orbifold model obtained by twisting the $SO(4)^3$ Narain lattice. In this compactification, the six-dimensional compactified space is the direct product of three simple two-tori, *i.e.*, $(T_2)^3$. This model has been investigated extensively in [25] in connection with M– and F–theory compactifications on special classes of Calabi–Yau threefolds that have been analyzed by Voisin [27] and Borcea [28]. They have been further classified by Nikulin [29] in terms of three invariants (r,a,δ), in terms of which $(h_{1,1},h_{2,1})=(5+3r-2a,65-3r-2a)$. Within this framework, the $(h_{1,1},h_{2,1})=(51,3)$ $Z_2\times Z_2$ orbifold model coincides with the Voisin–Borcea model with $(r,a,\delta)=(18,4,0)$. The dual relations between compactifications of this manifold on M–, F–theory compactifications and type IIB orientifolds have been demonstrated in [25].

The task ahead is clear. Naively, the (27,3) $Z_2 \times Z_2$ orbifold would correspond to a Voisin–Borcea model with $(r, a, \delta) = (14, 10, 0)$. However, such a Voisin–Borcea model is not (yet) known to exist. The problem is that in the Voisin–Borcea models the factorization of the six-dimensional manifold as a product of three disjoint manifolds is essential. However, our $(27, 3)$ $Z_2 \times Z_2$ model, being an orbifold of a $SO(12)$ lattice, is an intrinsically T^6 manifold, and, as such, factorization à la Voisin-Borcea is not possible. Therefore, the first step in trying to connect realistic free-fermion models to M– and F–theory compactifications is to construct the Calabi–Yau threefolds which correspond to the $Z_2 \times Z_2$ orbifold on the $SO(12)$ lattice. Although the relevant manifolds, or their Landau–Ginzburg potential realizations, are not yet known (at least not to us), we believe that they are not fundamentally different in nature from, or intrinsically more difficult than, the corresponding orientifolds related to the $Z_2 \times Z_2$ orbifold on $SO(4)^3$ lattice.

We now recap the current status of the effort to connect M– and F–theory compactifications to relevant phenomenological data. On the one hand, we have the appealing free-fermion models, in which one can address in detail many relevant phenomenological questions, and which provide promising candidates for a realistic superstring model. We have examined in detail in this paper the issue of proton stability, and shown how enhanced gauge symmetries which prevent fast proton can arise. Thus, we have exhibited a robust solution to the problem of the proton lifetime, which is of crucial relevance for M–theory compactifications.

As a step towards the elevation of these ideas to M and F theory, we have discussed the orbifold correspondences of these models. At the core of the realistic free-fermion models there is a $Z_2 \times Z_2$ orbifold on an $SO(12)$ lattice with $(h_{1,1}, h_{2,1}) = (27, 3)$. This is not the standard $Z_2 \times Z_2$ orbifold that has been discussed extensively in the literature, the more familiar one being that with $(h_{1,1}, h_{2,1}) = (51, 3)$. Nevertheless, the existence of duality relations between the $(51, 3)$ $Z_2 \times Z_2$ orbifold and M– and F–theory compactifications leads us to expect the existence of similar relations for the $(27, 3)$ $Z_2 \times Z_2$ orbifold. If the relevant Calabi-Yau threefold or its Landau–Ginzburg realization can indeed be found, the connection of M and F theory to relevant low-energy data would have made a major step forward, particularly with regard to proton stability. Such progress is not out of sight.

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