CERN-TH/97-224
DEMO-HEP-97/06
PSI-PR-97-26

ON THE DETERMINATION OF M_W AND TGCs IN W-PAIR PRODUCTION USING THE BEST MEASURED KINEMATICAL VARIABLES

F.A. Berends

Theoretical Physics Division, CERN
CH - 1211 Geneva 23
and
Instituut Lorentz, University of Leiden, P.O. Box 9506,
2300 RA Leiden, The Netherlands

C.G. Papadopoulos

Institute of Nuclear Physics, NCSR 'Democritos',
15310 Athens, Greece

and

R. Pittau

Paul Scherrer Institute, CH - 5232 Villigen-PSI

ABSTRACT

A study is made of the feasibility of maximum likelihood fits to determine M_W and triple gauge boson couplings using only those experimental kinematical variables that are well measured. A computational tool to calculate theoretical probabilities for those kinematical variables is discussed and then applied to samples of unweighted events produced by an event generator. Detailed results on the M_W determination for semileptonic final states in W-pair production show the feasibility of the method. For TGCs one result is presented as an illustration.

CERN-TH/97-224
DEMO-HEP-97/06
PSI-PR-97-26
September 1997

At LEP 2, measurements of W-pair production and the subsequent decay of the W-bosons are primarily used to determine the W-mass and the triple gauge boson coupling (TGC). The theoretical description of the four fermions produced in W-pair production leads to natural variables for these measurements: invariant masses M_1, M_2 of the $W^{\pm}$ decay products for the M_W determination and a set of five angles for the TGC determinations. The latter are the production angle Θ of W^+ in the laboratory system and the decay solid angles Ω_1, Ω_2 of an antifermion or fermion in the rest frames of a W^+ or W^-. On the other hand, from the experimental point of view the well-measured variables are energies and angles of charged leptons and angles of jets, all in the laboratory system. The experimental resolution of these variables is also well known. The set of these variables will henceforth be denoted as $\{\phi\}$. To this set one might want to add the total hadronic energy as an additional variable. Momenta of photons, mainly caused by initial-state radiation (ISR), are also usually not accurately known experimentally. So the reconstruction of the theoretically natural variables from the well-measured experimental quantities poses a problem. One often has to rely on some input from event generators for four-fermion production (for a review on these generators, see [1]). Detailed accounts of a direct reconstruction method for the M_W determination [2] and a maximum likelihood method for the determination of TGCs [3] have been given in the final report of the Workshop on Physics at LEP 2.

Besides the complication of reconstructing natural variables, the maximum likelihood method for TGCs poses another problem. The theoretical differential cross-section with respect to the above-mentioned five angles is a natural quantity when the TGC effects are evaluated in a zero-width approximation and without taking ISR into account. When these approximations are not made, the evaluation of this five-dimensional differential cross-section, or in other words the probability $P(\Theta, \Omega_1, \Omega_2, \{\alpha\})$ to find an event with $\Theta, \Omega_1, \Omega_2$ for a set $\{\alpha\}$ of anomalous TGCs, requires a fourfold integration. In fact, one has to integrate over the invariant masses M_1, M_2 and over the arguments x_1, x_2 of the $e^{\pm}$ structure functions for the ISR (for the case of ISR in W-pair production, see [4]).

This means that an efficient integration over these variables should be done. Recently, the strategy of the TGC determination [5] has been discussed in a working group at the Oxford LEP 2 Workshop. The above-mentioned problems of reconstructing natural variables and of required integrations led to a more general question: Can one successfully determine anomalous couplings $\{\alpha\}$ from a maximum likelihood fit, using for every event i, characterized by the experimentally well-determined values $\{\phi_i\}$ of the kinematical variables $\{\phi\}$, a theoretical probability $p(\{\phi_i\}, \{\alpha\})$? In order to answer this question, one first needs an efficient way to calculate for an input $\{\alpha\}$ and $\{\phi_i\}$ the multidifferential cross-section $d\sigma/d\phi$ and secondly one has to show that for variables $\{\phi\}$ one can actually determine $\{\alpha\}$ by a maximum likelihood fit.

It is the purpose of this letter to study the feasibility of this approach. Although strictly speaking only the TGC strategy was discussed in [5], it is clear that the maximum likelihood method could also be investigated for the M_W determination. In fact, the methods to calculate $p(\{\phi_i\}, \{\alpha\})$ for the TGC set $\{\alpha\}$ or $p(\{\phi_i\}, M_W)$ for a mass determination, are the same. The only difference is that for the first case one needs the matrix element as a function of

$\{\alpha\}$, whereas in the second case only the Standard Model matrix element is required. Thus the building of the analysing tool for the extraction of M_W or TGCs is in essence the same. Whether the tool gives a good determination of M_W or certain TGCs has to be established by analysing data from event generators. In this letter results for the M_W determination will be presented, whereas the detailed TGC results will be deferred to a later paper [6]. The reasons to focus now on M_W are threefold. In the first place, the maximum likelihood method for M_W offers an alternative to the reconstruction method. Secondly, it is the simplest application of the developed computer program. Thirdly, the variables $\{\phi\}$ and the integration algorithms are the same for the M_W and TGC determination.

In order to construct a tool for the evaluation of $p(\{\phi_i\}, M_W)$ one has to choose the set $\{\phi\}$ of accurately measured variables. Although one can always consider more sets $\{\phi\}$, the following choices seem reasonable in practice [5] for different four-fermion final states:

1. Semileptonic case: $q_1 q_2 \ell \nu$
   1a $\{\phi\} = (E_\ell, \Omega_\ell, \Omega_{q_1}, \Omega_{q_2})$
   1b $\{\phi\} = (E_\ell, \Omega_\ell, \Omega_{q_1}, \Omega_{q_2}, E_h)$, where E_h is the total energy of the jets.

2. Purely hadronic case: $q_1 q_2 q_3 q_4$
   $\{\phi\} = \{\Omega_{q_1} \Omega_{q_2} \Omega_{q_3} \Omega_{q_4}\}$.

3. Purely leptonic case: $\ell_1 \nu_1 \ell_2 \nu_2$
   $\{\phi\} = (E_{\ell_1}, \Omega_{\ell_1}, E_{\ell_2}, \Omega_{\ell_2}\}$

Since eight variables determine an event when no ISR is present, sets 1a and 3 would require one or two integrations, cases 1b and 2 none. Including ISR adds two integrations. When jets cannot be assigned to specific quarks a folding over the various possibilities should be included.

In order to make consistency checks, two programs have been developed to calculate $p(\{\phi_i\}, M_W)$, one program using parts of the event generator ERATO [7], another similarly based on EXCALIBUR [8]. Details will be published elsewhere [6]. For $\{\phi\}$ from set 1a, the program calculates

$$p(\{\phi_i\}, M_W) = \frac{1}{\sigma_{tot}} \; \frac{d\sigma}{dE_\ell d\Omega_\ell d\Omega_{q_1} d\Omega_{q_2}} \; . \tag{1}$$

When the experimental measurement range on $\{\phi\}$ is restricted, σ_{tot} should be calculated with the same restriction. For the other sets $\{\phi\}$ the analogous multidifferential cross-sections are used and calculated in the "analyser" program.

Now we want to show that the maximum likelihood determination of M_W works in the semileptonic case. To this end we generate 1600 unweighted events at a center-of-mass energy of 190 GeV, with either ERATO or EXCALIBUR, for a known input value

$$M_W = 80.230 \text{ GeV} \; . \tag{2}$$

With the analyser we calculate

$$\log L = \sum_i \log p_i(M_W) \; , \tag{3}$$

where i runs over the 1600 events and

$$p_i(M_W) = p(\{\phi_i\}, M_W) \ . \tag{4}$$

For the W-mass parameter M_W nine different values were chosen, leading to nine data points for $\log L$, the logarithm of the likelihood function. The integration needed to calculate $p_i(M_W)$ leads to a small error on those data points. A parabola is fitted to these points from which the reconstructed W-mass M_R is obtained with an error ΔM_R.

An example of this procedure is shown in Fig. 1. The 1600 events were generated in the whole phase space with a CC3 matrix element (containing only the three signal diagrams [4]) including the ISR. Also the analyser contained CC3 diagrams and ISR. The reconstructed mass is 80.238 ± 0.049. So within the errors (mainly determined by the number of experimental points, i.e. 1600) the correct W-mass has been recovered. The variable set 1a has been used in this analysis.

In order to get more insight in the reliability of the maximum likelihood procedure to find M_R and to study some physics effects, three different data sets of 1600 events were used, which were analysed in different ways. We label the various data sets with $j = 1, 2, 3$. All three include ISR, the first one is based on CC3, the second on CC10 and the last one on CC20. So the pure signal case is generated, the muon case with all diagrams and the electron case with all diagrams. The CC3 and CC10 events are produced in the whole phase space, while, for the CC20 set, the outgoing e^- has a $10°$ cut from the beam.

The various analysers are denoted by $i = 1, 5$ and are respectively CC3, CC3 + ISR, CC10 + ISR, CC20 + ISR, CC3 + ISR + folding. In the last case the average cross-section is used with the two possible quark assignments. The results for a number of (i, j) combinations are given in Table 1 for case 1a and in Table 2 for case 1b. The entries are the reconstructed masses.

From Table 1, the following conclusions can be drawn. Comparing cases (1,1) and (2,1) shows that neglect of ISR in the analysis causes a shift of 269 ± 49 MeV. Comparing (2,1) and (5,1) shows that folding gives within the errors a well-reconstructed M_R.

Comparing cases (2,2) and (3,2) tells us that the background diagrams, in the case the final state lepton is a muon, have no significant effect. An analysis with CC3 alone is sufficient, within the errors. The same happens for the case the final state lepton is an electron (see cases (2,3) and (4,3)). However, the latter statement depends on the error originating from just using 1600 events and on the phase space cuts. Here a $10°$ cut from the beam is applied on the outgoing e^-. Cutting less may give a larger background and a significant shift.

Similar conclusions can be drawn when the set 1b is used. The errors on the reconstructed M_R are better, since one less integration is needed. Also the shift in M_R from the ISR is smaller, now being 142 ± 33 MeV.

Variable set 3 is not as much efficient for the mass reconstruction. This is due to the fact that a large part of the kinematical information is actually missing. We have found that the determination error in this set of variables is twice as much as in the case 1a for the same number of generated unweighted events. The purely hadronic case can also be used. Since the folding complicates the calculation of p_i, we defer the discussion of this case to a future paper [6] .

The analyser gives the probability as a function of a limited set of kinematical variables $\{\phi\}$. It integrates over some undetermined variables and ISR. Whereas in event generators one wants to generate events with all information provided, also preferably ISR photon momenta, the analyser only has to integrate over the ISR. It is known that the structure function method gives a reliable integration over all emitted ISR photon momenta. Because of this ISR integration in the analyser program, the details of p_T distributions of ISR photons are expected to matter here less than for refined event generation, where they matter for the kinematics of each event. It should be noted that the unweighted events used for the analysis of Tables 1 and 2 are generated with ISR along either beam direction. In order to test the effect of transverse photon momenta of the ISR, another event sample generated by KORALW [9] was analysed. The inclusion of transverse photon momenta did not give results different from the ones in the tables, at least within the errors of the analysis. From this point of view, the maximum likelihood method for determining M_W may have an advantage over the direct reconstruction method.

In conclusion, the maximum likelihood method for determining M_W from well-measured experimental variables, with well-known resolutions, seems to offer an alternative to the direct reconstruction method.

Besides the incorporation of the ISR, the method also easily takes into account the full set of diagrams for each channel. Beforehand one can test whether a CC3 + ISR analysis is sufficient, or whether background diagrams should be included.

Although further results concerning the full set of TGC parameters, as well as the consideration of all possible production channels, will be presented elsewhere [6], we give here one indication of its feasibility. We have performed an analysis based on the $\alpha_{W\phi}$ parameter [3]. More precisely the 1600 events generated with CC3+ISR have been analysed in the case 1a, with the result:

$$\alpha_{W\phi} = 0.002 \pm 0.026 \ .$$

We also generated 1600 events with CC3 but without ISR and analysed them using the set of variables described in 1b. In that case, where no ISR is present, we expect to have a more or less ideal situation, since the full kinematical information is available. The fitted value is now

$$\alpha_{W\phi} = 0.006 \pm 0.024 \ .$$

Both numbers agree very well with the results presented in [3], where in the so-called ‘ideal’ case a determination error of 0.018 has been found by fitting a sample of 2600 events.

Summarizing, the feasibility of the method has been demonstrated from a theoretical point of view. Further studies with detector simulation would be required to see the results of the method in an experimental environment.

Acknowledgements: Discussions with the members of the Oxford TGC working group are gratefully acknowledged. The KORALW sample of events was kindly provided to us by Z. Was.

References

[1] D. Bardin et al., in Physics at LEP 2, CERN 96-101 (1996), eds. G. Altarelli, T. Sjöstrand and F. Zwirner, Vol. 2, p. 3, `hep-ph/9709270`.

[2] Z. Kunszt et al., ibidem, Vol. 1, p. 141, `hep-ph/9602352`.

[3] G. Gounaris et al., ibidem, Vol 1, p. 525, hep-ph/9601233; see also M. Gintner, S. Godfrey and G. Couture, Phys. Rev. D52 (1995) 6249 .

[4] W. Beenakker et al., ibidem, Vol. 1, p. 79, `hep-ph/9612260`.

[5] P. Clarke et al., to be published in Proceedings Oxford LEP 2 Workshop.

[6] F. A. Berends, C. G. Papadopoulos and R. Pittau, to be published.

[7] C.G. Papadopoulos, *Phys. Lett.* **B352** (1995) 144; *Comput. Phys. Commun.* **101** (1997) 183.

[8] F.A. Berends, R. Kleiss and R. Pittau, *Nucl. Phys.* **B424** (1994) 308; *Nucl. Phys.* **B426** (1994) 344; *Comput. Phys. Commun.* **85** (1995) 437.

[9] M. Skrzypek, S. Jadach, W. Plaszek and Z. Was, CERN-TH-95-205.

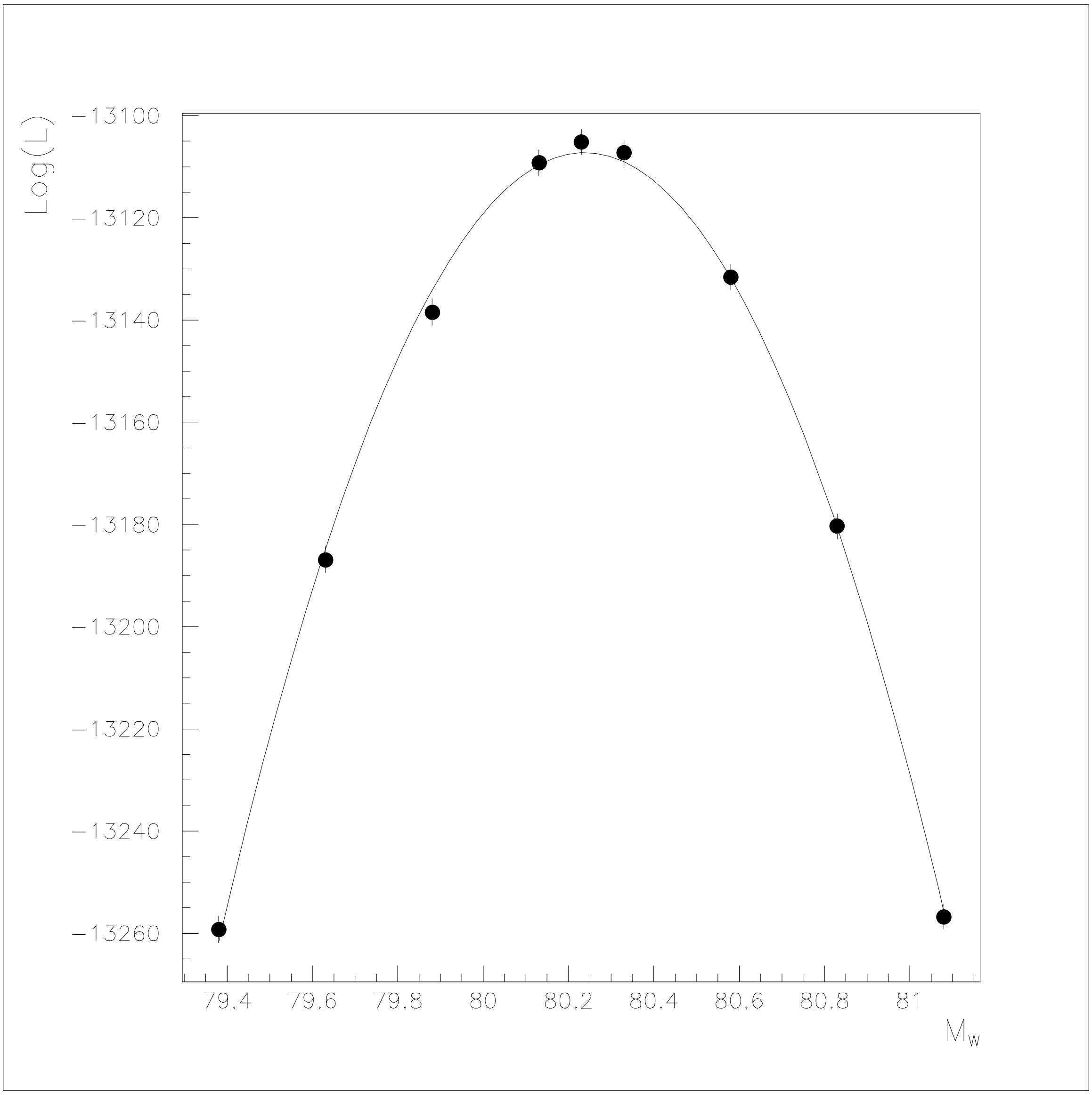

Figure 1: Example of likelihood curve for case $i = 2,\ j = 1$ in Table 1.

$j =$ → $i =$ ↓	1 CC3 + ISR	2 CC10 + ISR	3 CC20 + ISR
1 CC3	80.507 ±0.045		
2 CC3 + ISR	80.238 ±0.049	80.224 ± 0.048	80.280 ± 0.047
3 CC10 + ISR		80.277 ± 0.048	
4 CC20 + ISR			80.231 ± 0.049
5 CC3 + ISR + folding	80.249 ±0.048		

Table 1: The reconstructed $M_R \pm \Delta M_R$ from three different data samples of 1600 unweighted events ($j = 1, 2, 3$). The analyser has been used for five different treatments of the cross-section ($i = 1, 5$). Sets $j = 1$ and $j = 2$ have no cuts, while, for set $j = 3$, the outgoing e^- has a 10° cut from the beam. The experimental variables are those of set 1a.

$j =$ → $i =$ ↓	1 CC3 + ISR	2 CC10 + ISR	3 CC20 + ISR
1 CC3	80.380 ±0.032		
2 CC3 + ISR	80.238 ±0.033	80.238 ± 0.032	80.269 ± .032
3 CC10 + ISR		80.240 ± 0.032	
4 CC20 + ISR			80.240 ± 0.033
5 CC3 + ISR + folding	80.239 ±0.033		

Table 2: The reconstructed $M_R \pm \Delta M_R$ from three different data samples of 1600 unweighted events ($j = 1, 2, 3$). The analyser has been used for five different treatments of the cross-section ($i = 1, 5$). Sets $j = 1$ and $j = 2$ have no cuts, while, for set $j = 3$, the outgoing e^- has a 10° cut from the beam. The experimental variables are those of set 1b.