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Supersymmetry breaking in M-theory and quantization rules

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Abstract

We analyze in detail supersymmetry breaking by compactification of the fifth dimension in M-theory in the compactification pattern $11d \to 5d \to 4d$ and find that a superpotential is generated for the complex fields coming from $5d$ hypermultiplets, namely the dilaton S and the complex structure moduli. Using general arguments it is shown that these fields are always stabilized such that they don't contribute to supersymmetry breaking, which is completely saturated by the Kähler moduli coming from vector multiplets. It is shown that this mechanism is the strong-coupling analog of the Rohm-Witten quantization of the antisymmetric tensor field strength of string theories. The effect of a gaugino condensate on one of the boundaries is also considered.

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1 Introduction

It is largely believed nowdays that the strongly coupled regime of the heterotic string is described within the M-theory and, in particular, the strongly coupled $E_8 \times E_8$ heterotic string, traditionally considered as the most relevant one for phenomenology, can be described, in the low energy limit, by the eleven-dimensional supergravity with the two E_8 gauge factors living each on a 10-d boundary [1]. The radius of the eleventh dimension is related to the string coupling by $R_{11} \sim \lambda_{st}^{2/3}$. So in the strongly coupled regime, R_{11} has to be large and possibly larger than the typical radius of the other six compact dimensions [2], [3]. Describing four-dimensional physics from $E_8 \times E_8$ heterotic strongly coupled string should thus be equivalent to compactifing the eleven-dimensional supergravity on a Calabi-Yau manifold and then compactifing the fifth dimension on S^1/Z_2.

Recently [4], [5], [6] (see also [7] in the type II context), attention was paid to the $4d$ supersymmetry breaking by the compactification from $5d$ to $4d$, by using the field-theoretical Scherk–Schwarz mechanism [8]. It was argued in [5] that the results look like non-perturbative from the perturbative heterotic string point of view. For the simplest truncation $11d \to 5d$ corresponding to no complex structure moduli, a superpotential generation for S was obtained. The corresponding model has spontaneously broken supersymmetry with a zero cosmological constant, the invariance $S \to 1/S$ and a minimum that is reached for $S = 1$. We give in Section 2 the next non-trivial example involving one complex structure modulus and then more general results are given by using generic arguments. Then the effect of a gaugino condensate added on one of the boundaries is studied; the picture which emerges is shown to be consistent. In Section 3 we show that the Scherk-Schwarz mechanism in this context describes in strong coupling regime the Rohm-Witten quantization [9] of the antisymmetric tensor field strength [2] , used in the early days of string phenomenology [10] in the context of gaugino condensation for fixing the dilaton with a (tree-level) zero cosmological constant.

[2]I would like to thank Jean-Pierre Derendinger for discussions which lead me to study this analogy.

2 The spontaneous breaking of $N = 1$ supersymmetry in four dimensions by compactification

To be specific, in the following we discuss the $5d \to 4d$ compactification. The Scherk–Schwarz mechanism is a generalized dimensional reduction that allows for the fields a dependence in the compact coordinates. This dependence must satisfy some properties: it has to be in a factorizable form and has to correspond to an R-symmetry of the theory. If we denote generically by Φ all the (boson and fermion) fields of the theory , we have the following decomposition

$$\Phi(x, x_5) \;\; = \;\; U(x_5)\; \Phi(x), \tag{1}$$

where x_5 denotes the compact coordinate and x non-compact ones. This tensor decomposition is stable under product and exterior derivation. For $-\pi \leq x_5 \leq \pi$, defining

$$\Phi(\pi) = \mathcal{U}\Phi(-\pi)\ , \tag{2}$$

consistency ask for $\mathcal{U}$ to be a symmetry of the $5d$ theory.

This extended dimensional reduction generates a potential for the scalar fields corresponding to the kinetic terms in the compact space. The requirement for this scalar potential to be positive imposes further restrictions on the form of U. A solution was proposed by Scherk and Schwarz, by taking

$$U \;\; = \;\; e^{Mx_5}\ , \tag{3}$$

where M is an antihermitian matrix depending on the field representation. When applied to the kinetic term for the $5d$ spin-$\frac{3}{2}$ field, masses for the resulting $4d$ gravitinos are also generated.

The Horava-Witten projection Z_2^{HW} acts as

$$Z_2^{HW}\phi(-x_5) = \eta\phi(x_5)\ ,\ Z_2^{HW}\Psi(-x_5) = \eta\gamma_5\Psi(x_5)\ , \tag{4}$$

where ϕ denote bosonic fields and Ψ fermionic fields and $\eta = 1$ for $g_{\mu\nu}, g_{IJ}, C_{5IJ}$, $C_{5\mu\nu}, \Psi_\mu, \Psi_I$, and $\eta = -1$ for $g_{5\mu}, C_{IJK}, C_{\mu\nu\rho}, \Psi_5$. The consistency of the Scherk-Schwarz mechanism with Z_2^{HW} asks for the condition [11]

$$\{Z_2^{HW}, M\} = 0\ . \tag{5}$$

It was proven in [5] that, at the field theory level, the only symmetry that is compatible with the projection Z_2^{HW} is a $U(1)$ subgroup of the $SU(2)_R$ symmetry appearing in the $N=2$ supersymmetry algebra. We quikly remind the results obtained within the simplest truncation [12], corresponding to an $SU(3)$ invariance in the compactified space of volume $e^{3\sigma}$ and $h_{(2,1)}=0$. In this case, in $5d$ the only matter multiplet is the universal hypermultiplet $(e^{3\sigma}, C_{ijk}=\epsilon_{ijk}a, C_{\mu\nu\rho})$, whose scalar fields parametrize the coset $\frac{SU(2,1)}{SU(2)\times U(1)}$ [13]. This structure can be simply viewed from $4d$ by a direct truncation (*without* the Z_2^{HW} projection). The lagrangian of the universal hypermultiplet can be derived from the Kähler potential [13]

$$\mathcal{K} = -\ln(S+S^\dagger - 2a^\dagger a) \ , \tag{6}$$

where the Hodge duality in $5d$ is

$$\sqrt{2}e^{6\sigma}G_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma\delta}(\partial^\delta a_1 + i a^\dagger \overleftrightarrow{\partial^\delta} a) \tag{7}$$

and $S = e^{3\sigma} + a^\dagger a + i a_1$. The $SU(2)$ symmetry acts linearly on the redefined fields

$$z_1 = \frac{1-S}{1+S} \ , \ z_2 = \frac{2a}{1+S} \ , \tag{8}$$

which form a doublet (z_1, z_2). The Z_2^{HW} projection acts as $Z_2^{HW} S = S$, $Z_2^{HW} a = -a$, which translates on the $SU(2)$ doublet in the obvious way:

$$Z_2^{HW} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} . \tag{9}$$

The Scherk–Schwarz decomposition in this case reads explicitly [5]

$$\begin{pmatrix} \hat{z}_1 \\ \hat{z}_2 \end{pmatrix} = \begin{pmatrix} \cos m x_5 & \sin m x_5 e^{i\theta} \\ -\sin m x_5 e^{-i\theta} & \cos m x_5 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} , \tag{10}$$

corresponding to the matrix defined in (3) $M = im(cos\theta\sigma_2 + sin\theta\sigma_1)$ and where m is a real mass parameter and θ a phase.[3] Notice that, thanks to the anticommutation relation (5), which is clearly verified, the fields $\hat{z}_i$ have

[3] The possibility of adding this phase was noticed in the second reference in [6]. However, this phase add no new physical freedom and will be put to zero in the following.

the same Z_2^{HW} parities as the fields z_i. The resulting scalar potential in $4d$ in Einstein metric is computed from the kinetic terms of the $(\hat{z}_1, \hat{z}_2)$ fields derived from (6). After making $z_2 = 0$, corresponding to the projection Z_2^{HW}, it is easily worked out and can be seen as a superpotential generation for S. The $4d$ theory is completely described by

$$\begin{aligned} K &= -\ln(S + S^\dagger) - 3\ln(T + T^\dagger) \ , \ W = 2m(1+S) \ , \\ V &= \frac{4m^2}{(S+S^\dagger)(T+T^\dagger)^3} |1 - S|^2 \ . \end{aligned} \tag{11}$$

The resulting model is of a no-scale type [14]. This is a general result for models obtained by the Scherk-Schwarz mechanism. Notice that in the z_1 variable the superpotential is just a constant $W = 2\sqrt{2}m$. The minimum of the scalar potential is $S = 1$ and gives a spontaneously broken supergravity model with a zero cosmological constant. The order parameter for supersymmetry breaking is the gravitino mass $m_{3/2}^2 = e^{\mathcal{K}} |W|^2 = 8m^2/(T + T^\dagger)^3$.

The next non-trivial example corresponds to a compactification with $h_{(2,1)} = 1$, which can be obtained for example with Z_6, Z_8 or Z_{12} projections in the compactified space. The $5d$ theory contains two hypermultiplets (S, a_S), with $C_{ijk} = \epsilon_{ijk} a_S$ and (U, a_U), with $C_{ij\bar{k}} = a_U$ and two vector multiplets. The hypermultiplets scalars span the coset $U(2,2)/U(2) \times U(2)$. The corresponding Kähler potential is

$$K = -\ln det(\mathcal{T} + \mathcal{T}^\dagger) \ , where \ \mathcal{T} \ = \ \begin{pmatrix} S & 2a_S \\ 2a_U & U \end{pmatrix} \ . \tag{12}$$

The $SU(2)_R$ symmetry acts linearly on the matrix $z = z_0(1 - \mathcal{T})(1 + \mathcal{T})^{-1}$, in terms of which the Kähler potential is (up to a Kähler transformation) $K = -Tr\ln(z_0 z_0^\dagger - z z^\dagger)$. Here z_0 is an arbitrary, fixed matrix which, without loosing generality can be put to $z_0 = 1$ in the following. Imposing the Horava-Witten projection means $a_S = a_U = 0$ and the $4d$ theory contains the moduli S, U coming from $5d$ hypermultiplets and T_1, T_2, T_3 moduli, two coming from the two $5d$ vector multiplets and one (the overall volume) from the $5d$ gravitational multiplet . The Scherk-Schwarz decomposition reads then

$$\hat{z} \ = \ \begin{pmatrix} \frac{1-S}{1+S} & 0 \\ 0 & \frac{1-U}{1+U} \end{pmatrix} \begin{pmatrix} \cos mx_5 & \sin mx_5 \\ -\sin mx_5 & \cos mx_5 \end{pmatrix} \ . \tag{13}$$

The $4d$ potential is computed as

$$V = K_{ij,\bar{k}\bar{l}} \partial_5 \hat{z}^{ij} \partial^5 \hat{z}^{\dagger \bar{k}\bar{l}} = m^2 e^{-3\gamma} \frac{|z_{11}|^2 + |z_{22}|^2}{(1-|z_{11}|^2)(1-|z_{22}|^2)} \qquad (14)$$

in $4d$ SUGRA units, where $g_{55} = e^{2\gamma} \equiv t^2$ and $g_{\mu\nu}^{(5)} = e^{-\gamma} g_{\mu\nu}^{(4)}$. As in the previous example, the result coresponds to a superpotential generation $W = 2\sqrt{2}m$. In the S, U variables, the lagrangian is

$$\begin{aligned} K &= -\ln(S+S^\dagger) - \ln(U+U^\dagger) - \sum_{i=1}^{3} \ln(T_i + T_i^\dagger) \ , W = \sqrt{2} m (1+S)(1+U), \\ V &= \frac{2m^2}{(S+S^\dagger)(U+U^\dagger)\prod_i (T_i+T_i^\dagger)} \left[|1-S|^2 |1+U|^2 + |1+S|^2 |1-U|^2 \right] \end{aligned} \qquad (15)$$

The vacuum corresponds to $S = U = 1$. It is easy to compute the physical masses and to check that $m_S^2 = m_U^2 = m_{3/2}^2$, in accordance with the Scherk-Schwarz mechanism. Notice that the auxiliary fields $G_S = G_U = 0$ don't contribute to supersymmetry breaking. If we integrate-in the field U and put $T_i = T$ we recover the previous example (11). This is valid for any generalization by integrating-in the complex structure moduli U_α.

General results can be obtained by noticing that the Kähler moduli T_i coming from $5d$ vector multiplets are described by special Kähler geometry [15] and therefore their Kähler potential is

$$K = -\ln \mathcal{F} \ \ , \ \mathcal{F} = \frac{1}{6} c^{ijk} (T_i + T_i^\dagger)(T_j + T_j^\dagger)(T_k + T_k^\dagger) \ , \qquad (16)$$

where c^{ijk} are the intersection numbers of the Calabi-Yau manifold and $\mathcal{F}$ is related to the prepotential $F = \frac{i}{6} c^{ijk} T_i T_j T_k$ (for example, $F = iT^3$ for the first model and $F = iT_1 T_2 T_3$ for the second model in this paragraph). On the other hand, as the $SU(2)_R$ symmetry doesn't act on the vector moduli, there is no induced superpotential in $4d$ and so the whole vector moduli lagrangian is described by $\mathcal{F}$. Then, by using $t_i \mathcal{F}^i = 3\mathcal{F}$, where $2t_i = T_i + T_i^\dagger$, it is easy to show that [16] $G^i G_i = 3$ (where $G = K + \ln |W|^2$) and the breaking of supersymmetry is saturated by Kähler moduli. With these considerations, it is easy to determine the goldstino direction. Indeed, by compactification, as the hypermultiplets don't contribute to supersymmetry breaking, the goldstino should be a combination of Ψ_5 (fifth component of a

Majorana gravitino) and chiral fermions coming from $5d$ vector multiplets. The $SU(2)_R$ acts explicitly on the last ones and they become massive. On the other hand, due to the special structure of the $5d$ gravitino kinetic term

$$L_{kin} = -\frac{1}{2}\bar{\Psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \Psi_\rho \ , \tag{17}$$

the component Ψ_5 which survives after the Horava-Witten projection cannot acquires a mass, therefore it must be identified with the goldstino. [4]

Using [16] $G^i = -2t_i$ and denoting by χ_i the fermion associated to the moduli T_i, we get the expression for the goldstino

$$g = e^{\frac{G}{2}} G_i \chi^i \sim \ -\frac{m_{3/2}}{t^3} c^{ijk} t_i t_j \chi_k \sim \Psi_5 \ , \tag{18}$$

where Ψ_5 is, as we said, the fifth component of the gravitino, whose counterpart Ψ_μ is projected by Z_2^{HW}. In order to compute (18) we used the fact that $G_\alpha = 0$ for complex structure U_α fields. The last equality in (18) is just the supersymmetric partner of the relation $t^3 = 1/6 c^{ijk} t_i t_j t_k$, where t is the overall volume moduli coming from the $5d$ gravitational multiplet $g_{55} = t^2$. Therefore, for general models with complex structure moduli U_α, zero cosmological constant (which is always obtained at tree level in the Scherk-Schwarz mechanism) asks for $G_S = G_\alpha = 0$. This constraint, together with $m_S^2 = m_\alpha^2 = m_{3/2}^2$ can be used in order to construct effective lagrangians directly in $4d$. One example of model obtained along these lines corresponds to $h_{(1,1)} = h_{(2,1)} = 3$. The lagrangian is a simple generalization of (15)

$$K = -\ln(S + S^\dagger) - \sum_{\alpha=1}^{3} \ln(U_\alpha + U_\alpha^\dagger) - \sum_{i=1}^{3} \ln(T_i + T_i^\dagger) \ , \tag{19}$$

$$W = \frac{m}{\sqrt{2}}(1 + S) \prod_\alpha (1 + U_\alpha) \ . \tag{20}$$

In all the examples we worked out (including other U moduli fields), *the generated superpotential $W(S, U_\alpha)$ is a constant in the variables where the $SU(2)_R$ symmetry acts linearly*, so we conjecture that this is a general result.

[4]After our argument was completed, we learned that this was recently noticed also in the second ref. in [6].

An important question is of course the possible values the parameter m can take. To answer this question, we use the result of Hull and Townsend [17] which states that generically any global symmetry is broken to the corresponding discrete subgroup by the Dirac charge quantization. In particular, the $SU(2,R)_R$ symmetry should be broken to the $SL(2,Z)_R$ subgroup. Consequently, we must impose the condition

$$\mathcal{U} \ = \ \begin{pmatrix} cos2\pi m & sin2\pi m \\ -sin2\pi m & com2\pi m \end{pmatrix} \in SL(2,Z) \ , \tag{21}$$

which has as solutions $m = \pm 1/2, \pm 1/4$ in units of M_P.

It is useful to express the gravitino mass in a more usual way by using relations obtained in [5]. We redefine $m = nM_P$, n being then a pure number. Then we get, for all the models discussed above

$$m_{3/2}^2 = \frac{8n^2 M_P^2}{(T+T^\dagger)^3} = \frac{n^2 M_{11}^2}{t^2} = \frac{n^2}{\rho^2} \ , \tag{22}$$

where ρ is the fifth radius and we used the relation [5] $tM_{11}^2 = M_P^2$, which is the analog of the relation $sM_s^2 = M_P^2$ of the perturbative heterotic string[5]. The final expression (22) is of course the usual expression for Kaluza-Klein type masses. Notice that we can also rewrite the gravitino mass as

$$m_{3/2} = n \frac{M_{11}^3}{M_P^2} \ , \tag{23}$$

in a way that will become very transparent later on, when we will couple a gaugino condensate on one boundary of the system. As explained in [6], the matter fields feel the supersymmetry breaking only through radiative corrections and get soft masses which are generically $m_{soft} \sim m_{3/2}^2/M_P$.

A next step in understanding the dynamics of the strongly coupled heterotic string is the coupling of the five dimensional bulk to the two $4d$ boundaries containing the matter and the gauge fields.[6] In this paper, we neglect the problems (discussed in [2]) due to the modified Bianchi identity

$$dG \sim \sum_i \delta(x_5 - x_i)(trF_i^2 - \frac{1}{2} trR^2) dx^{11} \ , \tag{24}$$

[5]In [5] it was considered the possibility that $m \sim M_{11}$, in which case the gravitino mass was smaller compared to (22).

[6]This issue was recently considered from a complementary point of view in [20], [21].

where $x_i = 0, \pi$ are the positions of the two boundaries, because of our computational limitation in solving exactly the supersymmetry conditions in the compactified space. While this is an important omission, we believe including its effects will not qualitatively change our results.

We incorporate the effect of a gaugino condensate on one (strongly coupled) boundary, let's say at $x_5 = 0$. It was noticed in [18] that, in complete analogy with the weakly coupled case, the $11d - 10d$ lagrangian contains the perfect square

$$\frac{-1}{12k_{11}^2} \int d^{11}x\sqrt{g}\left[G_{ABC11} - \frac{\sqrt{2}}{16\pi}(\frac{k_{11}}{4\pi})^{2/3}\delta(x^{11})\bar{\lambda}^a\Gamma_{ABC}\lambda^a\right]^2 , \qquad (25)$$

where k_{11} is the $11d$ gravitational constant and λ^a are gauginos living on the boundary. We consider only the simplest case $h_{(2,1)} = 0$. Then the compactification of the above term (25) to $5d$ changes the field strength G_{5ijk}. This corresponds to the shift in the kinetic term for the universal complex field a as

$$\partial_5 a \to \partial_5 a - \beta\delta(x_5)\bar{\lambda}\lambda , \qquad (26)$$

where β is related to the coefficient of the fermion bilinear in (25) and $\bar{\lambda}\lambda$ is the $4d$ gaugino condensate. According to [1], $\delta(x_5)$ is defined here to transform as a scalar under diffeomorphisms. This means it implicitly contain a vierbien $e_5^5 = e^\gamma$ in its definition; also, the 11 index in G_{ABC11} is a Lorentz one and for doing the computation it must be changed into an Einstein index, a fact which turns out to be crucial in the following. The rest of the $5d$ lagrangian is unchanged and we can apply again the Scherk-Schwarz mechanism. The scalar potential compared to the case without gaugino condensate is shifted according to

$$V \to V - \beta e^{-\gamma}[e^{-3\hat{\sigma}}\partial_5(\hat{a} + \hat{a}^\dagger)]_{(x_5=0)}\bar{\lambda}\lambda + \beta^2 e^{-3\sigma}\delta(0)(\bar{\lambda}\lambda)^2 , \qquad (27)$$

where $e^{-\gamma}$ factor originates from the inverse vierbein necessary to convert a Lorentz index into an Einstein one, as discussed above. Using (10) for doing the computation and then going into $4d$ SUGRA ($g_{\mu\nu}^{(5)} = e^{-\gamma}g_{\mu\nu}^{(4)}$ and $\lambda \to e^{\frac{3\gamma}{4}}\lambda$) units we get the result

$$V = \frac{1}{(S + S^\dagger)}\left[4m^2\frac{|1 - S|^2}{(T + T^\dagger)^3} - \frac{2\sqrt{2}m\beta}{(T + T^\dagger)^{3/2}}(2 - S - S^\dagger)\bar{\lambda}\lambda + 2\beta^2\delta(0)(\bar{\lambda}\lambda)^2\right] . (28)$$

The last term in (28) is of course ill-defined and can hardly help us in finding the correct result. On the other hand, the first two terms have a clear tendency of forming a perfect square and therefore the correct result should be

$$V = \frac{1}{(S+S^\dagger)} | \frac{2m(1-S)}{(T+T^\dagger)^{3/2}} - \sqrt{2}\beta\bar{\lambda}\lambda|^2 \ . \tag{29}$$

Puting back the mass units in our result, we see that we found (for $m \sim M_P$, as we argued earlier)

$$< \bar{\lambda}\lambda > \sim \frac{M_P^3}{t^{3/2}} = M_{11}^3 \ , \tag{30}$$

where in the last step we used again the formula $tM_{11}^2 = M_P^2$. We therefore found explicitly the result conjectured in [6], namely the gaugino condensation scale is the M-theory scale M_{11}. Now the formulae (23),(30) are of course simply explained in perfect analogy with the gaugino condensation scenario in SUGRA [19]

$$m_{3/2} \sim \frac{< \bar{\lambda}\lambda >}{M_P^2} \ . \tag{31}$$

In the presence of the condensate, the vev of S is shifted. Notice that, here S is the volume of compactified space on the boundary containing the condensate and therefore is related to the gauge coupling of the strongly coupled hidden sector. We now compute the volume of the compactified space on the other (observable) boundary. The result is

$$\mathcal{V}(\pi) \equiv \hat{S}(\pi) = \frac{\mathcal{V}(0)}{|cos^2\frac{m\pi}{2} + sin^2\frac{m\pi}{2}S|^2} \ . \tag{32}$$

In the absence of the gaugino condensate, $S = 1$ and therefore $\mathcal{V}(\pi) = \mathcal{V}(0)$, the two boundaries being perfectly symmetric. In the presence of the gaugino condensate, $S < 1$ and we get $\mathcal{V}(\pi) > \mathcal{V}(0)$ such that the observable world has a smaller gauge coupling. The picture is therefore consistent and reminds us the situation described in [2], where it was shown that, due to the modified Bianchi identity (24) a similar phenomenon occurs.

Finally, we note that, if we don't impose the Horava-Witten projection, the resulting N=2 model in $4d$ has interesting properties, too. Namely, the

Scherk-Schwarz masses, given by the parameter m which becomes now the N=2 central charge, are BPS saturated and in consequence the tree level result is actually exact. Indeed, the $5d$ supersymmetry algebra becomes in $4d$, in a Weyl notation

$$\left(\begin{array}{cc} \{Q_2, Q_1\} & \{Q_2, \bar{Q}_2\} \\ \{\bar{Q}_1, Q_1\} & \{\bar{Q}_1, \bar{Q}_2\} \end{array} \right) \quad = \quad 2 \left(\begin{array}{cc} P_5 & \sigma^\mu P_\mu \\ \bar{\sigma}^\mu P_\mu & -P_5 \end{array} \right) , \tag{33}$$

where Q_1, Q_2 are the two supersymmetry charges in $4d$ and P_5 is the fifth momentum, which is therefore the central charge. The usual argument gives here for the mass operator $M^2 \geq P_5^2$. It is now straightforward to check the BPS relation $M^2 = P_5^2$ by using Scherk-Schwarz decompositions of type (10) and the spectrum of masses computed from the scalar potential.

3 Quantization of S and of the complex structure moduli.

It was shown [9] in the context of string theory that the field strength of the antisymmetric tensor B, $H = dB - \frac{\alpha'}{2}\omega_{3Y}$ satisfies the quantization rule

$$\int_{C_3} H = \frac{2\pi}{T_2} p \ , \tag{34}$$

where T_2 is the string tension, C_3 is a closed three-manifold and p is an integer. In particular, this predicted in components $H_{ijk} = c\epsilon_{ijk}$, with c a quantized parameter which was used in [10] in the gaugino condensation scenario in order to break supersymmetry with a stabilized dilaton and zero cosmological constant. Rohm and Witten argued that c depends generically on the complex structure moduli.

We claim now that similar quantization rules appear in our version of the Scherk-Schwarz mechanism with an interpretation very similar in spirit to that of Rohm and Witten. More precisely, we show by an explicit computation that despite the fact that the three form components $C_{ijk}, C_{ij\bar{k}}, C_{\mu\nu\rho}, \mu, \nu,$ $\rho = 1, 2, 3, 4$ are odd under Z_2^{HW} and therefore have no zero modes in $4d$, the corresponding field strengths have background values leading to quantization rules. The generalization of (34) in strong coupling regime is

$$\int_{S^1/Z_2 \times C_3} G = \frac{2\pi}{T_3} p \ , \tag{35}$$

where $G = 6dC + a\delta(x^5)dx^5\omega_{3Y}$ (a is given in [1]) and T_3 is the membrane tension. We neglect in this paper the Chern-Simmons possible contribution to G. Their consequence can be discussed along the lines of ref. [9], but this is beyond our goal here. In components, the field strength which interest us are $G_{5ijk}, G_{5ij\bar{k}}$. The components C_{ijk} and $C_{ij\bar{k}}$ depend in a non-trivial way on x^5, so we get the result

$$\int dx^5 \hat{G}_{5ijk} = \epsilon_{ijk}[\hat{a}(\pi) - \hat{a}(-\pi)] = 2\epsilon_{ijk}\hat{a}(\pi) \ , \tag{36}$$

exactly because of the twist in the boundary conditions asked by the Scherk-Schwarz mechanism. This is equivalent with the presence of a five-brane as a magnetic source for G. Similar quantization rules appear from $G_{5ij\bar{k}}$. The right-hand side of (36) can be easily calculated by using the results of the preceding paragraph. The result is

$$\hat{G}_{5ijk} = 3m\epsilon_{ijk}\frac{(S-1)(cos^2\frac{mx_5}{2} - sin^2\frac{mx_5}{2}S)}{(cos^2\frac{mx_5}{2} + sin^2\frac{mx_5}{2}S)^2} \ , \ \hat{a}(\pi) = \frac{S-1}{ctg\frac{m\pi}{2} + tg\frac{m\pi}{2}S} \tag{37}$$

for $h_{(2,1)} = 0$ and

$$\begin{aligned} (\hat{G}_S)_{5ijk} &= 3m\epsilon_{ijk}\frac{(S-1)(1+U)(cos^2\frac{mx_5}{2} - sin^2\frac{mx_5}{2}SU)}{(cos^2\frac{mx_5}{2} + sin^2\frac{mx_5}{2}SU)^2} \ , \\ (\hat{G}_U)_{5ijk} &= 3m\epsilon_{ijk}\frac{(1+S)(U-1)(cos^2\frac{mx_5}{2} - sin^2\frac{mx_5}{2}SU)}{(cos^2\frac{mx_5}{2} + sin^2\frac{mx_5}{2}SU)^2} \ , \\ 2\hat{a}_S(\pi) &= \frac{(S-1)(1+U)}{ctg\frac{m\pi}{2} + tg\frac{m\pi}{2}SU} \ , 2\hat{a}_U(\pi) = \frac{(1+S)(U-1)}{ctg\frac{m\pi}{2} + tg\frac{m\pi}{2}SU} \end{aligned} \tag{38}$$

for $h_{(2,1)} = 1$. For $U = 1$ the second set of quantization rules (38) coincide with (37), as it should. As promised, the quantization conditions involve explicitly the complex structure moduli and the dilaton S. We stress that the parameter m was already quantized before, so the quantized quantities are really the vev's of the moduli fields. Notice that at the global minimum values, the quanta of charge are zero. The quantization conditions tell us that the classical minimum is valid at a non-perturbative level. Another, more intriguing possibility, would to get a quantum minimum which satisfy the quantization rules with a large S, in order to accomodate the low energy observed phenomenology.

Another quantity of interest which appears here is a background value of $G_{\mu\nu\rho\sigma}$[7]. For simplicity reasons, we are computing it in the most simple case, $h_{(2,1)} = 0$. We use (10) in order to find

$$\hat{S} = \frac{1 - cosmx_5 z_1}{1 + cosmx_5 z_1} \ , \hat{a} = \frac{-sinmx_5 z_1}{1 + cosmx_5 z_1} \ , \hat{a}_1 = i\frac{(z_1 - z_1^\dagger)cosmx_5}{|1 + cosmx_5 z_1|^2} \ . \qquad (39)$$

Then, the Hodge duality (7) gives us the remarkably simple result

$$\hat{G}_{\mu\nu\rho\sigma} = -\frac{m}{\sqrt{2}} e^{-3\sigma} a_1 sinmx_5 \epsilon_{\mu\nu\rho\sigma} \ . \qquad (40)$$

The $4d$ lagrangian, neglecting for the moment the Scherk-Schwarz potential, is invariant under discrete shifts of the axion field a_1. This means, by using (40) that, on the $x_5 = \pi$ boundary, $G_{\mu\nu\rho\sigma}$ is quantized too. It is already known that, in any case, G is quantized [22] due to the modified Bianchi identity (24), so we showed that this holds true in the context discussed here. In complete analogy with the discussion concerning $G_{5ijk}, G_{5ij\bar{k}}$, the scalar potential (11) reach its minimum for $a_1 = 0$, corresponding to a zero quantum of charge.

4 Conclusions

This paper studies a class of $4d$ models obtained by compactifying from $11d \to 5d \to 4d$ the M-theory of Horava and Witten and twisting the boundary conditions in the fifth dimension à la Scherk-Schwarz. General features of such models are given, based on geometrical properties of the $5d$ theory. The presence of a gaugino condensate on one of the boundaries is also included, showing that indeed this corresponds to a larger gauge coupling on this boundary, still keeping the cosmological constant to zero. We showed explicitly that M_{11} is the scale of the gaugino condensation and that $m_{3/2} \sim < \bar{\lambda}\lambda > /M_P^2$, as conjectured in [6].

It is shown that, due to the twisted boundary conditions, magnetic charges appear in the system giving quantization rules similar to that discussed by Rohm and Witten in the case of perturbative string theories. We claim that the Scherk-Schwarz mechanism here is a manifestation in strong coupling

[7] I would like to thank Pierre Binétruy for collaboration in this part of the paper.

regime of the Rohm-Witten mechanism. The addition of the gaugino condensate leads to a physical picture which is very close to that discussed in [10]. More precisely, our picture is not that the Scherk-Schwarz mechanism is the gaugino condensation in the M-theory context, but that the gaugino condensation combine with the Scherk-Schwarz mechanism in the same way the gaugino condensation combines with the quantization of the antisymmetric field strength in the perturbative strings. On the other hand, in the M-theory case, the phenomenological perspectives are certainly better, since we naturally get a condensation scale of order M_{11}, compared to a scale of order M_P in the perturbative case, asked by the Rohm-Witten quantization rules.

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