Supersymmetry breaking in M-theory[*]

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Abstract

We describe the breaking of supersymmetry in M-theory by coordinate dependent (Scherk-Schwarz) compactification of the eleventh dimension. Supersymmetry is spontaneously broken in the gravitational and moduli sector and communicated to the observable sector, living at the end-point of the semicircle, by radiative gravitational interactions. This mechanism shares the generic features of non-perturbative supersymmetry breaking by gaugino condensation, in the presence of a constant antisymmetric field strength, in the weakly coupled regime of the heterotic string, which suggests that both mechanisms could be related by duality. In particular an analysis of supersymmetric transformations in the infinite-radius limit reveals the presence of a discontinuity in the spinorial parameter, which coincides with the result found in the presence of gaugino condensation, while the condensate is identified with the quantized parameter entering the boundary conditions.

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We describe the breaking of supersymmetry in M-theory by coordinate dependent (Scherk-Schwarz) compactification of the eleventh dimension. Supersymmetry is spontaneously broken in the gravitational and moduli sector and communicated to the observable sector, living at the end-point of the semicircle, by radiative gravitational interactions. This mechanism shares the generic features of non-perturbative supersymmetry breaking by gaugino condensation, in the presence of a constant antisymmetric field strength, in the weakly coupled regime of the heterotic string, which suggests that both mechanisms could be related by duality. In particular an analysis of supersymmetric transformations in the infinite-radius limit reveals the presence of a discontinuity in the spinorial parameter, which coincides with the result found in the presence of gaugino condensation, while the condensate is identified with the quantized parameter entering the boundary conditions.

1. Introduction

Models of particle physics derived from the 10-dimensional (10D) $E_8 \times E_8$ heterotic string, compactified on an appropriate 6D internal manifold, are the most attractive candidates for describing the observed low-energy world. In particular, compactification on a Calabi-Yau (CY) manifold leads to a 4D $N = 1$ supersymmetric theory that can accommodate the gauge group and matter content of the standard model. One difficulty is the mismatch between the gauge coupling unification scale, $M_G \sim 10^{16}$ GeV, and the heterotic string scale M_H, which is determined in terms of the Planck mass, $M_p \sim 10^{19}$ GeV, as

$$M_H = (\alpha_G/8)^{1/2} M_p \sim 10^{18} \text{ GeV,} \qquad (1)$$

where $\alpha_G \sim 1/25$ is the unification gauge coupling. However, the perturbative relation $M_G = M_H$ does not hold if the compactification scale $V^{-\frac{1}{6}}$ [compactification volume$\equiv (2\pi)^6 V$] $\ll M_H$, in which case the 10D theory is strongly interact-

ing,

$$\lambda_H = 2(\alpha_G V)^{1/2} M_H^3 \gg 1 \qquad (2)$$

and the scales mismatch can be given an interesting solution.

2. Strongly coupled $E_8 \times E_8$ heterotic string

The strong coupling limit of the heterotic $E_8 \times E_8$ superstring compactified on a CY manifold is believed to be described by the eleven-dimensional M-theory compactified on $CY \times S^1/Z_2$, where the semicircle has a radius ρ [1]. The relations between the eleven- and ten-dimensional parameters are [2]:

$$M_{11} = M_H \left(\frac{\sqrt{2}}{\lambda_H} \right)^{1/3} \qquad \rho^{-1} = \frac{1}{\sqrt{2}\lambda_H} M_H , \quad (3)$$

where we have defined the eleven-dimensional scale $M_{11} = 2\pi (4\pi \kappa_{11}^2)^{-1/9}$ [3] and κ_{11} is the 11D gravitational coupling. When the ten-dimensional heterotic coupling is large ($\lambda_H \gg 1$), the radius of the semi-circle is large and M-theory is weakly coupled on the world-volume.

Using eqs. (1) and (2), one can express M_{11} and ρ in terms of the four-dimensional parameters:

$$M_{11} = (2\alpha_G V)^{-1/6} \qquad \rho^{-1} = \frac{4}{\alpha_G} M_{11}^3 M_p^{-2} . \quad (4)$$

In this regime, the value of the unification scale M_G becomes $\sim M_{11}$, which is lower than M_H (and can be fixed to the desired value because of V), while the radius ρ of the semicircle is at an intermediate scale $\rho^{-1} \sim 10^{12}$ GeV, and for isotropic CY the compactification scale $V^{-1/6}$ is of the order of M_{11}[4]. Fortunately, this is inside the region of validity of M-theory, $\rho M_{11} \gg 1$ and $(2\pi)^6 V \kappa_{11}^{-4/3} \gg 1$. As a result, the effective theory above the intermediate scale behaves as 5-dimensional, but only in the gravitational and moduli sector; the gauge sectors coming from $E_8 \times E_8$ live at the 4D boundaries of the semicircle.

3. Compactification of M-theory on $CY \times S^1/\mathbf{Z}_2$

Here, we review the main properties of M-theory compactification in four dimensions on a seven-dimensional internal space, which is the product of a Calabi–Yau manifold with the semicircle S^1/Z_2. Proceeding in two steps, we will first consider the five-dimensional theory on a Calabi–Yau threefold with Hodge numbers $h_{(1,1)}$ and $h_{(2,1)}$ leading to $N = 1$ 5D space-time supersymmetry [5]. In addition to the gravitational multiplet,

$$\{e_N^M(5), \ \psi_M(8), \ A_M(3)\} \qquad (5)$$

$(M, N = 1, \cdots, 5)$ where we have indicated in parenthesis the corresponding number of degrees of freedom, the massless spectrum consists of $n_V = h_{(1,1)} - 1$ vector multiplets

$$\{A_M(3), \ \phi(1), \ \psi(4)\} \qquad (6)$$

and $n_H = h_{(2,1)} + 1$ hypermultiplets. The gauge group is abelian, $U(1)^{n_V + 1}$, where the additional factor counts the graviphoton. Starting with the eleven-dimensional fields,

$$\{e_I^J(44), \ A_{IJK}(84), \ \psi_I(128)\} \qquad (7)$$

$(I, J, K = 1, \cdots, 11)$, and splitting the Lorentz indices as $(M, i, \bar{j})$ with $M = 1, \ldots, 5$ and $i, \bar{j} = 1, 2, 3$, the $h_{(1,1)}$ gauge fields are given by $A_{Mi\bar{j}}$ while the $h_{(1,1)} - 1$ vector moduli correspond to $g_{i\bar{j}}$ with unit determinant. Moreover, the hypermultiplet moduli are given by the $h_{(2,1)}$ complex

scalar pairs $(g_{ij}, A_{ij\bar{k}})$, along with the universal scalars $(\det g_{i\bar{j}}, A_{MNP}, A_{ijk} = a\epsilon_{ijk})$.

The second step consists in the compactification of the previous 5D theory down to four dimensions on S^1/Z_2, where the Z_2 acts as an inversion on the fifth coordinate $y \to -y$ and changes the sign of the 11D 3-form potential $A \to -A$ [1]. As a result, one obtains $N = 1$ supersymmetry in four dimensions together with $h_{(1,1)} + h_{(2,1)} + 1$ massless chiral multiplets. The corresponding scalar moduli are the $h_{(1,1)}$ real pairs $(g_{i\bar{j}}, A_{5i\bar{j}})$, the $h_{(2,1)}$ complex scalars g_{ij} and the universal real pair $(g_{55}, A_{5\mu\nu})$.

On top of the massless states, there is the usual tower of their Kaluza–Klein excitations with masses

$$M^2 = \frac{n^2}{\rho^2} \quad ; \quad n = 0, \pm 1, \ldots \qquad (8)$$

corresponding to the fifth component of the momentum, p_5, which is quantized in units of the inverse radius of S^1, $1/\rho$. Because of the Z_2 projection, only the symmetric combination of their excitations $|n\rangle + |-n\rangle$ survive. On the other hand, the Z_2-odd states that were projected away at the massless level, give rise to massive excitations corresponding to the antisymmetric combination $|n\rangle - |-n\rangle$. It follows that all states of the 5D theory appear at the massive level.

In addition to these untwisted fields, there are twisted states located at the two end-points of the semicircle, giving rise to the gauge group and to ordinary matter representations. In the case of the standard embedding, there is an E_8 sitting at one of the end-points and an E_6 with $h_{(1,1)}$ **27** and $h_{(2,1)}$ $\overline{\mathbf{27}}$ matter chiral multiplets sitting at the other. Of course, in any realistic model, E_6 should be further broken down to the standard model gauge group, for instance by turning on (discrete) Wilson lines.

4. Supersymmetry breaking by Scherk-Schwarz on the eleventh dimension

The $N = 1$ supersymmetry transformations in the 5D theory are [6]:

$$\delta e_M^m \ = \ -\frac{i}{2} \overline{\mathcal{E}} \Gamma^m \Psi_M$$

$$\delta \Psi_M \quad = \quad D_M \mathcal{E} + \cdots \qquad (9)$$

where e_M^m is the fünfbein, $\Gamma^m = (\gamma^\mu, i\gamma_5)$ are the Dirac matrices, Ψ_M is the gravitino field, $\mathcal{E}$ the spinorial parameter of the transformation, and the dots stand for non-linear terms. Similar transformations hold for the components of vector multiplets and hypermultiplets for which our subsequent analysis can be generalized in a straightforward way.

All fermions in eq. (9) can be represented as doublets under the $SU(2)$ R-symmetry whose components are subject to the (generalized) Majorana condition; in a suitable basis [7]:

$$\Psi \equiv \left(\begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right) = \left(\begin{array}{c} \gamma_5 \psi_2^* \\ -\gamma_5 \psi_1^* \end{array} \right) , \qquad (10)$$

where Ψ describes any generic (Dirac) spinor. It is convenient to decompose the spinors with respect to the Γ_5 chirality. Using the relations $\gamma_5^2 = 1$ and $\gamma_5^* = -\gamma_5$, which are valid in the above basis, it follows that $\gamma_5 \psi_1 = \pm\psi_1$ implies $\gamma_5 \psi_1^* = \mp\psi_1^*$. We can then define:

$$\Psi_{L,R} \equiv \left(\begin{array}{c} \psi_{L,R} \\ \pm\psi_{R;L}^* \end{array} \right); \quad \Psi = \Psi_L + \Psi_R , \qquad (11)$$

in terms of the 4D chiral spinors $\psi_{L,R} = \pm\gamma_5 \psi_{L,R}$, where $\psi_{R,L}^* \equiv (\psi_{L,R})^*$. This decomposition amounts, in terms of $SU(2)_R$ doublets, to the condition

$$\Gamma_5 \Psi_{L,R} = \pm\Psi_{L,R}; \quad \Gamma_5 = \left(\begin{array}{cc} \gamma_5 & 0 \\ 0 & -\gamma_5 \end{array} \right) . \quad (12)$$

We can now define the $\mathcal{R}$-chirality in terms of the spinors defined in eq. (11),

$$\mathcal{R}\Psi_{L,R}(x^\mu, y) = \pm\eta\Psi_{L,R}(x^\mu, -y) . \qquad (13)$$

with $\eta = 1$ for $\Psi = \Psi_\mu$ and $\eta = -1$ for $\Psi = \Psi_5$. The $\mathbf{Z}_2$ projection is defined by keeping the states that are even under $\mathcal{R}$. It follows that the remaining massless fermions are the left-handed components of the 4D gravitino $\Psi_{\mu L}$, as well as the right-handed components of Ψ_{5R}. Taking into account the $\mathbf{Z}_2$ action in the bosonic sector, which projects away the off-diagonal components of the fünfbein (e_μ^5), the above massless spectrum is consistent with the residual $N = 1$ supersymmetry transformations at $D = 4$ given by eq. (9) with a

fermionic parameter $\mathcal{E}$ reduced to its left-handed component $\mathcal{E}_L$.

In order to spontaneously break supersymmetry, we apply the Scherk-Schwarz mechanism on the fifth coordinate y [8]. For this purpose, we need an R-symmetry, which transforms the gravitino non-trivially, and impose boundary conditions, around S^1, which are periodic up to a symmetry transformation:

$$\Psi_M(x^\mu, y + 2\pi\rho) = e^{2i\pi\omega Q}\Psi_M(x^\mu, y) , \qquad (14)$$

where Q is the R-symmetry generator and ω the transformation parameter. The continuous symmetry is in general broken by the compactification to some discrete subgroup, leading to quantized values of ω. For instance, in the case of $\mathbf{Z}_N$ one has $\omega = 1/N$ and $Q = 0, \ldots, N-1$. For generic values of ω, eq. (14) implies that the zero mode of the gravitino acquires an explicit y-dependence:

$$\Psi_M(x^\mu, y) = U(y)\Psi_M^{(0)}(x^\mu) + \cdots; \ U = e^{i\frac{\omega}{\rho}yQ} (15)$$

where the dots stand for Kaluza-Klein (KK) modes.

Consistency of the theory requires that the matrix U commutes with the reflection $\mathcal{R}$, which defines the $N = 1$ projection [9,10]. From eq. (13) one then finds:

$$\Gamma_5 U(-y) = U(y)\Gamma_5 , \qquad (16)$$

implying that the generator Q anticommutes with Γ_5, $\{Q, \Gamma_5\} = 0$. Notice that condition (16) guarantees that the $\mathcal{R}$-chirality of a spinor, $\Psi_{L,R}(x^\mu, y)$, in the sense of eq. (13), coincides with the Γ_5-chirality of its zero-mode $\Psi_{L,R}^{(0)}(x^\mu)$, in the sense of eq. (12). In this way one can write the decomposition (15) for the chiral components of Ψ, i.e. $\Psi_{L,R}(x^\mu, y) = U(y)\Psi_{L,R}^{(0)}(x^\mu)$. A solution is given by [11] (for the general solution see [12]):

$$Q = \sigma_2; \quad U = \cos\frac{\omega y}{\rho} + i\sigma_2 \sin\frac{\omega y}{\rho} , \qquad (17)$$

where σ_i are the Pauli matrices representing $SU(2)_R$ generators.

For the particular value $\omega = 1/2$ there is an additional solution to eq. (16) [12],

$$Q = 1 \quad ; \quad U = -\cos\frac{y}{2\rho} , \qquad (18)$$

which, acting on the 5D fields, consists on $\exp\{i\pi Q\} = (-)^{2s}$, changing the sign of fermions and leaving bosons invariant. This solution involves both $n = 0$ and $n = -1$ KK-modes, which makes the effective field theory description of the spontaneous supersymmetry breaking more complicated. For this reason we restrict the analysis in this section to the solution (17).

Inspection of the supersymmetry transformations (9), together with the requirement that the fünfbein zero mode does not depend on y, $e_M^m = e_M^m(x^\mu)$, shows that the y-dependence of the supersymmetry parameter is the same as that of the gravitino zero-mode [8], i.e.

$$\mathcal{E}(x^\mu, y) = U(y)\mathcal{E}^{(0)}(x^\mu) . \tag{19}$$

Supersymmetry in the 4D theory is then spontaneously broken, with the goldstino being identified with the fifth component of the 5D gravitino, $\Psi_5^{(0)}$. Indeed, for global supersymmetry parameter, $D_\mu \mathcal{E}^{(0)} = 0$, its variation is [Note that the operator $U^{-1}\partial_y U$ turns a left-handed spinor, in the sense of eq. (12), into a right-handed one.]:

$$\delta\Psi_5^{(0)} = i\sigma_2 \frac{\omega}{\rho}\mathcal{E}^{(0)} + \cdots \tag{20}$$

while no other fermions can acquire finite constant shifts in their transformations.

The above arguments are also valid in the $N = 1$ theory, obtained by applying the $\mathbf{Z}_2$ projection defined through the $\mathcal{R}$-reflection (13). The y-dependence of the remaining zero modes is always given by eq. (15). Supersymmetry is spontaneously broken:

- The goldstino is identified as the right-handed component

$$\text{goldstino} \equiv \psi_{5R}^{(0)}, \tag{21}$$

which, from eq. (20), transforms as:

$$\delta\psi_{5R}^{(0)} = \frac{\omega}{\rho}\varepsilon_R^{(0)*} + \cdots \tag{22}$$

- The surviving gravitino is $\Psi_{\mu L}^{(0)}$ in the notation of eq. (11). Its mass is given by

$$m_{3/2} = \frac{\omega}{\rho} \tag{23}$$

- In the limit $\rho \to \infty$, supersymmetry is locally restored: $m_{3/2} \to 0$, $\delta\psi_{5R}^{(0)} \to 0$.

Note, however, that the above analysis in the $N = 1$ case is valid, strictly speaking, for values of y inside the semicircle, obtained from the interval $[-\pi\rho, \pi\rho]$ through the identification $y \leftrightarrow -y$. This leads to a discontinuity in the transformation parameter $\mathcal{E}$ around the end-point $y = \pm\pi\rho$, since $U(-\pi\rho) = U^{-1}(\pi\rho)$:

$$\mathcal{E}(-\pi\rho) \neq \mathcal{E}(\pi\rho) . \tag{24}$$

This discontinuity survives even in the large-radius limit $\rho \to \infty$ where the gravitino mass vanishes and supersymmetry is restored locally. This phenomenon is reminiscent of the one found in ref. [13], where the discontinuity at the weakly coupled end $y = \pi\rho$ is due to the gaugino condensate of the hidden E_8 formed at the strongly coupled end $y = 0$. In fact the two results become identical for the transformation parameter $\mathcal{E}$ in the neighbourhood $y \sim \pi\rho$, in the limit $\rho \to \infty$:

$$\varepsilon_L(y) \sim \cos\pi\omega\, \varepsilon_L^{(0)} + \epsilon(y)\sin\pi\omega\, \varepsilon_R^{(0)*} . \tag{25}$$

On the other hand, it is easy to see that the goldstino variation vanishes in this limit, since the discontinuity in $\partial_y\varepsilon(y)_L$ is proportional to $\delta(y)\sin(y\omega/\rho)$. The transformation parameter $\varepsilon_L(y)$ is thus identified with the spinor η' of ref. [13], which solves the unbroken supersymmetry condition $\delta\psi_{5R} = 0$.

5. Supersymmetry breaking in the observable sector

At the lowest order, supersymmetry is broken only in the five-dimensional bulk (gravitational and moduli sector), while it remains unbroken in the observable sector. The communication of supersymmetry breaking is then expected to arise radiatively, by gravitational interactions. This issue is studied below in the particular case $\omega = 1/2$ and $\exp\{i\pi Q\} = (-)^{2s}$ [14], though more general cases can be equally computed [15].

5.1. Scalar masses

At the one-loop level, the diagrams that contribute to the scalar masses in the observable sec-

tor were studied in ref. [14], where the vertices come from the kinetic terms.

$$\mathcal{G}^{\varphi\varphi}_{z(n)} \, \phi\overline{\psi}_\varphi \gamma_\mu D^\mu \chi^{(n)} + \cdots \; . \qquad (26)$$

Fields from the boundary, generically denoted by (φ, ψ_φ), always appear in pairs, as dictated by the Z_2 invariance. Moreover, in the effective field theory limit $\rho M_{11} \gg 1$, their couplings to fields from the bulk are the same for all Kaluza–Klein excitations. The latter are the moduli $(z^{(n)}, \chi^{(n)})$ and the graviton $(g^{(n)}_{\mu\nu}, \psi^{(n)}_\mu)$ supermultiplets. Moduli and matter field indices are dropped for notational simplicity.

After adding the contribution of diagrams related by supersymmetry, we obtain the following expression for the scalar masses:

$$m^2_{\varphi\bar\varphi} \sim G^{-1}_{\varphi\bar\varphi} \left(G^{i\bar{j}} R_{i\bar{j}\varphi\bar\varphi} - G_{\varphi\bar\varphi} \right) \frac{m^4_{3/2}}{M^2_p} \mathcal{J} \; , \qquad (27)$$

where we used the relation (23) for $m_{3/2}$, and $\mathcal{J}$ is a constant given by:

$$\mathcal{J} = \int_0^\infty \frac{dx}{x^3} \left(\frac{\pi}{x} \right)^{1/2} \left[\theta_3 \left(\frac{i\pi}{x} \right) - \theta_4 \left(\frac{i\pi}{x} \right) \right] \quad (28)$$

where θ_i are the Jacobi theta-functions and we have used the Poisson resummation formula. In eq. (27), $G_{i\bar{j}}$ and $G_{\varphi\bar\varphi}$ are the moduli and matter metrics, while $R_{i\bar{j}\varphi\bar\varphi}$ is the moduli-matter mixed Riemann tensor. The factor $G^{-1}_{\varphi\bar\varphi}$ comes from the wave function renormalization and the two terms in the bracket correspond to the contributions of the moduli and graviton supermultiplets. As a result, we find the scalar masses $m_{\varphi\bar\varphi} = \mathcal{O}(10^{-1}) \, m^2_{3/2}/M_p \sim 10^4 \, \mathrm{GeV}$ generically. This is only a rough estimate, since besides the moduli dependent prefactor in eq. (27), the result is very sensitive to the value of M_{11}. In fact, eqs. (4) and (23) show that $m_{\varphi\bar\varphi}$ scales as M^6_{11} and, thus, a modest factor of 2 in M_{11} changes the scalar masses by almost two orders of magnitude.

A similar analysis can be applied to compute the masses of the scalar moduli. The evaluation of the corresponding diagrams yields [14]:

$$m^2_{z\bar{z}} \sim 5 \, G^{-1}_{z\bar{z}} \left(R_{z\bar{z}} - G_{z\bar{z}} \right) \frac{m^4_{3/2}}{M^2_p} \mathcal{J} \; , \qquad (29)$$

where $R_{z\bar{z}}$ is the moduli Ricci tensor and the constant $\mathcal{J}$ is given in eq. (28). Thus, all moduli scalars obtain masses of the same order as the scalar masses in the observable sector, $\mathcal{O}(10)$ TeV.

The fact that all scalar squared mass splittings are of order $m^4_{3/2}/M^2_p$ is a consequence of the absence of quadratic divergences in the effective supergravity. Inspection of eq. (28) shows that cancellation of quadratic divergences arises non-trivially. Indeed, any single excitation n of the sum gives a contribution to the integral, which is quadratically divergent at $x = 0$ as dx/x^2, so that after introducing an ultraviolet cutoff $\propto 1/M^2_p$ one would get a contribution of order $m_{3/2}$ to the mass. However, after summing over all modes and performing the Poisson resummation, one finds that the integrand has an exponentially suppressed (non-analytic) ultraviolet behaviour as $e^{-\pi^2/x}/x^{5/2}$. One can also compute the effective potential as a function of the background ρ:

$$V_{\mathrm{eff}} = -\frac{N\mathcal{J}}{32\pi^2} \frac{1}{\rho^4} \; , \qquad (30)$$

where N is the number of massless multiplets from the bulk. This result explicitly shows the vanishing of $\mathrm{Str}M^2$ after supersymmetry breaking.

5.2. Gaugino masses

Gaugino masses also receive radiative gravitational contributions. At the one-loop level they lead to individual contributions:

$$m_{\lambda\lambda} \propto \frac{m^3_{3/2}}{M^2_p} \; , \qquad (31)$$

where we followed the same steps as in the case of scalar masses.

The above result shows that the one-loop gravitational contributions to gaugino masses are too small for phenomenological purposes. This is a general problem, which has been known for a long time [16,17]. A possible solution exists if there are massive matter fields transforming non-trivially under the gauge group. Then, their mass splittings generate gaugino masses by one-loop diagrams involving gauge interactions. The latter

lead to finite contributions given by [18]:

$$m_{\lambda\lambda} \sim N_s \frac{\alpha}{2\pi} \mu f\left(\frac{m_s}{\mu}\right) \ , \qquad (32)$$

where μ is the supersymmetric mass and m_s^2 the squared mass splitting of those matter fields; N_s denotes their multiplicity, α is the corresponding gauge coupling, and the function $f(x)$ is nearly constant for $x \gtrsim 2$ while it behaves as x for $x \lesssim 1$. Thus, the gaugino masses are of the order of $(\alpha/2\pi)N_s \min(\mu, m_s)$.

It is easy to see that when μ is below the intermediate scale ρ^{-1}, the evaluation of the scalar masses (27), (28) remains valid up to $\mathcal{O}(\mu/m_{3/2})$ corrections. It follows that the gaugino masses are approximately one order of magnitude lower than the scalar masses if $\mu \gtrsim m_s$. Although this mechanism can give acceptable masses to charginos and neutralinos, provided that the Higgs supersymmetric parameter μ is large enough, gluino masses would require the Standard Model particle content to be extended by the presence of extra colour multiplets in vector-like representations such as triplets or leptoquarks. Of course, in this case, unification requires that the extra matter appears in complete $SU(5)$ representations, e.g. $(\mathbf{5} + \overline{\mathbf{5}})$ or $(\mathbf{10} + \overline{\mathbf{10}})$. Otherwise, in the absence of extra matter, this scenario leaves open the possibility of having light gluinos with masses of order $(\alpha_3/2\pi)\, m_{\text{top}} = \mathcal{O}(1)$ GeV [19].

To summarize, the mass spectrum we obtained in the observable sector originates from local supersymmetry breaking, with a gravitino mass $m_{3/2}$ at an intermediate scale defined by the size of the eleventh dimension of M-theory. All scalars then acquire masses of order $m_{3/2}^2/M_p$, while gaugino masses are of order $m_{3/2}^3/M_p^2$. This situation is again identical to the case where supersymmetry is broken in the heterotic string by gaugino condensation stabilized by a VEV of the antisymmetric tensor field strength [16,17]. As we saw, the problem of having very light gauginos can be solved by means of gauge interactions involving extra fields and providing gaugino masses of order $(\alpha/2\pi)m_{3/2}^2/M_p$. Therefore, this scenario predicts a hierarchy of supersymmetric mass spectrum where scalars are much heavier than gauginos.

Finally, in the old analysis of gaugino condensation, based on the heterotic string tree-level effective supergravity, it was found that scalars remained massless at the one-loop order, because of the dilaton properties of the Kähler potential [16]. In fact, it is easy to see that the term in the brackets of eq. (27) vanishes when the Kähler potential has for instance the no-scale $SU(1,n)$ form $K = -3\ln(z + \bar{z} - |\varphi|^2)$ and φ's have zero VEVs. This can lead to an alternative scenario where gauginos, with masses of order $m_{3/2}^3/M_p^2 \sim 1$ TeV (for $m_{3/2} \sim 10^{14}$ GeV), seed supersymmetry breaking in the rest of the observable sector by gauge interactions. Since in this case the corresponding diagrams are logarithmically divergent, all supersymmetric masses turn out to be of the same order of magnitude. However, this scenario is expected (and was explicitly shown [17]) to be unstable under higher-order loop corrections, as the dilaton symmetry is in general broken at the quantum level.

6. Relation with gaugino condensation

Supersymmetry breaking by Scherk-Schwarz compactification of the eleventh dimension reproduces the main features (at least in the simplest case) of gaugino condensation in the weakly coupled heterotic string. Then, it is natural to conjecture that it provides a dual description of gaugino condensation in the strongly coupled regime [14,12].

6.1. The weakly coupled heterotic string

On the heterotic side, one expects that (local) supersymmetry can be broken by gaugino condensation effects in the hidden E_8, at least in the (10D) weakly coupled regime [20]. Let us briefly describe the main features of this mechanism. The physical picture is that the condensate $\langle\lambda\lambda\rangle$ develops at a scale Λ_c, where the gauge coupling of the hidden E_8 becomes strong:

$$\langle\lambda\lambda\rangle \sim \Lambda_c^3 = \mu^3 e^{-\frac{2\pi}{c_8 \alpha_8(\mu)}} \ , \qquad (33)$$

with $c_8 = 30$ being the quadratic Casimir of E_8 and $\alpha_8(\mu)$ its coupling constant at the scale μ.

This phenomenon can be described by introducing a chiral supermultiplet U whose vacuum expectation value (VEV) reproduces the condensate (33) [21]. The effective non-perturbative superpotential is determined by consideration of the anomalous Ward identities:

$$W_{\rm np} \propto U \left(\frac{1}{\alpha_W} + \frac{c_8}{2\pi} \ln \frac{U}{\mu^3} \right) \ , \qquad (34)$$

where α_W is the Wilsonian effective coupling (at the scale μ), which depends holomorphically on the moduli [22]. It is related to the physical coupling by:

$$\frac{1}{\alpha_8} = {\rm Re} \frac{1}{\alpha_W} + \frac{c_8}{4\pi}(-K + 2\ln(S + \bar{S})) \ , \qquad (35)$$

where K is the Kähler potential and S is the heterotic dilaton whose VEV determines the 4D string coupling constant, ${\rm Re}S = 1/\alpha_G$.

Minimization of the effective potential with respect to U implies to leading order in Λ_c/M_p the condition $\partial_U W_{\rm np} = 0$ [23], which gives

$$U = \mu^3 e^{-\frac{2\pi}{c_8 \alpha_W} - 1} \qquad ; \qquad W_{\rm np} \propto U \ . \qquad (36)$$

Using this result together with eqs. (33) and (35), it is straightforward to obtain the value of the gravitino mass:

$$m_{3/2} = |W_{\rm np}|e^{K/2} \propto \frac{1}{\alpha_G}\Lambda_c^3 M_p^{-2} \ . \qquad (37)$$

The effective potential should also be minimized with respect to the dilaton field S. Unfortunately, its runaway behaviour drives the theory to the supersymmetric limit with vanishing coupling, $S \to \infty$. A possible stabilization mechanism was initially proposed by means of a VEV for the field-strength of the antisymmetric tensor field along the compact directions, which shifts the superpotential by a constant [20]. However, this constant was found to be quantized, so that $W_{\rm np}$ becomes of order one at the minimum [24]. Then, eq. (37) implies that the only way to obtain a hierarchy for the gravitino mass is by making $e^{K/2}$ small, or equivalently by having a large compactification volume $V \sim e^{-K}$. As a result, we obtain the following scaling relations (in M_p units):

$$m_{3/2} \sim V^{-1/2}; \quad \Lambda_c \sim V^{-1/6} \ . \qquad (38)$$

Assuming that eqs. (37) and (38) hold in the strong coupling regime, a comparison with the duality relations (4) implies the identification of the condensation scale Λ_c with the M-theory scale M_{11} and the inverse radius of the semicircle ρ^{-1} with the gravitino mass:

$$\Lambda_c \sim M_{11} \quad ; \qquad m_{3/2} \sim \rho^{-1} \ . \qquad (39)$$

6.2. M-theory

In the strongly coupled regime:

- The relation $m_{3/2} \sim \rho^{-1}$ is provided by the Scherk-Schwarz mechanism as we have seen in previous sections.

- In the description of gaugino condensation by the Scherk-Schwarz mechanism, the condensation scale is identified with the M-theory scale M_{11}. This implies that the hidden E_8 is strongly coupled and should not contain any massless matter in the perturbative spectrum. Consistency then requires that the corresponding gauge coupling be large, $\alpha_8(M_{11}) \gtrsim 1$.

On the M-theory side this provides a constraint that naively fixes the 4D unification coupling α_G to be in a non-perturbative regime. Fortunately, there are important M-theory threshold effects that invalidate this conclusion. These effects can be understood from the lack of factorization of the 7-dimensional internal space as a direct product of the semicircle with a Calabi-Yau manifold, ${\rm CY} \times S^1/{\bf Z}_2$ [2]. As a result, the Calabi-Yau volume V becomes a function of ρ and takes different values at the two end-points of the semicircle. In the large-radius limit, one finds [2]:

$$V(0) = V(\pi\rho) - \frac{1}{32\pi^2}\rho M_{11}^{-3} \qquad (40)$$

$$\left| \int_{\rm CY} \frac{\omega}{4\pi^2} \wedge ({\rm tr}F' \wedge F' - {\rm tr}F \wedge F) \right| \ ,$$

where ω is the Kähler form of CY and F' (F) is the field strength of the strongly (weakly) coupled E_8 sitting at the end-point $y = 0$ ($y = \pi\rho$). The integral in the r.h.s. is a linear function of the $h_{(1,1)} - 1$ Kähler class moduli for unit volume, which belong to 5D vector multiplets. Its natural

value is M_{11}^{-2} up to a proportionality factor of order 1 [2].

Following eq. (4), the gauge coupling constants at the two end-points are related to the corresponding volumes as [1]:

$$\frac{1}{\alpha_G} = 2M_{11}^6 V(\pi\rho); \quad \frac{1}{\alpha_8(M_{11})} = 2M_{11}^6 V(0), \quad (41)$$

Imposing now the constraint

$$\alpha_8(M_{11}) \gtrsim 1; \quad M_{11} \sim \Lambda_c \quad (42)$$

and using eqs. (40) and (41), one finds $\rho \sim \rho_{\rm crit}$ where $\rho_{\rm crit}$ corresponds to the critical value at which the volume at the strongly coupled end vanishes and the hidden E_8 decouples from the low-energy spectrum:

$$\rho_{\rm crit}^{-1} \sim \frac{\alpha_G}{16\pi^2} M_{11} \simeq 2 \times 10^{-4} M_{11} \quad (43)$$

Note that this condition can also be thought of as resulting from a minimization of the (positive semi-definite) 4D gaugino condensation potential, which is proportional to $V(0)$ and, thus, vanishes at zero volume. It is remarkable that the above relation provides the hierarchy necessary to fix ρ^{-1} at the intermediate scale $\sim 10^{12}$ GeV, when one identifies the M-theory scale M_{11} with the unification mass $\sim 10^{16}$ GeV inferred by the low-energy data [2,4].

- The $\rho \to \infty$ limit

We have already mentioned that in the $\rho \to \infty$ limit both gaugino condensation and Scherk-Schwarz mechanisms lead to similar conclusions on supersymmetry breaking. In fact the proportionality constant $\sin \pi\omega$ in eq. (25) plays the role of the gaugino condensate in the dual description and vanishes only for integer values of ω for which the Scherk-Schwarz mechanism becomes trivial. In general ω is quantized, as we discussed earlier, which is consistent with the quantization of the gaugino condensate through its equation of motion that relates it with the VEV of the antisymmetric tensor field strength [24].

In the presence of gaugino condensation, the discontinuity in the function $\epsilon(y)$ (24) was interpreted as a topological obstruction that signals supersymmetry breaking when effects of finite radius would be taken into account [13]. Here we have shown that the same discontinuity, in the infinite-radius limit, is reproduced by the Scherk-Schwarz mechanism.

7. Conclusion

- The eleventh dimension of the M-theory seems an interesting candidate to perturbatively break supersymmetry in the gravitational and moduli sector.
- The Scherk-Schwarz mechanism of M-theory is not a single model but a framework where many different models can be accomodated. If for instance we use (a $U(1)$ subgroup of) the $SU(2)_R$, then all fermions of the vector multiplets and all complex scalars of the hypermultiplets transform in a similar fashion as the gravitino [11]. However, if we use $(-)^{2s}$ ($\omega = 1/2$), it is acting non-trivially only on the fermions of both vector multiplets and hypermultiplets [14].
- This mechanism provides an alternative "perturbative" explanation of the gauge hierarchy, where the smallness of the ratio $m_{\rm susy}/M_p \sim 10^{-16}$ is provided by powers of the unification coupling $\sim (\alpha_G/16\pi^2)^4$ instead of the conventional non-perturbative suppression $\sim e^{-1/\alpha_G}$. Of course in both cases, the remaining open problem is to determine the actual value of the gauge coupling α_G. In the present context of M-theory, this amounts to fixing the volume of the Calabi-Yau manifold $V(\pi\rho)$.
- The features of supersymmetry breaking by the Scherk-Schwarz mechanism are similar to (some) models of non-perturbative supersymmetry breaking by gaugino condensation in the weakly coupled heterotic string.
- One of the main open problems is to find the general features of the low-energy effective theory describing the mechanism of supersymmetry breaking, and the proposed equivalence between the *perturbative* breaking of supersymmetry in the M-theory, by the Scherk-Schwarz mechanism on the eleventh dimension [11], and the *non-perturbative* breaking by gaugino condensation in the heterotic string [25,26].

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