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MASSIVE STRING MODES AND NON-SINGULAR PRE-BIG-BANG COSMOLOGY

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Abstract

Perturbative α' corrections to the low energy string effective action have been recently found to have a potentially regularizing effect on the singularity of the lowest order pre-big-bang solutions. Whether they actually regularize it, however, cannot be determined working at any finite order in a perturbative expansion in powers of the string constant α', because of scheme dependence ambiguities. Physically, these corrections are dominated by the integration over the first few massive string states. Very massive string modes, instead, always have a regularizing effect which is non-perturbative in α' and which basically comes from the fact that in a gravitational field with Hubble constant H they are produced with an effective Hawking temperature $T = H/(2\pi)$, and an infinite production rate would occur if this temperature exceeded the Hagedorn temperature. Depending on which regularizing mechanism takes place first, different cosmological scenarios are possible in the string phase of the model, in which the Hubble parameter either approaches asymptotically a constant value or gets close to a maximum value and then bounces back.

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1 Introduction

String theory is an appropriate framework for discussing the singularities of general relativity and in particular the big-bang singularity, and a large number of works have been devoted to the study of cosmological solutions of the low energy effective action of string theory (see e.g. [1-5]). At lowest order in the string constant α', the solutions of the equations of motion still reach a singularity at a finite value of time, say $t = 0$, when we evolve the present state of the Universe backward in time [2]. This is not surprising since, in the lowest order effective action, α' only enters as an overall constant and therefore drops from the equations of motion; therefore, there is no scale at which the singularity can be regularized. For homogeneous fields, the low energy action has a symmetry, scale factor duality, which relates different solutions of the equations of motions [1, 6]. In the simplest case of an isotropic Friedmann-Robertson-Walker (FRW) metric with scale factor $a(t)$, and of a vanishing antisymmetric tensor field $B_{\mu\nu}$, it reads

$$a \to \frac{1}{a}, \quad \phi \to \phi - 2d \log a\,, \tag{1.1}$$

where ϕ is the dilaton field and d is the number of spatial dimensions. This symmetry can be generalized to a global $O(d, d)$ symmetry in the more general case of non-diagonal metrics and non-vanishing $B_{\mu\nu}$. Since it involves the dilaton field, it is a string symmetry, with no counterpart in Einstein gravity. Combining scale factor duality with time reversal, it is possible to associate with every 'post-big-bang' solution defined for $t > 0$ a 'pre-big-bang' solution defined for $t < 0$, through the transformation $a(t) \to 1/a(-t)$. Of course, at lowest order in α', the pre-big-bang solution also runs into a singularity as $t \to 0^-$. It is therefore natural to ask whether the inclusion of corrections allows a smooth transition between the pre-big-bang and the post-big-bang solutions, providing a non-singular cosmological model.

The effective action of string theory has two different types of perturbative corrections: higher orders in α', which are genuinely stringy corrections related to the finite extent of the string, and loop corrections, which carry higher powers of e^{ϕ}. Perturbative α' corrections provide a scale for the regularization of the singularity and have been studied in [7], where it has been found that the equations of motion in the case of constant curvature and linear dilaton reduce, at all orders in α', to a set of $(d+1)$ algebraic equations in $(d+1)$ unknowns. If these algebraic equations have a real solution, this can act as late time attractor of the lowest order pre-big-bang solution, as has been shown on specific examples at $O(\alpha')$. The singularity is then replaced by a phase of De Sitter inflation with linearly growing dilaton. To complete the transition to the standard radiation-dominated FRW model (the graceful exit problem) $O(e^{\phi})$ corrections must play a crucial role [8, 9, 10].

In this paper we examine again the role of α' corrections in string cosmology. In sect. 2, after recalling the results of ref. [7] and extending them to the case of compact dimensions (sect. 2.1), we discuss in some detail the scheme dependence of the results (sect. 2.2) and the relation with scale factor duality (sect. 2.3). Unfortunately, to verify that the mechanism proposed in [7] does take place would require the knowledge of a beta function at all orders in α'. Therefore, in sect. 3 we try to obtain a better understanding of the physics behind the perturbative corrections and of the general mechanism of smoothing of singularities in string theory. We will relate the perturbative α' corrections to the integration over the first few excited string states (sect. 3.1); instead, the mechanism of singularity regularization is related to the exponential growth of the density of states; it is therefore basically due to very massive string modes.

In sect. 4, using the result of a computation by Lawrence and Martinec [11], we study the effect on the lowest order solution of the backreaction due to very massive string modes produced by the gravitational field, and we find that it always regularizes the singularity. If both the mechanism suggested in [7] and this one are operative, the cosmological model depends on which one takes place first. If perturbative α' corrections regularize the solution, and if this happens to take place at a low value of the curvature, when the production of massive modes is still small, then the cosmological scenario of ref. [7] is appropriate and the solution approaches asymptotically a stage of De Sitter inflation, until $O(e^{\phi})$ corrections set in. But even if perturbative α' corrections do not regularize the solution, the mechanism involving very massive string modes is still operative and does regularize the singularity. In this case, rather than a phase with constant Hubble parameter H, we find that H increases up to a critical value and then suddenly bounces back. In sect. 5 we present the conclusions and we discuss possible phenomenological implications of the second scenario.

2 Perturbative α' corrections

2.1 The constant curvature solution

Let us recall the results of ref. [7]. Including first order corrections in α', a possible form of the effective action, in the string frame, is [12, 13]

$$S = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{-g}\, e^{-\phi} \left[R + (\nabla\phi)^2 - \frac{k\alpha'}{4} R^2_{\mu\nu\rho\sigma} \right] , \tag{2.1}$$

where λ_s is the string length, $\lambda_s \sim \sqrt{\alpha'}$, $k = 1, 1/2$ for the bosonic and heterotic string, respectively, and we have neglected the antisymmetric tensor field. For type II strings $k = 0$, and the first correction starts at order $R^4_{\mu\nu\rho\sigma}$ [14]. Performing the field redefinition $g_{\mu\nu} \to g_{\mu\nu} + (k\alpha')\delta g_{\mu\nu}$, $\phi \to \phi + (k\alpha')\delta\phi$ with

$$\delta g_{\mu\nu} = R_{\mu\nu} - \partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}(\nabla\phi)^2, \quad \delta\phi = \frac{1}{4}\left[R + (2d-3)(\nabla\phi)^2 \right] \tag{2.2}$$

and truncating at first order in α' gives the action

$$S = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{-g}\, e^{-\phi} \left[R + (\nabla\phi)^2 - \frac{k\alpha'}{4} \left(R^2_{GB} - (\nabla\phi)^4 \right) \right] , \tag{2.3}$$

where $R^2_{GB} = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$ is the Gauss-Bonnet term. This form is particularly convenient because higher order derivatives in the α' correction cancel after integrations by part [15]. We will discuss in some detail field redefinitions in sect. 2.2.

We now restrict ourselves to a homogeneous and isotropic *FRW* background, $ds^2 = N^2(t)dt^2 - a^2(t)d\mathbf{x}^2$ and homogeneous dilaton $\phi = \phi(t)$. Varying the action with respect to a, ϕ, we get two dynamical equations of motion. Introducing $H = \dot{a}/a$ and specializing to $d = 3$ for notational simplicity, they read (in units $k\alpha' = 1$):

$$-6\dot{H}(1 + H^2) + 2\ddot{\phi}\left(1 + \frac{3}{2}\dot{\phi}^2\right) - 12H^2 + 6H\dot{\phi} - \dot{\phi}^2 - \frac{3}{4}\dot{\phi}^4 - 6H^4 + 3H\dot{\phi}^3 = 0 \tag{2.4}$$

$$-12\dot{H}(1 - H\dot{\phi}) + 6\ddot{\phi}(1 + H^2) - 18H^2 + 12H\dot{\phi} - 3\dot{\phi}^2 + \frac{3}{4}\dot{\phi}^4 + 12H^3\dot{\phi} - 6H^2\dot{\phi}^2 = 0\,. \tag{2.5}$$

The variation with respect to the lapse function N gives, instead, a constraint on the initial values:

$$6H^2 - 6H\dot{\phi} + \dot{\phi}^2 - 6H^3\dot{\phi} + \frac{3}{4}\dot{\phi}^4 = 0\,. \tag{2.6}$$

The constraint is conserved by the dynamical equations of motion, and is therefore satisfied at any time if it is satisfied at the initial time. We can now try to solve eqs. (2.4-2.6) with the ansatz $H = \text{const} = y$, $\dot{\phi} = \text{const} = x$. The ansatz reduces the three differential equations to algebraic equations in x, y. At first sight we have three independent equations for two unknown variables. However, reparametrization invariance gives one relation between these equations such that if the constraint equation and, say, the equation obtained with a variation with respect to a (or, in the general anisotropic case, the d equations obtained with a variation with respect to a_i, $i = 1, \ldots, d$) are satisfied, then the equation obtained with a variation with respect to ϕ is automatically satisfied.

In the generic anisotropic case with scale factors a_i ($i = 1, \ldots, d$), the ansatz therefore reduces the system of $(d+2)$ differential equations to $(d+1)$ algebraic equations in $(d+1)$ variables $H_i = y_i, \dot{\phi} = x$, and this is true at all orders in α'.

These algebraic equations are nothing but the requirement that there is a zero in the beta functionals of the underlying sigma model, when the background is specialized to the form of our ansatz, and we will therefore write them as $\beta_i(\mathbf{g}) = 0$, where $i = 1, \ldots, (d+1)$ and $\mathbf{g} = (x, y_1, \ldots, y_d)$.

It is easy to generalize this result to the case of compact dimensions. Let us first consider the case in which the compact space is spatially curved, e.g. consider four-dimensional space M^4 times S^2, with metric

$$ds^2 = N^2(t)dt^2 - \sum_{i=1}^{3} a_i^2(t)dx_i^2 - b^2(t)\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{2.7}$$

(The extension to more general cases will be obvious.) Writing down the equations of motion we see that now b enters not only in the combination $\dot{b}/b$, but also through terms $\sim 1/b^2$, which are due to the curvature of the sphere. Therefore the ansatz $\dot{a}_i/a_i = \text{constant}$, $\dot{\phi} = \text{constant}$ can only be consistent if $b = \text{const}$, or $\dot{b}/b = 0$. Again for the ansatz $\dot{a}_i/a_i = y_i, \dot{\phi} = x, b = \text{const}$ the relation between equations of motion derived from reparametrization invariance eliminates one equation and we have a number of algebraic equations equal to the number of variables y_i, x, b.

The same happens if, instead, we compactify on a torus. In this case the explicit dependence on the scale factor, which forces it to be constant, comes from the winding modes, whose energy grows with the scale factor.

In conclusion, the existence of a solution with $H_i, \dot{\phi}$ constant (and b constant, for compact dimensions) depends on whether the algebraic equations discussed above have real solutions. For the action (2.3) this is indeed the case, and the solution turns out to be an attractor of the lowest order pre-big-bang solution, which is therefore regularized [7]. However, the inclusion of higher orders in α' or field redefinitions of the type used in eq. (2.2) produce different algebraic equations, which may or may not have real solutions. These issues, which were already noted in ref. [7], will be further discussed in the next section.

2.2 Scheme dependence of the results

In principle, we would like to know the beta functions $\beta_i(\mathbf{g})$ exactly, while what we have is a perturbative expansion in powers of α'. Somewhat optimistically, one might still hope to find a zero which, in units $k\alpha' = 1$, is located at $g_i \ll 1$, thus justifying a perturbative treatment. Unfortunately, an even more fundamental obstacle stands in the way. The problem is that, if we work at finite order in α', the coefficients of the algeraic equation, or of the expansion of the functions β_i, are subject to ambiguities. A straightforward way to understand this point is to observe that we can perform fields redefinitions that mix different orders in α'. The most general form of such redefinitions, at order α', is [16] $g_{\mu\nu} \to g_{\mu\nu} + (k\alpha')\delta g_{\mu\nu}$, $\phi \to \phi + (k\alpha')\delta\phi$, with

$$\delta g_{\mu\nu} = a_1 R_{\mu\nu} + a_2 \partial_\mu\phi\partial_\nu\phi + a_3 g_{\mu\nu}(\nabla\phi)^2 + a_4 g_{\mu\nu} R + a_5 g_{\mu\nu}\Box\phi \,, \tag{2.8}$$

$$\delta\phi = b_1 R + b_2(\nabla\phi)^2 + b_3\Box\phi \,. \tag{2.9}$$

It is not necessary to include a term $\nabla_\mu\partial_\nu\phi$ in eq. (2.8) because it can be reabsorbed by a general coordinate transformation [16]. After this redefinition, the new action, truncated at order α', is (in units $k\alpha' = 1$)

$$S = -\int d^{d+1}x\sqrt{-g}\, e^{-\phi}\Big[R + (\nabla\phi)^2 - \frac{1}{4}R^2_{\mu\nu\rho\sigma} + c_1 R_{\mu\nu}R^{\mu\nu} + c_2 R^2 + c_3(\nabla\phi)^4 + \\ + c_4 R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + c_5 R(\nabla\phi)^2 + c_6 R\Box\phi + c_7\Box\phi(\nabla\phi)^2 + c_8(\Box\phi)^2\Big] \,, \tag{2.10}$$

with the coefficients c_i functions of a_i, b_i. (We have eliminated terms that can be reduced to the above terms after integrations by part or use of Bianchi identity). Despite the fact that we have eight coefficients c_i and eight parameters $a_1, \dots, a_5, b_1, b_2, b_3$, we cannot fix the

c_i at arbitrary values since, from the explicit expression of the c_i as functions of $a_1, \dots, b_3$, one finds that they satisfy a relation $c_2 + c_3 + c_7 + c_8 = c_5 + c_6$. Within this constraint, however, the c_i can be chosen at will with the appropriate field redefinitions. The existence of a relation between the c_i means that there is a one-parameter family of field redefinitions which, at order α', leaves the action invariant. It is readily found to be

$$\delta g_{\mu\nu} = \zeta g_{\mu\nu} \left(R - (\nabla\phi)^2 + 2\Box\phi \right) , \tag{2.11}$$

$$\delta\phi = \zeta \left(\frac{d-1}{2} R - \frac{d+1}{2} (\nabla\phi)^2 + d\Box\phi \right) , \tag{2.12}$$

with ζ a real parameter. We now ask whether the existence of a zero of the functions β_i, at a given order in α', is affected by the field redefinitions, and the answer is positive. For instance, we can fix $c_1 = 1, c_2 = -1/4, c_3 = 1/4$, and vary c_4 (which does not enter the relation between the c_i) setting all others c_i to zero. For $c_4 = 0$ we have the action used in [7], and there is a zero of the beta functions. In $d = 3$, for the isotropic case, it is located at $x \simeq 1.404, y \simeq 0.616$. Increasing c_4 this zero disappears (escaping at infinity in the (x, y) plane) at a critical value $c_4 \simeq 0.05$.

Therefore, while we expect that, at all orders in α', the existence of a zero in the beta functions is independent of field redefinitions, this is not true for the truncation at any finite order in α'.

There are other useful ways to understand the existence of ambiguities in the perturbative expansion of the functions β_i, and these different points of view are believed to be equivalent. First, from the point of view of the underlying sigma model, they are due to the dependence of the perturbative coefficients of the beta functionals on the renormalization scheme. This dependence starts at two loops, i.e. from the terms $\sim R^2$ in the action, eq. (2.10).

The effective low energy action, on the other hand, is constructed in such a way as to reproduce the string theory S-matrix elements. From this point of view, the ambiguities in the coefficients come from the fact that some coefficients cancel in the computation of on-shell amplitudes [17, 16, 18]. Suppose for instance that we want to fix the coefficients of the operators $R^2_{\mu\nu\rho\sigma}, R^2_{\mu\nu}$ and R^2 in the effective action. We would then expand these operators around the flat metric, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and compute their contribution to the 3-point and 4-point amplitudes. However, the $\sim h^3$ part of $\sqrt{-g}R_{\mu\nu}R^{\mu\nu}$ and of $\sqrt{-g}R^2$ vanish; therefore, the coefficients c_1, c_2 in the action (2.10) cannot be determined from the knowledge of the string amplitude with three on-shell gravitons. Both $\sqrt{-g}R_{\mu\nu}R^{\mu\nu}$ and $\sqrt{-g}R^2$ are non-vanishing when expanded at order h^4. However, the string amplitude includes one-particle reducible graphs with graviton exchange; to reproduce it we must therefore sum the contact and the exchange terms derived from the effective action. In the sum, the coefficients c_1, c_2 cancel (see [18] for a detailed computation), and so they cannot be determined from the comparison with string amplitudes. The coefficient of $R^2_{\mu\nu\rho\sigma}$ is instead fixed by this procedure, at the value $-1/4$.

2.3 Relation with scale factor duality

Scheme dependence should not change physical results if we had the exact expression for the beta functions. It enters into play when we truncate at finite order in α'. When scheme dependence appears, some scheme will be 'better' than others, in the sense that the results obtained at finite order in this scheme will be closer to the exact result. Since scale factor duality plays an important role in motivating the cosmological model that we are discussing, one might hope that a scheme that respects scale factor duality at a given order in α' will be better, in the above sense.

Scale factor duality has been generalized to $O(\alpha')$ in refs. [19, 20]. The analysis of ref. [19] is very general, and refers to the full $O(d, d)$ symmetry. The result of ref. [19] is that there is one, and only one action invariant under scale factor duality at order α', and it is given by the action (2.10) with $c_1 = 1, c_2 = -1/4, c_3 = 1/4, c_4 = 1, c_5 = -1/2, c_6 = 0, c_7 = -1/2, c_8 = 0$. At the same time, the duality transformation must acquire α' corrections. If we restrict to *FRW* metrics and vanishing antisymmetric tensor field, then $a_i \to 1/a_i$ (or $\log a_i \to -\log a_i$) must be generalized as

$$\log a_i \to -(\log a_i) - k\alpha' \left(\frac{\dot{a}_i}{a_i}\right)^2 . \tag{2.13}$$

The transformation of the dilaton is fixed from the condition that $\phi - \sum_i \log a_i$ is invariant. In $d = 3$, for isotropic metric, the equations $\beta_i(x, y) = 0$ for this dual action have two real solutions[2] at $(x \simeq 1.526, y \simeq 1.913)$ and at $(x \simeq 6.201, y \simeq 1.931)$ (plus the solutions $(-x, -y)$, which are always associated with the solution (x, y)). The existence of a pair of solutions (plus the sign-reversed pair) is related to the scale factor duality of the action.

[2]Real solutions for this action exist for any d. This can be proved numerically up to large values of d, where the results matches with a large d expansion.

However, for the action that we are considering, scale factor duality is valid only at order α', and not exactly, since the transformation (2.13), when applied to the $O(\alpha')$ terms of the action, generates $O(\alpha'^2)$ terms that are truncated. Thus the two solutions are not exactly dual to each other.

Integrating the equations of motion numerically, one finds [7] that these solutions do not act as a late time attractor of the lowest order solution, which still diverges at some finite value of time.

To further explore the relation with duality we have tried a different generalization of duality at $O(\alpha')$. In fact, the action found in [19] and the form of the duality transformation (2.13) are uniquely fixed only if we consider the general case of non-isotropic metrics. If however we restrict to isotropic metrics, a transformation

$$\log a \rightarrow -(\log a) - \lambda k\alpha' \left(\frac{\dot{a}}{a}\right)^2 , \tag{2.14}$$

with arbitrary λ, leaves the action (2.10) invariant at order α' if the coefficients c_i are $c_1 = 1, c_2 = -1/4, c_3 = 1/4 + (\lambda-1)/18, c_4 = 1 + (\lambda-1)/9, c_5 = -c_4/2, c_6 = 0, c_7 = -2c_3, c_8 = 0$. (The relation between the c_i is still satisfied.) For $\lambda = 1$ we recover the action and the transformation of ref. [19]. However, for every transformation (2.14) with given λ we now have one action that is invariant.[3] In a sense, this enlarged family of transformations is less interesting than the transformation found in [19], because the latter is the only one that can be generalized to the anisotropic case and to the inclusion of the antisymmetric tensor field. However, this family of transformations also includes the case $\lambda = 0$, i.e. the transformation $a \rightarrow 1/a$, without α' corrections, which is appealing for its simplicity. Furthermore, since this transformation does not generate higher orders in α', it is an exact invariance of the action (2.10), with the appropriate choice of c_i, rather than an invariance at $O(\alpha')$.

We have therefore studied the equations of motion for the action with $\lambda = 0$ duality. We have found that the functions $\beta_i(x, y)$ have a pair of zeros (plus the sign reversed pair), and these zeros are related by exact duality invariance, as it should. But, again, these solutions do not act as late time attractors of the lowest order solution, which instead runs into a singularity. The type of singularity is however different from the one encountered for the

[3]Basically, we have a much less rigid structure because many expressions which are independent in the anisotropic case, such as $\sum_i H_i^2$ and $(\sum_i H_i)^2$, collapse to the same expression in the isotropic case, and we can arrange cancellations between them.

case $\lambda = 1$. It is interesting to understand in some detail what happens. The dynamical equations of motion can be written in matrix form

$$A \begin{pmatrix} \ddot{\phi} \\ \dot{H} \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \tag{2.15}$$

where A is a 2×2 matrix that depends on $H, \dot{\phi}$, and f_1, f_2 are functions of $H, \dot{\phi}$. At a finite value of time we find that $\det A = 0$, and therefore the equations become singular. As a simpler example of a similar situation, consider the differential equation $(1 - h)\dot{h} = h$ for some function $h(t)$. For small h the solution grows like e^t, until the 'determinant' $(1 - h)$ vanishes, so the situation is similar to our case. Here however we can integrate the equation exactly, and find $t(h) = \log h - h$. Above a critical value $t = -1$, this relation cannot be inverted while, below this value, there are two branches $h_+(t)$ and $h_-(t)$ that merge at $t = -1$.

This suggests that what happens in our case is that the doubling of solutions due to duality transformations with $\lambda = 0$ produces pairs of solutions of the dynamical equations of motion, which merge at some critical value of time, and thereafter move into the complex plane. (These solutions are not dual to each other. One corresponds to a small initial value of H and $\dot{H} > 0$, and the other to H large, and $\dot{H} < 0$, while duality changes $H \to -H$). If this conjecture is correct, then it appears that duality works against the regularization mechanism. In any case, neither a scheme that respects duality with $\lambda = 1$ nor a scheme that respects duality with $\lambda = 0$ provides a realization of the regularization mechanism, at $O(\alpha')$.

3 The physics behind the regularization of the singularity

The conclusion of the previous sections is that perturbative α' corrections can in principle regularize the singularity; however, to determine whether this actually happens we should know the functions β_i at all orders in α', because results at finite order are scheme dependent. The attempt to fix the scheme using scale factor duality as a guiding principle does not give encouraging results. We therefore ask whether there are general physical principles that suggest that the singularity is indeed regularized. Generally speaking, we expect that string theory eliminates all unwanted singularities. However, is the physical mechanism that eliminates the singularity related to perturbative α' corrections? Or should we look for a different type of corrections? In this section we discuss this question.

3.1 Effective action and massive string modes

As it is well known, the low energy action of string theory can be obtained either by computing the beta functions of the sigma model or from the requirement of reproducing the S-matrix elements. The two methods are equivalent, as was proved in [21]. Still, the physical pictures underlying the two computations are quite different. In particular, in the first method the massive string modes seem to play no role at all.

In fact, what one does in order to include massive modes at the sigma model level is the following. The sigma model action is written including the interaction with the background fields representing the massive modes [13, 22],

$$S = \frac{1}{\alpha'} \int d^2\sigma \sqrt{-h} \Big[g_{\mu\nu}(X) h^{ij} \partial_i X^\mu \partial_j X^\nu + B_{\mu\nu}(X) \epsilon^{ij} \partial_i X^\mu \partial_j X^\nu + \phi(X) {}^{(2)}R + \\ + \frac{1}{\alpha'} F_{\mu\nu\rho\sigma}(X) h^{ij} h^{kl} \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho \partial_l X^\sigma + \ldots \Big] , \qquad (3.1)$$

where together with the metric, dilaton and antisymmetric field, we have displayed an example of a field at the next mass level. Once we include massive fields at the first excited level, the whole tower of massive fields must be included, since they are associated with non-renormalizable interactions from the point of view of the 2-dimensional theory, and all possible terms are in principle generated by the sigma model loop (i.e. α') expansion.

However, when we compute higher loops in the beta functionals of the massless modes, the massive modes play no role: the operators associated with massive modes are non-renormalizable and do not mix with the massless sector. More generally, to renormalize background fields at a given mass level, we need only fields at the same or lower mass level [23].

However, since in string theory we have an infinite tower of massive modes, while the effective low energy action is written in terms of massless fields only, it is clear that we have somehow integrated over the massive modes. This physical interpretation is confirmed by the computation of the effective action through the comparison with S-matrix elements. In this case the effective action, at all orders in α' but without e^ϕ corrections, is obtained by requiring that it reproduces the string amplitudes at genus zero, with an arbitrary number of insertions of vertex operators for the massless fields. In the field theory language, we expect that the α' corrections to the amplitudes can be represented as a sum of tree level exchange graphs with massive string modes in the intermediate state. This is indeed the case, as can be read from the computation of Gross and Witten [14] of graviton scattering in type II superstring theory. In this case the tree amplitude is [24, 14]

$$A = \frac{G_N^2}{128} \left[\frac{\Gamma(-s/8)\Gamma(-t/8)\Gamma(-u/8)}{\Gamma(1+\frac{1}{8}s)\Gamma(1+\frac{1}{8}t)\Gamma(1+\frac{1}{8}u)} \right] K(\epsilon^{(i)}, k^{(j)}) \tilde{K}(\epsilon^{(i)}, k^{(j)}) \,, \tag{3.2}$$

where G_N is the gravitational constant, s, t, u the Mandelstam variables and $K, \tilde{K}$ are kinematical factors.

Expanding the term in brackets for small momenta, one gets a leading term $-2^9/(stu)$, which, when inserted back in eq. (3.2), gives the tree level scattering amplitude derived from the lowest order effective action; and a correction term $-2\zeta(3)$, which gives the leading correction to the effective action; from the kinematical factors $K, \tilde{K}$ one finds that it corresponds to a term in the effective action quartic in the Riemann tensor [14] (remember that for type II superstrings the coefficient k in eq. (2.1) is zero, i.e. there is no $\sim R^2_{\mu\nu\rho\sigma}$ correction).

Writing the Riemann zeta function as $\zeta(3) = \sum_{n=1}^{\infty} n^{-3}$, we see that the correction can be interpreted as a sum over an infinite tower of intermediate states, and that states at level n give a contribution $\sim 1/n^3$.

This means that the α' corrections are dominated by the first few massive string states. For instance, summing over the first 10 excited states, we get $\sum_{n=1}^{10} 1/n^3 \simeq 1.1975\ldots$, to be compared with the value of the Riemann zeta function $\zeta(3) \simeq 1.2020\ldots$.

Having obtained a physical picture of the mechanism that is responsible for the perturbative α' corrections to the effective action (integration over the first few massive modes), we proceed in the next section to discuss the physical mechanism that is responsible for the regularization of the singularity.

3.2 Small distances vs. large energy singularities

It can be useful to distinguish between small distance singularities and large energy singularities. In field theory small distances means large energies, and the distinction is therefore meaningless. In string theory, however, the behavior of amplitudes in high energy scattering suggests the existence of a generalized uncertainty principle of the form [25]

$$\Delta x \geq \frac{1}{\Delta p} + \text{const.} G_N \Delta p \,. \tag{3.3}$$

(Such an uncertainty principle is also suggested by general properties of quantum black holes [26].) Therefore in string theory the connection between high energies and small distances is not completely trivial and it is useful to consider the two cases separately.

The most basic reason why we expect strings to regulate singularities at small distances is that the very notion of invariant point-like event is not meaningful in string theory. While in quantum field theory an invariant event can be defined through the splitting of a particle in two, no such concept exist for strings, since the point in space-time where a string splits in two depends on the Lorentz frame used [27].

At large energies, the regulating mechanism is instead the exponential growth of the density of states. The asymptotic density of states has the form

$$d(M) \sim \left(\frac{M}{M_0}\right)^{-b} e^{M/M_0} \,, \tag{3.4}$$

where $M_0 = c/\sqrt{\alpha'}$ and b, c are numerical constants, which depend on the string theory under consideration. An example of how the density of states prevents quantities with dimension of energy from growing arbitrarily large is given by the fact that we cannot raise the temperature of a system beyond the Hagedorn temperature, $T_{\rm HAG} = M_0$, because otherwise the canonical partition function diverges.

Another interesting example is given by the existence of a limiting value of the electric field for an open bosonic string. In the presence of an external electric field E, the rate for charged-string pair production is in fact [28]

$$w \sim \sum_S q_S \sum_{k=1}^{\infty} (-1)^k \left(\frac{|\epsilon|}{k}\right)^{(d+1)/2} \exp\left\{-\frac{\pi k}{|\epsilon|}(M_S^2 + \epsilon^2)\right\} \,. \tag{3.5}$$

The sum goes over all physical string states S, with mass M_S, and q_S is a factor that depends on the charge of the state S; in the weak field limit, $\epsilon \simeq eE + o(E^3)$, where e is the total electric charge of the string, and one recovers the Schwinger result. As long as ϵ is finite, the factor $\exp(-\pi k M_S^2/|\epsilon|)$ ensures the convergence of the rate, even if the sum over the states S, for large masses, is an integral with the exponentially growing density of states (3.4).

However, at a critical value of the electric field, ϵ goes to infinity and the rate therefore diverges.

The cosmological singularity is a large energy singularity: the Hubble constant, and therefore the curvature or the energy density, diverges on the lowest order solutions of the equations of motion. We therefore expect that the regulating mechanism has to do with the existence of an infinite set of massive states; in fact, we expect something similar to what happens in eq. (3.5), since the existence of a maximum electric field and of a maximum gravitational field do not seem to be two fundamentally different problems.

So, the first conclusion is that we should not necessarily expect that perturbative α' corrections regulate the singularity. As we discussed in sect. 3.1, these corrections can be accounted for with an accuracy of less than one per cent summing over the first ten mass levels. The regularization of the singularity, instead, must be crucially related to the existence of an infinite tower of massive states. Of course, this does not mean that the mechanism proposed in [7] cannot regulate the singularity. It simply means that we do not have any *a priori* argument that allows us to confidently state that the mechanism will operate.

The second point suggested by the above considerations is to look for massive modes production in a gravitational field. This will be discussed in the next section.

4 Production of highly excited modes

A time varying gravitational field produces particles. The general mechanism is the existence of a non-trivial Bogoliubov transformation between the in and the out vacuum; the best known example of this phenomenon is the Hawking radiation from black holes. The production rate of massive string modes, in a gravitational field with characteristic variation scale H, has been computed by Lawrence and Martinec [11] (see also [29] for related ideas on singularity regularizations). Using string field theory, the result for the Bogoliubov coefficients β_k turns out to be essentially identical to the production in the quantum field theory limit,

$$|\beta_k|^2 = e^{-2\pi E(k)/H}\,; \tag{4.1}$$

therefore, the rate of energy density production during the expansion is approximately

$$\dot{\rho} \;\sim\; \frac{1}{\lambda_s^d}\int_{M_0}^{\infty} dM\, M d(M) e^{-2\pi M/H} \sim$$

$$\sim \frac{M_0^2}{\lambda_s^d} \int_1^\infty dx\, x^{1-b} e^{-\gamma x}\,, \tag{4.2}$$

where

$$\gamma = \frac{2\pi M_0}{H} - 1 = \frac{2\pi c}{H\sqrt{\alpha'}} - 1\,. \tag{4.3}$$

Although a number of approximations, discussed in ref. [11], have been used in deriving eq. (4.1), its physical meaning appears clear and compelling. Equation (4.1) says that when the typical frequency of the gravitational field becomes comparable to the mass M, the production of a mode of this mass is no longer suppressed. This formula, which has been suggested in [11] as valid for all the mass levels, is also qualitatively confirmed by the explicit computation of the Bogoliubov coefficients for the graviton production, i.e. at mass level zero [2, 30]. In this case, in fact, one finds that $|\beta_k|^2$ is a number of order 1 for energies of the order of the string mass (red-shifthing the frequency at the present epoch, this means that $|\beta_k|^2 = O(1)$ if the physical frequency, as seen today, is of order 10 GHz), and has an exponential cutoff for larger frequencies. Actually, the spectrum also shows some deviations from an exact black body spectrum (it goes like ω^3 for low frequencies but as $\omega^{3-2\mu}$, where μ depends on details of the string phase, for intermediate frequencies.) These deviations, however, are not very relevant to the present discussion.

Following ref. [11], we will therefore assume that eq. (4.2) captures the correct physics, although the details should not be taken too literally.

Equation (4.2) clearly has a regularizing effect on the growth of H. When H exceeds a critical value H_c, given by $H_c/(2\pi) = M_0$, the production rate diverges. This result can also be interpreted by noting that $H/(2\pi)$ is the Hawking temperature in the De Sitter space, and therefore the condition on H_c is a requirement that the Hawking temperature does not exceed the Hagedorn temperature, $T_{\rm HAG} = M_0$.

The effect that we are discussing is non-perturbative in α'. In fact, for $\gamma > 0$ the integral in eq. (4.2) is an incomplete gamma function, and from its known asymptotic expansion we find that, in the limit $\alpha' \to 0$,

$$\dot\rho \sim \left(\frac{1}{\alpha'}\right)^{\frac{d}{2}+1} \exp\left\{-\frac{2\pi c}{H\sqrt{\alpha'}}\right\}\,, \tag{4.4}$$

and it is therefore non-analytic in α' at $\alpha' = 0$.

To understand in more detail the evolution of the cosmological model, we tentatively use the expression (4.2) for the energy density produced as a backreaction term in the cosmological equations. The equations of motion, without perturbative α' corrections, and including matter sources with $T_\mu^\nu = (\rho(t), -\delta_i^j p(t))$, are [2, 5] (in the isotropic case, in d spatial dimensions)

$$\dot{\bar{\phi}}^2 - dH^2 = 2\lambda_s^{d-1} e^{\bar{\phi}} \bar{\rho} \tag{4.5}$$

$$2\ddot{\bar{\phi}} - \dot{\bar{\phi}}^2 - dH^2 = 0 \tag{4.6}$$

$$\dot{H} - H\dot{\bar{\phi}} = \lambda_s^{d-1} e^{\bar{\phi}} \bar{p} \tag{4.7}$$

where $\bar{\phi} = \phi - d \log a, e^{\bar{\phi}}\bar{\rho} = e^{\phi}\rho, e^{\bar{\phi}}\bar{p} = e^{\phi}p$. We use eq. (4.2) for the time derivative of ρ; the pressure p is determined by the conservation of the energy momentum tensor, which gives

$$\dot{\rho} + dH(\rho + p) = 0\,. \tag{4.8}$$

This is equivalent to requiring that the constraint equation (4.5) is conserved by the dynamical equations of motion (4.6) and (4.7). Since we know $\dot{\rho}$ rather than ρ, p, these are in principle integro-differential equations. However, we can combine eq. (4.7) and eq. (4.5) so that only the combination $\rho + p$ appears, which is expressed through $\dot{\rho}$ using eq. (4.8), and we are left with ordinary differential equations. Writing everything in terms of ϕ rather than $\bar{\phi}$, the dynamical equations of motion can then be written as

$$2\dot{H} + \dot{\phi}^2 - 2(d+1)H\dot{\phi} + d(d+1)H^2 = -\frac{2}{d}\lambda_s^{d-1} e^{\phi} \frac{\dot{\rho}(H)}{H}\,, \tag{4.9}$$

$$2\ddot{\phi} - 2d\dot{H} - \dot{\phi}^2 + 2dH\dot{\phi} - d(d+1)H^2 = 0\,. \tag{4.10}$$

The constraint equation

$$\dot{\phi}^2 + d(d-1)H^2 - 2dH\dot{\phi} = 2\lambda_s^{d-1} e^{\phi} \rho \tag{4.11}$$

is still an integro-differential equation. However, we impose the constraint at the initial time $t_0 \ll -1$, where the right-hand side is negligible. The constraint is then automatically preserved by the evolution (we use its conservation as a check of the numerical integration). We have then integrated these equations numerically. The results are qualitatively the same for any d and any value of b, c in eqs. (4.2) and (4.3) that we have tried. For numerical reasons, the integration works better for large values of c. The plots that we present refer to $d = 3$ and to the values $b = 10, c = 10$ (this value of c is responsible for the values $\sim O(10)$ of the H-scale in Fig. 1). Figure 1 shows the behavior of the Hubble parameter versus cosmic time t. At large negative values of t we recover of course the lowest order solution, since the production of massive string states is exponentially small; $H(t)$ then reaches a maximum value and then very abruptly bounces off and goes to zero at a finite value of t. At this point the numerical integration stops.[4] The behavior of $\dot{\phi}$ is shown in fig. 2, while fig. 3 shows $\dot{\rho}(t)$. Lowering c towards smaller values produces an even steeper decrease after the maximum, until the numerical integration becomes unreliable. We could not reach values as small as $c = [(2+\sqrt{2})\pi]^{-1}$, which is the value for the heterotic string.

We can make the following comments. (i) When H goes to zero, the derivative $\dot{H}$ is very large. Therefore in this regime the behavior of the solution will be completely dominated by higher-order perturbative corrections in α', which introduce higher-order derivatives of H and higher powers of $\dot{H}$. This means that we should not take seriously the fact that $H \to 0$ at some value of time. In this regime the evolution is completely dominated by other contributions which are not present in our equations. (ii) At the point where H reaches a maximum, we find that both e^{ϕ} and $e^{\phi}\rho$ are very large. This means that we cannot neglect higher order e^{ϕ} corrections, and that a perturbative treatment of the massive modes production as a back-reaction is not adequate. Note that, differently from what happens in the study of perturbative α' corrections, we cannot make e^{ϕ} small in the interesting region just by taking its initial value sufficiently small. Lowering the initial value of e^{ϕ} now delays the time at which the regularizing term $e^{\phi}\dot{\rho}$ becomes important, and therefore gives ϕ more time to grow.

In conclusion, none of the details of this numerical solution can be taken seriously. We can only draw some very general conclusion:

1. The solution is regularized, and H does not diverge. This follows from the expression of $\dot{\rho}$ and is correct as long as this expression captures the relevant physics, independently of numerical details.

2. The solution does not approach asymptotically a state $H = \text{const} \simeq H_c$, contrary to what happens when perturbative α' corrections are the regularization mechanism. Figures (1) and (3) show that $\dot{\rho}$ and H have a very similar behavior in the interesting

[4] This is probably only a problem of the numerical integration, since $\dot{\rho}/H$ is not singular but rather goes to zero when $H \to 0$. However, as we will discuss below, the equation is not valid in this regime, and there is not much point in trying to improve the numerical integration.

region, and a large H implies a large rate of increase in the energy density of the massive modes produced. This process certainly must stop, since the gravitational field cannot produce an arbitrarily large energy density in massive modes. This forces H to decrease, after it has reached a maximum value close to the critical value H_c. This conclusion also seems to be quite general and independent of the specific details of the equations used.

3. Around the maximum, e^{ϕ} and perturbative α' corrections are both important for determining the detailed evolution of the solutions. The dynamics in this region is therefore very complicated.

5 Conclusions

As we saw, there are two distinct mechanisms that are relevant to a discussion of the singularity in string cosmology. The one considered in [7] is classical, in the sense that it does not depend on e^{ϕ} but only on the finite extent of the string. Although it naively appears that it can be analysed order by order in perturbation theory, its assessment really requires the knowledge of an exact beta function. At present, we have no argument to establish whether it does take place or not.

The second mechanism, discussed in this paper, is instead a quantum effect, since it involves quantum creation of massive string modes, and enters the equations with a factor e^{ϕ}. It is non-perturbative in α', as is shown by eq. (4.4). In spite of the fact that its computation requires a number of assumptions, it has a clear physical interpretation, which shows that it always regularizes the lowest order solution.

If the perturbative α' corrections do regularize the singularity, and if this takes place at a value of $H \ll H_c$, then the production of massive modes is exponentially small and the cosmological scenario is the one discussed in [7], with a De Sitter phase that should end when e^{ϕ} corrections finally become large (see [10] for an explicit realization of this graceful exit mechanism with some toy model e^{ϕ} corrections).

If, instead, the beta functions $\beta_i(\mathbf{g})$ do not have a zero, or if this zero is not an attractor of the lowest order solution, the regularization mechanism is the one discussed in this paper, and rather than a long De Sitter phase we expect a bounce in H. We expect that the bounce will take place also if both mechanisms are operative and the value of H at which the beta functions vanish is not much smaller than the critical value H_c for massive mode production. This critical value, in units $\alpha' = 1$, is $H_c = 2\pi c$; for the heterotic string this gives $H_c = 2/(2+\sqrt{2})$, while for type II superstrings $H_c = 1/\sqrt{2}$. In any case, it is a number of order 1.

We conclude by pointing out briefly some possible phenomenological consequences of this second scenario. Massive string modes correspond to long strings, and are a possible seed for the generation of density perturbations. This suggestion has been considered in the context of string cosmology in [31], where it has been shown that during a stringy phase of the model these seed fields can be amplified and grow exponentially.

Another interesting point concerns gravitational radiation. The massive string states produced will eventually decay. The decay of the excited string levels has been studied in ref. [32]. The result is that, for large level number N, the width is dominated by decays in which one of the products is massless, i.e. by transitions of the form $N \to N-1$ with emission of a graviton or another massless particle. From the mass formula $M_N^2 \sim N$, the level spacing is $\Delta M_N \sim 1/M_N$. If we assume that each string at an excited level N decays at level $N-1$ with emission of a graviton of frequency $\omega = \Delta M_N$, and, if we use the density of states (3.4), we get a spectrum of the form

$$\frac{dP}{d\omega} \sim \omega^{\alpha} \exp\left\{-\frac{\omega_0}{\omega}\right\}, \qquad \omega_0 \sim \frac{1}{\sqrt{\alpha'}}\left(\frac{H_c}{H_{\max}} - 1\right), \tag{5.1}$$

where $H_{\max}$ is the maximum value of H actually reached, $H_{\max} < H_c$. The massive modes have $M_N \gtrsim 1/\sqrt{\alpha'}$, and the maximum value of ΔM_N and therefore of ω is also of the order of $1/\sqrt{\alpha'}$. For larger values of the frequency the spectrum is cutoff exponentially. The interesting feature of eq. (5.1) is that, because of the exponential factor depending on $1/\omega$, there is a sharp cut off also at small frequencies. Compared with a black body radiation, therefore, the spectrum (5.1) is much more concentrated around the maximum value, and it might even look as a relatively narrow line, with a peak value larger than the peak of a black body spectrum with the same total energy density. The most naive estimate of the peak frequency is obtained by red-shifting the string mass from a string time to the present epoch, which gives a frequency $\omega_{\text{today}}/(2\pi)$ of the order of 10 GHz, which is also the typical cutoff value of the gravitons produced by quantum vacuum fluctuations [2, 30, 33]. However, this radiation is produced within the string phase, rather than at the transition between the string phase and the *FRW* phase, and a further red-shift must be included, to take into account the expansion from the time of production to the onset of a standard *FRW* cosmology. We cannot compute it, in the absence of a knowledge of the mechanism that terminates the string phase. There is, however, the interesting possibility that this gravitational radiation might be concentrated in a relatively narrow band accessible to the LIGO/Virgo gravitational wave detectors, which are expected to operate in the 6 Hz–1 kHz region.

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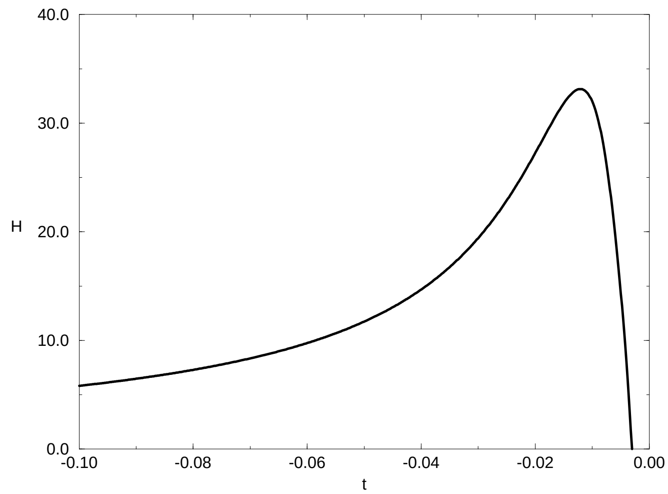

Figure 1: *The Hubble parameter H as a function of time. The initial conditions for the numerical integration are $t_0 = -20, \phi(t_0) = -10, H(t_0) = -1/(t_0\sqrt{3})$; $\dot{\phi}(t_0)$ is determined by the constraint equation.*

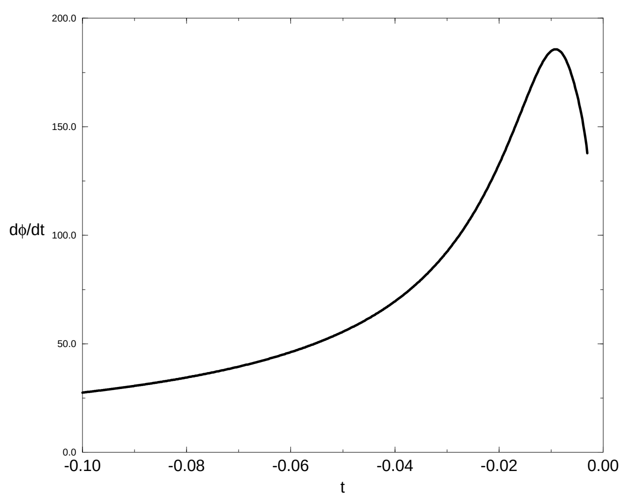

Figure 2: *$\dot{\phi}$ as a function of time. Same initial conditions as in fig. 1. The integration stops when H reaches zero.*

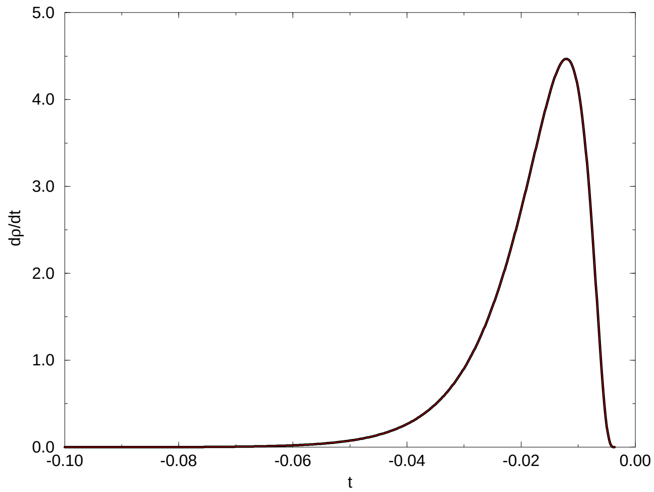

Figure 3: $\dot{\rho}$ *as a function of time. Same initial conditions as in fig. 1.*