Tailoring T-Duality Beyond the First Loop

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ABSTRACT

In this article we review a recent calculation of the two-loop σ-model corrections to the T-duality map in string theory. Using the effective action approach, and focusing on backgrounds with a single Abelian isometry, we give the $O(\alpha')$ modifications of the lowest-order duality transformations. The torsion plays an important role in the theory to $O(\alpha')$, because of the Chern-Simons couplings to the gauge fields that arise via dimensional reduction.

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1   Introduction

It is widely believed that General Relativity requires certain alterations in order to be brought into accord with quantum mechanics. The theory does not yield a satisfactory account of phenomena at very small scales even in the context of classical physics. As an example, one can take the singularity theorems of Hawking and Penrose, which show that a gravitating system contains singular regions, where the theory breaks down. The strongest contender to date for the extension of General Relativity into the quantum realm is string theory. It has brought about the belief that matter consists of very small extended objects, strings and p-branes, the size of which is of the order of Planck length. The finite size of the elementary building blocks in string theory could lead to dramatic modifications of small scale physics, resolving some of the problems faced by General Relativity.

In the last few years, we have witnessed a rapid and profound development of string theory, leading to the establishment of an interconnection of different string constructions. The principal tool and guide in the course of this unification of string theory was the concept of duality. In technical terms, duality arises because of the considerably richer symmetries of string theory than in ordinary General Relativity. In the string spectrum, in addition to the graviton, there appear other degrees of freedom, such as the scalar dilaton and the antisymmetric torsion tensor (or the Kalb-Ramond field), with precisely determined couplings. String duality symmetries arise from the invariance of the theory under the exchange of the degrees of freedom in the string spectrum. Dualities provide the natural maps between seemingly different string theories, and not only different solutions in the phase space of a single theory [1]). This has led to the uniqueness proof of string theory, whereby all consistent string constructions have been recognized as the facets of a single fundamental theory, labeled the M-theory.

A duality symmetry we have studied in [2] was the so-called T-duality, or string scale-factor duality. At the level of the world-sheet σ-model, it has been identified by Buscher [3] as a simple Hodge duality of a cyclic target coordinate. This duality has been also investigated, and generalized, in [4]-[10]. Most of these investigations have been conducted at the one-loop level of the effective action approach to string theory, where the action is truncated down to the terms of the second order in derivatives. However, it has been argued, and in some special situations proven, that this symmetry is exact order-by-order in perturbation theory [5, 9, 7, 11]. It has also been shown that the lowest-order form of the on-shell T-duality map remains unaffected by higher order α' corrections when viewed as a relation between specific conformal field theories (CFT's) [12], some dual solutions in two dimensions [13] and some special supersymmetric solutions [14]. In these cases, the proof relied on special properties of the solutions studied - there either existed an exact, nonperturbative CFT formu-

lation, or the solutions were highly symmetric, which protected them from acquiring quantum corrections. A picture which has emerged from these examples is that the T-duality map can be expanded as a perturbative series in the inverse string tension α'.

Since dualities play such an important role in the new formulation of string theory, the question of their validity beyond the first loop[1] is very important one. A step towards the inclusion of higher-order corrections has been taken in [15], where T-duality map has been determined to two loops on backgrounds with all but one cyclic coordinates. Later, in [2] we have generalized this approach to include the gauge fields which couple to the winding and momentum modes of the string, and were not considered in [15]. The situations turned out to be quite a bit more subtle than in the simpler case, because of the highly nontrivial role of the torsion field.

2 $O(\alpha'^0)$

Here we review the one-loop results, in order to show the explicit lowest-order T-duality. The lowest-order term in the effective action of any string theory truncated to only the model-independent zero mass modes is (throughout this paper we use the string frame with $e^{-2\phi}$ out front, since the symmetry is most simply realized there)

$$\Gamma^{(0)} = \int d^{d+1}x\sqrt{\bar{g}}\; e^{-2\bar{\phi}}\left\{\bar{R}(\bar{g}) + 4(\bar{\nabla}\bar{\phi})^2 - \frac{1}{12}\bar{H}^2\right\}. \tag{2.1}$$

Our convention for the signature of the metric is $(-, +, \ldots, +)$, the Riemann curvature is $\bar{R}^\mu{}_{\nu\rho\sigma} = \partial_\rho\bar{\Gamma}^\mu{}_{\nu\sigma} - \ldots$, and the torsion field strength is the antisymmetric derivative of the torsion potential: $\bar{H}_{\mu\nu\rho} = \nabla_\mu\bar{B}_{\nu\rho}+\text{cyclicpermutations}$. The overbar denotes the quantities in the original, $d+1$-dimensional, frame, before we carry out the Kaluza-Klein reduction. Note that the definition of the torsion field strength $\bar{H}$ encodes the torsion potential gauge invariance: if we shift the $\bar{B}$-field according to $\bar{B} \to \bar{B} + d\Lambda$, where Λ is an arbitrary one-form, the theory remains unchanged. As a consequence, the Bianchi identity for $\bar{H}$ takes a very simple form: $d\bar{H} = 0$.

In the presence of a single Killing isometry, we can carry out the Kaluza-Klein dimensional reduction down to d dimensions. The most natural way to carry out the reduction is to ensure that gauge symmetries are manifest at every step of the calculation. The reduced action will feature two additional gauge fields coupled to the graviton, axion and dilaton, and an additional scalar field, which is the breathing mode of the cyclic coordinate. The reduction ansätz for the metric and dilaton, $g_{\mu\nu}$

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[1]We consider here the loop expansion of the world-sheet σ-model in the field-theoretic sense, where α' is the loop counting variable. From the string theory point of view, these corrections are classical, since α' measures the effects of the extended structure of strings.

and ϕ respectively, is

$$d\bar{s}^2 = g_{\mu\nu}dx^\mu dx^\nu + \exp(2\sigma)(dy + V_\mu dx^\mu)^2 \qquad \phi = \bar{\phi} - \frac{1}{2}\sigma \qquad (2.2)$$

All the lowering and raising of the indices in the rest of this work will be done with respect to the reduced metric $g_{\mu\nu}$. The vector field V_μ is the standard Kaluza-Klein gauge field, which couples to the momentum modes of the theory. The reduction of the axion field has to be done with more care because of the anomaly which appears in it. In the naive decomposition of the two-form $\bar{B} = (1/2)\bar{B}_{\mu\nu}dx^\mu \wedge dx^\nu + W_\mu dx^\mu \wedge dy$ (here $W_\mu = \bar{B}_{\mu y}$ is the other gauge field, arising from the "off-diagonal" components of the torsion, and which couples to the winding modes), the space-time components $\bar{B}_{\mu\nu}$ contribute to the reduced torsion, but they are not invariant under the translations along y. When $y \to y' = y - \omega(x)$ and $V_\mu \to V'_\mu + \partial_\mu\omega$ (s.t. the cyclic einbein $E = dy + V_\mu dx^\mu$ is gauge-invariant), we find $\bar{B}_{\mu\nu} \to \bar{B}'_{\mu\nu} = \bar{B}_{\mu\nu} + W_\mu\partial_\nu\omega - W_\nu\partial_\mu\omega$. One could try $\hat{B}_{\mu\nu} = \bar{B}_{\mu\nu} - (W_\mu V_\nu - W_\nu V_\mu)$, which in fact does not change by the local translations along y, but when $W_\mu \to W'_\mu = W_\mu + \partial_\mu\lambda_y$, $\hat{B}_{\mu\nu} \to \hat{B}'_{\mu\nu} = \hat{B}_{\mu\nu} - (\partial_\mu\lambda_y V_\nu - \partial_\nu\lambda_y V_\mu)$. So the two gauge symmetries of the reduced theory are not decoupled, because the reduced torsion potential cannot be simultaneously invariant under both of them. The three-form field strength however must be gauge invariant, and so following [10], we can define the reduced torsion by $B_{\mu\nu} = \bar{B}_{\mu\nu} - (1/2)(W_\mu V_\nu - W_\nu V_\mu)$. Then, the gauge-invariant field strength can be written as $H = dB - (1/2)W\,dV - (1/2)V\,dW$, using the form notation. This expression is manifestly T-duality invariant to the lowest order, and hence is the correct stepping stone towards extending duality to two loops and beyond. As we will see below, this field strength actually appears in the dimensionally reduced action, and hence is indeed the correct choice for the reduced dynamical variable.

With the ansätz (2.2), we can now carry out the reduction of (2.1). Here we will skip the details, and just give the reduced action; it is (after dividing $\Gamma_R^{(0)}$ by the y-volume $\int dy$):

$$\Gamma_R^{(0)} = \int d^dx \sqrt{g}\, e^{-2\phi} \left\{ R + 4(\nabla\phi)^2 - (\nabla\sigma)^2 - \frac{1}{4}e^{2\sigma}Z - \frac{1}{4}e^{-2\sigma}T - \frac{1}{12}H^2 \right\}. \qquad (2.3)$$

with B and H given by

$$B_{\mu\nu} = \bar{B}_{\mu\nu} - \frac{1}{2}\left(W_\mu V_\nu - W_\nu V_\mu\right) \qquad (2.4)$$

and

$$H_{\mu\nu\lambda} = \nabla_\mu B_{\nu\lambda} - \frac{1}{2}W_{\mu\nu}V_\lambda - \frac{1}{2}V_{\mu\nu}W_\lambda + cyclic\ permutations \qquad (2.5)$$

The T-duality map is (apart from trivial rescalings) the transformation $\sigma \leftrightarrow -\sigma$, $V_\mu \leftrightarrow W_\mu$, and it is obvious that the action (2.3) is invariant under it. The equations

of motion which are obtained from varying the action (2.3) (and are simply related to the string β-functions, in this order) are covariant under T-duality: the β-functions of the dilaton, reduced metric and torsion are symmetric, the β-function of the modulus is antisymmetric, and the β-functions of the gauge fields get interchanged, as expected from the world-sheet σ-model realization of T-duality as a map which exchanges the momentum and winding modes (the effect of T-duality on the σ-model β functions has also been discussed in [16]).

3 $O(\alpha'^1)$

We are now ready to discuss the two-loop corrections. The appearance of the $O(\alpha')$ corrections in the effective action can be changed by finite renormalizations of the string world-sheet couplings [17, 18]. In a sense, this blurs the notion of string theory quantities as functions of α' - instead of a single set of solutions, one ends up with equivalence classes, specified by field redefinitions. In order to do any calculations, one has to adopt a concrete scheme, thus fixing the form of the counterterms in the effective action and the functional dependence on α'. A scheme may be specified by simultaneously requiring manifest unitarity in perturbation theory and linear realization of duality [15] (for the definition of this scheme, see also [19, 20]). For this action, the relationship between the β-functions and the functional derivatives turns out to be local, to $O(\alpha')$. Hence the covariance of the β-functions and the invariance of the action in this scheme, to order $O(\alpha')$, are equivalent. Here we will review the explicit form of the corrections. The effective action to two loops is

$$
\begin{aligned}
\Gamma \ = \ & \int d^{d+1}x \sqrt{\bar{g}} e^{-2\bar{\phi}} \left\{ \bar{R}(\bar{g}) + 4(\bar{\nabla}\bar{\phi})^2 - \frac{1}{12}\bar{H}^2 \right. \\
& + \alpha'\lambda_0 \left[-\bar{R}_{GB}^2 + 16\left(\bar{R}^{\mu\nu} - \frac{1}{2}\bar{g}^{\mu\nu}\bar{R}\right)\bar{\nabla}_\mu\bar{\phi}\bar{\nabla}_\nu\bar{\phi} - 16\bar{\nabla}^2\bar{\phi}(\bar{\nabla}\bar{\phi})^2 + 16(\bar{\nabla}\bar{\phi})^4 \right. \\
& + \frac{1}{2}\left(\bar{R}_{\mu\nu\lambda\rho}\bar{H}^{\mu\nu\alpha}\bar{H}^{\lambda\rho}{}_\alpha - 2\bar{R}^{\mu\nu}\bar{H}_{\mu\nu}^2 + \frac{1}{3}\bar{R}\bar{H}^2\right) - 2\left(\bar{\nabla}^\mu\bar{\nabla}^\nu\bar{\phi}\bar{H}_{\mu\nu}^2 - \frac{1}{3}\bar{\nabla}^2\bar{\phi}\bar{H}^2\right) \\
& \left. \left. - \frac{2}{3}\bar{H}^2(\bar{\nabla}\bar{\phi})^2 - \frac{1}{24}\bar{H}_{\mu\nu\lambda}\bar{H}^\nu{}_{\rho\alpha}\bar{H}^{\rho\sigma\lambda}\bar{H}_\sigma{}^{\mu\alpha} + \frac{1}{8}\bar{H}_{\mu\nu}^2\bar{H}^{2\mu\nu} - \frac{1}{144}(\bar{H}^2)^2 \right] \right\}. \quad (3.6)
\end{aligned}
$$

where we introduce a useful shorthand notation:

$$
\bar{H}_{\mu\nu}^2 = \bar{H}_{\mu\alpha\beta}\bar{H}_\nu{}^{\alpha\beta}, \quad \text{and} \quad \bar{H}^2 = \bar{H}_{\mu\alpha\beta}\bar{H}^{\mu\alpha\beta} \quad (3.7)
$$

The parameter λ_0 allows us to move between different string theories: $\lambda_0 = -1/8$ for heterotic, $-1/4$ for bosonic, and 0 for superstring. In this sense, our calculations are completely general (although, they are of course trivial in the case of the superstring - where the $O(\alpha')$ terms vanish identically and hence the lowest-order T-duality does not acquire any corrections; curiously, in this context T-duality is not a map between

the solutions of a theory but instead a map between different theories). The $\bar{R} - \bar{H}$ terms also include the Lorentz-Chern-Simons terms which emerge in the heterotic theory to two loops. The curvature squared terms are arranged in the Gauss-Bonnet combination, $\bar{R}_{GB}^2 = \bar{R}_{\mu\nu\lambda\sigma}^2 - 4\bar{R}_{\mu\nu}^2 + \bar{R}^2$. While the dimensional reduction of this action may appear difficult at first, it is manageable when carried out on the tangent space. The details of the dimensional reduction can be found in [2]. The contributions to the reduced action contain terms which are invariant under the one-loop level duality transformations $\sigma \leftrightarrow -\sigma$, $V_\mu \leftrightarrow W_\mu$ and also terms which are not symmetric under this map. It is these latter terms which we are interested in here. They are the ones forcing us to correct the one-loop duality map with $O(\alpha')$ contributions. In order to separate the two-loop contributions into one-loop duality-invariant and duality-noninvariant parts, we can apply the transformations, and work out $\Gamma_{inv} = (1/2)(\Gamma_2 + T\Gamma_2)$ and $\Gamma_{ninv} = (1/2)(\Gamma_2 - T\Gamma_2)$. Using this and the results of [2], the duality-violating sector of the reduced $O(\alpha')$ action is

$$
\begin{aligned}
\Gamma_{ninv}^{(2)} &= \alpha' \lambda_0 \int d^d x \sqrt{g} e^{-2\phi} \Big\{ -4\nabla_\mu \sigma \nabla^\mu (\nabla \sigma)^2 - \nabla_\mu \sigma \nabla^\mu \left[e^{-2\sigma} T + e^{2\sigma} Z \right] \\
&\quad + \frac{1}{8} \left[e^{-4\sigma} T^2 - e^{4\sigma} Z^2 \right] + \frac{1}{4} H_{\alpha\beta\sigma} H_{\mu\nu}{}^\sigma \left[W^{\alpha\beta} W^{\mu\nu} e^{-2\sigma} - V^{\alpha\beta} V^{\mu\nu} e^{2\sigma} \right] \quad (3.8) \\
&\quad + \left[\nabla^\mu V^{\nu\lambda} W_\mu{}^\sigma - \nabla^\mu W^{\nu\lambda} V_\mu{}^\sigma \right] H_{\nu\lambda\sigma} + \frac{1}{2} \left[V^{\nu\lambda} W^{\mu\sigma} - W^{\nu\lambda} V^{\mu\sigma} \right] \nabla_\lambda H_{\nu\mu\sigma} \\
&\quad + 2\nabla_\gamma \sigma V_{\mu\nu} W^\gamma{}_\lambda H^{\mu\nu\lambda} + 2\nabla_\gamma \sigma W_{\mu\nu} V^\gamma{}_\lambda H^{\mu\nu\lambda} \Big\}
\end{aligned}
$$

Given the noninvariant terms (3.8) and the one-loop level duality $\sigma \leftrightarrow -\sigma$, $V_\mu \leftrightarrow W_\mu$, the natural way to reconcile them is to interpret (3.8) as the $O(\alpha')$ terms in the expansion of the exact T-duality map, which presumably exists in a complete quantum theory, which admits various string theories as its limits. Hence, one ought to be able to incorporate these terms by redefining the duality-invariant one-loop action - in effect shifting the one-loop level fields by amounts proportional to α', while preserving any other symmetries the one-loop level theory has. With this in mind, we only need to ensure that the noninvariant terms (3.8) be absorbed to $O(\alpha')$. Any deviation away from the perturbative form of T-duality, at any higher loop order, can be safely ignored from the point of view of the effective action, in this order. Although this may appear limited to the two loop level, it conforms with the idea that T-duality is perturbatively exact. At higher orders, we expect that the terms violating the two-loop form of duality, that will emerge from similar calculations, can be absorbed away as further corrections of the $O(\alpha'^0)$ and $O(\alpha')$ sectors of the action. In fact, we expect that such analytical cancellations will go on *ad infinitum*, yielding a Taylor-series expression for the complete T-duality map.

To find the corrections of the T-duality map, we define the shifts of the one-loop degrees of freedom as follows:

$$
\sigma \to \hat{\sigma} + \alpha' \delta\sigma \quad V_\mu \to \hat{V}_\mu + \alpha' \delta V_\mu \quad W_\mu \to \hat{W}_\mu + \alpha' \delta W_\mu \quad H_{\mu\nu\lambda} \to \hat{H}_{\mu\nu\lambda} + \alpha' \delta H_{\mu\nu\lambda} \quad (3.9)
$$

Starting from the one-loop action (2.3), the shifts induce the $O(\alpha')$ correction

$$\delta\Gamma = \alpha' \int d^d x \sqrt{g}\, e^{-2\phi} \left\{ \; -2\nabla_\mu \sigma \nabla^\mu (\delta\sigma) - \frac{e^{2\sigma}}{2}[\delta\sigma V_{\mu\nu}^2 + V^{\mu\nu}\delta V_{\mu\nu}] \right.$$
$$\left. + \frac{e^{-2\sigma}}{2}[\delta\sigma W_{\mu\nu}^2 - W^{\mu\nu}\delta W_{\mu\nu}] - \frac{1}{6}H^{\mu\nu\lambda}\delta H_{\mu\nu\lambda} \right\} \quad (3.10)$$

In this formula we have replaced the shifted degrees of freedom (denoted by a hat in (3.9)) by the original one-loop ones, in the spirit of the active interpretation of symmetries. As we see from the above, the torsion field needs to be corrected too, although at the lowest order it is a singlet under T-duality. It may look puzzling why it is necessary to correct a field which is only a passive spectator in the arena of T-duality, in order to restore the symmetry. Here we must remember that the reduced torsion couples to the gauge fields via the mixed Chern-Simons term. Since the vectors transform nontrivially under duality, their corrections must play a key role in the restoration of duality at the two-loop level. Their coupling to the torsion via the Chern-Simons terms however also induces the corrections of the torsion field. Thus the torsion assumes the role of a custodian field. Its corrections arise because of gauge symmetries and are needed to restore duality. The form of the corrections however is determined by the need to preserve the anomaly terms and gauge invariance. These two conditions require that [2]

$$\delta B_{\mu\nu} = cW_{\lambda[\mu}V^\lambda{}_{\nu]} + \delta W_{[\mu}V_{\nu]} + \delta V_{[\mu}W_{\nu]} \quad (3.11)$$

and

$$\delta H_{\mu\nu\lambda} = 3c\nabla_{[\mu}(W_\nu{}^\rho V_{\lambda]\rho}) - 3V_{[\mu\nu}\delta W_{\lambda]} - 3W_{[\mu\nu}\delta V_{\lambda]} \quad (3.12)$$

where c is a constant to be determined by duality restoration. Note that up to this constant, the torsion correction is completely determined by the gauge sector of the theory. Substituting (3.12) into (3.10), and requiring that (3.8) and (3.10) cancel each other (i.e. that the $O(\alpha')$ corrections in (3.8) are absorbed by (3.10)), determines the functional form of the shifts. We have found the unique explicit form of these corrections

$$\delta\sigma \;=\; -2\lambda_0(\nabla\sigma)^2 - \frac{\lambda_0}{4}e^{2\sigma}Z - \frac{\lambda_0}{4}e^{-2\sigma}T$$
$$\delta V_\alpha \;=\; 2\lambda_0 V_{\alpha\rho}\nabla^\rho\sigma - \frac{\lambda_0}{2}H_{\alpha\beta\gamma}W^{\beta\gamma}e^{-2\sigma}$$
$$\delta W_\alpha \;=\; 2\lambda_0 W_{\alpha\rho}\nabla^\rho\sigma + \frac{\lambda_0}{2}H_{\alpha\beta\gamma}V^{\beta\gamma}e^{2\sigma} \quad (3.13)$$

with the constant $c = -2\lambda_0$ [2]. The resulting two terms in the action, the renormalized one-loop part and the invariant two-loop part (which we did not give here for brevity's sake, but which can be found in [2]), then are manifestly invariant under the one-loop level form of the T-duality map $\sigma \leftrightarrow -\sigma$, $V_\mu \leftrightarrow W_\mu$, acting on the corrected

fields. If we return to the original one-loop level degrees of freedom (unhatted ones in the equation (3.9)), and interpret the $O(\alpha')$ shifts as the two-loop corrections to the T-duality map, we can rewrite the new two-loop T-duality map as

$$
\begin{aligned}
\sigma \;\; &\rightarrow \;\; -\sigma - 4\alpha'\lambda_0(\nabla\sigma)^2 - \frac{\alpha'\lambda_0}{2}[e^{2\sigma}Z + e^{-2\sigma}T] \\
V_\mu \;\; &\rightarrow \;\; W_\mu - 4\alpha'\lambda_0 W_{\mu\nu}\nabla^\nu\sigma - \alpha'\lambda_0 H_{\mu\nu\lambda}V^{\nu\lambda}e^{2\sigma} \\
W_\mu \;\; &\rightarrow \;\; V_\mu - 4\alpha'\lambda_0 V_{\mu\nu}\nabla^\nu\sigma + \alpha'\lambda_0 H_{\mu\nu\lambda}W^{\nu\lambda}e^{-2\sigma} \\
H_{\mu\nu\lambda} \;\; &\rightarrow \;\; H_{\mu\nu\lambda} - 12\alpha'\lambda_0\nabla_{[\mu}(W_\nu{}^\rho V_{\lambda]\rho}) - 12\alpha'\lambda_0 V_{[\mu\nu}W_{\lambda]\rho}\nabla^\rho\sigma \\
&\qquad -12\alpha'\lambda_0 W_{[\mu\nu}V_{\lambda]\rho}\nabla^\rho\sigma - 3\alpha'\lambda_0(e^{2\sigma}V^{\rho\sigma}V_{[\mu\nu} - e^{-2\sigma}W^{\rho\sigma}W_{[\mu\nu})H_{\lambda]\rho\sigma}
\end{aligned}
\tag{3.14}
$$

The full reduced action, containing all one- and two-loop contributions is invariant under (3.14) to order α', as one can check by directly applying these transformation rules. This is our final result.

4   Conclusion

In summary, we have presented here the $O(\alpha')$ two-loop corrections to the lowest-order T-duality map in string theory. We have started with the effective field theory of the model-independent zero mass sector, which included two-loop corrections in the manifestly unitary form. Focusing on the string backgrounds with a single isometry, we have shown that the theory is invariant under the two-loop corrected T-duality map. We have arrived at the form of the corrections by an iterative reformulation of the α' expansion: those $O(\alpha')$ terms which violate the one-loop form of duality induce $O(\alpha')$ corrections in the original duality map. The terms linear in α' should be thought of as the first subleading terms of the Taylor expansion of the duality map in α'. One unusual feature of the scheme we have used is that the BPS solutions of the lowest-order action do not retain their form when the two-loop terms are included. As a result, when our duality corrections are applied to BPS states, they contain nonvanishing terms to $O(\alpha')$. While this may sound odd, given the current lore [14], one should remember that while for BPS states there exists a scheme in which the classical solutions are exact to all orders in the α' expansion, this of course need not be true in any scheme. Hence, those terms among our two-loop corrections which do not vanish on BPS backgrounds should be removed by string field redefinitions. We will not delve on the details here. Suffice it to say that, in some sense, these terms behave like gauge degrees of freedom.

Another interesting feature of our calculation is that the torsion field strength plays a crucial role in restoring two-loop duality. This should not come as a complete surprise. As has been pointed out by Maharana and Schwarz, who discovered the Chern-Simons terms in the definition of the torsion field strength in the reduced

theory [10], the anomaly was essential in rendering the one-loop theory T-duality invariant. This role of the anomaly seems to persist to two loops, and raises an interesting possibility that the concepts of the anomaly and T-duality may somehow be related in the full quantum theory beyond the effective action limit (M-theory). Checking if such a relationship exists demands a nonperturbative approach, because of the complexity of higher-loop counterterms.

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