CERN-TH/97-198
hep-th/9708115

Adjoint QCD_2 and the Non-Abelian Schwinger Mechanism

S. DALLEY [1]

Theory Division, CERN
CH-1211 Geneva 23, Switzerland

Abstract

Massless Majorana fermions in the adjoint representation of $SU(N_c)$ are expected to screen gauge interactions in $1+1$ dimensions, analogous to a similar Higgs phenomena known for $1+1$-dimensional $U(1)$ gauge theory with massless fundamental fermions (Schwinger model). Using the light-cone formalism and large-N_c limit, a non-abelian analogue of the Schwinger boson is shown to be responsible for the screening between heavy test charges. This adjoint boson does not exist simply as a physical state, but boundstates are built entirely from this particle.

[1] CERN Fellow, on leave from Department of Applied Mathematics and Theoretical Physics, Cambridge University, Silver Street, Cambridge CB3 9EW, England.

1 Introduction

In a classic paper [1], Schwinger showed that the photon of two-dimensional QED acquires a propagating longitudinal component of mass $e/\sqrt{\pi}$ by a dynamical version of the Higgs mechanism involving massless electrons. This solvable model has been a source of much inspiration in the development of gauge theories of particle physics. It is natural to ask if there is a similarly tractable model involving non-abelian gauge theory. In this paper we will show that by adding additional matter representations in the adjoint representation to QCD in $1+1$ dimensions (QCD_2), one has a very close analogue of the Schwinger mechanism. Such models are interesting because one may study exactly some of the non-perturbative effects of the zero transverse momentum modes of gluons or gluinos from $3+1$ dimensions. Even when exact results are not possible, the models are useful for testing numerical algorithms which can then be applied in higher dimensions.

Recent interest in the problem of adjoint QCD_2 began with the light-cone quantisation of the large-N_c limit performed by Klebanov and the author [2]. Numerical solution of the light-cone Schrodinger equation for singlet boundstates of adjoint quanta revealed repeated 'Regge trajectories' — a kind of glueball analogue of the single meson trajectory found by 't Hooft for large-N_c QCD_2 with fundamental fermions [3]. In the low-lying spectrum these trajectories could be accurately classified by the number of adjoint quanta in a boundstate, but since the number of these particles in not conserved, even at large N_c, it was found that at higher mass this simple picture broke down for light quanta. Further numerical and analytic work [4, 5, 6, 7] confirmed these conclusions and it was also suggested that generically the density of bound states rises exponentially, leading to a Hagedorn transition [4, 8].

New insight into the problem involving massless adjoint fermions came from an observation of Kutasov and Schwimmer [9], who showed that for massless fermions in two-dimensional gauge theory (not necessarily large N_c) the massive physics is largely independent of the representations present, provided they make up the same value of the chiral anomaly in each of the two chiral sectors, so that the total anomaly cancels. This result is especially clear in the light-cone formalism, where massless left moving fermions decouple (for a quantisation surface $x^+ = (x^0 + x^1)/\sqrt{2}$ = const.) from the Hilbert space of massive boundstates. Physical results are thus insensitive to most of the details of the left-moving representations. Choosing instead the quantisation surface $x^- = (x^0 - x^1)/\sqrt{2}$ = const., one arrives at the same conclusion for the right-moving fermions and therefore the entire theory. One consequence of this universality is that the massive physics of massless adjoint Majorana fermions is the same as that for $N_f = N_c$ flavours of massless fundamental Dirac fermions.

Gross et al. [10] have emphasized that this should imply screening of fundamental sources in the presence of dynamical massless adjoint fermions. Such screening obviously occurs in the presence of dynamical fundamental fermions (unless $N_c >> N_f$) since the flux line can break, but it is far from obvious that this should occur in the adjoint case. Although the universality results of ref.[9] constitute a (physicist's) proof, one would nevertheless like to understand the screening behaviour of massless adjoint fermions from a more direct and physical viewpoint. Various arguments were advanced in ref.[10] in evidence of this conclusion, and the conclusion that screening disappears if the adjoint fermions are given a mass, when the equivalence with fundamental fermions no longer holds. The most powerful of these arguments showed that the Wilson loop obeys the appropriate area or perimeter law according to whether the adjoint fermions are massive or massless respectively. It was also emphasized that the Schwinger model [1], where fractional charges can be screened by massless integer charges, was an abelian prototype of this behaviour.

In this paper, the correspondence with the Schwinger mechanism — a two-dimensional dynamical version of the Higgs phenomenon — is made more explicit. In the large-N_c QCD_2 with adjoint fermions, the vacuum polarization of the gluon is calculated in light-cone Tamm-Dancoff formalism, showing that adjoint fermions screen the linear Coulomb potential between heavy fundamental sources when massless but not when massive. A composite bosonic state transforming in the adjoint representation of global colour symmetry is found to be responsible for this non-abelian Schwinger mechanism, in rather direct analogy with Schwinger's massive photon. For massless adjoint fermions, the singlet spectrum of single-particle states is built entirely out of these bosons. The light-cone analysis has many similarities with that of the usual Schwinger model [11].

The large-N_c limit is used in this paper since it illustrates particularly clearly the phenomena in question. There has been a large amount of recent work on the vacuum properties at finite N_c, mostly for $SU(2)$, a (probably incomplete) list of which is refs.[12].

2 Adjoint QCD_2 and Screening.

The action for $1+1$ dimensional $SU(N_c)$ gauge theory coupled to Majorana fermions Ψ of mass m in the adjoint representation is

$$S = \int d^2x \, \mathrm{Tr} \, \left\{ \mathrm{i}\overline{\Psi}\gamma_\alpha D^\alpha \Psi - m\overline{\Psi}\Psi - \frac{1}{4g^2} F_{\alpha\beta}F^{\alpha\beta} \right\} \ . \tag{1}$$

The conventions of ref.[2] will largely be followed (in particular with regard to normal-ordering). In the light-cone formalism x^+ is treated as 'time' and x^- as 'space' and we use the light-cone gauge $A_- = (A_0 - A_1)/\sqrt{2} = 0$. Then A_+ and the left-moving components of Ψ are eliminated by their

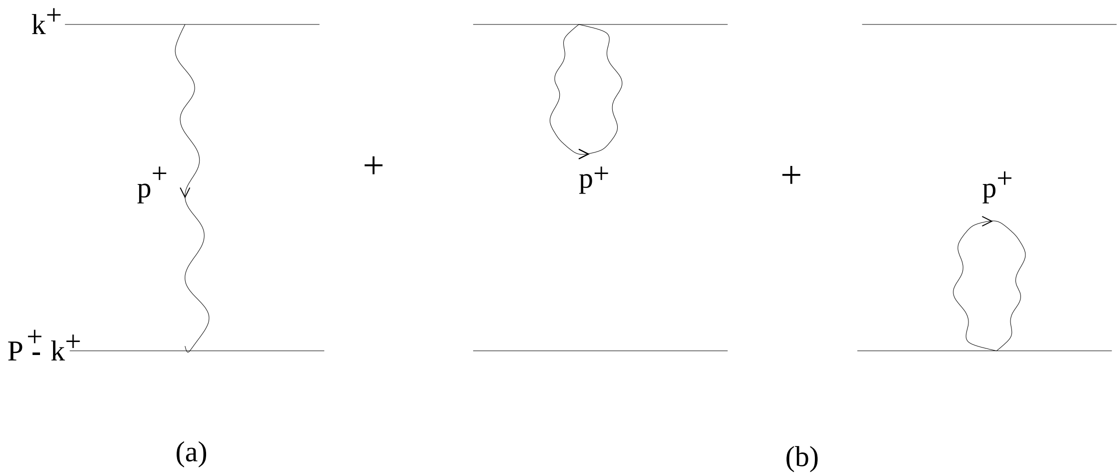

Figure 1: Elementary processes of light-cone perturbation theory contributing to the boundstate equation (4). Wavey lines are instantaneous gluons, plain lines are fundamental fermions. Evolution in x^+ to the right is understood. The (bare) gluons propogate in x^- but not in x^+; they are instantaneous. The self-energy diagrams (b), corresponding to the last term in eq.(4), effect the principal-value nature of the Coulomb exhange process.

constraint equations of motion to yield a light-cone hamiltonian

$$P^- = \int dx^- \ \mathrm{Tr} \ \left\{ \frac{m^2}{2} \psi \frac{1}{\mathrm{i}\partial_-} \psi + \frac{g^2}{2} J^+ \frac{1}{(\mathrm{i}\partial_-)^2} J^+ \right\} \ . \tag{2}$$

The traceless hermitian fermionic matrices ψ_{ij} are the propogating right-moving components of Ψ while the current $J^+_{ij} = 2\psi_{ik}\psi_{kj}$, with $i \in \{1, \cdots, N_c\}$ (at large N_c we are justified in not subtracting the appropriate Traces to distinguish $SU(N_c)$ from $U(N_c)$). The only remnants of the constrained degrees of freedom are the instantaneous propagators $1/\partial_-$ and $1/(\partial_-)^2$ of the left-movers and A_+ respectively, which appear in the mass term and current term respectively in (2). The Hilbert space at fixed light-cone time x^+ is constructed from the Fourier modes of ψ,

$$\psi(x^-) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dp^+ \tilde{\psi}(p^+) \mathrm{e}^{-\mathrm{i}p^+ x^-} \ , \tag{3}$$

by applying creation operators $\tilde{\psi}(p^+)$, $p^+ < 0$, to a Fock vacuum $|0>$ annihilated by $\tilde{\psi}(p^+)$, $p^+ > 0$. To any finite order in a Tamm-Dancoff truncation on the number of fields $\tilde{\psi}$ in the Hilbert space, only singlet states under the residual global colour transformation $U^\dagger \psi U$, $U \in SU(N_c)$, are annihilated by $\tilde{J}^+(p^+ = 0)$ and so avoid the $1/(p^+)^2$ singularity of (2) at $p^+ = 0$. There are also zero modes $\tilde{\psi}(0)$ which form representations of an $SU(N_c)$ affine Lie algebra when applied to the Fock vacuum $|0>$. Ignoring them is valid at large N_c and $m = 0$ for the bosonic single-particle boundstates [9], and also likely to be a good approximation for large m since the endpoint of the wavefunction in momentum space is suppressed.

One may easily show that the instantaneous gluon propogator $1/(\partial_-)^2$ corresponding to A_+ gives rise to a linear potential between two heavy coloured sources. The light-cone Schrodinger equation for the wavefunction $\phi(k^+, P^+ - k^+)$ of a pair of fundamental fermions of mass m_F and momentum k^+ and $P^+ - k^+$ is 't Hooft's equation [3]

$$\begin{aligned} 2P^-\phi(k^+, P^+ - k^+) &= m_F^2 \left(\frac{1}{k^+} + \frac{1}{P^+ - k^+} \right) \phi(k^+, P^+ - k^+) \\ &\quad - g_N^2 \int_{-k^+}^{P^+ - k^+} \frac{dp^+}{(p^+)^2} [\phi(k^+ + p^+, P^+ - k^+ - p^+) - \phi(k^+, P^+ - k^+)] \quad (4) \end{aligned}$$

where $g_N^2 = g^2 N_c/\pi$ is held fixed in the large N_c limit. The elementary processes which comprise this equation are illustrated by the diagrams of light-cone perturbation theory in Figure 1. When m_F is large the non-relativistic (equal-time) Schrodinger equation may be derived from (4) by expanding in powers of velocity and $1/m_F$ [13]. We have

$$\begin{aligned} k^+ &= \sqrt{m_F^2 + (k^1)^2} + k^1 \\ &= m_F + k^1 + \frac{(k^1)^2}{2m_F} + \cdots \end{aligned} \quad (5)$$

The wavefunction is peaked at $k^+ = P^+/2$ and one may derive an equation in terms of the relative equal-time momentum $q^1 \approx 2m_F(1 - 2(k^+/P^+)) << m_F$ of the pair of fermions;

$$\frac{(q^1)^2}{4m_F}\phi(q^1) - g_N^2 \int_{-\infty}^{\infty} \frac{dp^1}{(p^1)^2} [\phi(p^1 + q^1) - \phi(q^1)] = E\ \phi(q^1)\ . \quad (6)$$

Here, $p^1 = 4m_F p^+/P^+$,

$$\phi(q^1) = \phi\left(P^+\left(\frac{1}{2} + \frac{q^1}{4m_F}\right), P^+\left(\frac{1}{2} - \frac{q^1}{4m_F}\right)\right) \quad (7)$$

and $E = \sqrt{2P^+P^-} - 2m_F$ is the binding energy. In terms of the position space wavefunction $\phi(x^1)$ this becomes

$$\left[-\frac{1}{4m_F}(\partial_{x^1})^2 + V(x^1)\right]\phi(x^1) = E\phi(x^1)\ , \quad (8)$$

with $V(x^1) = \pi g_N^2 |x^1|$. Note that the small p^1 region in (6), hence the small p^+ region in (4), governs the asymptotic behaviour of V as $|x^1| \to \infty$.

If we now add adjoint fermions to the problem, Figure 2 shows the expansion of the full A_+ propagator $G(x)$ in terms of the bare instantaneous propogator g_N^2/x^2 and adjoint fermion loop corrections. We have introduced the momentum fractions $x = p^+/P^+$ transferred and $y = k^+/P^+$ flowing through the external fundamental fermion, where P^+ is the total momentum flowing through the system. There is also an analogous self-energy equation (see fig.1(b)). In light-cone Hamiltonian formalism one must remember to divide by $(M^2 - \sum_a m_a^2/x_a)$ for every distinct intermediate state, where m_a and x_a are masses and momentum fractions of the physical quanta appearing in the intermediate state and $M^2 = 2P^+P^-$ is the invariant mass of the system. The one-loop approximation

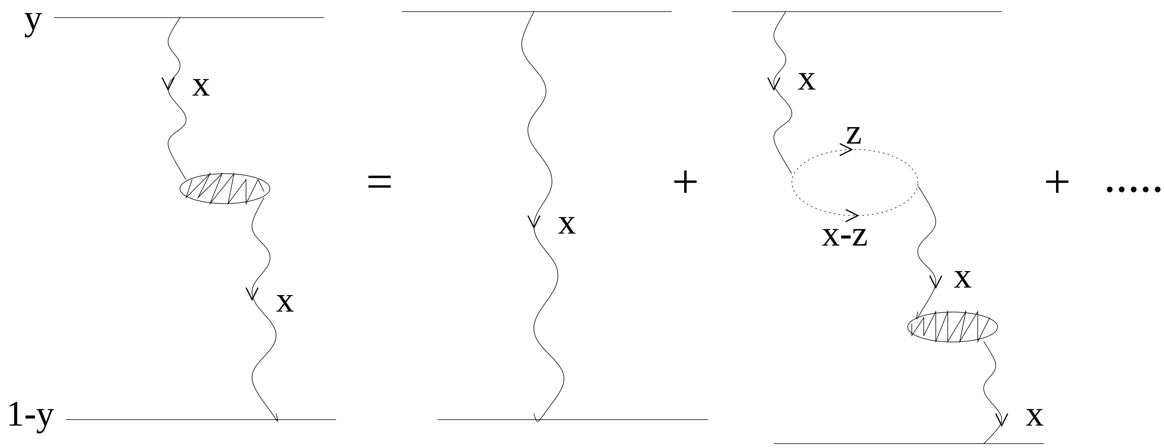

Figure 2: Expansion in light-cone perturbation theory of the full A_+ propagator. Dotted lines are adjoint fermions.

to fig. 2 sums fermion bubbles;

$$G(x) = \frac{g_N^2}{x^2} + \frac{g_N^2}{x^2} \cdot \int_0^x dz \frac{1}{M^2 - \frac{m_F^2}{y-x} - \frac{m_F^2}{1-y} - \frac{m^2}{z} - \frac{m^2}{x-z}} \cdot G(x) \ . \tag{9}$$

Now

$$M^2 - \frac{m_F^2}{y-x} - \frac{m_F^2}{1-y} \approx m_F E - \frac{(q^1)^2}{4} - 4m_F^2 x \tag{10}$$

for large m_F because $x << y$. But since $x = p^1/4m_F$, $p^1 \sim q^1$, and $E \sim (q^1)^2/m_F$, the first two terms may be neglected compared to the third. Evaluating the z-integral one then finds

$$G(x) = \frac{g_N^2}{x^2 + C(x)} \tag{11}$$

where

$$C(x) = \frac{g_N^2}{4m_F^2}\left(1 + \frac{m^2}{2m_F^2 x^2 (r_+ - r_-)} \log\left[\frac{|r_-|}{r_+}\right]\right) \tag{12}$$

$$r_\pm = \frac{1 \pm \sqrt{1 + m^2/m_F^2 x^2}}{2} \ . \tag{13}$$

If $m \neq 0$ then $G(x \to 0) \to g_N^2/[x^2(1 + g_N^2/6m^2)]$, so that the non-relativistic potential is still asymptotically linear, but with reduced slope. If $m = 0$ the string tension vanishes and the dressed gluon propogator changes to

$$G(x)|_{m=0} = \frac{g_N^2}{x^2 + g_N^2/4m_F^2} \ . \tag{14}$$

The pole in the gluon propogator has been cancelled by another pole coming from the propagation of two adjoint fermions produced with a constant wavefunction $\phi(z, x-z) = \text{const.}$. The corresponding non-relativistic potential is easily found to be

$$V(x^1) = \frac{g_N \pi}{2}\left(1 - \mathrm{e}^{-2g_N|x^1|}\right) \tag{15}$$

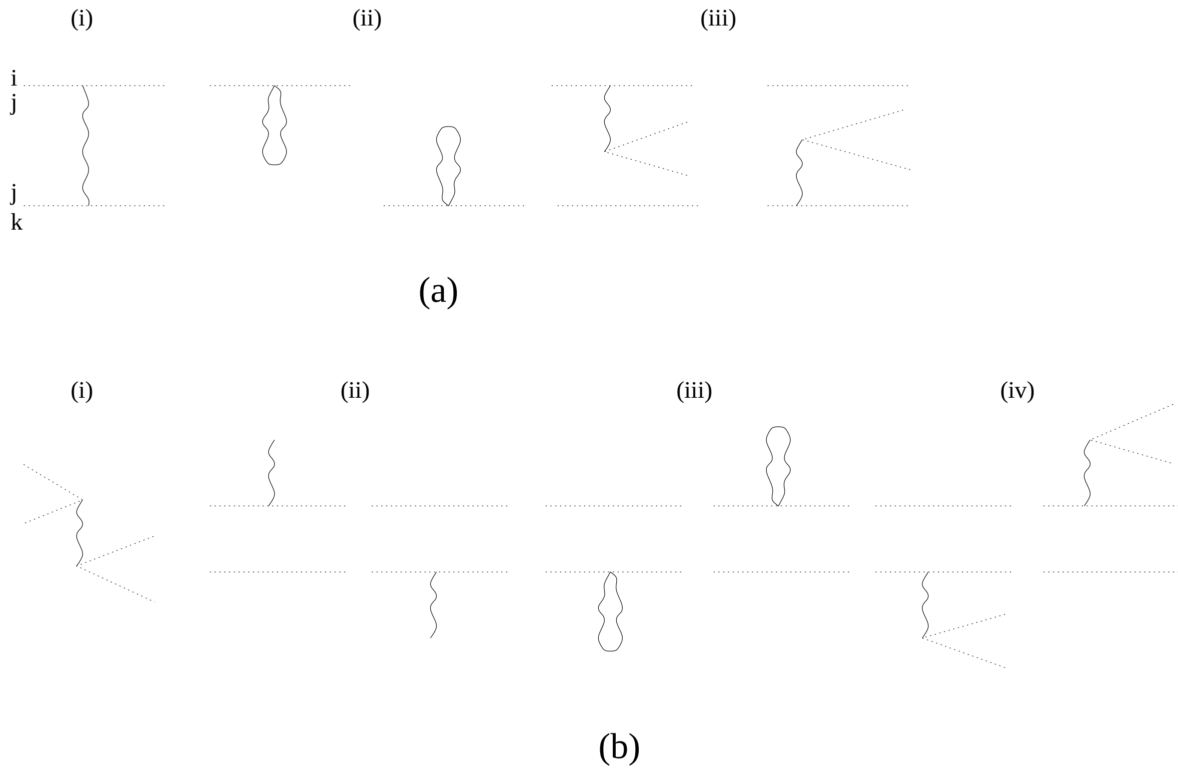

Figure 3: This figure lists the elementary processes acting on $\tilde{\psi}_{ij}\tilde{\psi}_{jk}$ for example: (a) internal processes, which only involve the contracted index j; (b) external processes, which involve one or both of the uncontracted indices i and k. Generalization to $2n$ fermions in the initial state is straightforwardly obtained by adding the appropriate spectators, on the outside of figs.(a)(i)(ii)(iii) and on the inside of figs.(b)(ii)(iii)(iv); fig.(b)(i) may become either internal or external when spectators are added.

At small distances the Coulomb potential $V(x^1 \to 0) = \pi g_N^2 |x^1|$ is recovered while at large distances the potential tends to a constant $V(x^1 \to \infty) \to g_N \pi/2$. Potentials similar to (15) have been found from abelian [10] and non-abelian [14] static classical solutions of the gluon effective action, but the physical mechanism underlying it was not apparent.

The $m = 0$ result Eq.(14) is actually much more general than 1-loop since it includes *all* further diagrams inside the fermion loop. The reason is that all these processes cancel. The planar topology of the large N_c limit allows us to understand this result by writing a meaningful boundstate equation for adjoint states,[2]

$$|\Psi>_{\rm adj} = \sum_{n=1}^{\infty} \int_0^1 \prod_{b=1}^{2n} dw_b \frac{\delta(\sum_b w_b - 1)}{N^n} \phi_{2n}(w_1, w_2, \ldots, w_{2n}) \{\tilde{\psi}(-w_1)\tilde{\psi}(-w_2)\cdots\tilde{\psi}(-w_{2n})\}_{ik}|0> , \tag{16}$$

provided we keep only the 'internal' processes involving contracted colour indices (Figure 3 (a)). In

[2]Here, the w_b are fractions of the fraction x.

this case the corresponding light-cone Schrodinger equation $2P^+P^-|\Psi>_{\rm adj}= M^2_{\rm adj}|\Psi>_{\rm adj}$ has an exact $M^2_{\rm adj}=0$ two-particle solution with constant wavefunction $\phi_2=$ const., $\phi_{2n}=0$ for $n>1$. It is straightforward to verify this on the boundstate integral equations [5] for $|\Psi>_{\rm adj}$ restricted to the processes of fig.3(a); the non-trivial equations are

$$M^2_{\rm adj}\phi_2(w_1,1-w_1)=g_N^2\int_{-w_1}^{1-w_1}\frac{dw}{w^2}[\phi_2(w_1,1-w_1)-\phi_2(w_1+w,1-w-w_1)]\ , \tag{17}$$

$$\begin{aligned} &M^2_{\rm adj}\phi_4(w_1,w_2,w_3,1-w_1-w_2-w_3)=0\\ &=\frac{g_N^2}{(w_2+w_3)^2}\phi_2(w_1,1-w_1)-\frac{g_N^2}{(w_2+w_3)^2}\phi_2(w_1+w_2+w_3,1-w_1,w_2,w_3)\ . \end{aligned}\tag{18}$$

The constant two-particle wavefunction remains an eigenstate if we add the process of fig.3(b)(i), which allows it to mix with the longitudinal component of the gluon. We must add

$$g_N^2\int_0^1 dw\ \phi_2(w,1-w) \tag{19}$$

to eq.(17). The mass eigenvalue is then shifted to $M^2_{\rm adj}=g_N^2$ (the Higgs mechanism). Since other solutions of the adjoint boundstate equation are orthogonal to $\phi_2=$ const., they do not contribute as intermediate states in the vacuum polarization. The constant wavefunction two-particle adjoint state is the non-abelian analogue of Schwinger's boson. Unlike Schwinger's massive photon, it does not occur simply as a physical state because it is coloured. Its definition required us to drop 'external' processes involving the uncontracted colour indices (Figure 3(b)). In the Tamm-Dancoff approximations these involve uncancelled infinities unless the adjoint boson is inserted inside an overall singlet. These processes cause it to interact with other coloured states through Coulomb exchange (fig.3(b)(ii)(iii)) and creation of further bosons (fig.3(b)(iv)).

Another way to see why the constant wavefunction two-particle adjoint state plays a privileged role when $m=0$ is to note that it is simply the current $\tilde{J}^+_{ik}(-p^+)|0>$ acting on the vacuum. As emphasized in ref.[9], the hamiltonian (2) is expressed entirely in terms of the $SU(N_c)$ currents $J^+=J^{+a}\mathbf{T}^a$ when $m=0$, which satisfy an affine Lie algebra

$$[\tilde{J}^{+a}(p),\tilde{J}^{+b}(k)]=kN_c\delta_{p+k,0}\delta^{ab}+{\rm i}f^{abc}\tilde{J}^{+c}(p+k)\ , \tag{20}$$

and the boundstate problem can be framed algebraicly in terms of the bosonic basis of $\tilde{J}^+$'s rather than the fermionic basis of $\tilde{\psi}$'s.[3] Single-particle bosonic physical states are of the form

$$\sum_{n=2}^{\infty}\int_0^{P^+}\prod_{a=1}^{n}dk_a^+\delta(\sum_a k_a^+-P^+)h_n(k_1^+,\dots,k_n^+)\,{\rm Tr}\{\tilde{J}^+(-k_1^+)\cdots\tilde{J}^+(-k_n^+)\}|0>\ . \tag{21}$$

[3] In general at $m=0$ the basis of $\tilde{\psi}$'s will contain multi-particle combinations of the basis of $\tilde{J}^+$'s [9].

The light-cone boundstate integral equations for h_n have been worked out in detail in ref.[15], and are similar to the corresponding ones in the fermionic basis [5]. There are also fermionic boundstates, related by a softly broken supersymmetry [4, 17].

When $m > 0$, the non-abelian Schwinger boson is no longer an exact eigenstate of the adjoint boundstate equation defined above, and many other intermediate states contribute to the vacuum polarization in a complicated way. At large $m >> g^2 N$ we should recover the bare Coulomb amplitude. There is evidence that confinement is lost at high temperature however [4, 16], at least for large m.

3 Summary.

It has been shown how a non-abelian analogue of the Schwinger boson is responsible for screening of heavy sources in large-N_c QCD_2 with massless adjoint Majorana fermions. The heavy source potential was calculated from vacuum polarization of the gluon; every gluon propogator may be replaced by the screened version (14). When the adjoint fermions are massive, although there are additional states contributing to the vacuum polarization, which were not included in the calculation beyond one-loop, the confining result found here is probably a good guide to the exact behaviour. Since Schwinger's work, physicists have come up with innumerable ways to look at the Schwinger model and no doubt many of them have an analogue in Adjoint QCD_2. These investigations are left for the future, but we mention that it would be interesting to understand how the results relate to the those found in the bosonized formalism [18] and also to see explicitly what happens to gauge invariance.

Acknowledgements: I thank D.Kutasov for a discussion.

References

[1] J. Schwinger, Phys. Rev. **D128** 2425 (1962) .

[2] S. Dalley and I. R. Klebanov, Phys. Rev. **D47** (1993) 2517.

[3] G. 't Hooft, Nucl. Phys. **B75**, 461 (1974).

[4] D. Kutasov, Nucl. Phys. **B414** (1994) 33.

[5] G. Bhanot, K. Demeterfi, and I. R. Klebanov, Phys. Rev. **48** (1993) 4980.

[6] K. Demeterfi, I. R. Klebanov, and G. Bhanot, Nucl. Phys. **B418** (1994) 15.

[7] F. Antonuccio and S. Dalley, Nucl. Phys. B **461** 275 (1996).

[8] J. Boorstein and D. Kutasov, Lecture by DK at *New Non-Perturbative Methods and Quantization on the Light-Cone*, Les Houches, France, 24 Feb - 4 Mar 1997 (to appear in proceedings).

[9] D. Kutasov and A. Schwimmer, Nucl. Phys. **B442** (1995) 447.

[10] D. J. Gross, I. R. Klebanov, A. V. Matytsin, and A. Smilga, Nucl. Phys. **B461** (1996) 109.

[11] H. Bergknoff, Nucl. Phys. **B122** (1977) 215.

[12] I. Kogan, Phys. Rev. **D49** (1994) 6799; A. Smilga, Phys. Rev. **D49** (1994) 6836, Phys. Rev. **D54** (1996) 7757; F. Lenz, M. Shifman, and M. Thies, Phys. Rev. **D51** (1995) 7060; S. S. Pinsky and A. C. Kalloniatis, Phys. Lett. **B365** (1996) 225; A. Zhitnitsky, Phys. Rev. **D53** (1996) 5821; S. Pinsky and D. Robertson, Phys. Lett. **B379** (1996) 169; I. Kogan and A. Zhitnitsky, Nucl. Phys. **B465** (1996) 99; L.D. Paniak, G. W. Semenoff, and A. R. Zhitnitsky, Nucl. Phys. **B487** (1997) 191, preprint hep-ph/9701270; S. Pinsky and R. Mohr, Int. J. Mod. Phys. **A12** (1997) 1063; G. McCartor and D. G. Robertson, preprint hep-th/9612083; S. Pinsky, preprint hep-th/9612073; T. Fugleberg, I. Halperin, and A. Zhitnitsky, preprint hep-ph/9705281.

[13] G. P. Lepage and S. J. Brodsky, Phys. Rev. **D22** (1980) 2157.

[14] Y. Frishman and J. Sonnenschein, preprint hep-th/9701140; A. Armoni and J. Sonnenschein, preprint hep-th/9703114.

[15] A. Armoni and J. Sonnenschein, Nucl. Phys. **B457** (1995) 81.

[16] G. Semenoff, O. Tirkkonen, and K. Zarembo, Phys. Rev. Lett. **77** (1996) 2174.

[17] J. Boorstein and D. Kutasov, Nucl Phys. **B421** (1994) 263.

[18] Y. Frishman, A. Hanany, and J. Sonnenschein, Nucl. Phys. **B429** (1994) 75;