Quenched Spectroscopy for the $N = 1$ Super-Yang–Mills Theory

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We present results for the Quenched SU(2) $N = 1$ Super-Yang–Mills spectrum at $\beta = 2.6$, on a $V = 16^3 \times 32$ lattice , in the OZI approximation. This is a first step towards the understanding of the chiral limit of lattice $N = 1$ SUSY.

1. Introduction

Softly broken $N = 1$ supersymmetry gives a natural solution to the long-standing hierarchy problem arising from the Standard Model, considered as a low-energy effective model of a more fundamental theory which includes gravity. Moreover, analytical solutions have been found in a wide class of supersymmetric gauge theories [1]. The most remarkable results have been obtained in the context of extended supersymmetry ($N > 1$), where the renormalization properties are severely constrained. Thus, the study of supersymmetric gauge theories on the lattice is of great importance for the confirmation of the predicted non-perturbative behaviour for $N > 1$ and the extension of our understanding to the phenomenologically relevant softly broken $N = 1$ case.

The simplest non-abelian supersymmetric gauge theory is $N = 1$ SU(2) Super-Yang–Mills (SYM): this theory contains gluons and gluinos (Majorana fermions in the adjoint representation) and no matter fields. A lattice formulation of the $N = 1$ SYM theory was given in ref.[2]. The path integral is

$$Z = \int DU e^{-S_g} det(\Delta)^{1/2} e^{-\bar{\eta}^a [\Delta^{-1}]_{ab} \eta^b} \qquad (1)$$

where S_g is the usual sum over plaquettes; Δ_{ab} is the fermionic matrix (containing a Wilson term) and $a, b = 1, \ldots, N_c^2 - 1$ are color indices. The Majorana nature of the fermion fields is reflected in the power 1/2 of the fermion determinant [3]. Some preliminary unquenched results for gauge observables have been presented in refs.[3,4].

Supersymmetry imposes that the gluon and the gluino should be degenerate in mass (hence, due to gauge symmetry, both should be massless). However, both chiral symmetry and supersymmetry are explicitly broken on the lattice by the Wilson term, which gives mass to the gluino. Moreover, supersymmetry is also broken by the space-time discretization. In order to recover the desired continuum theory, both symmetries should be restored. In ref.[2] it has been shown that the chiral and the supersymmetric Ward Identities (WI) are simultaneously satisfied in the continuum limit, by tuning the bare gluino mass m_0 to the critical value m_c for which the renormalized gluino mass vanishes.

As in QCD, due to confinement, the low-energy spectrum is expected to consist of colorless composite fields of gluons and gluinos, which can be described by a low-energy effective supersymmetric lagrangian. Due to supersymmetry, the gluino-gluon bound belong to supermultiplets. The lowest-lying chiral supermultiplet is [5]

$$\Phi = (\tilde{\pi}, \tilde{\sigma}, \tilde{\chi}, F, G), \qquad (2)$$

where $\tilde{\pi} = \bar{\lambda}^a \gamma_5 \lambda^a$ is a pseudoscalar, $\tilde{\sigma} = \bar{\lambda}^a \lambda^a$ is

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a scalar and $\tilde{\chi} = \frac{1}{2}\sigma_{\mu\nu}F^a_{\mu\nu}\lambda^a$ is a fermion. The fields F and G (scalar and pseudoscalar respectively) are auxiliary fields with no dynamics. In the supersymmetric limit, these particles should be degenerate in mass [5].

The composite $\tilde{\pi}$ pseudoscalar is analogous to the QCD singlet η', since a single flavour of gluinos is present in the theory. Its two-point correlation function $C_{\tilde{\pi}}(t)$ contains connected as well as disconnected diagrams. In [5] it has been suggested that, in the OZI approximation (where only the connected part of $C_{\tilde{\pi}}(t)$ is retained), $\tilde{\pi}$ should behave like the QCD pion. This means that in this approximation $\tilde{\pi}$ should become massless at some critical value m_c of the bare gluino mass, whereas $\tilde{\sigma}$ and $\tilde{\chi}$ should remain massive. In the full theory, the disconnected part of $C_{\tilde{\pi}}(t)$ should give mass to $\tilde{\pi}$ and restore the predicted mass degeneracy between all the particles belonging to the Φ supermultiplet (since chiral symmetry and supersymmetry are simultaneously restored at m_c [2]).

2. Quenched Super-Yang-Mills theory

Since supersymmetry requires an exact balance of the bosonic and fermionic degrees of freedom, the quenched approximation is not necessarily a good approximation to a supersymmetric theory. Nevertheless, it is useful to study the quenched spectrum as a preliminary step towards the true SYM.

We present results for the supermultiplet Φ for $\tilde{\pi}, \tilde{\sigma}$ and $\tilde{\chi}$, obtained at $\beta = 2.6$ on a $V = 16^3 \times 32$ lattice in the OZI approximation. All statistical errors have been computed with the jacknife method. Some preliminary quenched results for $\tilde{\chi}$ have been presented in [6] (see also [7]).

We obtain the pseudoscalar and scalar masses from a sample of 950 gluino propagators at four different values of the Wilson hopping parameter: $K = 0.174, 0.178, 0.182, 0.184$. These values have been chosen in order to simulate relatively light gluinos. In the case of the $\tilde{\pi}$ two-point correlation function, the effective mass has not settled fully to a plateau; the lattice is somewhat short in the time-direction. We extract a reliable $\tilde{\pi}$ mass by performing two-mass fits of the correlation function. In the case of the $\tilde{\sigma}$ correlation function a plateau is observed within the statistical errors (much bigger than for $\tilde{\pi}$). Hence, we can extract the $\tilde{\sigma}$ mass from a one-mass fit. We stress that, in contrast to the measurements of the QCD scalar, no smearing has been required. The results for the $\tilde{\pi}$ and $\tilde{\sigma}$ masses are presented in fig.2. The $\tilde{\pi}$ mass shows the expected dependence on the Wilson hopping parameter: namely, $m^2_{\tilde{\pi}}$ decreases linearly with K. The critical value where the gluino becomes massless is

$$K_c = 0.18752(9) \tag{3}$$

(with one-mass fits we obtain $K_c = 0.18759(2)$).

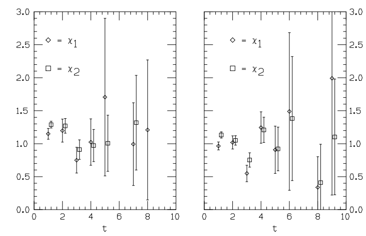

Figure 1. Effective masses for $\tilde{\chi}$ with $n_s = 4$ (left) and $n_s = 8$ (right) as a function of time at $K = 0.182$. Diamonds refer to $C_1(t)$ and squares to $C_2(t)$.

A numerically independent measure of K_c can be extracted from the quenched OZI chiral Ward Identity. In a theory with two flavours of gluinos, the non-singlet axial current $A^{ns}_\mu = \bar{\lambda}^a\gamma_\mu\gamma_5\lambda^a$ satisfies the usual PCAC relation:

$$Z_A \partial_\mu A^{ns}_\mu = 2(m_0 - \bar{m})P^{ns}, \tag{4}$$

where $P^{ns} = \bar{\lambda}^a\gamma_5\lambda^a$ is the non-singlet pseudoscalar density, Z_A the finite renormalization constant of A^{ns}_μ and $m_0 - \bar{m} = 0$ at K_c. In the quenched approximation, the following identities hold:

$$\begin{aligned} \langle \partial_\mu A^s_\mu P^s \rangle|_{OZI} &= \langle \partial_\mu A^{ns}_\mu P^{ns} \rangle, \\ \langle P^s P^s \rangle|_{OZI} &= \langle P^{ns} P^{ns} \rangle, \end{aligned} \tag{5}$$

where A^s_μ, P^s are the singlet axial current and pseudoscalar density respectively. Hence,

$$2\rho = \left.\frac{\langle \partial_\mu A^s_\mu P^s \rangle}{\langle P^s P^s \rangle}\right|_{OZI} = \frac{2(m_0 - \bar{m})}{Z_A}. \tag{6}$$

The quantity 2ρ should vanish in the chiral limit. We obtain in this way

$$K_c = 0.18762(4) \tag{7}$$

in perfect agreement with eq.(3). For more details, see [8].

Using its properties under the discrete symmetries, it is easy to show [8] that the two-point correlation function of $\tilde{\chi}$ is described by just two form factors:

$$C_{\tilde{\chi}}(t) = C_1(t)I + C_2(t)\gamma_0. \tag{8}$$

Moreover, $C_{\tilde{\chi}}(t)$ is OZI-blind. In order to reduce contact terms in the time-direction (note that $\tilde{\chi}$ is an extended object, being proportional to $\sigma_{\mu\nu}F^a_{\mu\nu}$) we use the spatial operator $\tilde{\chi}^s = \frac{1}{2}\sigma_{ij}F^a_{ij}\lambda^a$, whose correlation function should behave as $C_{\tilde{\chi}}(t)$ at large times. We implemented the smearing of ref.[9] in F^a_{ij} by replacing the link $U_\mu(x)$ with $U_\mu(x)$ plus the staple multiplied by a tuning parameter ϵ (in our case, ϵ has been fixed at 0.5). This smearing has been repeated $n_s = 4$ and 8 times. In fig.1 we present results obtained for the effective mass of $\tilde{\chi}$ with $n_s = 4$ (500 gluino propagators) and $n_s = 8$ (450 propagators) at a fixed K. We show results from both $C_1(t)$ and $C_2(t)$. A short plateau appears at very small times. We extract the $\tilde{\chi}$ effective mass from the $C_2(t)$ form factor at $n_s = 4$. No signal at all was observed without smearing.

In fig.2 we present the masses of $\tilde{\sigma}$ and $\tilde{\chi}$, the squared $\tilde{\pi}$ mass and the 2ρ parameter. Linear extrapolations at K_c are also shown. It can be seen that the $\tilde{\sigma}$ and $\tilde{\chi}$ masses at K_c are similar in magnitude. This could be a first signal of the recovery of supersymmetry at K_c, but must be carefully verified in the context of a full unquenched non-OZI simulation.

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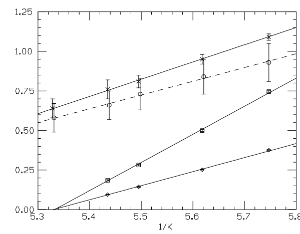

Figure 2. Results for the $\tilde{\sigma}$ mass (crosses), the $\tilde{\chi}$ mass (circles), the $\tilde{\pi}$ squared mass (squares) and the 2ρ parameter (diamonds) as a function of $1/K$.

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