CERN-TH/97-188
ROME 97/1175
TUM-HEP-279/97

CHARMING-PENGUIN ENHANCED B DECAYS

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Abstract

Long-distance contributions of operators of the effective non-leptonic weak Hamiltonian containing charmed currents have been recently studied. Penguin-like contractions of these operators, denoted as charming penguins, have been shown to be relevant for several B decays into two pseudoscalar mesons. In particular, they are expected to give large enhancements to processes which would be otherwise Cabibbo suppressed. Their contributions easily lead to values of $BR(B^+ \to K^0\pi^+)$ and $BR(B_d \to K^+\pi^-)$ of about 1×10^{-5}, as recently found by the CLEO collaboration. In this paper, we show that such large branching fractions cannot be obtained without charming penguins. This holds true irrespectively of the model used to compute the non-leptonic amplitudes and of the relevant parameters of the CKM mixing matrix. We use the experimental measurements of the $B^+ \to K^0\pi^+$ and $B_d \to K^+\pi^-$ decay rates to constrain the charming-penguin amplitudes and to predict BRs for a large set of other two-body decay channels where their contributions are also important. These include several pseudoscalar-vector and vector-vector channels, such as $B \to \rho K$ and $B \to \rho K^*$, which have not been measured yet.

CERN-TH/97-188
ROME 97/1175
TUM-HEP-297/97
July 1997
Revised Oct. 1997

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1 Introduction

In a recent paper [1] it has been shown that penguin-like contractions of operators containing charmed quarks, denoted as charming penguins, are able to enhance the $B^+ \to K^0\pi^+$ and $B_d \to K^+\pi^-$ decay rates with respect to the values predicted with factorization. By assuming reasonable values for the charming-penguin contributions, the corresponding branching ratios are of the order of $(1–2) \times 10^{-5}$, larger than the expected $BR(B_d \to \pi^+\pi^-)$. This observation is particularly interesting because, in absence of charming-penguin diagrams, the $B^+ \to K^0\pi^+$ and $B_d \to K^+\pi^-$ rates turn out to be rather small either because there is a Cabibbo suppression or because the non-Cabibbo suppressed terms come from penguin operators which have rather small Wilson coefficients [1]. The recent CLEO measurements [2]

$$\begin{aligned} BR(B_d \to K^+\pi^-) &= (1.5^{+0.5+0.1}_{-0.4-0.1} \pm 0.1) \times 10^{-5} \\ BR(B^+ \to K^0\pi^+) &= (2.3^{+1.1+0.2}_{-0.9-0.2} \pm 0.2) \times 10^{-5} \\ BR(B^+ \to \eta' K^+) &= (7.1^{+2.5}_{-2.1} \pm 0.9) \times 10^{-5} \\ BR(B_d \to \eta' K^0) &= (5.3^{+2.8}_{-2.2} \pm 1.2) \times 10^{-5} \end{aligned} \tag{1}$$

allow an estimate of charming-penguin amplitudes and call for more quantitative studies.

In this paper, by using the experimental information obtained from the measured decay channels, we determine the parameters of charming penguins and predict a large set of BRs which have not been measured yet. The main results of our study, which will be described in detail below, are summarized in table 1. In several cases we find that the value of the BR is strongly enhanced by charming penguins. The most interesting channels are $B_d \to \rho^- K^+$, $B^+ \to \rho^+ K^0$ and $B_d \to \omega K^{(*)0}$, for which the contribution of charming penguins is larger than the theoretical uncertainties and the corresponding BRs are close to the present experimental upper limits. This means that they will eventually be measured, and compared to our predictions, in the near future. Within larger theoretical uncertainties, the BRs of other channels, such as $B \to \pi K^*$, $B^+ \to \omega K^{(*)+}$ and $B \to \phi K^{(*)}$, are also appreciably enhanced by charming penguins and close to the present limits. Large enhancements are also expected for $B \to \eta^{(\prime)} K$ decays. In these cases, however, our predictions are rather poor because of the presence of contributions, related to the anomaly, which are very difficult to evaluate. For $B_d \to \pi^0\pi^0$, we typically obtain a BR of about $(5–10) \times 10^{-7}$. Values as large as $(2–3) \times 10^{-6}$ remain, however, an open possibility. Finally, we also predict the BRs of several $B \to Dh$ ($h = \pi$, ρ, etc.) decay channels, for which only upper bounds exist, see table 2.

The plan of the paper is the following: in section 2, we recall some basic facts about charming penguins and describe the main features of the present analysis; in section 3 we discuss the theoretical uncertainties stemming from the parameters of the CKM mixing matrix, the choice of the Wilson coefficients of the weak Hamiltonian and the models used in the calculation of the relevant amplitudes; in section 4 the procedure used in the determination of the important hadronic parameters is illustrated; sections 5 and 6 contain a brief discussion of the results and the conclusions respectively.

[1] This is true unless the corresponding matrix elements are much larger than those predicted with factorization.

For the definition of the various parameters used in this study and the notation, the reader should refer to [1]; the experimental data have been taken from refs. [2, 3, 4].

2 Charming penguins

"Charming penguins" denote penguin-like contractions in matrix elements of operators containing charmed quarks, the contribution of which would vanish using factorization [1]. Their effects are enhanced in decays where emission diagrams are Cabibbo suppressed with respect to penguin diagrams. Similar effects can also be obtained by a breaking of factorization in the matrix elements of the penguin operators, Q_3–Q_{10}, or assuming large chromo-magnetic contributions. Indeed all penguin-like effects, including the chromo-magnetic ones, have the same quantum numbers as charming penguins. For this reason they cannot be disentangled experimentally. Only an explicit non-perturbative theoretical calculation of the operator matrix elements, done consistently in a given renormalization scheme (and missing to date), can distinguish the different terms. On the other hand, our parametrization of charming penguins effectively accounts for the other penguin effects.

In absence of a consistent calculation of the matrix elements, penguin effects can also be enhanced by choosing a procedure where large "effective" Wilson coefficients of the penguin operators are obtained and factorization is used to compute the hadronic amplitudes. Attempts to reproduce the experimental data along this line have been recently made, see for example refs. [5, 6] (the results of ref. [6] will be discussed more in detail in section 4). In these approaches, the physical mechanism for the enhancement is similar to the one of ref. [1], but the procedure used in the calculations is completely different: penguin diagrams involving charmed quarks are computed perturbatively on external quark states, and their effects included in the effective Wilson coefficients of the penguin operators Q_3–Q_{10}. It can be shown, however, that perturbation theory cannot be used to evaluate these effects: the relevant kinematical range of momenta involved in these calculations [5, 6] corresponds to a region where charm-quark threshold effects, which are largely responsible for the enhancement of the $B \to K\pi$ rates, are important and the perturbative approach ought to fail [7]. For these reason, following ref. [1], we parametrize charming-penguin effects in terms of the non-perturbative parameters η_L and δ_L, rather than make any attempt to compute them using perturbation theory and factorization.

We now illustrate the main features of our new analysis:

i) in ref. [1], all the amplitudes were normalized to the disconnected emission diagrams, without computing them in any specific model. This makes the results model independent but only allows predictions of ratios of partial rates. In the present work we compute the disconnected-emission diagrams in the factorization approximation, which allows us to predict absolute branching ratios. The values of the hadronic parameters ξ and δ_ξ, relevant for connected-emission diagrams, are extracted from a fit to decay channels which are expected to be dominated by emission diagrams, namely $B \to \pi D^{(*)}$, $B \to \rho D^{(*)}$, $B \to K^{(*)} J/\Psi$ and $B \to DD^{(*)}$; we also allow for some breaking of factorization, by varying the disconnected-emission amplitudes by 15% with respect to their factorized value;

ii) we do not fit the Wilson coefficients of the relevant operators of the weak Hamiltonian, in the QCD combinations a_1 and a_2, see for example ref. [8]. We rather compute them at leading (LO) and next-to-leading (NLO) order in perturbation theory, at several values of the renormalization scale μ. In this way we estimate the uncertainty given by the scale and renormalization prescription dependence;

iii) the values of the charming-penguin parameters η_L and δ_L are constrained by fitting the first two decay channels in eq. (1). We do not attempt to fit the $B \to \eta' K$ decay rates since these channels receive contributions, due to the anomaly, which are difficult to estimate. For these channels we will present in the following a rough estimate of the branching ratio, obtained by including non-anomalous contributions only.

iv) using the values of the parameters as determined in i) and iii), a large set of BRs for two-body decay channels are predicted, including pseudoscalar-vector and vector-vector final states which were not considered in ref. [1]. The results are also compared with the available experimental bounds;

v) a comparative analysis of two-body BRs with and without charming penguins is performed. We show that there is strong evidence of a large charming penguin contribution in the data. As the experimental errors will be reduced, and more channels will be measured, this evidence will eventually find more support;

vi) given its relevance in the extraction of the CP violating angle α [9], particular attention is devoted to $BR(B_d \to \pi\pi)$ decay. We will give an average value of the $BR(B_d \to \pi^0\pi^0)$, corresponding to central values of our parameters, and a maximum value, obtained by varying annihilation, GIM-penguin and charming-penguin contributions [1] to this channel, with the constraint that the predictions of the measured channels remain compatible with the data at the 2-σ level.

In our analysis, we are mostly concerned with large enhancements induced by charming penguins, rather than with effects of the order of 20–30%. Moreover our study is not focused in testing the factorization hypothesis. For these reasons, since we obtain without any special effort satisfactory results for those decay channels which are dominated by emission diagrams, we have neither tried to optimize the parameters in this sector nor to estimate an error on the theoretical predictions. For these channels, we will only present results obtained with different models and Wilson coefficients. The spread of the results is representative of the theoretical error.

On the other hand, the present experimental information on charming-penguin dominated decays is not sufficient to fix precisely the relevant non-perturbative parameters. This is not a problem specific to our approach: in all cases where charming penguins are important, there are many smallish contributions (due to annihilations, GIM-penguins etc. [1]) which cannot be precisely estimated by any non-perturbative method. Thus we will present a band of expected values for those channels where the enhancement is very large.

3 Uncertainties of the theoretical predictions

In order to assess the relevance of the charming penguins and make sensible predictions, a central issue is checking the stability of the results. We have studied their dependence on several experimental and theoretical parameters which enter the calculation of the decay amplitudes. In particular:

1. most of the theoretical predictions for two-body decays of heavy mesons are based on factorized formulae [10]–[12]. Although we do not assume factorization, we normalize the amplitudes to the factorized values of the disconnected-emission diagrams [1]. The latter depend on the particular model used to evaluate the relevant form factors [13]–[15]. We have selected a set of representative models which are consistent with the scaling laws derived from the HQET, namely lattice QCD [15] (LQCD), the quark model of ref. [16] (QM) and the most recent results from QCD sum rules [17] (QCDSR). We also consider the model used in ref. [12] (ABLOPR), because a detailed analysis of the ratio $BR(B_d \to K^+\pi^-)/BR(B_d \to \pi^+\pi^-)$ was presented there. We analyse the stability of the values of the fitted parameters (ξ, δ_ξ, η_L and δ_L) and of the predicted BRs for different choices of the model used for computing the form factors;
2. we vary the value of CKM-parameter σ and the CP violating phase δ ($\sigma e^{-i\delta} = \rho - i\eta$ in the standard Wolfenstein parametrization [18]). We choose the ranges $\sigma = 0.24$–0.48 and $-0.7 \leq \cos\delta \leq +0.7$, although it can be argued on the basis of several analyses [19, 20] that $\cos\delta \gtrsim 0$;
3. we check the stability of our results against other effects which are suppressed in the factorization hypothesis. The contribution of annihilation diagrams and GIM penguins, as defined in ref. [1], has been studied.

Before presenting our results, it is appropriate to discuss the evaluation of the relevant form factors used in the calculation of the disconnected emission amplitudes.

Heavy-heavy matrix elements are related to the Isgur-Wise (IW) function $\xi(\omega)$. HQET allows the calculation of $\xi(\omega = 1)$ corresponding to $q^2 \sim q^2_{max}$. In most of the cases considered here, the range of momenta which is used is instead around $q^2 \sim 0$. Since the experimental measurements support a linear dependence of ξ on q^2, the relevant parameter to evaluate the form factors is the slope of the IW function $\hat{\rho}^2$ (a small extrapolation in q^2 is indeed needed since data exist down to rather small value of the momentum transfer). We find that the χ^2 of our fit to the non-leptonic BRs is a steep function of $\hat{\rho}^2$. The data can be fitted reasonably well only if $\hat{\rho}^2 = 0.55$–0.75 (with a preferred value of 0.65), which lies within the allowed experimental range [21]. For more detailed and upgraded discussions on factorization and its tests, see [7, 8].

For the heavy-light form factors, HQET predicts scaling laws with the heavy quark mass only for $q^2 \sim q^2_{max}$ [22]. The evaluation of the form factors at the values of q^2 needed for the factorized non-leptonic amplitudes requires further theoretical inputs, which are provided by a variety of non-perturbative approaches. The state of the art for the set of models used here is summarized in fig. 1. We find that the various determinations of the heavy-to-light form factors by the QM of ref. [16], QCDSR [17], and LQCD [15] agree reasonably well in a broad range of values of q^2. For this reason, the values of the BRs obtained using these model are very similar, see tables 1, 2 and 3. On the contrary, ABLOPR systematically gives values of the form factors 1.5–2.5 times larger than the other models. As a consequence most of the BRs computed with ABLOPR for B going into two light mesons are much larger than in all other cases. This fact has several consequences that will be discussed in sections 4 and 5.

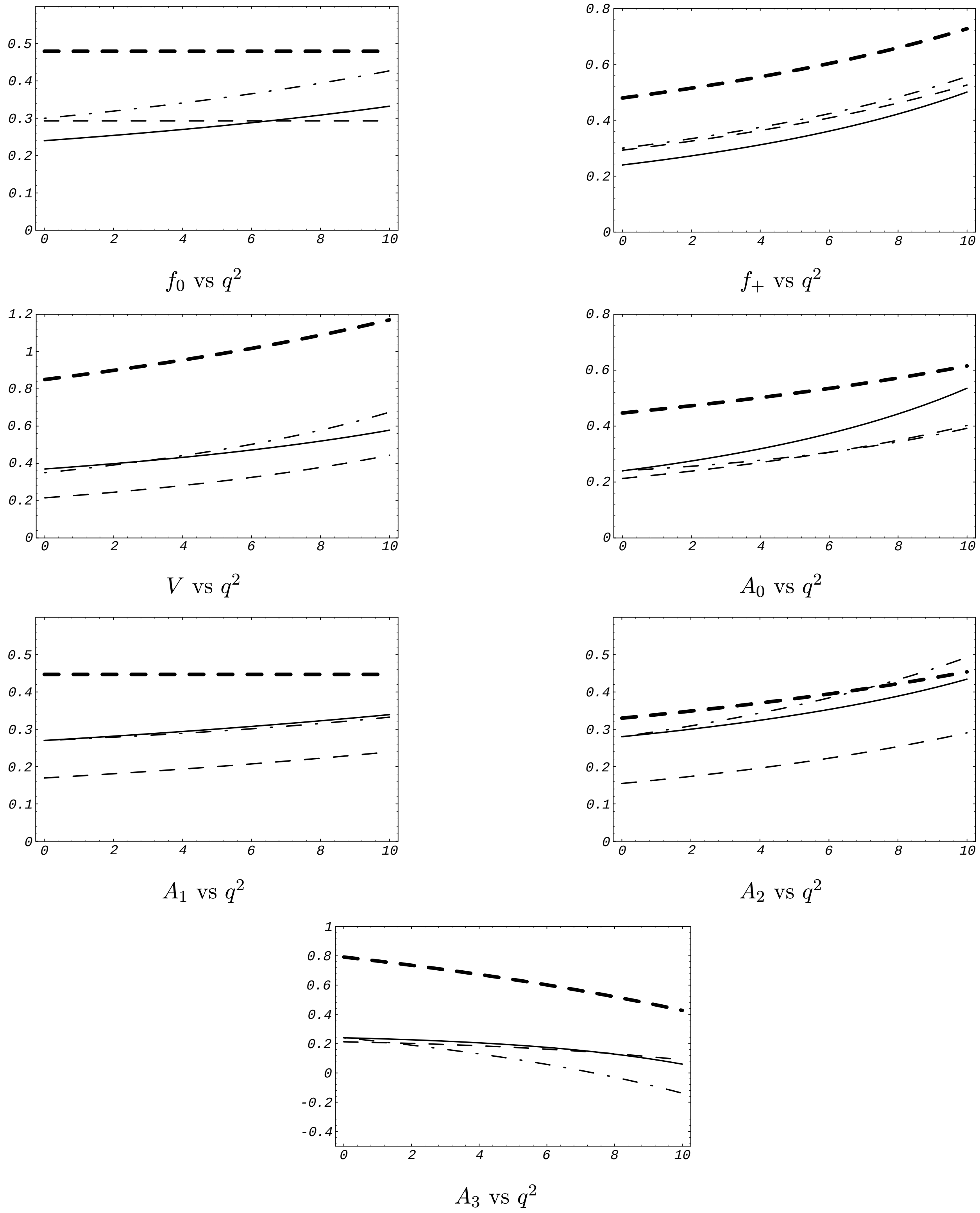

Figure 1: *Heavy–light form factors vs q^2 (GeV^2) from LQCD (solid), QM (dashed), QCDSR (dot-dashed) and ABLOPR (bold-dashed).*

4 Fixing ξ, δ_ξ, η_L and δ_L

We now start the discussion of the results of our analysis. We stress again that, given the large uncertainties present in the evaluation of the BRs, our attitude is to focus mainly on large effects induced by charming-penguins. Although we do not even try to evaluate the theoretical errors, still we vary the input parameters, in order to check the stability of our predictions.

The decay channels which have been used to fix the most important hadronic parameters are presented in tables 3 and 4. In each table we show a list of decay channels calculated with and without charming penguins [in square brackets] [2]. The parameters of the charming-penguin contributions, i.e. η_L and δ_L have been chosen as explained further on in this section. For each decay, we consider the results obtained using different models for the form factors, namely the QM of ref. [16], QCDSR[17], LQCD [15] and ABLOPR [12]. The spread of the results obtained in the different cases can be taken as an estimate of the error due to the model dependence. These results have been obtained for "central values" of the parameters: the Wilson coefficients are computed at the LO at a scale $\mu = 5$ GeV (these coefficients are taken from ref. [19] to which the reader can refer for details); $\cos\delta = 0.38$ and $\sigma = 0.36$; the disconnected emission diagrams are computed using factorization; GIM-penguin and annihilation contributions are put to zero. In the following the results obtained with this choice of the parameters will be denoted as CV (Central Values).

Table 3 contains measured decay channels that are dominated by emission diagrams (and for which the dominant contributions are of $O(\lambda^2)$), where λ is the Cabibbo angle in the Wolfenstein parametrization. They are used for fitting ξ and δ_ξ. Reasonable values of the parameters are obtained in all cases, with fairly good χ^2 values. The channels dominating the fit, namely those with the smaller relative experimental error (the BRs of which are given above the horizontal line in the table), are essentially independent of the charming-penguin contributions and of the CKM parameters $\cos\delta$ and σ. For this reason, both the χ^2 and the fitted parameters have negligible dependence on these quantities. The experimental values of the BRs, as well as the values of the χ^2 and the results of the fit, for each model, are also given. For the different models, different determinations of ξ and δ_ξ are obtained. The larger ABLOPR form factors prefer an appreciably smaller value of ξ and have some problem in reproducing the measured $BR(B \to K^* J/\Psi)$.

In table 3, for completeness, we also present (for QCDSR only) a band of results obtained in the following way: we vary at the LO the renormalization scale from $\mu = 5$ to $\mu = 2$ GeV; we take the Wilson coefficients computed at the NLO in the NDR scheme (but only at $\mu = 5$ GeV); we change the value of the disconnected-emission diagrams by $\pm 15\%$ with respect to their factorized value; we allow the value of the GIM-penguin and of the annihilation diagrams to vary within the intervals defined in ref. [1] and take $-0.7 \le \cos\delta \le +0.7$ and $0.24 \le \sigma \le 0.48$. The spread of values given in the table is representative of the theoretical uncertainty due to the parameters listed above. The band of results obtained by varying the parameters as explained above is denoted as the Band Value (BV).

With the exception of ABLOPR, very similar results are obtained with all the

[2] Only in those cases where charming-penguin contractions contribute.

models considered in our study. For this reason, in table 4 we only discuss the results obtained using QCDSR, as a representative case, and ABLOPR. In this table, we give a detailed "map" of the results obtained for $BR(B^+ \to K^0\pi^+)$ and $BR(B_d \to K^+\pi^-)$, which have been used to fit the charming-penguin parameters η_L and δ_L. We also present, for the same values of the parameters, $BR(B_d \to \pi^+\pi^-)$. The values in square brackets are computed without charming penguins. To illustrate the effects of a large coefficient for the most important penguin operator Q_6, we also give the results with the coefficients computed at the NLO in NDR: in this regularization scheme the relevant penguin coefficients turn out to be larger at NLO than at LO. The values of ξ and δ_ξ are taken from the fit to the data of table 3, which only involve heavy-heavy and heavy-light meson final states. For light-light meson final states, it would have been possible, of course, to choose values of ξ and δ_ξ different from those fitted on the heavy-heavy and heavy-light decay channels. Different choices of ξ for pseudoscalar-pseudoscalar, vector-pseudoscalar and vector-vector final states are also possible in principle. The inflation of parameters would have only complicated the analysis without changing the basic conclusions.

When charming penguins are included, QCDSR (as well as with LQCD or the QM), give good results for $BR(B^+ \to K^0\pi^+)$ and $BR(B_d \to K^+\pi^-)$ and at the same time respect the experimental upper bound on $BR(B_d \to \pi^+\pi^-)$. This is true almost irrespectively of the choice of the Wilson coefficients and of $\cos\delta$ [3] and of σ: only for large values of σ, and very small values of $\cos\delta$, there is a problem with the upper limit on $BR(B_d \to \pi^+\pi^-)$. There is a general tendency to have $BR(B^+ \to K^0\pi^+)$ close to $BR(B_d \to K^+\pi^-)$ instead of larger, as found experimentally. Given the experimental errors (and the uncertainties coming from GIM-penguins and annihilation diagrams) we believe that this difference is at present not significant. As far as the last of the BRs in eq. (1) is concerned, with charming penguins we typically obtain $BR(B^+ \to \eta' K^+) \sim (2.5–3.0) \times 10^{-5}$, slightly less than one-half of the central experimental value (without charming penguins $BR(B^+ \to \eta' K^+) \sim (0.2–1.5) \times 10^{-5}$). Since the difference probably comes from anomalous contributions, which are not directly related to charming penguins and are very difficult to evaluate, this discrepancy does not affects our main conclusions.

With charming penguins, ABLOPR may find even better agreement with the data, in that it easily reproduces the pattern $BR(B^+ \to K^0\pi^+) \sim 1.5\, BR(B_d \to K^+\pi^-)$ in a wide range of $\cos\delta$ and σ. However, it violates the bound on $BR(B_d \to \pi^+\pi^-)$ unless special values of the parameters are assumed. If we demand good agreement for all the three BRs, only a low value of σ is acceptable. In this case, with a larger value of the coefficient of Q_6, as obtained at the NLO, ABLOPR finds agreement with the data even without charming penguins.

With the exception of ABLOPR, which we discuss separately below, we conclude that charming-penguin contributions are essential for reproducing the data. For all the other models that we have studied (QM, LQCD and QCDSR), theoretical estimates obtained using factorization without charming penguins predict too small BRs. In particular $BR(B^+ \to K^0\pi^+)$ is always a factor $\sim$ 4–10 smaller than its measured value. Although the experimental errors are still rather large, we believe that these data already show evidence of substantial charming-penguin contributions to $B \to K\pi$

[3]Notice that with or without charming penguins $BR(B^+ \to K^0\pi^+)$ is practically independent of $\cos\delta$.

decays.

ABLOPR deserves a separate discussion. By using some specific set of Wilson coefficients (corresponding to a large value of the coefficient of the penguin operator Q_6), this model may predict correctly the experimental values for $BR(B_d \to K^+\pi^-)$ and $BR(B^+ \to K^0\pi^+)$ even without charming-penguin contributions. This is the case, for example, if one computes the coefficients at the NLO, or at a low value of the renormalization scale μ. The reason is that the values of the form factors in ABLOPR are quite atypical, see fig. 1. The requirement that the experimental values of $BR(B_d \to K^+\pi^-)$ and $BR(B^+ \to K^0\pi^+)$ are reproduced without violating other experimental bounds, in particular $BR(B_d \to \pi^+\pi^-)$, demands, however, a special choice of the Wilson coefficients of the operators of the effective Hamiltonian and of the values of σ, which must be small, or $\cos\delta$, which must be negative (see table 4). Other problems afflict this model, particularly in the vector sector. For example, many experimental upper bounds, see table 1, are violated by using ABLOPR. This model has also problems with semileptonic decays because it gives a very large semileptonic $BR(B \to \rho)$ (and a relatively large $BR(B \to \pi)$): one finds $BR(B \to \rho)/BR(B \to \pi) = 2.98$, to be compared to the experimental number $1.4^{+0.6}_{-0.4} \pm 0.3 \pm 0.4$ [21] and, as far as V_{ub} is concerned, $V_{ub}/V_{cb} = 0.047$ from $B \to \rho$ and $V_{ub}/V_{cb} = 0.070$ from $B \to \pi$.

At present, we cannot completely exclude that, by using ABLOPR, it is possible to find a very special set of parameters satisfying the following requirement: large values of $BR(B_d \to K^+\pi^-)$ and $BR(B^+ \to K^0\pi^+)$, without invoking charming-penguin effects, and respect of the experimental bounds, notably those on $BR(B_d \to \pi^+\pi^-)$ and on BRs of channels containing vector mesons in the final states. Considering also the problems of ABLOPR with semileptonic decays, and the fact that it is so different from all the other models, we find this interpretation of the data rather remote.

We now discuss the results of ref. [6] where the $B \to K\pi$, $B \to \eta' K$ and $B \to \pi\pi$ branching ratios have also been studied. The authors of this paper conclude that is possible to describe the data by using a specific set of effective coefficients, which include charm-quark penguin effects computed in perturbation theory, and factorized hadronic amplitudes.

On the theoretical validity of the approach followed in ref. [6], we have already commented in section 2. Concerning the phenomenological analysis, our main observation is that, whenever (any kind of) penguins are important, the values of the BRs are strongly correlated, as shown in table 4. As an example, a very small value of ξ, as required to reproduce $BR(B^+ \to \eta' K^+)$, leads to a violation of the experimental bound on $BR(B_d \to \pi^+\pi^-)$. For this reason, in this table, we have decided to present the BRs for several choices of the main parameters separately. From the analysis of ref. [6], one may conclude that, for any given decay channel, there are points in the parameter space where agreement between theory and data can be achieved. However this is no longer the case when the correlations among different predictions, induced by using the *same* set of parameters for all the light-light channels, are taken into account. In particular, on the basis of our experience, we think that it is very difficult to reproduce the experimental $BR(B^+ \to \eta' K^+)$ without violating the bound on $BR(B_d \to \pi^+\pi^-)$. Clearly, extra-contributions coming from the anomaly may change this conclusion.

5 Predictions for yet unmeasured BRs

Having determined the hadronic parameters from the measured BRs, we use them to predict those which have not been measured yet. These include many channels where the charming-penguin contributions are Cabibbo-enhanced with respect to emission diagrams, together with some Cabibbo-suppressed channels (of $O(\lambda^3)$), which are physically interesting, such as $B \to \pi\pi$ and $B \to \pi\eta'$. The results are collected in table 1. As expected, charming-penguin-dominated channels are strongly enhanced: typically by a factor of 3–5; in some case, the enhancement can be as large as one order of magnitude. $B \to \rho K^{(*)}$, $B \to \omega K^{(*)}$ and $B \to \eta^{(\prime)} K^{(*)}$ are the channels most sensitive to charming-penguin effects. In some case, such as $B \to \eta K$ and $B_d \to \rho^0 K^0$, the BRs remain small, at the level of 10^{-6}, in spite of the large enhancement, and may be difficult to measure.

By looking to the column QCDSR-BV, we observe that the typical uncertainty is about a factor of 2, although, for some channel, it can reach even factors of 10 or more. For this reason, for the Cabibbo-suppressed channels (corresponding to the BRs below the horizontal line), the errors are so large that predictions with and without charming penguins largely overlap. This (hélas!) occurs also in some charming-penguin-enhanced channels (the BRs of which are given above the horizontal line). All predictions involving η' and η suffer from the poor theoretical control that we have on anomalous contributions, which are never included in our calculations.

The best candidates to test charming penguins are $B \to \rho K$ decays (in the different charge combinations) and $B_d \to \omega K^{(*)0}$. For these channels, the enhancement is larger than the estimated uncertainties, and the expected BRs are not far from the present experimental limits, so that one can expect that they will soon be measured. Within larger uncertainties, other channels are also appreciably enhanced by charming penguins, e.g. $B \to \pi K^*$, $B^+ \to \omega K^{(*)+}$ and $B \to \phi K^{(*)}$. With QCDSR, LQCD and the QM the experimental bounds are generally respected; in many cases, using ABLOPR, they are instead violated, often by a large factor, even in absence of charming-penguin contributions.

The Cabibbo-suppressed $B \to \pi\pi$ decay plays a fundamental role in the extraction of unitary-triangle angle α. In ref. [1], it has been shown that charming-penguin contributions in $B_d \to \pi^+\pi^-$ decays do not allow the extraction of this parameter using only the asymmetry measured in this channel. Still, it may be possible to extract α by using the isospin analysis of ref. [23]. This method requires $BR(B_d \to \pi^0\pi^0)$ to be large enough. In ref. [1], by normalizing the rates to some conventional value for $BR(B_d \to \pi^+\pi^-)$, it was shown that values of this BR as large as $(2\text{–}3) \times 10^{-6}$ were possible. In the present case, we typically obtain, within large uncertainties, an average value of $(5\text{–}10) \times 10^{-7}$. We then include other contributions, such as annihilation diagrams and GIM penguins [1], and try to maximize the $BR(B_d \to \pi^0\pi^0)$ as a function of the hadronic parameters, allowing χ^2-values twice larger than those found in the fit. In this case, by stretching all the parameters, we find that a maximum value $BR(B_d \to \pi^0\pi^0) \sim (2\text{–}3) \times 10^{-6}$ is still allowed. Therefore the extraction of α from the $B_d \to \pi^+\pi^-$ CP-asymmetry may be a very difficult, hopefully not impossible, task.

Finally, in table 2, our predictions of other emission-dominated channels, for which only upper bound have been measured, are also presented. We see that all the models satisfy the bounds without any problem.

6 Conclusions

In this paper, by using the present experimental measurements and constraints, we have determined some non-perturbative hadronic parameters necessary to predict the BRs for two-body B-meson decays. Using this information, we have predicted a large set of yet unmeasured BRs and discussed the corresponding uncertainties. The latter have been estimated by using different models, different Wilson coefficients and by varying the CKM parameters within the ranges allowed by experiments.

We found that, in several cases, notably $B \to \rho K$, charming-penguin effects are larger than the theoretical error and the expected value of the corresponding BRs are close to the experimental upper bound. Therefore these channels are the ideal playground to test non-factorizable charming-penguin effects.

Acknowledgements

We thank A. Ali, P. Ball, V. Braun, J. Charles, J. Flynn, C. Greub, M. Neubert, O. Pène and A. Soni for useful discussions on the subject of this paper. L.S. acknowledges the support of "Fondazione A. Della Riccia". We acknowledge the partial support by the EC contract CHRX-CT93-0132 and by M.U.R.S.T.

References

[1] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, CERN-TH/97-30, hep-ph/9703353.

[2] CLEO Coll., D.M. Asner et al., Phys. Rev. D53 (1996) 1039;
J. Fast, preprint CLEO TALK 97-3;
P. Pomianowski, preprint CLEO TALK 97-11;
C.P. Jessop et al., preprint CLEO 96-24, hep-ex/9702013;
F. Wurtheim, preprint CALT-68-2121, hep-ex/9706010;
CLEO Coll., S. Anderson et al., CLEO CONF 97-22a;
CLEO Coll., M.S. Alam et al., CLEO CONF 97-23;
CLEO Coll., A. Anastassov et al., CLEO CONF 97-24.

[3] PDG, Phys. Rev. D54 (1996) 1.

[4] T.E. Browder, K. Honscheid and D. Pedrini, Ann. Rev. Nucl. Part. Sci. 46 (1996) 395.

[5] N.G. Deshpande and X.-G. He, Phys. Lett. B336 (1994) 471;
R. Fleischer, Phys. Lett. B341 (1994) 205;
A.J. Buras and R. Fleischer, Phys. Lett. B341 (1995) 379;
G. Kramer, W.F. Palmer and H. Simma, Z. Phys. C66 (1995) 429;
R. Fleischer and T. Mannel, preprint TTP97-17, hep-ph/9704423.

[6] A. Ali and C. Greub, preprint DESY 97-126, hep-ph/9707251.

[7] M. Ciuchini, R. Contino, E. Franco, G. Martinelli and L. Silvestrini, in preparation.

[8] M. Neubert and B. Stech, preprint CERN-TH-97-099, to appear in second edition of Heavy Flavours, ed. by A.J. Buras and M. Lindner, World Scientific, hep-ph/9705292.

[9] see e.g. I.I. Bigi and A.I. Sanda, Nucl. Phys. B281 (1987) 41;
Y. Nir and H.R. Quinn, Ann. Rev. Nucl. Part. Sci. 42 (1992) 211.

[10] Deandrea et al., Phys. Lett. B318 (1993) 549.

[11] G. Kramer and W.F. Palmer, Phys. Rev. D52 (1995) 6411.

[12] R. Aleksan et al., Phys. Lett. B356 (1995) 95.

[13] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C29 (1985) 637; C34 (1987) 103;
N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. D39 (1989) 799;
N. Isgur and D. Scora, Phys. Rev. D40 (1989) 1491;
F.J. Gilman and R.R. Singleton, Phys. Rev. D41 (1990) 142.

[14] V.M. Belayev et al., Phys. Rev. D51 (1995) 6177;
P. Ball, Phys. Rev. D48 (1993) 3190;
P. Ball and V.M. Braun, Phys. Rev. D54 (1996) 2182.

[15] ELC Collaboration, As. Abada et al., Nucl. Phys. B416 (1994) 675;
APE Collaboration, C.R. Allton et al., Phys. Lett. B345 (1995) 513;
UKQCD Collaboration, D.R. Burford et al., Nucl. Phys. B447 (1995) 425;
UKQCD Collaboration, J.M. Flynn et al., Nucl. Phys. B461 (1996) 327; B476 (1996) 313;
J. Flynn, to appear in the proceedings of the 14th International Symposium on Lattice Field Theory, St. Louis, 4-8 June 1996, hep-lat/9610010;
G. Martinelli, Nucl. Instrum. Meth. A384 (96) 241 and refs. therein.

[16] D. Melikhov, Phys. Lett. B394 (1997) 385.

[17] P. Ball and V.M. Braun, Phys. Rev. D55 (1997) 5561;
A. Khodjamirian and R. Ruckl, preprint WUE-ITP-96-020, presented at 28th International Conference on High-energy Physics (ICHEP 96), Warsaw, Poland, 25-31 Jul 1996, hep-ph/9610367;
P. Ball, V.M. Braun, A. Khodjamirian and R. Ruckl, BaBar Yellow Book (March 1997).

[18] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[19] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, Z. Phys. C68 (1995) 239.

[20] A. Buras, M. Jamin and M.E. Lautenbacher, Phys. Lett. B389 (1996) 749;
A. Ali and D. London, Nucl. Phys. Proc. Suppl. 54A (1997) 297;
M. Ciuchini, preprint CERN-TH/97-2, to appear in the Proceedings of the 4th KEK Topical Conference on "Flavor Physics", KEK, Japan, 29-31 October 1996, hep-ph/9701278;
A. Buras, preprint TUM-HEP-259-96, Proceedings of 28th International Conference on High-energy Physics (ICHEP 96), Warsaw, Poland, 25-31 Jul 1996, and refs. therein, hep-ph/9610461.

[21] L. Gibbons, invited talk at 28th International Conference on High-energy Physics (ICHEP 96), Warsaw, Poland, 25-31 Jul 1996; hep-ex/9704017;
CLEO collaboration, J.P. Alexander et al., Phys. Rev. Lett. 77 (1996) 5000.

[22] N. Isgur and M.B. Wise, Phys. Rev. D42 (1990) 2388.

[23] M. Gronau and D. London, Phys. Rev. Lett. 65 (1990) 3381.

Channel	QCDSR–CV $BR \times 10^5$	QCDSR–BV $BR \times 10^5$	Lattice–CV $BR \times 10^5$	QM–CV $BR \times 10^5$	ABLOPR–CV $BR \times 10^5$	Experiment $BR \times 10^5$
$B_d \to \eta' K^0$	2.45 [0.19]	1.97–3.55 [0.12–0.67]	2.49 [0.10]	2.39 [0.19]	4.48 [0.75]	$\star$
$B_d \to \eta' K^{*0}$	0.75 [0.01]	0.30–2.41 [0.00–0.10]	1.41 [0.00]	0.47 [0.01]	2.56 [1.06]	< 9.9
$B_d \to \eta K^0$	0.07 [0.01]	0.01–0.26 [0.00–0.02]	0.07 [0.00]	0.06 [0.01]	0.10 [0.02]	–
$B_d \to \eta K^{*0}$	0.04 [0.02]	0.01–0.22 [0.01–0.06]	0.05 [0.01]	0.03 [0.02]	0.87 [0.83]	< 3.3
$B_d \to \omega K^0$	0.73 [0.04]	0.13–1.70 [0.03–0.38]	0.80 [0.02]	0.70 [0.06]	0.64 [0.02]	–
$B_d \to \omega K^{*0}$	1.85 [0.11]	1.01–3.26 [0.08–0.58]	2.76 [0.12]	1.43 [0.08]	6.26 [0.41]	< 3.8
$B_d \to \phi K^0$	1.48 [0.11]	0.68–2.29 [0.10–0.75]	1.77 [0.08]	0.92 [0.12]	2.59 [0.92]	< 4.2
$B_d \to \phi K^{*0}$	3.27 [0.09]	1.90–6.73 [0.07–0.62]	5.08 [0.11]	1.65 [0.05]	10.3 [0.29]	< 2.2
$B_d \to \pi^0 K^0$	0.65 [0.04]	0.49–0.95 [0.01–0.26]	0.68 [0.03]	0.64 [0.03]	0.94 [0.13]	< 4.1
$B_d \to \pi^0 K^{*0}$	0.40 [0.03]	0.29–0.60 [0.01–0.13]	0.61 [0.02]	0.26 [0.03]	0.66 [0.09]	< 2.0
$B_d \to \pi^- K^{*+}$	3.13 [0.17]	2.05–5.07 [0.05–1.01]	3.47 [0.11]	3.16 [0.16]	2.84 [0.44]	< 6.7
$B_d \to \rho^- K^+$	0.63 [0.02]	0.21–0.67 [0.00–0.11]	1.19 [0.02]	0.53 [0.02]	1.89 [0.09]	< 3.3
$B_d \to \rho^- K^{*+}$	2.21 [0.12]	1.44–3.57 [0.03–0.72]	4.15 [0.13]	1.75 [0.09]	7.57 [1.18]	–
$B_d \to \rho^0 K^0$	0.44 [0.03]	0.07–1.63 [0.00–0.09]	0.81 [0.01]	0.35 [0.03]	1.33 [0.43]	< 3.0
$B_d \to \rho^0 K^{*0}$	1.01 [0.01]	0.50–2.45 [0.01–0.13]	1.86 [0.02]	0.60 [0.01]	4.39 [0.16]	< 46
$B^+ \to \eta' K^+$	2.31 [0.20]	2.05–3.14 [0.11–0.92]	2.36 [0.10]	2.27 [0.20]	3.80 [0.70]	$\star$
$B^+ \to \eta' K^{*+}$	4.14 [0.04]	1.45–11.22 [0.00–0.57]	4.82 [0.02]	4.18 [0.03]	85.0 [1.59]	< 29
$B^+ \to \eta K^+$	0.12 [0.02]	0.02–0.46 [0.00–0.08]	0.13 [0.01]	0.11 [0.01]	0.25 [0.05]	–
$B^+ \to \eta K^{*+}$	0.27 [0.04]	0.04–1.19 [0.01–0.22]	0.29 [0.03]	0.25 [0.04]	3.13 [1.09]	< 24
$B^+ \to \omega K^+$	0.60 [0.02]	0.06–1.39 [0.00-0.66]	0.65 [0.04]	0.60 [0.03]	0.39 [0.12]	$\star$
$B^+ \to \omega K^{*+}$	1.50 [0.08]	0.84–2.48 [0.03–1.12]	2.24 [0.16]	1.20 [0.06]	3.64 [0.59]	< 11
$B^+ \to \phi K^+$	1.54 [0.12]	0.70–2.38 [0.10–0.78]	1.84 [0.09]	0.96 [0.13]	2.69 [0.96]	< 0.53
$B^+ \to \phi K^{*+}$	3.35 [0.09]	1.96–6.90 [0.08–0.64]	5.21 [0.11]	1.70 [0.06]	10.6 [0.30]	< 4.1
$B^+ \to \pi^0 K^+$	0.94 [0.18]	0.67–1.03 [0.09–0.74]	0.86 [0.10]	0.95 [0.18]	0.94 [0.45]	< 1.6
$B^+ \to \pi^0 K^{*+}$	1.70 [0.14]	1.15–2.42 [0.06–0.71]	1.81 [0.09]	1.71 [0.13]	1.55 [0.35]	< 8.0
$B^+ \to \pi^+ K^{*0}$	0.92 [0.13]	0.73–1.15 [0.08–0.37]	1.29 [0.07]	0.62 [0.12]	1.54 [0.40]	< 3.9
$B^+ \to \rho^+ K^0$	2.67 [0.00]	0.88–7.28 [0.00–0.36]	3.05 [0.00]	2.62 [0.00]	27.3 [0.00]	< 6.4
$B^+ \to \rho^+ K^{*0}$	2.21 [0.09]	1.42–4.01 [0.06–0.38]	3.82 [0.09]	1.34 [0.07]	9.50 [1.06]	–
$B^+ \to \rho^0 K^+$	0.39 [0.06]	0.18–0.61 [0.00–0.21]	0.61 [0.03]	0.35 [0.06]	0.81 [0.53]	< 1.4
$B^+ \to \rho^0 K^{*+}$	1.26 [0.15]	0.81–1.62 [0.06–0.63]	2.18 [0.14]	1.00 [0.11]	4.31 [1.18]	< 90
$B_d \to \pi^+ \pi^-$	0.99 [0.63]	0.39–2.63 [0.15–1.55]	0.71 [0.39]	0.95 [0.60]	2.37 [1.71]	< 1.5
$B_d \to \pi^0 \pi^0$	0.04	0.01–0.12 [0.01–0.11]	0.004 [0.002]	0.06 [0.05]	0.01	< 0.93
$B_d \to \rho^0 \pi^0$	0.08 [0.11]	0.00–0.21 [0.02–0.21]	0.01 [0.07]	0.12 [0.13]	0.00 [0.03]	< 1.8
$B_d \to \rho^+ \pi^-$	2.67 [1.71]	1.02–7.51 [0.45–4.15]	1.93 [1.07]	2.55 [1.63]	6.29 [4.52]	< 8.8
$B_d \to \pi^+ \rho^-$	0.59 [0.38]	0.22–1.75 [0.11–0.91]	0.66 [0.37]	0.46 [0.30]	1.92 [1.38]	–
$B_d \to \rho^0 \rho^0$	0.16 [0.18]	0.00–0.58 [0.04–0.46]	0.02 [0.18]	0.21 [0.19]	0.10 [0.19]	< 28
$B_d \to \rho^+ \rho^-$	1.81 [1.16]	0.69–5.10 [0.31–2.82]	2.19 [1.22]	1.37 [0.88]	16.6 [11.9]	< 220
$B_d \to \omega \pi^0$	0.06 [0.01]	0.03–0.42 [0.00–0.29]	0.08 [0.005]	0.06 [0.02]	0.06 [0.00]	–
$B_d \to \omega \rho^0$	0.10 [0.01]	0.05–1.22 [0.01–0.73]	0.14 [0.01]	0.08 [0.01]	0.36 [0.01]	< 3.4
$B_d \to \phi \pi^0$	0.00 [0.00]	0.00–0.01 [0.00–0.01]	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	< 0.65
$B_d \to \eta' \pi^0$	0.48 [0.36]	0.04–2.14 [0.01–2.91]	0.21 [0.22]	0.44 [0.33]	1.05 [0.98]	< 2.2
$B^+ \to \pi^+ \pi^0$	0.53	0.16–1.18	0.36	0.51	1.07	< 2.0
$B^+ \to \rho^+ \pi^0$	1.12	0.30–2.62	0.80	1.05	2.63	< 7.7
$B^+ \to \pi^+ \rho^0$	0.52	0.19–1.08	0.47	0.47	1.01	< 5.8
$B^+ \to \rho^+ \rho^0$	0.94	0.23–2.11	1.08	0.73	7.44	< 100
$B^+ \to \omega \pi^+$	0.84 [0.50]	0.38–2.35 [0.08–1.64]	0.94 [0.45]	0.70 [0.45]	1.69 [0.99]	$\star$
$B^+ \to \omega \rho^+$	1.81 [0.94]	0.80–5.57 [0.19–3.40]	2.44 [1.04]	1.36 [0.72]	12.9 [7.23]	–
$B^+ \to \phi \pi^+$	0.00 [0.00]	0.00–0.02 [0.00–0.02]	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	< 0.56
$B^+ \to \pi^+ \eta'$	1.09 [0.76]	0.01–3.33 [0.13–3.36]	0.87 [0.58]	0.97 [0.65]	3.09 [2.53]	< 4.5
$B_d \to \pi^0 J/\Psi$	2.27	2.06-2.36	1.82	2.70	0.41	< 6
$B_d \to \rho^0 J/\Psi$	4.44	4.04–4.63	5.05	3.61	1.43	< 25
$B^+ \to \rho^+ J/\Psi$	9.22	8.39–9.62	10.48	7.49	2.97	< 77
$B_d \to K^0 \bar{K}^0$	0.09 [0.01]	0.05–0.56 [0.00–0.41]	0.09 [0.005]	0.08 [0.01]	0.13 [0.03]	< 1.7
$B_d \to K^+ K^-$	0.00 [0.00]	0.00–0.50 [0.00–0.50]	0.00 [0.00]	0.00 [0.00]	0.00 [0.00]	< 1.7
$B^+ \to K^+ \bar{K}^0$	0.09 [0.01]	0.05–0.58 [0.00–0.43]	0.09 [0.005]	0.08 [0.01]	0.13 [0.03]	< 2.1

Table 1: *Predictions for yet unmeasured BRs. The channels above the horizontal line are those enhanced by charming penguins. In square brackets, the BRs obtained without charming penguins are shown.* $BR(B^+ \to \omega K^+)$, $BR(B^+ \to \omega\pi^+)$, $BR(B^+ \to \eta' K^+)$ *and* $BR(B_d \to \eta' K^0)$, *for which preliminary measurements exist* $((1.5^{+0.7}_{-0.6} \pm 0.3) \times 10^{-5}$, $(1.1^{+0.6}_{-0.5} \pm 0.2) \times 10^{-5}$, $(7.1^{+2.5}_{-2.1} \pm 0.9) \times 10^{-5}$ *and* $(5.3^{+2.8}_{-2.2} \pm 1.2) \times 10^{-5}$ *respectively), have been given in this table since they have not been used in the fit of* η_L *and* δ_L.

Channel	QCDSR–CV $BR \times 10^5$	Lattice–CV $BR \times 10^5$	QM–CV $BR \times 10^5$	ABLOPR–CV $BR \times 10^5$	Exp. upper bound $BR \times 10^5$
$B_d \to \pi^0 \bar{D}^0$	17	11	17	4	< 33
$B_d \to \pi^0 \bar{D}^{*0}$	28	19	35	6	< 55
$B_d \to \rho^0 \bar{D}^0$	8	10	10	3	< 55
$B_d \to \rho^0 \bar{D}^{*0}$	32	40	28	19	< 117
$B_d \to \omega \bar{D}^0$	8	10	9	3	< 57
$B_d \to \omega \bar{D}^{*0}$	32	40	27	19	< 120
$B_d \to \eta' \bar{D}^0$	8	5	8	2	< 33
$B_d \to \eta' \bar{D}^{*0}$	12	8	14	48	< 50

Table 2: *Predictions for emission-dominated decay channels for which only experimental upper bounds exist.*

Channel	QCDSR–CV $BR \times 10^5$	QCDSR–BV $BR \times 10^5$	Lattice–CV $BR \times 10^5$	QM–CV $BR \times 10^5$	ABLOPR–CV $BR \times 10^5$	Experiment $BR \times 10^5$
$B_d \to \pi^+ D^-$	301	292–344	293	308	318	310 ± 44
$B_d \to \pi^+ D^{*-}$	323	313–369	314	317	341	280 ± 41
$B_d \to \rho^+ D^-$	794	770–907	771	802	839	840 ± 175
$B_d \to \rho^+ D^{*-}$	994	964–1136	966	916	1051	730 ± 153
$B^+ \to \pi^+ \bar{D}^0$	508	498–527	491	498	437	500 ± 54
$B^+ \to \pi^+ \bar{D}^{*0}$	605	595–619	595	604	504	520 ± 82
$B^+ \to \rho^+ \bar{D}^0$	1015	988–1103	1078	1022	1015	1370 ± 187
$B^+ \to \rho^+ \bar{D}^{*0}$	1396	1364—1496	1553	1269	1479	1510 ± 301
$B_d \to K^0 J/\Psi$	81	80–81	65	96	103	85 ± 14
$B_d \to K^{*0} J/\Psi$	164	163–165	185	120	51	132 ± 24
$B^+ \to K^+ J/\Psi$	84	84	68	100	107	102 ± 11
$B^+ \to K^{*+} J/\Psi$	171	170–172	192	126	54	141 ± 33
$B_d \to D_s^+ D^-$	816 [880]	710–1202 [769–940]	801 [855]	792 [864]	935 [931]	740 ± 284
$B_d \to D_s^{*+} D^-$	824 [887]	723–1253 [844–996]	807 [861]	802 [871]	842 [938]	1140 ± 505
$B_d \to D_s^+ D^{*-}$	643 [691]	565–994 [677–796]	629 [671]	626 [579]	734 [731]	940 ± 332
$B_d \to D_s^{*+} D^{*-}$	2358 [2535]	2067–3584 [2412–2848]	2309 [2462]	1992 [2165]	2693 [2682]	2000 ± 729
$B^+ \to D_s^+ \bar{D}^0$	847 [913]	736–1247 [797–975]	831 [887]	822 [896]	970 [966]	1360 ± 433
$B^+ \to D_s^{*+} \bar{D}^0$	857 [922]	752–1304 [877–1035]	840 [895]	833 [906]	979 [975]	940 ± 386
$B^+ \to D_s^+ \bar{D}^{*0}$	669 [718]	588–1034 [704–828]	655 [697]	651 [706]	763 [760]	1180 ± 462
$B^+ \to D_s^{*+} \bar{D}^{*0}$	2448 [2632]	2146–3721 [2505–2956]	2397 [2556]	2067 [2247]	2796 [2785]	2700 ± 1045
Results of the fit						
ξ	0.47	(0.40–0.57)	0.50	0.50	0.29	
δ_ξ	0.42	(0.21–0.68)	0.00	0.53	0.00	
χ^2/dof	1.00	0.99–1.63	1.64	0.67	1.97	

Table 3: *Theoretical predictions for several BRs obtained by using different models for the semileptonic heavy-heavy and heavy-light form factors. For each model, ξ and δ_ξ are fitted by minimizing the χ^2/dof computed using the BRs above the horizontal line only. The fitted values of these parameters are also shown, together with the corresponding χ^2/dof. For QCDSR-CV, we also give in parentheses the value of ξ and χ^2 obtained by putting $\delta_\xi = 0$. Charming penguin contributions, using η_L and δ_L as determined from $BR(B^+ \to K^0 \pi^+)$ and $BR(B_d \to K^+ \pi^-)$, are included. In square brackets, the BRs obtained without charming penguins are shown.*

	$\cos\delta$	σ	$BR(B^+ \to K^0\pi^+)$ $BR \times 10^5$	$BR(B_d \to K^+\pi^-)$ $BR \times 10^5$	$BR(B_d \to \pi^+\pi^-)$ $BR \times 10^5$	η_L	δ_L
Experiment			$2.3^{+1.1+0.2}_{-0.9-0.2} \pm 0.2$	$1.5^{+0.5+0.1}_{-0.4-0.1} \pm 0.1$	< 1.5		
	-0.7	0.48	1.49 [0.18]	1.61 [0.48]	2.09 [0.93]	-0.26	1.73
	0.38	0.48	1.62 [0.19]	1.60 [0.19]	1.59 [1.10]	-0.29	0.38
	+0.7	0.48	1.68 [0.20]	1.59 [0.11]	1.49 [1.15]	-0.30	0.06
	-0.7	0.36	1.36 [0.19]	1.62 [0.40]	1.30 [0.50]	-0.25	1.71
QCDSR–CV	0.38	0.36	1.49 [0.19]	1.61 [0.19]	0.99 [0.63]	-0.28	0.32
LO $\mu = 5$ GeV	+0.7	0.36	1.54 [0.20]	1.61 [0.12]	0.94 [0.67]	-0.29	0.004
	-0.7	0.24	1.23 [0.19]	1.63 [0.33]	0.69 [0.21]	-0.24	1.62
	0.38	0.24	1.37 [0.19]	1.62 [0.19]	0.52 [0.30]	-0.27	0.18
	+0.7	0.24	1.42 [0.19]	1.62 [0.14]	0.51 [0.32]	-0.27	-0.10
	-0.7	0.48	1.43 [0.53]	1.62 [0.97]	2.24 [0.96]	-0.34	1.43
	0.38	0.48	1.75 [0.56]	1.58 [0.49]	1.71 [1.26]	-0.28	0.25
	+0.7	0.48	1.85 [0.57]	1.57 [0.35]	1.64 [1.34]	-0.26	-0.08
	-0.7	0.36	1.30 [0.53]	1.63 [0.85]	1.30 [0.51]	-0.32	1.25
QCDSR–CV	0.38	0.36	1.62 [0.56]	1.60 [0.49]	1.04 [0.73]	-0.25	0.15
NLO $\mu = 5$ GeV	+0.7	0.36	1.71 [0.56]	1.59 [0.39]	1.02 [0.80]	-0.24	-0.16
	-0.7	0.24	1.22 [0.54]	1.63 [0.75]	0.47 [0.20]	-0.25	0.65
	0.38	0.24	1.50 [0.55]	1.61 [0.51]	0.53 [0.35]	-0.22	-0.4
	+0.7	0.24	1.57 [0.56]	1.60 [0.44]	0.54 [0.40]	-0.21	-0.31
	-0.7	0.48	2.30 [0.54]	1.50 [1.27]	4.67 [2.50]	1.07	-0.23
	0.38	0.48	2.30 [0.57]	1.50 [0.52]	3.55 [2.97]	-0.40	0.43
	+0.7	0.48	2.30 [0.58]	1.50 [0.30]	3.41 [3.11]	-0.43	0.14
	-0.7	0.36	2.03 [0.55]	1.55 [1.05]	2.81 [1.36]	1.01	-0.23
ABLOPR–CV	0.38	0.36	2.19 [0.17]	1.52 [0.49]	2.37 [1.71]	-0.33	0.82
LO $\mu = 5$ GeV	+0.7	0.36	2.23 [0.57]	1.51 [0.33]	2.27 [1.81]	-0.38	0.50
	-0.7	0.24	1.78 [0.55]	1.59 [0.87]	1.41 [0.56]	0.93	-0.25
	0.38	0.24	1.93 [0.56]	1.57 [0.49]	1.21 [0.80]	0.28	-0.89
	+0.7	0.24	1.97 [0.57]	1.56 [0.38]	1.17 [0.87]	0.16	-1.35
	-0.7	0.48	2.29 [1.42]	1.50 [2.50]	5.09 [2.51]	1.43	-0.22
	0.38	0.48	2.30 [1.50]	1.50 [1.26]	3.71 [3.29]	-0.24	0.64
	+0.7	0.48	2.30 [1.52]	1.50 [0.89]	3.60 [3.52]	-0.18	-0.06
	-0.7	0.36	2.01 [1.43]	1.55 [2.20]	3.03 [1.34]	1.34	-0.23
ABLOPR–CV	0.38	0.36	2.30 [1.49]	1.50 [1.27]	2.56 [1.92]	-0.32	1.16
NLO $\mu = 5$ GeV	+0.7	0.36	2.30 [1.51]	1.50 [0.99]	2.33 [2.09]	-0.22	0.51
	-0.7	0.24	1.76 [1.44]	1.59 [1.93]	1.49 [0.54]	1.18	-0.26
	0.38	0.24	2.03 [1.48]	1.55 [1.31]	1.29 [0.92]	0.40	-0.68
	+0.7	0.24	2.09 [1.49]	1.54 [1.12]	1.25 [1.04]	0.23	-0.91

Table 4: *BRs for several values of the CKM parameters* $\cos\delta$ *and* σ *and different choices of the Wilson coefficients. The results are given for QCDSR and ABLOPR only. Results with LQCD and QM are very similar to those obtained with QCDSR. In square brackets we give the results obtained without charming penguins.*