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On the M-theory description of gaugino condensation

I. Antoniadis[a,b] and M. Quirós[c]

[a] *TH-Division, CERN, CH-1211 Geneva 23, Switzerland*

[b] *Centre de Physique Théorique, Ecole Polytechnique,*[†] *F-91128 Palaiseau, France*

[c] *Instituto de Estructura de la Materia, CSIC, Serrano 123, 28006 Madrid, Spain*

Abstract

We present an additional test of the recent proposal for describing supersymmetry breaking due to gaugino condensation in the strong coupling regime, by a Scherk-Schwarz mechanism on the eleventh dimension of M-theory. An analysis of supersymmetric transformations in the infinite-radius limit reveals the presence of a discontinuity in the spinorial parameter, which coincides with the result found in the presence of gaugino condensation. The condensate is then identified with the quantized parameter entering the modification of the Scherk-Schwarz boundary conditions. This mechanism provides an alternative perturbative explanation of the gauge hierarchy that determines the scale of low-energy supersymmetry breaking in terms of the unification gauge coupling.

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[†]Laboratoire Propre du CNRS UPR A.0014.

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The 10-dimensional (10D) $E_8 \times E_8$ heterotic string, compactified on an appropriate 6D internal manifold, is a good candidate for describing the observed low-energy world. In particular, compactification on a Calabi-Yau (CY) manifold leads to a 4D $N = 1$ supersymmetric theory that can accommodate the gauge group and matter content of the standard model. On the other hand, there is a mismatch between the gauge coupling unification scale, $M_G \sim 10^{16}$ GeV, and the heterotic string scale M_H, which is determined in terms of the Planck scale, $M_p \sim 10^{19}$ GeV, from the relation $M_H = (\alpha_G/8)^{1/2} M_p \sim 10^{18}$ GeV, where $\alpha_G \sim 1/25$ is the unification gauge coupling. However, this relation does not hold if the compactification scale is small compared to M_H, in which case the 10D theory is strongly interacting.

It is now believed that the strong coupling limit of the 10D heterotic string theory compactified on CY is described by the 11D M-theory compactified on CY$\times S^1/\mathbf{Z}_2$ upon the identification [1, 2]:

$$M_{11} \equiv 2\pi(4\pi\kappa_{11}^2)^{-1/9} = 2\pi(2\alpha_G \widehat{V})^{-1/6} \qquad ; \qquad \rho^{-1} = \frac{4}{\alpha_G} M_{11}^3 M_p^{-2} , \tag{1}$$

where κ_{11} is the 11D gravitational coupling, ρ is the radius of the semicircle $S^1/\mathbf{Z}_2$, and $\widehat{V}$ is the CY volume. In this regime, the value of the unification scale can become consistent with the M-theory scale $M_{11} \sim M_G$, if the radius ρ of the semicircle is at an intermediate scale $\rho^{-1} \sim 10^{12}$ GeV, while for isotropic CY the compactification scale $\widehat{V}^{-1/6}/2\pi$ is of the order of M_{11}. Fortunately, this is inside the region of validity of M-theory, $\rho M_{11} \gg 1$ and $\widehat{V}\kappa_{11}^{-4/3} \gg 1$. As a result, the effective theory above the intermediate scale behaves as 5-dimensional, but only in the gravitational and moduli sector; the gauge sectors coming from $E_8 \times E_8$ live at the 4D boundaries of the semicircle.

In recent works [3, 4, 5], the intermediate scale ρ^{-1} was related with the scale of supersymmetry breaking by means of a coordinate-dependent compactification of the eleventh dimension, analogue to the Scherk-Schwarz mechanism [6]. The observable world living at the boundaries remains unaffected and feels supersymmetry breaking only by gravitational interactions, which yield $m_{\rm susy} \sim \rho^{-2}/M_p$ [3, 5]. Moreover we suggested [5] that this (perturbative) mechanism describes ordinary (non-perturbative) gaugino condensation [7] in the strongly coupled heterotic string. In particular, assuming that the basic relations of gaugino condensation in the weakly coupled heterotic string, $m_{3/2} = |W|e^{K/2} \propto \Lambda_c^3$, also hold in the strong coupling regime, and using the fact that the superpotential W is of order 1 (in Planck units) at the minimum, we found simple scaling relations with the volume $\widehat{V} \sim e^{-K}$: $m_{3/2} \sim \widehat{V}^{-1/2}$ and $\Lambda_c \sim \widehat{V}^{-1/6}$. Comparison with the duality relations (1) yields the following identifications for the gravitino mass and the condensation scale: $m_{3/2} \sim \rho^{-1}$ and $\Lambda_c \sim M_{11}$ [5], respectively.

In this letter we further analyse the mechanism of supersymmetry breaking in M-theory by Scherk-Schwarz compactification on the eleventh dimension and present additional evidence that it provides a dual description of gaugino condensation in the strongly coupled heterotic string. We find that the goldstino is the (right-handed) fifth component of the 5D gravitino, while there is a discontinuity in the supersymmetric spinorial transformation parameter around the end-point $\pm\pi\rho$ of the semicircle. This discontinuity survives in the limit $\rho \to \infty$, where the gravitino mass vanishes and supersymmetry is locally restored, in agreement with the result previously found in the case of gaugino condensation in the strongly coupled heterotic string [8]. Furthermore, the quantization of the condensate is related to the quantized parameter entering the Scherk-Schwarz boundary conditions. Finally, consistency of the proposed dual description requires that the hidden E_8 be strongly coupled at the M-theory scale, which determines ρ^{-1} in terms of α_G at the desired intermediate scale $\sim 10^{12}$ GeV. Consequently, the large hierarchy between the supersymmetry-breaking scale in the observable sector and the Planck mass arises as a result of successive power suppressions of the gauge coupling of the form $(\alpha_G/16\pi^2)^4$.

We start by first considering the $N = 1$ 5D theory obtained from compactification of M-theory on a CY manifold with Hodge numbers $h_{(1,1)}$ and $h_{(1,2)}$ [9]. In addition to the gravitational multiplet, the massless spectrum contains $n_V = h_{(1,1)} - 1$ vector multiplets and $n_H = h_{(1,2)} + 1$ hypermultiplets. The $N = 1$ supersymmetry transformations in the 5D theory are [10]:

$$\begin{aligned}
\delta e_M^m &= -\frac{i}{2}\overline{\mathcal{E}}\Gamma^m\Psi_M \\
\delta\Psi_M &= D_M\mathcal{E} + \cdots \\
\delta\mathcal{X}^a &= -\frac{1}{2}v_\alpha^a(\partial\!\!\!/\phi^\alpha)\mathcal{E} + \cdots \\
\delta\phi^\alpha &= \frac{i}{2}v_a^\alpha\overline{\mathcal{E}}\mathcal{X}^a\,,
\end{aligned} \tag{2}$$

where e_M^m is the fünfbein, $\Gamma^m = (\gamma^\mu, i\gamma_5)$ are the Dirac matrices [1], Ψ_M is the gravitino field, $\mathcal{E}$ the spinorial parameter of the transformation, and the dots stand for non-linear terms. Finally, $\mathcal{X}^a$ and ϕ^α denote the fermionic and scalar components of vector multiplets while v_α^a is the vielbein of the corresponding moduli space. Similar transformations hold for the components of hypermultiplets for which our subsequent analysis can be generalized in a straightforward way.

All fermions in eq. (2) can be represented as doublets under the $SU(2)$ R-symmetry whose components are subject to the (generalized) Majorana condition; in a suitable basis [11]:

$$\Psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \gamma_5 \begin{pmatrix} \psi_2^* \\ -\psi_1^* \end{pmatrix}, \tag{3}$$

where Ψ describes any generic (Dirac) spinor of eq. (2) [2]. In this notation, it is understood that the γ-matrices act diagonally in the $SU(2)_R$ space. It is convenient to decompose the spinors with respect to the four-dimensional chirality. Using the relations $\gamma_5^2 = 1$ and $\gamma_5^* = -\gamma_5$, which are valid in the above basis, it follows that $\gamma_5\psi_1 = \pm\psi_1$ implies

[1]The Γ^m matrices are defined by their anticommutation rules, $\{\Gamma^m, \Gamma^n\} = 2\eta^{mn}$, where the space-time metric is $\eta^{mn} = \mathrm{diag}(1, -1, -1, -1, -1)$. They satisfy the relation $\Gamma^0 \cdots \Gamma^4 = 1$. The 4D matrices $\Gamma^\mu = \gamma^\mu$ ($\mu = 0, \ldots, 3$) are purely imaginary and $\Gamma^4 = i\gamma_5$ is real.

[2]Hereafter we will conventionally denote fermionic $SU(2)_R$ doublets with upper-case symbols and their corresponding components with lower-case ones.

$\gamma_5\psi_1^* = \mp\psi_1^*$. We can then define:

$$\Psi_L \equiv \begin{pmatrix} \psi_L \\ \psi_R^* \end{pmatrix} \qquad\qquad \Psi_R \equiv \begin{pmatrix} \psi_R \\ -\psi_L^* \end{pmatrix}, \tag{4}$$

in terms of the 4D chiral spinors $\psi_{L,R} = \pm\gamma_5\psi_{L,R}$, where $\psi_{R,L}^* \equiv (\psi_{L,R})^*$. This decomposition amounts, in terms of $SU(2)_R$ doublets, to the condition

$$\Gamma_5\Psi_{L,R} = \pm\Psi_{L,R} \qquad ; \qquad \Gamma_5 = \begin{pmatrix} \gamma_5 & 0 \\ 0 & -\gamma_5 \end{pmatrix}. \tag{5}$$

The "chiral" spinors $\Psi_{L,R}$ satisfy trivially the condition (3). Furthermore, it is easy to show the relations

$$\overline{\mathcal{E}}_L(\gamma^\mu)^{2n}\Psi_R = 2i\mathrm{Im}\left\{\bar{\varepsilon}_L(\gamma^\mu)^{2n}\psi_R\right\}, \qquad \overline{\mathcal{E}}_L(\gamma^\mu)^{2n+1}\Psi_L = 2i\mathrm{Im}\left\{\bar{\varepsilon}_L(\gamma^\mu)^{2n+1}\psi_L\right\},$$

$$\overline{\mathcal{E}}_L(\gamma^\mu)^{2n}\gamma_5\Psi_R = 2\mathrm{Re}\left\{\bar{\varepsilon}_L(\gamma^\mu)^{2n}\gamma_5\psi_R\right\}, \qquad \overline{\mathcal{E}}_L(\gamma^\mu)^{2n+1}\gamma_5\Psi_L = 2\mathrm{Re}\left\{\bar{\varepsilon}_L(\gamma^\mu)^{2n+1}\gamma_5\psi_L\right\},$$

which are also valid when L and R are interchanged.

Upon compactification to $D = 4$ on the semi-circle $S^1/\mathbf{Z}_2$ of radius ρ, one obtains an $N = 1$ supersymmetric theory that, for large ρ, describes the strong coupling limit of the heterotic string compactified on the same Calabi-Yau manifold as compactifies the 11-dimensional theory to $D = 5$ [1, 2]. The gauge group then appears at the two endpoints of the semicircle and consists of a subgroup of $E_8 \times E_8$ together with some matter representations depending on the particular embedding of the spin connection. For instance, for the standard embedding into a single E_8, one obtains an E_6 sitting at one end ($y = \pi\rho$) together with $h_{(1,1)}$ $\mathbf{27}$'s and $h_{(1,2)}$ $\overline{\mathbf{27}}$'s, and a pure E_8 sitting at the other end ($y = 0$).

The $\mathbf{Z}_2$ projection is defined through the reflection $\mathcal{R}$ of the fifth coordinate y parametrizing S^1, and has the following action on the 5D fermionic fields:

$$\mathcal{R}\Psi(x^\mu, y) \equiv \eta\Gamma_5\Psi(x^\mu, -y), \tag{6}$$

with $\eta = 1$ for $\Psi = \Psi_\mu$ and $\eta = -1$ for $\Psi = \Psi_5, \mathcal{X}$. In terms of the spinors defined in eq. (4),

$$\mathcal{R}\Psi_{L,R}(x^\mu, y) = \pm\eta\Psi_{L,R}(x^\mu, -y) \ . \tag{7}$$

The $\mathbf{Z}_2$ projection is defined by keeping the states that are even under $\mathcal{R}$. It follows that the remaining massless fermions are the left-handed components of the 4D gravitino $\Psi_{\mu L}$, as well as the right-handed components of Ψ_{5R} and $\mathcal{X}_R$. Taking into account the $\mathbf{Z}_2$ action in the bosonic sector, which projects away the off-diagonal components of the fünfbein (e_μ^5), the above massless spectrum is consistent with the residual $N = 1$ supersymmetry transformations at $D = 4$ given by eq. (2) with a fermionic parameter $\mathcal{E}$ reduced to its left-handed component $\mathcal{E}_L$.

In order to spontaneously break supersymmetry, we apply the Scherk-Schwarz mechanism on the fifth coordinate y [6]. For this purpose, we need an R-symmetry, which transforms the gravitino non-trivially and imposes boundary conditions, around S^1, which are periodic up to a symmetry transformation:

$$\Psi_M(x^\mu, y + 2\pi\rho) = e^{2i\pi\omega Q}\Psi_M(x^\mu, y) \, , \tag{8}$$

where Q is the R-symmetry generator and ω the transformation parameter. The continuous symmetry is in general broken by the compactification to some discrete subgroup, leading to quantized values of ω. For instance, in the case of $\mathbf{Z}_N$ one has $\omega = 1/N$ and $Q = 0, \ldots, N-1$. For generic values of ω, eq. (8) implies that the zero mode of the gravitino acquires an explicit y-dependence:

$$\Psi_M(x^\mu, y) = U(y)\Psi_M^{(0)}(x^\mu) + \cdots \quad ; \quad U = e^{i\frac{\omega}{\rho}yQ} \, , \tag{9}$$

where the dots stand for Kaluza-Klein (KK) modes.

Consistency of the theory requires that the matrix U commutes with the reflection $\mathcal{R}$, which defines the $N = 1$ projection [12, 13]. From eq. (6) one then finds:

$$\Gamma_5 U(-y) = U(y)\Gamma_5 \, , \tag{10}$$

implying that the generator Q anticommutes with Γ_5, $\{Q, \Gamma_5\} = 0$ [3] . The general solution is [4]:

$$Q = \sin\theta\sigma_1 + \cos\theta\sigma_2 \quad ; \quad U = \begin{pmatrix} \cos\frac{\omega y}{\rho} & \sin\frac{\omega y}{\rho} e^{i\theta} \\ -\sin\frac{\omega y}{\rho} e^{-i\theta} & \cos\frac{\omega y}{\rho} \end{pmatrix} , \tag{11}$$

where $\sigma_{1,2}$ are the Pauli matrices representing $SU(2)_R$ generators and θ is an arbitrary (real) parameter. For the particular value $\omega = 1/2$ there is an additional solution to eqs. (8) and (10) corresponding to [5]:

$$Q = \begin{pmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{pmatrix} \quad ; \quad U = \begin{pmatrix} e^{i\pi\theta_1} & 0 \\ 0 & e^{i\pi\theta_2} \end{pmatrix} \cos\frac{y}{2\rho} \qquad \left(\omega = \frac{1}{2}\right) , \tag{12}$$

with $\theta_{1,2}$ arbitrary constants. Note however that this solution involves both $n = 0$ and $n = -1$ KK-modes, which makes the effective field theory description of the spontaneous supersymmetry breaking more complicated. For this reason, we restrict our subsequent analysis to solution (11).

Inspection of the supersymmetry transformations (2), together with the requirement that the fünfbein zero mode does not depend on y, shows that the y-dependence of the supersymmetry parameter is the same as that of the gravitino zero-mode, i.e. $\mathcal{E}(x^\mu, y) = U(y)\mathcal{E}^{(0)}(x^\mu)$ [6]. Supersymmetry in the 4D theory is then spontaneously broken, with the goldstino being identified with the fifth component of the 5D gravitino, $\Psi_5^{(0)}$. Indeed, for global supersymmetry parameter, $D_\mu\mathcal{E}^{(0)} = 0$, its variation is [4]:

$$\delta\Psi_5^{(0)} = (U^{-1}\partial_y U)\mathcal{E}^{(0)} + \cdots \quad ; \quad U^{-1}\partial_y U = \frac{\omega}{\rho} \begin{pmatrix} 0 & e^{i\theta} \\ -e^{-i\theta} & 0 \end{pmatrix} , \tag{13}$$

while no other fermions can acquire finite constant shifts in their transformations. The reason is that the scalar components of the zero-mode supermultiplets are either inert

[3]Notice that condition (10) guarantees that the $\mathcal{R}$-chirality of a spinor, $\Psi_{L,R}(x^\mu, y)$, in the sense of eq. (7), coincides with the Γ_5-chirality of its zero-mode $\Psi^{(0)}_{L,R}(x^\mu)$, in the sense of eq. (5). In this way one can write the decomposition (9) for the chiral components of Ψ, i.e. $\Psi_{L,R}(x^\mu, y) = U(y)\Psi^{(0)}_{L,R}(x^\mu)$.

[4]Note that the operator $U^{-1}\partial_y U$ turns a left-handed spinor, in the sense of eq. (5), into a right-handed one.

under the R-symmetry, and therefore y-independent, or otherwise become massive, and y-dependent as $\phi(x^\mu, y) = v + U(y)\phi^{(0)}(x^\mu) + \cdots$, with v being a constant and the quantum fluctuation $\phi^{(0)}$ having zero vacuum expectation value (VEV). It is then clear from eq. (2) that the variation of their fermionic superpartners vanishes in the vacuum, $\delta\mathcal{X}^{(0)} = 0$.

The kinetic term of the 5D gravitino,

$$-i/2\overline{\Psi}_M \Gamma^{MNP} D_N \Psi_P,$$

where $\Gamma^{M_1 \dots M_n} \equiv \Gamma^{[M_1}\Gamma^{M_2}\dots\Gamma^{M_n]}$ is the totally antisymmetric product, gives rise to the following 4-dimensional Lagrangian for the gravitino zero modes:

$$e^{-1}\mathcal{L}^{(0)} = -\frac{i}{2}\overline{\Psi}^{(0)}_M \Gamma^{MNP} D_N \Psi^{(0)}_P - \frac{1}{2}\overline{\Psi}^{(0)}_\mu \left(U^{-1}\partial_y U\right)\Gamma^{\mu\nu}\gamma_5 \Psi^{(0)}_\nu\,, \tag{14}$$

where the matrix $U^{-1}\partial_y U$ is given in eq. (13). The first term in the r.h.s. of eq. (14) is the kinetic term of the gravitino zero modes, while the second is a mass term for the $\Psi^{(0)}_\mu$ component. Notice that the goldstino component $\Psi^{(0)}_5$ remains massless, as it should.

The above arguments are also valid in the $N = 1$ theory, obtained by applying the $\mathbf{Z}_2$ projection defined through the $\mathcal{R}$-reflection (6). The y-dependence of the remaining zero modes is always given by eq. (9). The goldstino is now the right-handed component $\psi^{(0)}_{5R}$, which, from eq. (13), transforms as:

$$\delta\psi^{(0)}_{5R} = \frac{\omega}{\rho} e^{i\theta} \varepsilon^{(0)*}_R + \cdots \tag{15}$$

The surviving gravitino is $\Psi^{(0)}_{\mu L}$ in the notation of eq. (4). Its kinetic term can be read off from the first term in the r.h.s. of eq. (14):

$$e^{-1}\mathcal{L}^{(0)}_{\rm kin} = -\frac{i}{2}\overline{\Psi}^{(0)}_{\mu L}\Gamma^{\mu\nu\rho} D_\nu \Psi^{(0)}_{\rho L} = -\frac{i}{2}\overline{\psi}^{(0)}_{\mu L}\Gamma^{\mu\nu\rho} D_\nu \psi^{(0)}_{\rho L} + \ h.c. \tag{16}$$

while the second term yields a mass for the gravitino zero mode equal to ω/ρ:

$$e^{-1}\mathcal{L}^{(0)}_m = -\frac{1}{2}\frac{\omega}{\rho}\,\overline{\Psi}^{(0)}_{\mu L}\begin{pmatrix} 0 & e^{i\theta} \\ -e^{-i\theta} & 0 \end{pmatrix}\Gamma^{\mu\nu}\gamma_5 \Psi^{(0)}_{\nu L} = \frac{1}{2}\frac{\omega}{\rho}\left[e^{i\theta}\overline{\psi}^{(0)}_{\mu L}\Gamma^{\mu\nu}\psi^{(0)*}_{\nu R} + \ h.c.\right]. \tag{17}$$

Note, however, that the above analysis in the $N = 1$ case is valid, strictly speaking, for values of y inside the semicircle, obtained from the interval $[-\pi\rho, \pi\rho]$ through the identification $y \leftrightarrow -y$. This leads to a discontinuity in the transformation parameter $\mathcal{E}$ around the end-point $y = \pm\pi\rho$, since $U(-\pi\rho) = U^{-1}(\pi\rho)$:

$$\mathcal{E}(-\pi\rho) \neq \mathcal{E}(\pi\rho)\,. \tag{18}$$

This discontinuity survives even in the large-radius limit $\rho \to \infty$ where the gravitino mass vanishes and supersymmetry is restored locally. This phenomenon is reminiscent of the one found in ref. [8], where the discontinuity at the weakly coupled end $y = \pi\rho$ is due to the gaugino condensate of the hidden E_8 formed at the strongly coupled end $y = 0$. In fact the two results become identical for the transformation parameter $\mathcal{E}$ in the neighbourhood $y \sim \pi\rho$, in the limit $\rho \to \infty$:

$$\lim_{\rho\to\infty} \varepsilon_L(y)\,|_{y\sim\pi\rho} = \cos\pi\omega\,\varepsilon_L^{(0)} + \epsilon(y)\sin\pi\omega\, e^{i\theta}\varepsilon_R^{(0)*}\,. \tag{19}$$

On the other hand, it is easy to see that the goldstino variation vanishes in this limit, since the discontinuity in $\partial_y\varepsilon(y)_L$ is proportional to $\delta(y)\sin(y\omega/\rho)$. The transformation parameter $\varepsilon_L(y)$ is thus identified with the spinor η' of ref. [8], which solves the unbroken supersymmetry condition $\delta\psi_{5R} = 0$.

Note that despite the change of 4D chirality, both terms in the r.h.s. of eq. (19) are invariant under the $\mathcal{R}$ reflection (6), which defines the $N = 1$ projection. Indeed, the second term containing the $\mathcal{R}$-odd right-handed spinor $\varepsilon_R^{(0)*}$ is multiplied by $\epsilon(y)$, which is also odd under $\mathcal{R}$. The proportionality constant $\sin\pi\omega$ plays the role of the gaugino condensate in the dual description and vanishes only for integer values of ω for which the Scherk-Schwarz mechanism becomes trivial. In general ω is quantized, as we discussed earlier, which is consistent with the quantization of the gaugino condensate through its equation of motion that relates it with the VEV of the antisymmetric tensor field strength [14].

In the presence of gaugino condensation, the discontinuity (18) was interpreted as a topological obstruction that signals supersymmetry breaking when effects of finite radius would be taken into account [8]. Here we have shown that the same discontinuity, in the infinite-radius limit, is reproduced by the Scherk-Schwarz mechanism. Moreover, in ref. [5] we have provided independent evidence that the finite-radius effects are also described by the Scherk-Schwarz mechanism on the eleventh dimension, in the region of validity of M-theory, $\rho M_{11} \gg 1$, where the 10D heterotic string remains strongly coupled.

In the description of gaugino condensation by the Scherk-Schwarz mechanism, the condensation scale is identified with the M-theory scale M_{11}. This implies that the hidden E_8 is strongly coupled and should not contain any massless matter in the perturbative spectrum. Consistency then requires that the corresponding gauge coupling be large, $\alpha_8(M_{11}) \gtrsim 1$. On the heterotic side, this condition follows from the minimization of the gaugino condensation potential, which relates the value of the condensate to the quantized VEV of the antisymmetric tensor field strength. On the M-theory side, this provides a constraint that naively fixes the 4D unification coupling α_G to be in a non-perturbative regime. Fortunately, there are important M-theory threshold effects that invalidate this conclusion. These effects can be understood from the lack of factorization of the 7-dimensional internal space as a direct product of the semicircle with a Calabi-Yau manifold, $\mathrm{CY}\times S^1/\mathbf{Z}_2$ [2]. As a result, the Calabi-Yau volume $\widehat{V}$ becomes a function of ρ and takes different values at the two end-points of the semicircle. In the large-radius limit, one finds:

$$\widehat{V}(0) = \widehat{V}(\pi\rho) - 2\pi^4 \rho M_{11}^{-3} \left| \int_{\mathrm{CY}} \frac{\omega}{4\pi^2} \wedge (\mathrm{tr}F' \wedge F' - \mathrm{tr}F \wedge F) \right| , \tag{20}$$

where ω is the Kähler form of CY and F' (F) is the field strength of the strongly (weakly) coupled E_8 sitting at the end-point $y = 0$ ($y = \pi\rho$). The integral in the r.h.s. is a linear function of the $h_{(1,1)} - 1$ Kähler class moduli for unit volume, which belong to 5D vector multiplets. Its natural value is M_{11}^{-2} up to a proportionality factor of order 1 [2].

Following eq. (1), the gauge coupling constants at the two end-points are related to the corresponding volumes as [1]:

$$\frac{1}{\alpha_G} = 2M_{11}^6 V(\pi\rho) \qquad ; \qquad \frac{1}{\alpha_8(M_{11})} = 2M_{11}^6 V(0)\,, \tag{21}$$

where α_G is the unification coupling of the weakly coupled gauge group and the reduced volumes are defined by $\widehat{V} \equiv (2\pi)^6 V$. Imposing now the constraint $\alpha_8(M_{11}) \gtrsim 1$ and using eqs. (21) and (20), one finds $\rho \sim \rho_{\rm crit}$ where $\rho_{\rm crit}$ corresponds to the critical value at which the volume at the strongly coupled end vanishes and the hidden E_8 decouples from the low-energy spectrum:

$$\begin{aligned} \rho_{\rm crit}^{-1} &= \frac{\alpha_G}{16\pi^2} M_{11}^3 \left| \int_{\rm CY} \frac{\omega}{4\pi^2} \wedge (\mathrm{tr}F' \wedge F' - \mathrm{tr}F \wedge F) \right| \\ &\sim \frac{\alpha_G}{16\pi^2} M_{11} \simeq 2 \times 10^{-4} M_{11} \end{aligned} \tag{22}$$

Note that this condition can also be thought of as resulting from a minimization of the (positive semi-definite) 4D gaugino condensation potential, which is proportional to $V(0)$ and, thus, vanishes at zero volume.

It is remarkable that the above relation provides the hierarchy necessary to fix ρ^{-1} at the intermediate scale $\sim 10^{12}$ GeV, when one identifies the M-theory scale M_{11} with the unification mass $\sim 10^{16}$ GeV inferred by the low-energy data [2, 3]. In fact, from eq. (17), ρ^{-1} determines the value of the gravitino mass and the scale of supersymmetry breaking for the gravitational and moduli sector in the 5-dimensional bulk. Supersymmetry breaking is then communicated to the observable sector, living at the boundary $y = \pi\rho$, by gravitational interactions yielding a low-energy supersymmetry breaking $m_{\rm susy} \sim \rho^{-2}/M_p$, which is at the TeV range [3, 5]. Thus, this mechanism provides an alternative "perturbative" explanation of the gauge hierarchy, where the smallness of the ratio $m_{\rm susy}/M_p \sim 10^{-16}$ is provided by powers of the unification coupling $\sim (\alpha_G/16\pi^2)^4$ instead of the conventional non-perturbative suppression $\sim e^{-1/\alpha_G}$. Of course in both cases, the remaining open problem is to determine the actual value of the gauge coupling α_G. In the present context of M-theory, this amounts to fixing the volume of the Calabi-Yau manifold $V(\pi\rho)$.

Note added

After completion of this work we have received the paper of ref. [15], where an independent analysis of supersymmetry breaking in M-theory is performed. There is however an important numerical difference in the estimate of the radius ρ of the semicircle due to the different definition of the unification scale. In fact, in ref. [15] M_G is identified with $\widehat{V}^{-1/6}$, instead of M_{11} which is also of the order of the mass of the first KK-excitation $V^{-1/6}$ (up to a factor of $(2\alpha_G)^{-1/6} \sim 1.5$). As a result, our determination of ρ^{-1} from the second relation of eq. (1) yields a value which is around three orders of magnitude below the one of ref. [15]. A similar difference holds for the determination of $\rho_{\rm crit}^{-1}$ in eq. (22) for the same reason. Another difference concerns the gaugino masses. We would like to stress that the vanishing of supersymmetry breaking in the observable sector is true only if this sector lives on a strict 4D boundary of the 5D world. This implies that there are no KK excitations along the eleventh (5th) dimension with quantum numbers of the Standard Model, and thus, there are no threshold corrections to the (observable) gauge couplings. This reduces to a condition on the compactification manifold implying, in particular, that the volume $V(\pi\rho)$ is independent of ρ. If this requirement is not satisfied, gaugino masses, as well as in general scalar masses, will be proportional to the gravitino mass [12, 13]. Such a scenario is however phenomenologically inconsistent since soft masses are pushed at the intermediate scale $\sim 1/\rho$ unless if the quantization condition of the Scherk-Schwarz parameter ω in eqs. (9,11), that corresponds to the gaugino condensate superpotential, can be avoided.

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