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LONGITUDINAL IMPEDANCE OF LHC VERSION-1 STRIPLINE BEAM POSITION MONITOR

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Abstract

The electrodes in the first version of the LHC beam position monitor are 50 Ω striplines. An enlarged aperture is required to keep the inner face of the electrodes in the shadow of the mechanical aperture of the machine. The longitudinal impedance of this device consists of two distinct components, one from the cavity and the other from the electrodes. The cavity part of the impedance can be reduced by inter-electrode shields as proposed by G. Lambertson. A complementary way of reducing this part of the impedance is to use tapered edges. The cavity wake potential of the beam position monitor is computed both with the 3D code MAFIA and analytically for a very short bunch (20 *mm*) and for a normal high energy LHC bunch (75 *mm*). The computation is done separately for the inter-electrode shields and for the tapered ends. The agreement between the two methods is very good, hence it can be concluded that the cavity impedance of the monitor cavity is well known up to the cut-off frequency of the LHC pipe (5.1 GHz). The impedance reduction by the shields and tapers can best be appreciated on the *Z/n* impedance plots. Finally, the wake field for the complete monitor which now also includes 4 striplines, is computed analytically for the two types of bunches.

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1. Introduction

The electrodes in the first version of the beam position monitors for the LHC are long 50 Ω striplines shorted at one end. Their dimensions are chosen in such a way that the sensitivity is sufficient to cope with the attenuation of long signal cables even for the very low intensity pilot bunch ($3\ 10^{9}$ *p.p.b.*). The design of this monitor is described in [1]. The aperture of the stripline monitor vacuum chamber must be larger than the aperture of the normal vacuum chamber. This is a consequence of the finite height of the stripline and the requirement that the electrode should be kept in the shadow of the normal machine aperture. The cross-section change that is unavoidable will contribute to the impedance of the machine. It has been suggested by G. Lambertson [2] that this impedance may be reduced by introducing 'fins' between the striplines. The effect of the abrupt cross-section change may also be reduced by tapering. In what follows three monitor cavities will be studied, one with inter-electrode fins or shields, one with tapered edges and the basic one with abrupt edges. The wake potential of the monitor bodies without electrodes will be computed with the code MAFIA [3] in time(space) domain and analytically in frequency domain. Finally the total impedance will be computed taking the electrode impedance into account assuming four orthogonal striplines.

2. Geometry of monitor vacuum chamber

The longitudinal cross-section of the basic shallow cavity structure is shown in Figure 1. The ends are abrupt and the structure is empty, i.e. without inter-electrode shields.

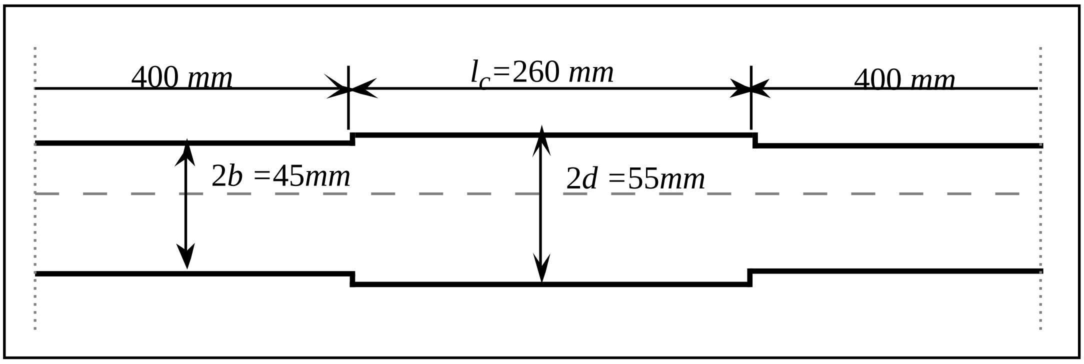

Figure 1 : Longitudinal cut of basic monitor vacuum chamber.

The dimensions of the structure were rounded to the nearest *mm* to avoid unnecessary problems with mesh sizes in MAFIA. Figure 2 shows the cross-section of the *empty* structure together with the cross-section of the *filled* one containing four inter electrode shields. A three-dimensional view of the structure with the shields produced by MAFIA is shown in Figure 3. The 4 niches for the striplines are clearly visible. For symmetry reasons a subtended angle of 45˚ was chosen for the niches and for the inter-electrode shields.

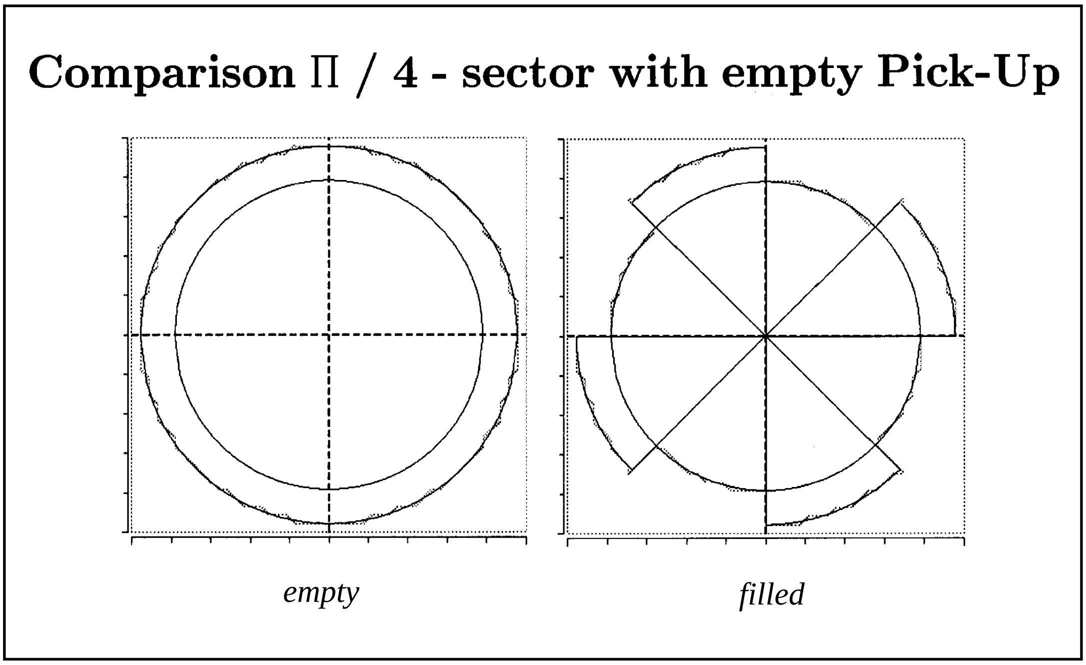

Figure 2 : Transverse cut through monitor body of *empty* and *filled* structure (MAFIA).

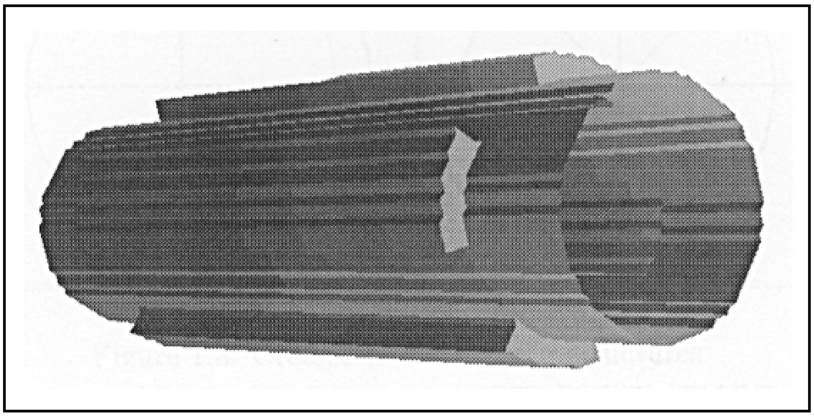

Figure 3 : Three-dimensional view of *filled* monitor structure (MAFIA).

3. Time (space) domain computation of BPM cavity wake potential

The wake potential was computed with MAFIA-3D both for the axial cylindrical symmetric *empty* structure with abrupt and tapered ends and for the *filled* one with abrupt edges. The calculation was done for two different bunches, one with *r.m.s.* length of 75 and one with 20 *mm*. The 75 *mm* bunch is the typical bunch length in the LHC at 7 TeV/c. The 20 *mm* bunch was chosen to probe the higher frequencies of the impedance. The angle θ of the taper is 30^{0}. The response of the structure to the excitation of the bunches is shown in Figures 4 and 5 for the long bunch and in 6 and 7 for the short bunch. The shaded area is the intensity profile of the exciting bunch. No specific vertical scale is associated with it.

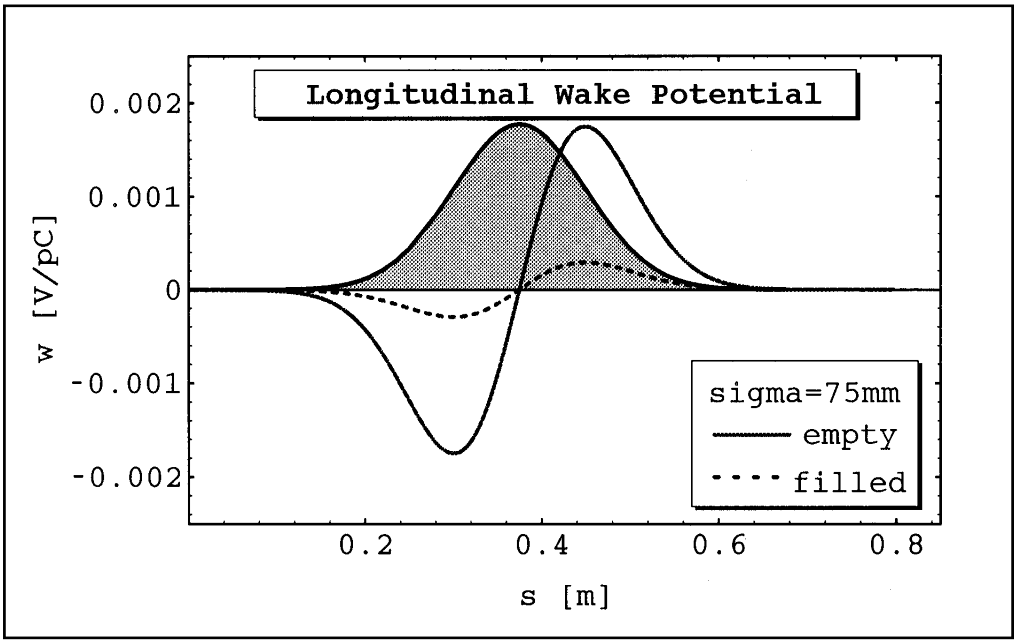

Figure 4 : Response of monitor cavity with and without inter-electrode shields to excitation of 75 *mm* bunch.

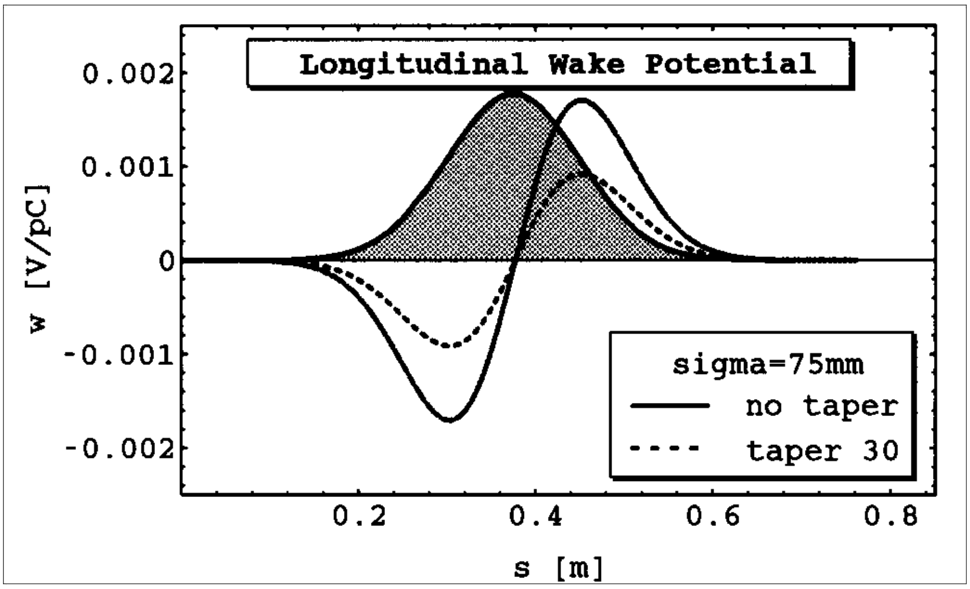

Figure 5 : Response of monitor cavity with and without 30^0 tapered ends to excitation of 75 *mm* bunch.

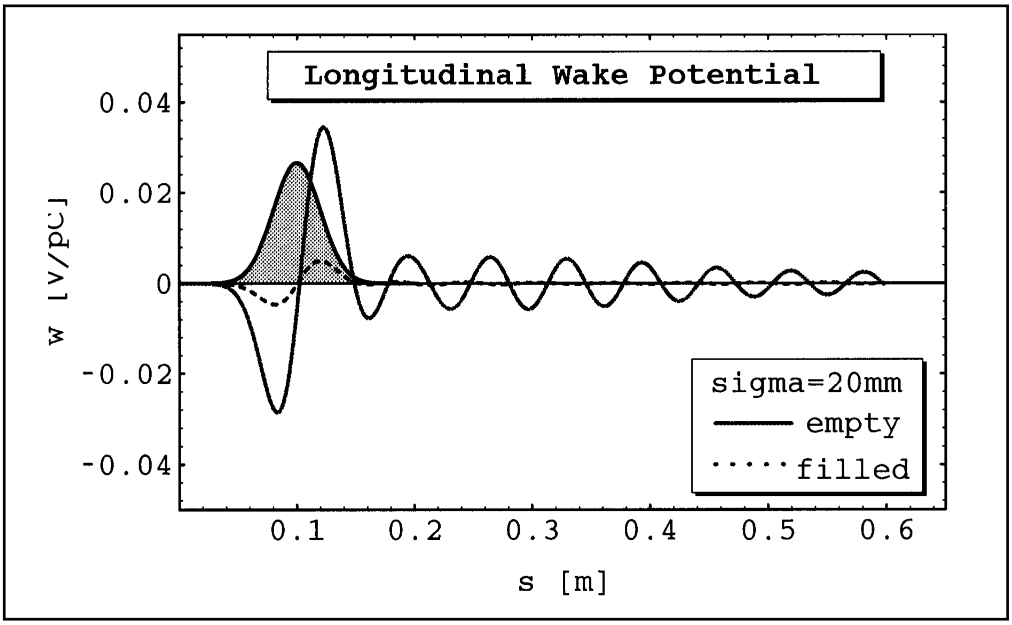

Figure 6 : Response of monitor cavity with and without inter-electrode shields to excitation of 20 *mm* bunch.

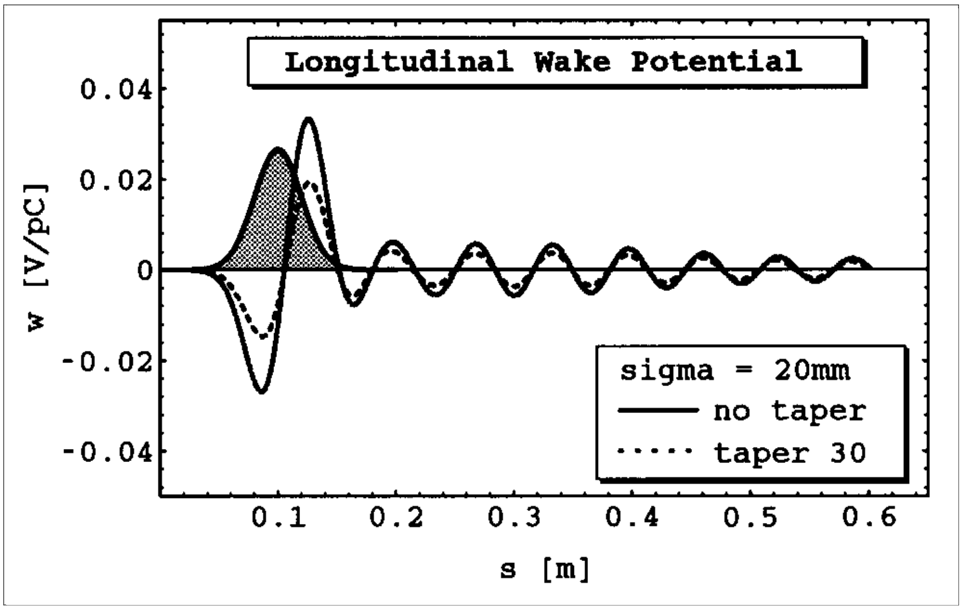

Figure 7 : Response of monitor cavity with and without 30^0 tapered ends to excitation of 20 *mm* bunch.

4. Analytic computation of BPM cavity wake potentials

The (inductive) impedance of a shallow cavity below cut-off frequency can be found with [4] :

$$Z_{c0} = \frac{Z_0}{2\pi} j \frac{\omega}{c} \ln\left(\frac{d}{b}\right) \frac{1-e^{-\gamma_0(\omega) l_c}}{\gamma_0(\omega)}, \tag{1}$$

where Z_0 is the impedance in vacuum, c the speed of light, b the radius of the pipe, d the radius of the cavity, l_c the length of the cavity and $\gamma_0(\omega)$ the propagation constant in the cavity.

$$\gamma_0(\omega) = \sqrt{\left(\frac{\tau_0}{d}\right)^2 - \left(\frac{\omega}{c}\right)^2}, \tag{2}$$

where τ_0=2.4 is the first root of the zero-th order Bessel function. The index $_c$ indicates the cavity part of the impedance, while the index $_0$ indicates the wave mode number (Bessel).

The impedance for frequencies higher than the cut-off frequency of the cavity but lower than the cut-off frequency of the pipe, can be found with :

$$Z_{c0} = \frac{Z_0}{2\pi} j \frac{\omega}{c} \ln\left(\frac{d}{b}\right) l_c \frac{\sin(\beta_0(\omega) l_c)}{\beta_0(\omega) l_c}, \tag{3}$$

where $\beta_0(\omega)$ is the imaginary part of the propagation constant $\gamma_0(\omega)$. Equations (1), (2) and (3) can be applied as such to the problem of the *empty* BPM cavity. An equation similar to (1) is proposed for the case of the *filled* BPM cavity based on the following argument. The *empty* cavity has perfect axial rotational symmetry while the *filled* one has a fourfold symmetry (Figure 2). The perturbed electro-magnetic fields will follow the same fourfold symmetry pattern. While the lowest mode of the fields is based on the first root of the zero order Bessel function J_0 (τ_0=2.4) for the *empty* cavity, it will be based on the first root of the fourth order Bessel function J_4 (τ_4=7.59) for the *filled* cavity. Physically the order of the Bessel function (0 or 4 in this case) is the number of cycles of variation of the longitudinal electrical field found when going around the cavity once. A factor 1/2 has to be introduced to take account of the fact that the reflective surface is reduced by that amount in the cavity *filled* with the shields. The propagation constant and impedance of the *filled* shallow cavity then become :

$$\gamma_4(\omega) = \sqrt{\left(\frac{\tau_4}{d}\right)^2 - \left(\frac{\omega}{c}\right)^2} \tag{4}$$

$$Z_{c4} = \frac{Z_0}{4\pi} j \frac{\omega}{c} \ln\left(\frac{d}{b}\right) \frac{1-e^{-\gamma_4(\omega) l_c}}{\gamma_4(\omega)}. \tag{5}$$

The cut-off frequency is higher by a factor τ_4/τ_0 and the equivalent of Eq. (3) is not needed since the bunch spectrum stays well below this limit.

Tapering the ends of the empty cavity will reduce the reflection of the edges as $\sin(\theta)$. This then yields the following expression for the impedance of the cavity with tapered ends :

$$Z_{ctap} = \frac{Z_0}{2\pi} j \frac{\omega}{c} \ln\left(\frac{d}{b}\right) l_c \frac{\sin(\beta_0(\omega) l_c)}{\beta_0(\omega) l_c} \sin(\theta) \tag{6}$$

The Fourier transform of the wake fields is the product of the impedance and the spectrum of the bunch. Assuming a charge of 1 *pC* yields :

$$\tilde{W}_c(\omega) = 10^{-12} Z_c(\omega) e^{-\frac{1}{2}\left(\frac{\omega\sigma_s}{c}\right)^2}, \tag{7}$$

where σ_s is the *rms* bunch length. The response of the excitation of the cavity by the bunch is found with the inverse Fourier integral :

$$W_c(s) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{W}_c(\omega) e^{j\frac{\omega}{c}s} d\omega \ . \tag{8}$$

The integration limits are finite in practice but they are chosen large in comparison with the *rms* bunch spectrum.

The results of the computations are shown in Figures 8 and 9.

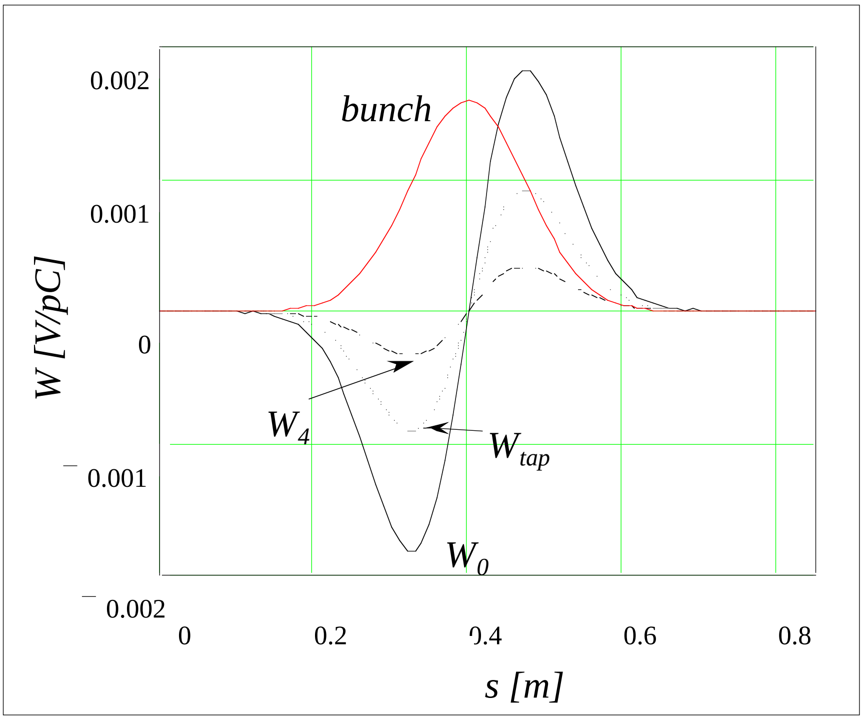

Figure 8 : Wake potential of 75 *mm* bunch in *empty* (W_0), *filled* (W_4) and *tapered* (W_{tap}) BPM monitor cavity.

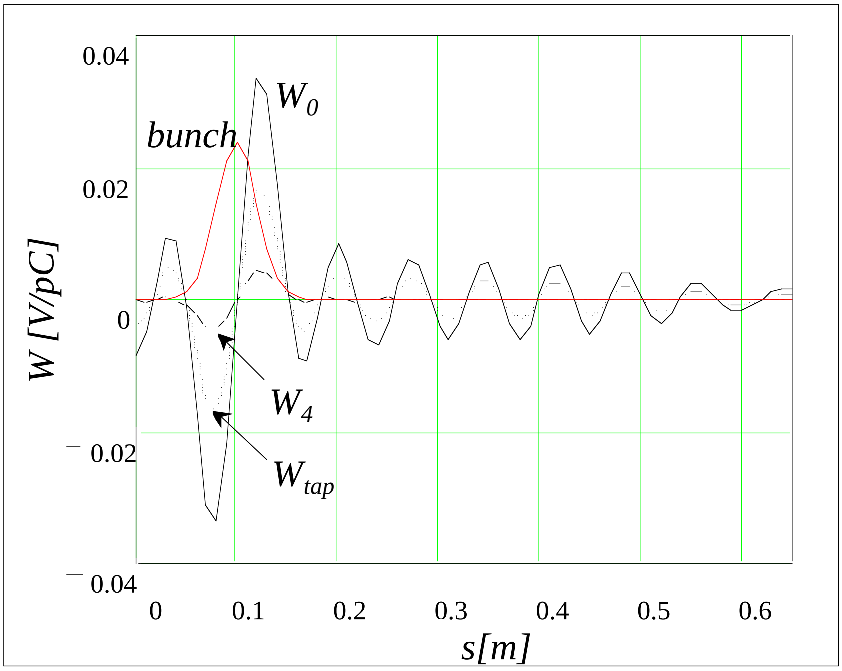

Figure 9 : Wake potential of 20 mm bunch in *empty* (W_0), *filled* (W_4) and *tapered* (W_{tap}) BPM monitor cavity.

The discrete nature of the Fourier transform that was used in the numerical computation leading to the results of Figures 8 and 9 is responsible for the symmetry in the response with respect to the center of the probing bunch.

The peak values of the wake potentials computed with MAFIA and analytically can be compared in Table 1.

σ_s	*mm*	75	20	75	20
$\hat{W}$ *not shielded*	*V/pC*	0.00175	0.0345	0.0018	0.034
$\hat{W}$ *shielded*	*V/pC*	0.00029	0.0049	0.00028	0.0039
$\hat{W}$ *30^0 taper*	*V/pC*	0.00091	0.0194	0.0009	0.0168
		MAFIA		*analytic*	

Table 1 : Peak values of wake potentials.

The results obtained with MAFIA are in good agreement with the results of the frequency analysis of the BPM cavity structures.

5. Total impedance of BPM

Electrodes are essential components in a beam position monitor. They will certainly modify the cavity-like impedance computed in the preceding section but more importantly, their load will make a contribution to the machine impedance which might be called useful impedance. It may be worthwhile to note that neither the changes of the cavity impedance due to the physical presence of the electrode volumes, nor the attenuations in the monitor due to finite conductivity of the material will be considered. The last item is certainly not negligible but it is taken into account in the resistive wall calculations elsewhere. The total impedance of the 4 striplines is given by:

$$Z_e = \frac{Z_0^2}{2\pi^2 Z_l}\left(\ln\left(\frac{d}{d-h}\right)\right)^2\left[1-\cos\left(\frac{2\omega l_e}{c}\right)+j\sin\left(\frac{2\omega l_e}{c}\right)\right], \tag{9}$$

where h is the height and l_e the length and Z_l the impedance (typically 50 Ω) of the stripline. The most striking feature is the fact that the reactive and resistive part of this impedance are of comparable importance.

Expressions for *Z/*n are obtained by multiplying Eq. (1), (3), (5), (6) and (9) by the factor Ω/ω, where Ω is the angular revolution frequency of the LHC. The results are plotted in Figures 10, 11 and 12. The influence of the electrodes can be appreciated by comparing Figure 10 with Figure 11. Figure 10 shows the reactive part of the impedance of the cavities *alone* and Figure 11 shows the impedance when they are equipped with 4 electrodes. The real part of the stripline or monitor impedance is shown in Figure 12.

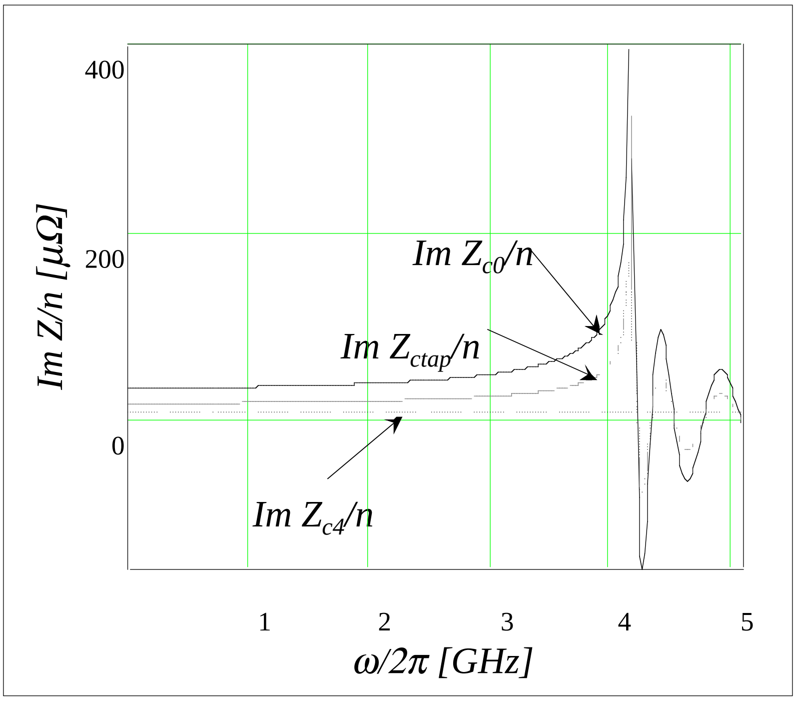

Figure 10 : Imaginary *Z/n* of cavity impedance

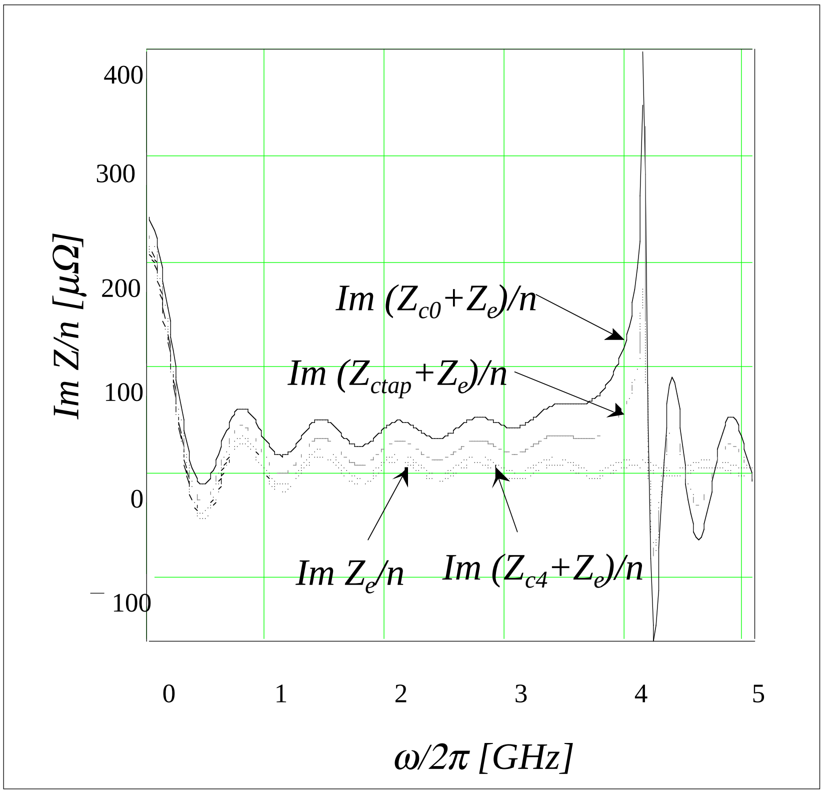

Figure 11 : Imaginary *Z/n* of cavity *and* stripline impedance

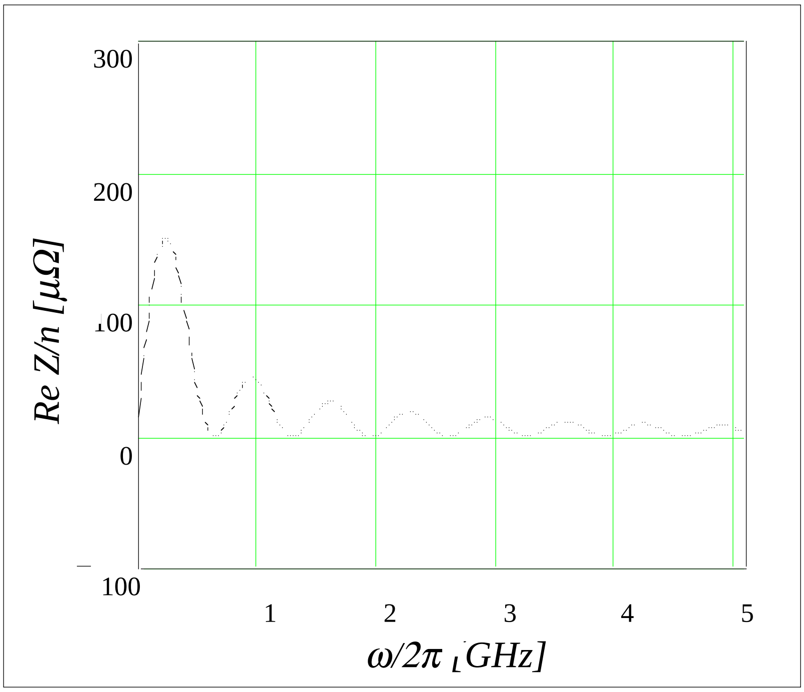

Figure 12 : Real *Z/n* of stripline impedance.

The response of the complete beam position monitor is shown in Figures 13 and 14 which should be compared with the responses of Figures 4 to 7 and 8 to 9.

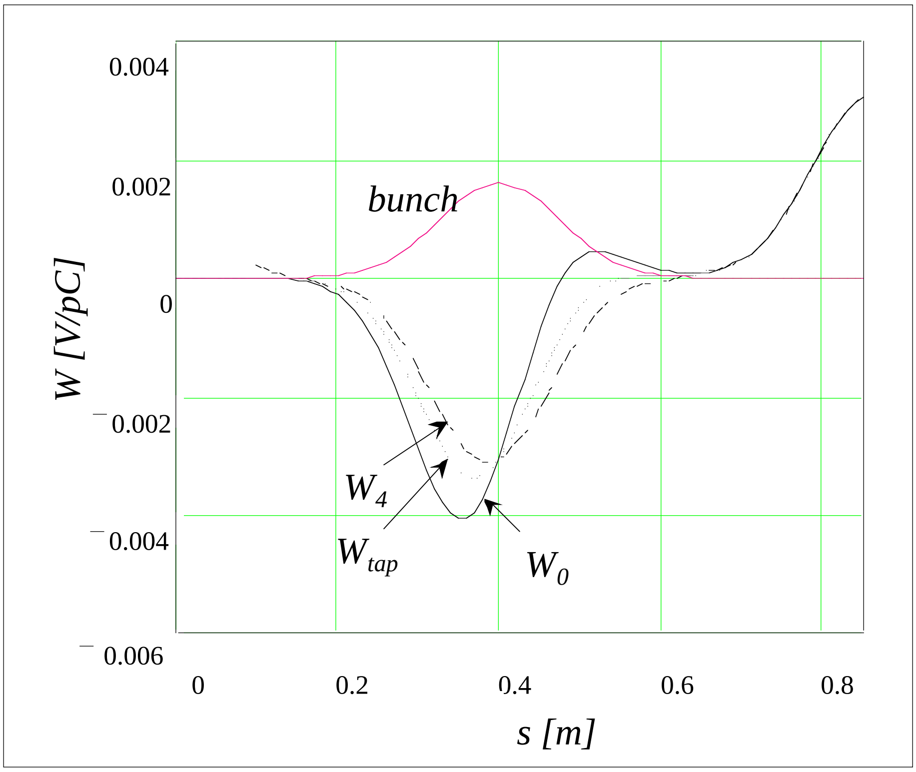

Figure 13 : Wake potential of 75 *mm* bunch in *empty* (W_0), *filled* (W_4) and *tapered* (W_{tap}) BPM monitor cavity with electrodes.

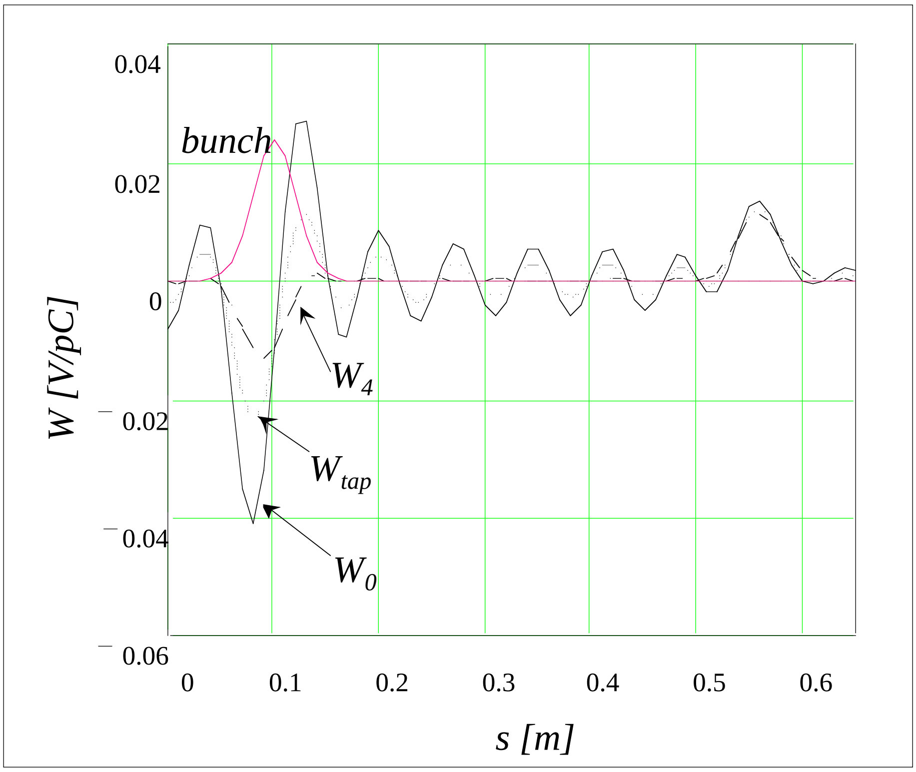

Figure 14 : Wake potential of 20 *mm* bunch in *empty* (W_0), *filled* (W_4) and *tapered* (W_{tap}) BPM monitor cavity with electrodes.

6. Conclusion

Two different methods of wake field calculation concerning a beam position monitor vacuum chamber, with a non-negligible degree of structural complication, lead to the same results. The successful mutual cross-check gives confidence that the impedance can correctly be computed for that kind of structure. It will be straightforward to repeat this exercise for newer monitor designs if the need arises. Inter-electrode shields are a very efficient way to reduce the monitor cavity impedance (a factor 6 is obtained if the shields occupy half the monitor circumference). An angle of 10^0 is required to obtain the same result with tapered ends. However, the fact should not be overlooked that the *electrode impedance* is dominant in the structures that were studied.

References

[1] J-P. Papis, L. Vos, *Design Basis for the LHC Beam Position Monitor,* CERN SL/91-6 (BI), 1991.

[2] G. Lambertson in Particle Accelerators, Volume 50, N 1-3 (1995)

[3] *MAFIA User's Guide Version 3.20*, CST GmbH, Darmstadt, Germany.

[4] L. Vos, *Calculations and Simulations concerning the Longitudinal Impedance of an Accelerator Vacuum Chamber,* SPS/DI-MST/Note/84-2, 1984.