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REVIEW OF V_{cb} AND V_{td}/V_{ts} §

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The current experimental status of the CKM matrix element V_{cb} and the ratio V_{td}/V_{ts} is reviewed. Knowledge of these elements has a strong impact on the unitarity triangle, of interest for studies of CP violation in the B system. The measurements of V_{cb} from both inclusive semileptonic b decays and the exclusive channel $B \to D^{(*)}\ell\nu$ are reaching high precision. For V_{td}/V_{ts} the strongest upper limit is derived from studies of B^0–$\overline{B}^0$ mixing.

1 Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix describes the mixing of quarks between their flavour- and mass-eigenstates: [1]

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1}$$

It can be parametrized[2] as an expansion in powers of the sine of the Cabibbo angle, $\lambda = \sin\theta_{\rm C} = 0.2205 \pm 0.0018$: [3]

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\ , \tag{2}$$

where three other parameters are introduced: A, ρ and the CP-violating parameter η. With this parametrization one sees that $|V_{ts}| \approx |V_{cb}| \sim 0.05A$, and $|V_{td}/V_{ts}| \sim \mathcal{O}(\lambda)$. Unitarity of the CKM matrix leads to relationships between the elements, including: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. This corresponds to a triangle in the (ρ,η) plane, the famous "unitarity triangle".

V_{cb} appears in the denominator of two sides of the triangle. It sets the scale of b decays, being the largest coupling of the b to lighter quarks; the other directly accessible decay $b \to u$ has a much weaker coupling: $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$.[4] V_{cb} is determined from the semileptonic b decays, which occur via the tree-level diagram shown in Fig. 1 (a). The inclusive and exclusive approaches to this measurement are described in the next section.

V_{td} measures the length of one side of the unitarity triangle: $|V_{td}/\lambda V_{cb}| = \sqrt{(1-\rho)^2+\eta^2}$. It is not directly accessible due to the small number of reconstructed top-quark events that are currently available, so instead it is determined indirectly via loop diagrams. Examples of such diagrams are given

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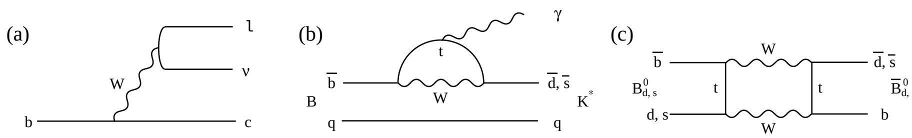

Figure 1: (a) Tree diagram for $b \to c\ell\nu$, (b) Penguin diagram for $b \to d/s\gamma$, (c) Box diagram for B^0–$\overline{B}^0$ oscillation.

in Fig. 1 (b) and (c): they include the radiative penguin diagrams, responsible for the $b \to s\gamma$ and $b \to d\gamma$ transitions; the penguin and box-type diagrams that contribute to the decay $K^+ \to \pi^+\nu\overline{\nu}$; and the box diagrams that are responsible for B^0–$\overline{B}^0$ oscillation. These processes are discussed in Section 3. The uncertainty in the calculation of such loop diagrams is dominated by the hadronization uncertainty, which largely cancels in the comparison of final states involving the s and d quarks; this is the motivation for determining the ratio V_{td}/V_{ts}.

2 Measuring V_{cb}

2.1 Inclusive measurement of $b \to c\ell\nu$

Treating the b quark as a free particle, its semileptonic partial width is given by:

$$\Gamma(b \to c\ell\nu) = \frac{G_{\mathrm{F}}^2\, m_b^5}{192\,\pi^3}\,\Phi\,|V_{cb}|^2 \equiv \alpha\,|V_{cb}|^2 = \frac{\mathcal{B}(b \to c\ell\nu)}{\tau_b} \qquad (3)$$

where Φ is a phase space factor. The theoretical uncertainty on extracting V_{cb} from this expression is dominated by the knowledge of the correction due to the binding of the b quark into a hadron, and the b-quark mass dependence, reduced by using constraints on $(m_b - m_c)$. Two recent predictions for the factor relating $\Gamma(b \to c\ell\nu)$ and $|V_{cb}|^2$ are $\alpha = 41\,\mathrm{ps}^{-1}$, and $43\,\mathrm{ps}^{-1}$.[5] Both quote a 10% theoretical uncertainty, but other commentators[6] have preferred 20%; I split the difference and use $(42 \pm 6)\,\mathrm{ps}^{-1}$.

The required experimental inputs are the semileptonic b branching ratio, and the inclusive b-hadron lifetime, τ_b. These topics are covered in more detail elsewhere.[7] For the semileptonic branching ratio, recent improved measurements use the charge and kinematic correlations in dilepton events to separate the $b \to \ell$ and $b \to c \to \ell$ components, giving reduced model dependence. A recent average of such measurements at the $\Upsilon(4S)$ gave $\mathcal{B}(B \to X\ell\nu) = (10.22 \pm 0.37)\%$.[8] The latest LEP average gives $\mathcal{B}(b \to X\ell\nu) = (11.15 \pm 0.20)\%$.[9]

Due to the presence of B_s^0 and Λ_b^0 hadrons in Z decays, these two results need not agree; however, assuming that the semileptonic branching ratios of the b hadrons scale with their lifetimes, and given the current knowledge of the b-hadron production fractions (discussed below), one expects $\mathcal{B}(b \to X\ell\nu) \approx 0.96\,\mathcal{B}(B \to X\ell\nu)$, i.e. a difference in the opposite sense to that observed. This is a long-standing discrepancy, of about two standard deviations. For the moment I will not combine the branching ratio results from LEP and the $\Upsilon(4S)$ experiments, but calculate values of V_{cb} separately for them.

The measured branching ratios need to be corrected for the $b \to u$ contribution. From a similar analysis to those of $b \to c$ decays:[10] $\Gamma(b \to u\ell\nu) = (60\,\mathrm{ps}^{-1})\,|V_{ub}|^2$, and hence:

$$\frac{\mathcal{B}(b \to u\ell\nu)}{\mathcal{B}(b \to c\ell\nu)} = 1.4 \left|\frac{V_{ub}}{V_{cb}}\right|^2 = (1.0 \pm 0.5)\% \ . \tag{4}$$

For the inclusive b lifetime the traditional technique is to study the impact parameter of high transverse-momentum leptons. Recent high-precision measurements have used topological vertex finding. The lepton based results should measure the average of the b-hadron lifetimes weighted by their production fractions and semileptonic branching ratios, which differs from the unbiassed average expected for topological vertex finding. Again, under the assumption that the semileptonic branching ratios scale with the lifetimes, one expects $\tau_b(\mathrm{leptons}) \approx 1.01\,\tau_b(\mathrm{topological})$. Neglecting this small effect in averaging the most recent results[11] I find $\tau_b = (1.566 \pm 0.017)\,\mathrm{ps}$. For the $\Upsilon(4S)$ I use $\tau_B = (\tau_{B_d} + \tau_{B^+})/2 = (1.60 \pm 0.03)\,\mathrm{ps}$. The resulting values of V_{cb} are $(41.0 \pm 0.4 \pm 2.9) \times 10^{-3}$ and $(38.8 \pm 0.8 \pm 2.8) \times 10^{-3}$ respectively, where the first error is experimental and the second theoretical.

2.2 Exclusive measurement of $B \to D^{(*)}\ell\nu$

The differential decay rate of $B \to D^{(*)}\ell\nu$, with respect to the boost $w = (m_B^2 + m_D^2 - q^2)/(2\,m_B\,m_D)$ of the D in the B rest frame, is given by:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}w} = \frac{G_\mathrm{F}^2}{48\,\pi^3}\kappa(m_B, m_D, w)\,|V_{cb}|^2 F^2(w) \ , \tag{5}$$

where κ is a known function and $F(w)$ is the hadronic form factor, which parametrizes the effects of the strong interaction on the decay. Following the approach of Heavy Quark Effective Theory (HQET), in the limit of large heavy-quark mass $m_Q \to \infty$ the light degrees of freedom in a meson are blind to the flavour and spin of the heavy quark. At the zero-recoil point of the meson decay, the heavy quark then changes flavour without perturbing its colour field, and thus the form factor is unity (i.e. the decay is not suppressed). This zero-recoil point corresponds to maximal four-momentum transfer to the lepton system, q^2, and thus $w = 1$ and $F(1) = 1$.

The decay $B \to D^* \ell \nu$ is favoured for the measurement, as the $1/m_Q$ correction is predicted to vanish,[12] and experimentally it has a large branching ratio and clean signal. At the $\Upsilon(4S)$ the B mesons are essentially at rest, which allows the boost to be measured accurately using $w = E_{D^*}/m_{D^*}$, with a resolution of $\sim 4\%$. At LEP the estimation of the q^2 is more difficult, and relies on the knowledge of the B flight direction determined by vertexing, with a resolution of typically $\sim 15\%$ on w; however, the large boost gives a higher efficiency for reconstructing the slow pion from the $D^{*+} \to D^0 \pi^+$ decay.

The decay rate vanishes as $w \to 1$, so one needs to extrapolate from the region of larger w. For the extrapolation one can expand $F(w)$:

$$F(w) = F(1) \left[1 - \rho^2 (w - 1) + c(w - 1)^2 + ...\right] \ . \tag{6}$$

Experiments traditionally use a linear fit, although recent work has been done to relate the curvature c to the slope ρ^2 (expressed as a square as it must be positive),[13] with the result $c \approx 0.66\,\rho^2 - 0.11$. The intercept and slope are strongly ($\sim 90\%$) correlated, as can be seen in Fig. 2, and this correlation needs to be accounted for when averaging results of different experiments. The results are also updated to use common assumptions on branching ratios and lifetimes, following the work of Gibbons.[8,14]

The result of the average is $F(1)\,|V_{cb}| = (34.2 \pm 1.6) \times 10^{-3}$, $\rho^2 = 0.71 \pm 0.08$. However, the quality of the fit is not perfect, due largely to the variation in slopes measured by the different experiments, with $\chi^2/\mathrm{dof} = 12.9/8$. Following the PDG prescription,[3] the error on the intercept is scaled by the square-root of this ratio, a factor 1.27. The combination is made using the results of linear fits to the w distribution of each experiment, but these have a slight bias as the curvature term is neglected. Assuming the true curvature is as given above, and for uniform population of the w distribution between 1.0 and 1.5, this bias would decrease the intercept by $\sim 2\%$, so the measured value is scaled by 1.02 ± 0.02. Finally the expected value of $F(1)$ is not exactly unity, but rather $\eta_A\,(1 + \delta)$, where $\eta_A = 0.960 \pm 0.007$ is a perturbative QCD correction, with the value given by a recent 2–loop calculation,[15] and $\delta = -0.055 \pm 0.025$ is a correction for the finite heavy-quark mass.[6,5,16] Hence $F(1) = 0.91 \pm 0.03$ is predicted, and thus $|V_{cb}| = (38.3 \pm 2.4 \pm 1.3) \times 10^{-3}$.

The decay $B \to D^+ \ell \nu$ can also be used to measure V_{cb} in a similar manner, and one can also test HQET by the comparison of the form factor $F(w)$ with that seen using $B \to D^* \ell \nu$ decays (as they are predicted to have the same shape in the heavy quark limit). Here there are fewer experimental results, as it is

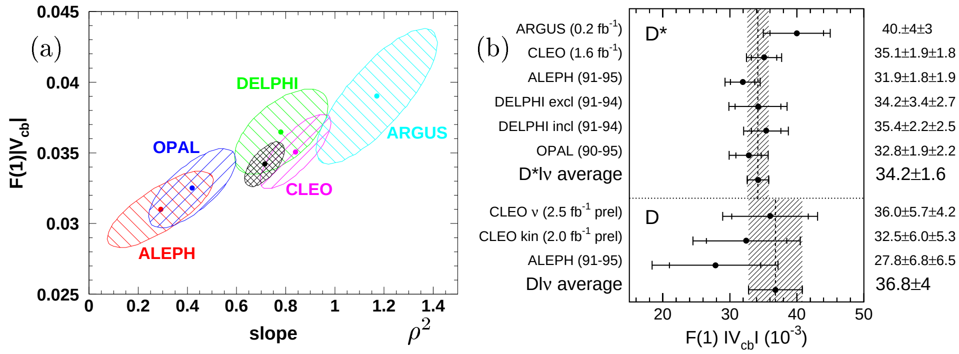

Figure 2: (a) One standard deviation error ellipses for the intercept and slope from the $D^*\ell\nu$ analyses; the doubly-hatched ellipse is the result of the fit. (b) Results for the intercept $F(1)|V_{cb}|$ from $D^*\ell\nu$ and $D^+\ell\nu$ analyses.

a more challenging mode, and the theoretical prediction $F(1) = 0.98 \pm 0.07$[17] has been less intensively scrutinized. The result is $|V_{cb}| = (38.5 \pm 4.5 \pm 2.8) \times 10^{-3}$. This is in good agreement with the result from the $D^*\ell\nu$ analysis, and also from the inclusive measurements, despite the different theoretical inputs. I average them all assuming that theoretical errors are fully correlated between the inclusive results, fully correlated between the exclusive results, and uncorrelated between inclusive and exclusive, to give $|V_{cb}| = (39.2 \pm 1.9) \times 10^{-3}$.

3 Measuring V_{td}/V_{ts}

3.1 Radiative penguins

Radiative penguin decays are extensively discussed elsewhere.[18] To summarize, CLEO see a strong signal for the decay $B \to K^*\gamma$ with a measured branching ratio of $(4.2 \pm 0.8 \pm 0.6) \times 10^{-5}$. However they see no evidence of the corresponding $b \to d\gamma$ modes, $B \to \rho^-/\rho^0/\omega\gamma$, and set a limit on the ratio of branching ratios:

$$\frac{\mathcal{B}(B \to \rho/\omega\gamma)}{\mathcal{B}(B \to K^*\gamma)} < 0.19 \quad (90\%\ \mathrm{CL})\ . \tag{7}$$

This can be converted into the limit $|V_{td}/V_{ts}| < 0.45$–0.56, where the range comes from different predictions for the SU(3)-breaking correction.[19] There remains a question-mark as to whether long-distance corrections are significant.[20]

3.2 $K^+ \to \pi^+ \nu\overline{\nu}$

The rare kaon decay $K^+ \to \pi^+ \nu\overline{\nu}$ gives a theoretically clean avenue to the extraction of $|V_{ts}^* V_{td}|$, as long-distance corrections are believed to be negligible.[21] However, experimentally it is challenging, as the branching ratio expected in the Standard Model is only $(0.6\text{–}1.2)\times 10^{-10}$.[22]

The measurement is being attempted by a dedicated experiment, E787 at Brookhaven,[23] in which a K^+ beam is stopped in an active target, and γ vetos are used to suppress the copious $K^+ \to \pi^+\pi^0$ decays. The range of the π^+ candidate and its kinetic energy are used to suppress the residual background from $K^+ \to \pi^+\pi^0$ and $K^+ \to \mu^+\nu$ decays, and no signal candidates are found from the data taken in 1989–91. A limit is set on the branching ratio: $\mathcal{B}(K^+ \to \pi^+\nu\overline{\nu}) < 2.4 \times 10^{-9}$ (90% CL), which does not (yet) reach the Standard Model expectation, so no significant constraint of the CKM matrix elements can be extracted. The analysis of an increased dataset is eagerly awaited.

3.3 B^0–$\overline{B}^0$ *oscillation*

The mass eigenstates of the B^0 system are a mixture of the flavour eigenstates B^0 $(\overline{b}q)$, $\overline{B}^0$ $(b\overline{q})$, and are given by $B_{1,2} = (B^0 \pm \overline{B}^0)/\sqrt{2}$ (neglecting CP violation). This mixture leads to an oscillation of the B^0 between particle and antiparticle state, with the probability density that an initially pure B^0 state decays as a $\overline{B}^0$ at proper time t given by:

$$\mathcal{P}(B^0 \to \overline{B}^0) = \frac{\Gamma}{4}\left(1 - \frac{\Delta\Gamma^2}{4\,\Gamma^2}\right) e^{-\Gamma t}\left(e^{\Delta\Gamma t/2} + e^{-\Delta\Gamma t/2} - 2\,\cos\Delta m\,t\right) \, . \quad (8)$$

Here the oscillation frequency $\Delta m = m(B_2) - m(B_1)$ is the mass difference of the two states, and $\Delta\Gamma = \Gamma(B_1) - \Gamma(B_2)$ is their width difference. A similar expression (with an oppositely-signed oscillatory term) gives the probability density that the initially pure B^0 state decays as a B^0, and the normalisation is such that $\int_0^\infty [\mathcal{P}(B^0 \to \overline{B}^0) + \mathcal{P}(B^0 \to B^0)]\, dt = 1$.

Such a formalism applies both to the B_d^0 and the B_s^0 mesons. The width difference is expected to be very small for the B_d^0, $(\Delta\Gamma/\Gamma)_d < 0.01$, whilst for the B_s^0 it may not be negligible, with a recent prediction of $(\Delta\Gamma/\Gamma)_s = 0.16^{+0.11}_{-0.09}$.[24] The measurement of this width difference would be interesting, as it can be related to Δm (albeit with significant hadronic uncertainty, at present): $\Delta m = (179 \pm 83)\ \Delta\Gamma$;[24] an upper limit on $\Delta\Gamma_s$ would then give an *upper* limit on Δm_s, whereas oscillation analyses (described below) have only so far provided a lower limit. One possible approach is to compare the lifetimes measured for the B_s^0 using the $D_s^+\ell\nu$ and $J/\psi\phi$ decay modes: the former should be an equal mixture of the two CP-eigenstates, whilst the latter is expected to be dominantly CP-even. However, with the current values from CDF[25] only a weak limit can be extracted, $(\Delta\Gamma/\Gamma)_s < 0.6$ or so.

Neglecting $\Delta\Gamma$, Eq. 8 reduces to $\mathcal{P}(B^0 \to \overline{B}^0) = e^{-t/\tau}(1 - \cos\Delta m\, t)/(2\,\tau)$. The measurement of such oscillations requires the tagging of the particle/anti-particle state of the B^0 at its production and decay. Many tagging techniques are now in use, including:

1. Charge of a lepton from the semileptonic decay $B^0 \to X\ell\nu$; this can also be used as a production state tag by relying on the $b\bar{b}$ production, and looking for the decay of the other b hadron in the opposite hemisphere. Lepton tagging has a high purity (low mistag rate $\sim 20\%$) but also a low efficiency ($\sim 10\%$) due to the semileptonic branching ratio.

2. Jet charge: the sum of the charges of tracks in a jet, weighted typically with some power of their momentum; this has a poorer mistag rate than a lepton tag ($\sim 35\%$) but a higher efficiency ($\sim 80\%$).

3. Charge of a same-side π or K, from fragmentation or B^{**} decay.

4. Charge of a reconstructed charmed hadron, from the B^0 decay.

The combination of available tags at LEP can achieve a mistag rate of about 27% for full efficiency.[26] One then measures the fraction of events with decay state different to their production state, as a function of the proper time. The significance of an oscillation signal is given by:

$$\mathcal{S} \approx \sqrt{\frac{N}{2}} P\,(1 - 2\eta)\, e^{-\Delta m^2 \sigma_t^2/2}\ , \tag{9}$$

where N is the number of candidates in the sample, P is the signal purity, η is the mistag rate, and the last term describes the damping due to finite resolution. For the B_s^0 in particular, where Δm_s is expected to be large, good proper-time resolution is clearly essential.

The reconstruction of the proper time of B^0 decays relies on the use of the high-precision silicon microvertex detectors, at LEP, CDF and SLD. There are two main classes of analyses: inclusive and semi-exclusive (fully exclusive analyses have insufficient statistics at present). The inclusive analyses use topological vertexing to measure the decay length of the B^0, often using a lepton track to seed the decay vertex search. The production vertex is found using knowledge of the beam spot position, which has transverse dimensions of $(150 \times 10)\,\mu\mathrm{m}^2$ at LEP, $(35 \times 35)\,\mu\mathrm{m}^2$ at CDF, and $(2 \times 1)\,\mu\mathrm{m}^2$ at SLD. The average b decay length L is about 3 mm at LEP and SLD, compared to 1–2 mm at CDF, and the typical resolution achieved $\sigma_L \approx 300\,\mu$m (but with tails). The proper time is then calculated as $t = L m_B / p_B$, where the B^0 momentum p_B is estimated using the sum of contributions from charged, neutral and missing energy. The proper time resolution is given by:

$$\sigma_t \sim \frac{\sigma_L}{\langle L \rangle} \tau \oplus \frac{\sigma_p}{p_B} t \ , \tag{10}$$

where the first term is typically ~ 0.2 ps, and the second term is $\sim 15\%$ times the proper time itself. The sample composition of such inclusive analyses is close to the unbiassed b-hadron production fractions, for which the latest values at LEP are: $f(B_d^0) = f(B^+) = (39.4^{+1.6}_{-2.0})\%$, $f(B_s^0) = (10.5^{+1.6}_{-1.5})\%$, $f(b\,\mathrm{baryon}) = (10.6^{+3.7}_{-2.7})\%$.[27]

Semi-exclusive analyses, on the other hand, are seeded with a fully-reconstructed charm decay: either a D^{*+} to enrich the B_d^0 content, or a D_s^+ to enrich the B_s^0 content. In this way $\sim 50\%$ purity can be achieved, but at the cost of low statistics, of order 100 events. However, the proper-time resolution and mistag rates are also improved, so such analyses tend to be competitive with the inclusive approach.

The many measurements of Δm_d are shown in Fig. 3. The LEP,[27] SLD[8] and CDF[28] averages are made accounting for correlated errors amongst the results. The time-integrated mixing measurements from the $\Upsilon(4S)$ allow the extraction of the dimensionless mixing parameter $x_d = \Delta m_d / \Gamma_d$, which can be converted into a value for Δm_d using the B_d^0 lifetime, $\tau_{B_d} = (1.55 \pm 0.04)$ ps.[11] Finally I calculate an overall world average $\Delta m_d = (0.460 \pm 0.018)\,\mathrm{ps}^{-1}$, assuming that the systematic errors of the individual averages are 50% correlated.

From the box diagram calculation, Δm_d can be related to the CKM matrix element V_{td}:

$$\Delta m_d = \frac{G_\mathrm{F}^2}{6\,\pi^2} m_t^2 \, S\!\left(\frac{m_t^2}{m_W^2}\right) \eta_B \, m_{B_d} \, f_{B_d}^2 B_{B_d} \, |V_{tb}^* V_{td}|^2 \tag{11}$$

where, following Buras,[22] the running top mass $m_t = (167 \pm 6)\,\mathrm{GeV}/c^2$ is used; $S(x)$ is a known function, given to good approximation by $0.784\,x^{-0.24}$; $\eta_B = 0.55 \pm 0.01$ is a QCD correction; f_{B_d} and B_{B_d} are the B_d^0 decay constant and non-perturbative "bag" factor, with $f_{B_d}\sqrt{B_{B_d}} = (200 \pm 40)$ MeV; and $|V_{tb}| \approx 1$. This gives: $|V_{td}| = (8.6 \pm 0.2 \pm 0.2 \pm 1.7) \times 10^{-3}$, where the errors are respectively due to Δm_d, m_t and $f\sqrt{B}$; clearly the overall error is dominated by the hadronic uncertainty.

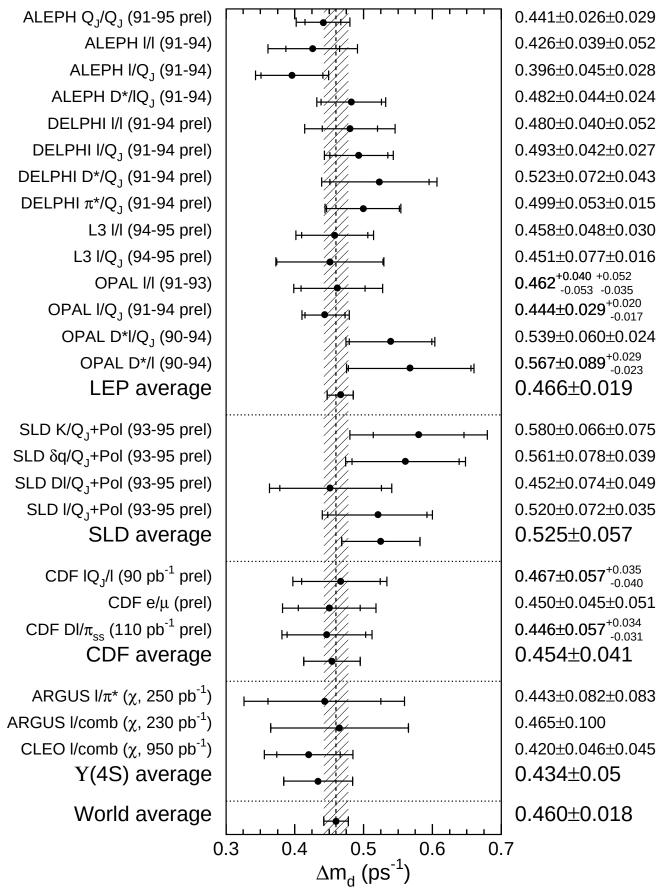

Figure 3: Results for the B_d^0 oscillation frequency Δm_d.

If Δm_s could be measured, then much of this uncertainty cancels in the ratio:

$$\frac{\Delta m_s}{\Delta m_d} = \xi^2 \frac{m_{B_s}}{m_{B_d}} \left| \frac{V_{ts}}{V_{td}} \right|^2 , \tag{12}$$

where $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}} = 1.15 \pm 0.05$ is the SU(3)-breaking term, estimated from lattice and QCD sum rules.[29] To determine the expected value of Δm_s in the Standard Model, one can fit for the position of the apex of the unitarity triangle, using experimental values for Δm_d, ϵ_K (the CP-violation

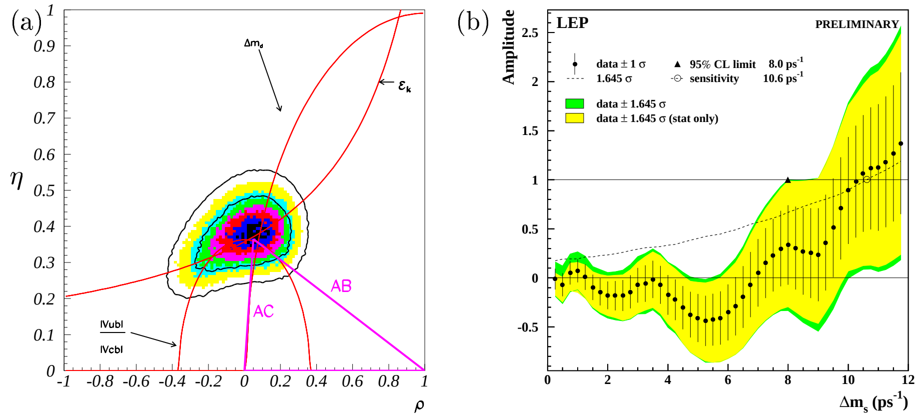

Figure 4: (a) The unitarity triangle, with apex (ρ, η). The result of a fit to the position of the apex using available experimental constraints (and assuming Gaussian theoretical errors) is shown superimposed, with contours of 68% and 95% CL. (b) The amplitude plot for the latest combination of LEP B_s^0 oscillation analyses.

parameter from the kaon system) and $|V_{ub}/V_{cb}|$ (assuming $B_K = 0.75 \pm 0.15$). The result of the fit is shown in Fig. 4 (a).[30] Note that Gaussian distributions have been assumed for the theoretical errors: this was the source of some controversy at the conference. Conflicting opinions were expressed: that "top hat" distributions should be used for the theoretical errors (this would be less conservative); that one should make separate plots for each possible combination of theoretical parameters within their allowed ranges (this would violate my page allocation); or that a confidence level should not be assigned to the likelihood contours (they should be labelled "conservative", and "even more conservative"!). Taking the fit with Gaussian errors at face value, the length of the right-hand side of the triangle can be used to predict Δm_s, from Eqs. 2 and 12. The resulting probability distribution is peaked at around $10\,\mathrm{ps}^{-1}$, with a 95% CL region of 6–18 ps^{-1}.

3.4 B_s^0 oscillation limits

No significant B_s^0 oscillation signal has yet been seen, so experiments use their data to set lower limits on the oscillation frequency Δm_s. The standard technique used has been to study the likelihood of the fit as a function of Δm_s. If the likelihood is "well-behaved" then a difference in negative log-likelihood of 1.92 units relative to the minimum would correspond to a 95% confidence level. However, typically there are multiple minima in the negative log-likelihood distribution, and it is necessary to calibrate the correspondence between the likelihood and confidence level, using samples of Monte Carlo events to simulate many repetitions of the measurement at each true value of Δm_s. This becomes heavy in computing resources, and makes the combination of results from different experiments impractical.

A new method for analysing oscillations was therefore developed,[31] inspired by Fourier transformation: instead of fitting for a proper-time oscillation, one looks for a peak in the frequency spectrum. The usual term $(1 - \cos \Delta m_s t)$ in the fitting function (for the fraction of events which are tagged as mixed) is replaced with $(1 - \mathcal{A} \cos \Delta m_s t)$, and the "amplitude" $\mathcal{A}$ is fitted for, at fixed frequency Δm_s. This is then repeated for different values of Δm_s. If the true value of Δm_s is assumed, then the amplitude should have a value of unity, whilst if one is far from the true value the amplitude should be zero; near to the true value one expects a Breit-Wigner dependence of amplitude on Δm_s (assuming constant resolution) with width $\sim 1/\tau_{B_s}$. The beauty of this approach is that the amplitude is measured with Gaussian errors at each value of Δm_s, so it is straight forward to combine the results of different experiments, by combining the amplitude measurements at each Δm_s. The error on $\mathcal{A}$ increases with increasing Δm_s, so there comes a point at which a value of $\mathcal{A} = 1$ can no longer be excluded. The lower limit at 95% confidence level on Δm_s is then taken as the value of Δm_s at which $\mathcal{A} + 1.645\, \sigma_{\mathcal{A}} = 1$. Note that the error on the amplitude is directly related to the significance of an oscillation signal, given in Eq. 9: $\sigma_{\mathcal{A}} = 1/\mathcal{S}$.

The latest amplitude plot from the combination of LEP results is shown in Fig. 4 (b).[27] This gives $\Delta m_s > 8.0\,\mathrm{ps}^{-1}$ (95% CL), corresponding to a limit on the dimensionless mixing parameter $x_s > 12$. This limit is in fact slightly lower than that presented at last summer's ICHEP conference in Warsaw,[8] despite the fact that more individual analyses are included in the combination. However, the sensitivity of the combined result has increased, to $10.6\,\mathrm{ps}^{-1}$; the sensitivity is defined as the value of Δm_s for which $1.645\, \sigma_{\mathcal{A}} = 1$, and is the point up to which, on average, one would expect to be able to exclude values of Δm_s. It is intriguing to note that the fact that the limit has not increased, despite the increased sensitivity, might be due to the first hint of a signal, since at around $11\,\mathrm{ps}^{-1}$ the value $\mathcal{A} = 0$ begins to be disfavoured, as would be expected if a signal was being seen; however, for now the significance is insufficient to claim observation of B_s^0 oscillation.

Using the limit on Δm_s and the measurement of Δm_d (fluctuated upwards by one standard deviation of its error, to be conservative) one can set a limit on the ratio: $\Delta m_s / \Delta m_d > 17$, and thus, using Eq. 12, $|V_{td}/V_{ts}| < 0.29$.

4 Conclusions

The CKM matrix elements V_{cb}, V_{td} and V_{ts} are fundamental constants of the Standard Model, and their measurement is actively pursued at LEP, CLEO, CDF and SLD. V_{cb} is determined from semileptonic b decay rates: either inclusive $b \to X\ell\nu$ or exclusive $B \to D^{(*)}\ell\nu$. The combined result is:

$$|V_{cb}| = (39.2 \pm 1.9) \times 10^{-3} \; . \tag{13}$$

One can then extract (from Eq. 2) the parameter $A = |V_{cb}|/\lambda^2 = 0.81 \pm 0.04$.

V_{td}/V_{ts} is probed by rare penguin decays, but the best limit to date is achieved from the study of B^0–$\overline{B}^0$ oscillation. The oscillation frequency of the B^0_d is now precisely measured, $\Delta m_d = (0.460 \pm 0.018)\,\mathrm{ps}^{-1}$, whilst for the B^0_s the combined limit from LEP is $\Delta m_s > 8.0\,\mathrm{ps}^{-1}$ at 95% CL. This gives a stronger constraint than the assumption of unitarity of the CKM matrix, and corresponds to:

$$\left|\frac{V_{td}}{V_{ts}}\right| < 0.29 \; . \tag{14}$$

The eventual observation of B^0_s–$\overline{B}^0_s$ oscillations (if not before, then certainly by LHC-B) will have a significant impact on the unitarity triangle analysis.

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