A Note on the D-2-Brane of the Massive Type IIA Theory and Gauged Sigma Models

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Abstract

Gauging the M-2-brane effective action with respect to an Abelian isometry in such a way that the invariance under gauge transformations of the 3-form potential is maintained (slightly modified) we obtain a fully covariant action with 11-dimensional target space that gives the massive D-2-brane effective action upon dimensional reduction in the direction of the gauged isometry.

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Introduction

Recently a dual effective action for the D-2-brane of the massive type IIA theory has been proposed in Ref. [1]. The worldvolume theory has an extra scalar instead of the Born-Infeld worldvolume field. In the massless limit, the scalar corresponds to the eleventh coordinate of the M-2-brane upon dimensional reduction. The equivalence between the usual action with Born-Infeld vector field and the dual action was established via an intermediate action that we present here with an auxiliary worldvolume metric for convenience:

$$S_{\mathrm{I}}\left[X^{\mu}, X, V_i, C_i\right] =$$

$$-\frac{T_{M-2}}{2} \int d^3\xi \ \sqrt{|\gamma|} \left\{ \gamma^{ij} \left[e^{-\frac{2}{3}\phi} g_{ij} - e^{\frac{4}{3}\phi} F_i F_j \right] - 1 \right\} \tag{1}$$

$$+\frac{T_{M-2}}{3!} \int d^3\xi \ \epsilon^{ijk} \left\{ C_{ijk} + 6\pi\alpha' D_i X \mathcal{F}_{jk} + 6m(\pi\alpha')^2 V_i \partial_j V_k \right\} .$$

Here

$$\begin{cases} F_i &= D_i X + A^{(1)}{}_i , \\[2mm] D_i X &= \partial_i X + C_i , \\[2mm] \mathcal{F}_{ij} &= 2\partial_{[i} V_{j]} - \frac{1}{2\pi\alpha'} B_{ij} . \end{cases} \tag{2}$$

This action has two extra worldvolume fields. If we eliminate C_i by using its equation of motion

$$F_i = -2\pi\alpha' e^{-\frac{4}{3}\phi} \star \mathcal{F}_i , \tag{3}$$

in the intermediate action, one gets

$$S\left[X^{\mu}, V_i\right] =$$

$$-\frac{T_{M-2}}{2} \int d^3\xi \ \sqrt{|\gamma|} \left\{ \gamma^{ij} e^{-\frac{2}{3}\phi} g_{ij} + 2(\pi\alpha')^2 e^{\frac{4}{3}\phi} \gamma^{ij} \gamma^{kl} \mathcal{F}_{ik} \mathcal{F}_{jl} - 1 \right\} \tag{4}$$

$$+\frac{T_{M-2}}{3!} \int d^3\xi \ \epsilon^{ijk} \left\{ C_{ijk} - 6\pi\alpha' A^{(1)}{}_i \mathcal{F}_{jk} + 6m(\pi\alpha')^2 V_i \partial_j V_k \right\} .$$

This is the usual action for the D-2-brane of the massive type IIA theory and here we clearly see that V_i is the characteristic Born-Infeld vector field and $\mathcal{F}$ is its field strength. The scalar X present in the intermediate action

has completely disappeared in favour of its dual V_i at the same time we eliminated C_i.

If instead we eliminate V_i using its equation of motion

$$\partial_{[i}V_{j]} = -\tfrac{1}{m\pi\alpha'}\partial_{[i}C_{j]}\,,\tag{5}$$

we get, up to a total derivative

$$\tilde{S}\left[X^\mu, X, C_i\right] =$$

$$-\tfrac{T_{M-2}}{2}\int d^3\xi\ \sqrt{|\gamma|}\left\{\gamma^{ij}\left[e^{-\frac{2}{3}\phi}g_{ij} - e^{\frac{4}{3}\phi}F_iF_j\right] - 1\right\}\tag{6}$$

$$+\tfrac{T_{M-2}}{3!}\int d^3\xi\ \epsilon^{ijk}\left\{C_{ijk} - 3D_iX B^{(1)}{}_{ij} - \tfrac{6}{m}C_i\partial_jC_k\right\}\,.$$

In this action, which, as we have shown, is, classically, completely equivalent to the usual action Eq. (4) V_i has completely disappeared in favour of its dual, the scalar field X, but an extra, auxiliary, worldvolume field, C_i remains. The main interest of this action is that it is very close to the action that one gets by direct dimensional reduction of the M-2-brane Ref. [2] and in fact it reduces to it when the massless limit is appropriately taken as explained in Ref. [1]. This could be helpful in finding the 11-dimensional origin of the massive type IIA theory. Here we will present an action with 11-dimensional target space that gives Eq. (6) upon direct dimensional reduction.

The fact that C_i is auxiliary can be read out from the symmetries that the above action enjoys. Apart from worldvolume and target-space reparametrizations we have gauge transformations of the RR potentials

$$\left\{\begin{array}{rcl} \delta_{\Lambda^{(1)}}X &=& -\Lambda^{(1)}(X^\nu)\,, \\[2mm] \delta_{\Lambda^{(1)}}A^{(1)}{}_\mu &=& \partial_\mu\Lambda^{(1)}\,, \\[2mm] \delta_\chi C_{\mu\nu\rho} &=& 3\partial_{[\mu}\chi_{\nu\rho]}\,, \end{array}\right.\tag{7}$$

local shifts of the scalar X^3

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[3]This is the symmetry gauged by C_i. Its presence is necessary in order to have the right number of bosonic degrees of freedom, namely 8. The scalar X can be completely eliminated from the action by using the equation of motion of the auxiliary vector field C_i: $F_i = \tfrac{4}{m}e^{-\frac{4}{3}\phi}\,{}^\star G_i$ where $G_{ij} = 2\partial_{[i}C_{j]} + \tfrac{m}{2}B^{(1)}{}_{ij}$ is the gauge-invariant field strength of C_i. When X is eliminated C_i becomes a dynamical field.

$$\begin{cases} \delta_\eta X &= \eta(\xi) \\[2mm] \delta_\eta C_i &= -\partial_i \eta \,, \end{cases} \tag{8}$$

and the "massive gauge transformations"

$$\begin{cases} \delta_\lambda A^{(1)}{}_\mu &= 2m\lambda_\mu \,, \\[2mm] \delta_\lambda C_i &= -m\lambda_i \,, \end{cases} \qquad \begin{cases} \delta_\lambda B^{(1)}{}_{\mu\nu} &= 4\partial_{[\mu}\lambda_{\nu]} \,, \\[2mm] \delta_\lambda C_{\mu\nu\rho} &= -3m\lambda_{[\mu}B^{(1)}{}_{\nu\rho]} \,. \end{cases} \tag{9}$$

Observe that the action is not invariant under gauge transformations of the NSNS 2-form $B^{(1)}_{\mu\nu}$. This is, however, expected because this field appears in the supergravity Lagrangian with a mass term that breaks the corresponding gauge invariance [3].

In Ref. [1] it was suggested that this auxiliary vector field could be interpreted in a similar fashion as the auxiliary vector field introduced in Ref. [4] to write a covariant action for the 11-dimensional Kaluza-Klein monopole. In the next section we will propose a different interpretation for the auxiliary C_i vector field which is, though, much in the same spirit.

1 Gauging the M-2-Brane Effective Action

The (bosonic part of) the M-2-brane is [5]

$$\begin{aligned} \hat{S}\,[\hat{X}^{\hat{\mu}}] &= -\tfrac{T_{M-2}}{2} \int d^3\xi \,\sqrt{|\gamma|} \left\{ \gamma^{ij}\partial_i \hat{X}^{\hat{\mu}}\partial_j \hat{X}^{\hat{\nu}}\hat{g}_{\hat{\mu}\hat{\nu}} - 1 \right\} \\[2mm] &\quad + \tfrac{T_{M-2}}{3!} \int d^3\xi \,\epsilon^{ijk}\partial_i \hat{X}^{\hat{\mu}}\partial_j \hat{X}^{\hat{\nu}}\partial_k \hat{X}^{\hat{\rho}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} \,. \end{aligned} \tag{10}$$

This action is invariant under worldvolume reparametrizations, 11-dimensional spacetime reparametrizations[4]

$$\begin{cases} \delta_{\hat{\epsilon}}\,\hat{X}^{\hat{\mu}} &= \hat{\epsilon}^{\hat{\mu}}(\hat{X}) \,, \\[3mm] \delta_{0\,\hat{\epsilon}}\,\hat{g}_{\hat{\mu}\hat{\nu}} &= -\pounds_{\hat{\epsilon}}\,\hat{g}_{\hat{\mu}\hat{\nu}} + \hat{\epsilon}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{g}_{\hat{\mu}\hat{\nu}} \,, \\[3mm] \delta_{0\,\hat{\epsilon}}\,\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= -\pounds_{\hat{\epsilon}}\,\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} + \hat{\epsilon}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} \,, \end{cases} \tag{11}$$

and gauge transformations of the 3-form potential

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[4]Here we write $\delta_0\phi = \phi'(x) - \phi(x) \neq \phi'(x') - \phi(x) = \delta\phi$.

$$\delta_{\hat{\chi}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3\partial_{[\hat{\mu}}\hat{\chi}_{\hat{\nu}\hat{\rho}]}\,. \tag{12}$$

Let us now consider the infinitesimal transformations

$$\begin{cases} \delta_{\eta}\hat{X}^{\hat{\mu}} & = & \eta(\xi)\hat{k}^{\hat{\mu}}(\hat{X})\,, \\[2mm] \delta_{\eta}\hat{g}_{\hat{\mu}\hat{\nu}} & = & \eta\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{g}_{\hat{\mu}\hat{\nu}}\,, \\[2mm] \delta_{\eta}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} & = & \eta\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}\,, \end{cases} \tag{13}$$

where η is an infinitesimal worldvolume scalar and $\hat{k}^{\hat{\mu}}$ an arbitrary space-time vector field. The action is not invariant under them (they are *not* reparametrizations) but transforms according to

$$\begin{aligned} \delta_{\eta}\hat{S} & = & -\tfrac{T_{M-2}}{2}\int d^3\xi\,\sqrt{|\gamma|}\gamma^{ij}\left\{2\partial_i\eta\hat{k}_i + \eta\partial_i\hat{X}^{\hat{\mu}}\partial_j\hat{X}^{\hat{\nu}}\pounds_{\hat{k}}\hat{g}_{\hat{\mu}\hat{\nu}}\right\} \\[2mm] & & +\tfrac{T_{M-2}}{3!}\int d^3\xi\,\epsilon^{ijk}\left\{3\partial_i\eta\left(i_{\hat{k}}\hat{C}\right)_{jk} + \eta\partial_i\hat{X}^{\hat{\mu}}\partial_j\hat{X}^{\hat{\nu}}\partial_k\hat{X}^{\hat{\rho}}\pounds_{\hat{k}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}\right\}\,, \end{aligned} \tag{14}$$

where

$$\left(i_{\hat{k}}\hat{C}\right)_{\hat{\mu}\hat{\nu}} = \hat{k}^{\hat{\rho}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}\,. \tag{15}$$

This transformation will be a symmetry if η is constant and $\pounds_{\hat{k}}\hat{g}_{\hat{\mu}\hat{\nu}} = \pounds_{\hat{k}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 0$. We will assume that the two latter conditions hold (so, in particular, $\hat{k}^{\hat{\mu}}$ is a Killing vector and the metric has an isometry) and will try to modify the action in order to make it invariant under the transformations with non-constant η, i.e. we will gauge the symmetry present for constant η.

The first step is to substitute everywhere in the original action (10) the partial derivatives of the $\hat{X}^{\hat{\mu}}$ fields by covariant derivatives:

$$D_i\hat{X}^{\hat{\mu}} = \partial_i\hat{X}^{\hat{\mu}} + C_i(\xi)\hat{k}^{\hat{\mu}}\,, \tag{16}$$

where C_i is an auxiliary (non-dynamical) worldvolume field that transforms as follows

$$\delta_{\eta}C_i = -\partial_i\eta\,, \Rightarrow \delta_{\eta}D_i\hat{X}^{\hat{\mu}} = \eta D_i\hat{X}^{\hat{\nu}}\partial_{\hat{\nu}}\hat{k}^{\hat{\mu}}\,. \tag{17}$$

We get

$$\hat{S}_{\text{gauged}_0} \ [\hat{X}^{\hat{\mu}}, C_i] \quad = \quad -\tfrac{T_{M-2}}{2} \int d^3\xi \ \sqrt{|\gamma|} \left\{ \gamma^{ij} D_i \hat{X}^{\hat{\mu}} D_j \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}} - 1 \right\}$$

$$+ \tfrac{T_{M-2}}{3!} \int d^3\xi \ \epsilon^{ijk} D_i \hat{X}^{\hat{\mu}} D_j \hat{X}^{\hat{\nu}} D_k \hat{X}^{\hat{\rho}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} . \tag{18}$$

While this substitution automatically makes the action invariant under the δ_η transformations, it completely breaks the gauge invariance of the 3-form potential[5]:

$$\delta_{\hat{\chi}} \hat{S}_{\text{gauged}_0} = \tfrac{T_{M-2}}{3!} \int d^3\xi \ \epsilon^{ijk} \left\{ 3C_i \left(\pounds_{\hat{k}} \hat{\chi} \right)_{jk} - 12 C_i \partial_j \lambda_k \right\} , \tag{19}$$

where

$$-\tfrac{1}{2} \hat{\lambda}_{\hat{\mu}} \equiv (i_{\hat{k}} \hat{\chi})_{\hat{\mu}} = \hat{k}^{\hat{\nu}} \hat{\chi}_{\hat{\mu}\hat{\nu}} . \tag{20}$$

While the first term in $\delta_{\hat{\chi}} \hat{S}_{\text{gauged}_0}$ can be set to zero by restricting the admissible gauge transformation to those with parameter $\hat{\chi}_{\hat{\mu}\hat{\nu}}$ whose Lie derivative with respect to $\hat{k}^{\hat{\mu}}$ vanish (which is understandable as a condition to preserve the previous condition $\pounds_{\hat{k}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 0$), it is not immediately clear how to cancel the second term. The solution, inspired by the dual D-2-brane action Eq. (6) is to make the auxiliary field to transform under $\delta_{\hat{\chi}}$:

$$\delta_{\hat{\chi}} C_i = -m \hat{\lambda}_i , \tag{21}$$

and to add a Chern-Simons (CS) term (which is automatically gauge-invariant under δ_η) to the Wess-Zumino (WZ) term of the action $\hat{S}_{\text{gauged } 0}$, getting

$$\hat{S}_{\text{gauged}} \ [\hat{X}^{\hat{\mu}}, C_i] \quad = \quad -\tfrac{T_{M-2}}{2} \int d^3\xi \ \sqrt{|\gamma|} \left\{ \gamma^{ij} D_i \hat{X}^{\hat{\mu}} D_j \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}} - 1 \right\}$$

$$+ \tfrac{T_{M-2}}{3!} \int d^3\xi \ \epsilon^{ijk} \left\{ D_i \hat{X}^{\hat{\mu}} D_j \hat{X}^{\hat{\nu}} D_k \hat{X}^{\hat{\rho}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} - \tfrac{6}{m} C_i \partial_j C_k \right\} . \tag{22}$$

The introduction of the parameter m with dimensions of inverse length is unavoidable to make the CS term dimensional correct. Its value is, from the classical point of view, immaterial. On the other hand, the effect of the introduction of the CS term is (as we will see) to add one degree of freedom to the action: without the CS term both C_i and one coordinate scalar could be eliminated by using the equation of motion of C_i (one degree of freedom is subtracted in the gauging). With the CS term C_i does not disappear in

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[5]In principle one could have made the discussion using the field strength as in Ref. [7].

doing this and the number of degrees of freedom is identical to that of the original action.

However, even though we now are able to cancel exactly the variation Eq. (19) we have introduced a new variation and we get new non-vanishing result with terms coming both from the kinetic[6] and WZ terms. Fortunately this can be cancelled without any further modification of the action by simply modifying the gauge transformation of $\hat{C}$ and making the metric transform under $\delta_{\hat{\chi}}$. Thus, to summarize, we find that the gauged action Eq. (22) is invariant under the $\delta_{\hat{\chi}}$ transformations

$$
\begin{cases}
\delta_{\hat{\chi}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= 3\partial_{[\hat{\mu}} \hat{\chi}_{\hat{\nu}\hat{\rho}]} + 3m\hat{\lambda}_{[\hat{\mu}} \left(i_{\hat{k}} \hat{C} \right)_{\hat{\nu}\hat{\rho}]}, \\[2ex]
\delta_{\hat{\chi}} \hat{g}_{\hat{\mu}\hat{\nu}} &= 2m\hat{\lambda}_{(\hat{\mu}} \hat{k}_{\hat{\nu})} = 2m\hat{\lambda}_{(\hat{\mu}} \left(i_{\hat{k}} \hat{g} \right)_{\hat{\nu})}, \\[2ex]
\delta_{\hat{\chi}} C_i &= -m\hat{\lambda}_i,
\end{cases}
\tag{23}
$$

and the δ_η transformations

$$
\begin{cases}
\delta_\eta \hat{X}^{\hat{\mu}} &= \eta(\xi)\hat{k}^{\hat{\mu}}(\hat{X}), \\[2ex]
\delta_\eta C_i &= -\partial_i \eta,
\end{cases}
\qquad
\begin{cases}
\delta_\eta \hat{g}_{\hat{\mu}\hat{\nu}} &= \eta\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}} \hat{g}_{\hat{\mu}\hat{\nu}}, \\[2ex]
\delta_\eta \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= \eta\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}},
\end{cases}
\tag{24}
$$

assuming the conditions

$$
\pounds_{\hat{k}} \hat{g}_{\hat{\mu}\hat{\nu}} = \pounds_{\hat{k}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \pounds_{\hat{k}} \hat{\chi}_{\hat{\mu}\hat{\nu}} = 0,
\tag{25}
$$

hold.

2 Direct Dimensional Reduction

It is easy to see that the above action Eq. (22) gives the dual action Eq. (6) of the D-2-brane of the massive type IIA theory proposed in Ref. [1] upon direct dimensional reduction in the direction X associated to the isometry we have gauged. Choosing coordinates adapted to the isometry $\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}x}$ we split the eleven coordinates $\hat{X}^{\hat{\mu}}$ into the ten 10-dimensional X^μ, $\mu = 0, \ldots, 9$ and the extra scalar $X = \hat{X}^{10}$.

Using the relations between the 11-dimensional and 10-dimensional fields

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[6] Observe that the covariant derivative transforms also covariantly under this transformation: $\delta_{\hat{\chi}} D_i \hat{X}^{\hat{\mu}} = -m D_i \hat{X}^{\hat{\nu}} \hat{\lambda}_{\hat{\nu}} \hat{k}^{\hat{\mu}}$.

$$\begin{cases} \hat{g}_{xx} &= -e^{\frac{4}{3}\phi}\,, \\[2ex] \hat{g}_{\mu x} &= \left(i_{\hat{k}}\hat{g}\right)_{\mu} = -e^{\frac{4}{3}\phi}A^{(1)}{}_{\mu}\,, \\[2ex] \hat{g}_{\mu\nu} &= e^{-\frac{2}{3}\phi}g_{\mu\nu} - e^{\frac{4}{3}\phi}A^{(1)}{}_{\mu}A^{(1)}{}_{\nu}\,, \end{cases} \qquad \begin{cases} \hat{C}_{\mu\nu\rho} &= C_{\mu\nu\rho}\,, \\[2ex] \hat{C}_{\mu\nu x} &= \left(i_{\hat{k}}\hat{C}\right)_{\mu\nu} = -B^{(1)}{}_{\mu\nu}\,, \end{cases}$$

$$(26)$$

$$\begin{cases} \hat{\chi}_{\mu\nu} &= \chi_{\mu\nu}\,, \\[2ex] \hat{\lambda}_{\mu} &= \lambda_{\mu}\,, \end{cases} \tag{27}$$

(observe that $\hat{\lambda}_x = \hat{k}^{\hat{\mu}}\hat{\lambda}_{\hat{\mu}} = 0$) it is immediate to recover both the dual action and all its symmetries. Observe that the transformations δ_χ and δ_λ become independent in ten dimensions.

3 Conclusion

We have constructed an effective action for a membrane with 11-dimensional target space whose direct dimensional reduction gives the dual action for D-2-brane of the massive 10-dimensional type IIA theory found in Ref. [1].

The action has a natural interpretation as a gauged sigma model in which the gauge invariance of the 3-form potential has been preserved in a slightly modified way. At the same time, the total number of scalar degrees of freedom has been preserved, making the supersymmetrization of the action (assuming κ-symmetry) possible.

On the other hand, the present action may help us in understanding the 11-dimensional origin of the massive type IIA theory. There is no cosmological 11-dimensional supergravity theory (see Ref. [8] and references therein) and our result may indicate how to accommodate the solitons of the massive 10-dimensional type IIA theory into the usual 11-dimensional supergravity theory. In this sense, gauged sigma models may proof to be a useful tool [6].

Acknowledgments

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