Dual Inflation

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(October 1, 1997)

We propose a new model of inflation based on the soft-breaking of N=2 supersymmetric SU(2) Yang-Mills theory. The advantage of such a model is the fact that we can write an exact expression for the effective scalar potential, including non-perturbative effects, which preserves the analyticity and duality properties of the Seiberg-Witten solution. We find that the scalar condensate that plays the role of the inflaton can drive a long period of cosmological expansion, produce the right amount of temperature anisotropies in the microwave background, and end inflation when the monopole acquires a vacuum expectation value. Duality properties relate the weak coupling Higgs region where inflation takes place with the strong coupling monopole region, where reheating occurs, creating particles corresponding to the light degrees of freedom in the true vacuum.

PACS numbers: 98.80.Cq Preprint CERN-TH/97-149, hep-th/9707059

The inflationary paradigm [1] not only provides a very elegant solution to the classical problems of the hot big bang cosmology, but also predicts an almost scale invariant spectrum of metric perturbations which could be responsible for the observed anisotropy of the cosmic microwave background (CMB). There are a dozen or so different models of inflation, motivated by distinct particle physics scenarios. In most of these models one has to make certain approximations: either they do not include large quantum corrections to their small parameters or they do not consider possibly strong non-perturbative effects.

One of the most robust aspects of inflation is the fact that, as long as there is an almost flat direction, the universe will expand quasi-exponentially, independently of the nature of the effective inflaton field. In most models, however, the inflaton is a fundamental scalar field and the scalar potential is put in by hand, except perhaps in Starobinsky model, where the field that drives inflation is related through a conformal transformation to higher derivative terms in the gravitational action, see Ref. [1].

In this letter we propose a new model, based on recent exact results in duality invariant supersymmetric Yang-Mills theories, where the inflaton field is a composite scalar field that appears in the theory. In the last couple of years there has been a true revolution in the understanding of non-perturbative effects in N=1 and N=2 supersymmetric quantum field theories, following the work of Seiberg and Witten [2] in N=2 supersymmetric SU(2). Duality and analyticity arguments allow one to write down an *exact* effective action for the light degrees of freedom in both the weak and in the strong coupling regions. However, our world is non-supersymmetric and therefore one should consider the soft breaking of N=2 supersymmetric SU(2) directly down to N=0. This was successfully done in Ref. [3], via the introduction of a dilaton spurion superfield which preserves the analyticity properties of the Seiberg-Witten solution. The result is a low energy effective scalar potential which is *exact*, i.e. includes all perturbative and non-perturbative effects. This potential presents an almost flat direction that could drive inflation. The results of Ref. [3] were studied in the context of low energy QCD. This letter simply draws the attention to the possibility that the exact scalar potential found in [3] could be consistently used at much higher energies, of order the GUT scale, and be responsible for cosmological inflation. The advantage with respect to other inflationary models based on N=1 supersymmetry is the complete control we have on the scalar potential, both along the quasi-flat direction and in the true vacuum of the theory, where reheating takes place.

In N=2 supersymmetric SU(2), the electromagnetic vector superfield A_μ^a has a scalar partner φ in the adjoint (obtained by succesive application of the two supersymmetries). We can construct a gauge invariant complex scalar field, $u = \mathrm{Tr}\,\varphi^2$, which parametrizes inequivalent vacua, see Ref. [5], and can be geometrically associated with a complex moduli space. It is this scalar condensate u which plays the role of the inflaton, once N=2 is softly broken to N=0 and the scalar potential acquires a minimum. In the Seiberg-Witten solution [2] one can distinguish two regions, the perturbative or weak-coupling Higgs region and the confining monopole/dyon region of strong coupling. In our model, the soft breaking of N=2 supersymmetry will give mass only to the monopole, and the dyon region will disappear [3]. Thanks to duality we will be able to consistently describe our low energy effective action in terms of the local degrees of freedom of the corresponding region. In particular, in the region where the monopole acquires a vacuum expectation value (VEV) the light degrees of freedom are the monopoles while the 'electrons' are confined into electric singlets. This will play an important role in reheating in this model, see [4].

The dynamical scale Λ of SU(2) is the only free parameter of the Seiberg-Witten solution, and can be related via $\Lambda = M\exp(iS)$ to the dilaton spurion superfield

$S = (\pi/2)\tau + \theta^2 F_0$ that breaks supersymmetry, where $\tau = 4\pi i/g^2 + \theta/2\pi$ is the generalized gauge coupling, see [3]. Here $M \equiv M_{\rm P}/\sqrt{8\pi}$ is the reduced Planck mass, which sets the fundamental scale in the problem. In our model we also have $f_0 = \langle F_0 \rangle$, the soft supersymmetry breaking parameter, which should be smaller than Λ in order to use the exact results of Seiberg-Witten. This exact solution is the first term in the expansion of the low-energy effective action, which because of supersymmetry can be related to an expansion in f_0, to order f_0^2.

The exact N=0 low-energy effective scalar potential satisfying the analyticity and duality properties of the Seiberg-Witten solution can be written as [3]

$$V(u) = -\frac{2}{b_{11}}\,\rho^4 - \frac{\det b}{b_{11}}\,f_0^2\,, \tag{1}$$

$$\rho^2 = -b_{11}|a|^2 + \frac{f_0}{\sqrt{2}}|b_{01}|\,, \qquad (\rho > 0) \tag{2}$$

in terms of the coordinates in the monopole region,

$$a(u) = \frac{4i\Lambda}{\pi}\,\frac{E' - K'}{k}\,, \qquad \tau_{11} = \frac{iK}{K'}\,, \tag{3}$$

$$\tau_{01} = \frac{2i\Lambda}{kK'}\,, \qquad \tau_{00} = \frac{8i\Lambda^2}{\pi}\left(\frac{E'}{k^2K'} - \frac{1}{2}\right)\,. \tag{4}$$

The functions $K(k)$ and $E(k)$ are complete elliptic functions of the first and second kind respectively, and $E'(k) \equiv E(k')$, where $k'^2 + k^2 = 1$. All these are functions of $k^2 = 2/(1 + u/\Lambda^2)$ in the complex moduli plane u. The functions $b_{ij} \equiv {\rm Im}\,\tau_{ij}/4\pi$. The scalar potential is thus a non-trivial function of the inflaton field.

In order to study the cosmological evolution of the inflaton under the potential (1) one should embed this model in supergravity. This was also performed in Ref. [3] and found that the only gravitational corrections to the scalar potential were proportional to $m_{3/2}^2 f_0^2$ and therefore completely negligible in our case, since inflation in this model turns out to occur at the GUT scale, see below, and thus much above the gravitino mass scale.

One has to take into account however the non-trivial Kähler metric for u [3]

$$ds^2 = {\rm Im}\,\tau_{11} da\, d\bar{a} = {\rm Im}\,\tau_{11}\left|\frac{da}{du}\right|^2 du\, d\bar{u}$$
$$\equiv \frac{1}{2}\mathcal{K}(u)\, du\, d\bar{u}\,, \tag{5}$$

where $\mathcal{K}(u)/2$ is the Kähler metric. The Lagrangian for the scalar field u in a curved background can then be written as

$$\mathcal{L} = \frac{1}{2}\mathcal{K}(u)\, g^{\mu\nu}\partial_\mu u\, \partial_\nu \bar{u} - V(u)\,, \tag{6}$$

$$= \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\, \partial_\nu\bar{\phi} - V(\phi)\,, \tag{7}$$

where $g_{\mu\nu}$ is the spacetime metric, and we have redefined the inflaton field through $d\phi \equiv \mathcal{K}(u)^{1/2} du$. We have plotted in Fig. 1 the scalar potential $V(\phi)$ as a function of the real and imaginary parts of ϕ. We have added a constant to the potential in order to ensure that the absolute minimum is at zero cosmological constant. In the absence of a working mechanism for the vanishing of this constant, we have to fix it by hand. Fortunately we can safely do this without affecting the consistency of the theory because we have started with a softly broken supersymmetric model.

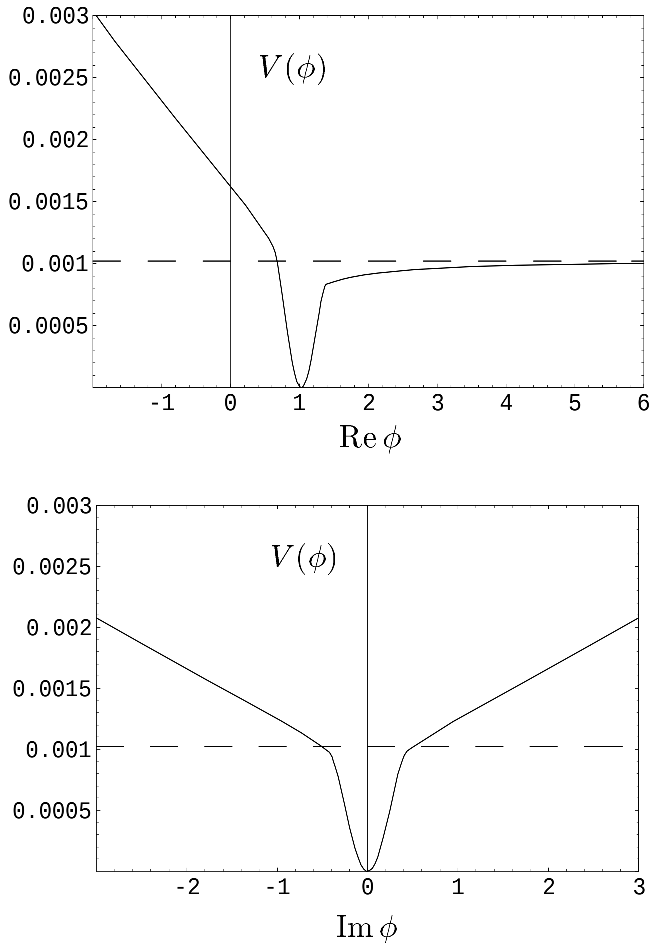

FIG. 1. The exact scalar potential $V(\phi)$ in moduli space. The top panel shows the potential as a function of ${\rm Re}\,\phi$ for ${\rm Im}\,\phi = 0$ and presents a minimum at $\phi_{\rm min} = \Lambda + f_0/\sqrt{8}$. The lower panel shows the potential as a function of ${\rm Im}\,\phi$ for ${\rm Re}\,\phi = \phi_{\rm min}$. The plateau (dashed line) has a vacuum energy density of $V_0 = f_0^2\Lambda^2/\pi^2$. We have plotted the figure in units of $\Lambda = 1$, for $f_0 = 0.1\Lambda$.

The flatness of this potential at ${\rm Im}\,\phi = 0, {\rm Re}\,\phi > \phi_{\rm min}$ looks like an excellent candidate for inflation. It was not included by hand but arised naturally from the soft-breaking of supersymmetry [3], albeit with a complicated functional form (1). There are two parameters in this model, the dynamical scale Λ and the symmetry breaking scale f_0. Non of these have to be fine tuned to be small in order to have successful inflation. Moreover, as we will show, the trajectories away from the positive real axis do not give inflation, which simplifies our analysis significantly. We will now consider the range of values of f_0 and Λ that give a phenomenologically viable model.

Let us write down the classical equations of motion for the homogeneous field $\phi \equiv \mathrm{Re}\phi$,

$$H^2 = \frac{1}{3M^2}\left[\frac{1}{2}\dot\phi^2 + V(\phi)\right], \tag{8}$$

$$\dot H = -\frac{1}{2M^2}\,\dot\phi^2, \tag{9}$$

$$\ddot\phi + 3H\dot\phi + V'(\phi) = 0\,. \tag{10}$$

Inflation will be ensured as long as $\epsilon \equiv -\dot H/H^2 < 1$. The end of inflation occurs at the value of ϕ for which $\dot\phi^2 = V(\phi)$. We can now apply the usual machinery to study inflationary cosmology with a scalar field potential [1,6]. It turns out that for all values of the parameters Λ and $f_0 < \Lambda$, the extreme flatness of the potential at $\phi > \phi_{\rm min}$ allows one to use the slow roll approximation in the Higgs region all the way to the monopole region, where the slope of the potential is so large that inflation ends and reheating starts as the condensate oscillates around the minimum. In the slow roll approximation we can construct the dimensionless parameters [6]

$$\epsilon = \frac{M^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \tag{11}$$

$$\eta = M^2\,\frac{V''(\phi)}{V(\phi)}, \tag{12}$$

$$N = \frac{1}{M^2}\int\frac{V(\phi)\,d\phi}{V'(\phi)}, \tag{13}$$

where N is the number of e-folds to the end of inflation.

In order to know what is the (approximately constant) rate of expansion during inflation, one has to find the value of the potential at the minimum, $V_0 \equiv |V(u_{\rm min})|$, see Eq. (1), as a function of the symmetry breaking parameter f_0. It can be shown, based on monodromy arguments [5], that in the monopole region close to the minimum, $a(u) \propto (u - \Lambda^2)$. This ensures that the deviation from Λ^2 of the position of the minimum $u_{\rm min}$ is linear in f_0, which then implies, to very good accuracy, that the minimum of the potential is quadratic in f_0,

$$u_{\rm min} = \Lambda^2\Big(1 + \frac{f_0}{2\sqrt2\Lambda}\Big), \tag{14}$$

$$V_0 = \frac{f_0^2\Lambda^2}{\pi^2}\Big(1 + \frac{f_0}{8\sqrt2\Lambda}\Big) \simeq \frac{f_0^2\Lambda^2}{\pi^2}\,. \tag{15}$$

These expressions are excellent approximations in the range of values of $0 \le f_0 \le 0.5$, see Fig. 1.

In the Higgs region, along the positive real axis, it is possible to write in a compact way the Kähler metric and the scalar potential

$$\mathcal{K}(u) = \frac{2k^2}{\Lambda^2\pi^2}\,KK', \tag{16}$$

$$V(u) = \frac{f_0^2\Lambda^2}{\pi^2}\left[1 - 2\Big(\frac{K-E}{k^2K} - \frac{1}{2}\Big)\right]. \tag{17}$$

We can now address the issue of what are the parameters f_0 and Λ which ensure at least 60 e-folds of inflation in order to solve the classical problems of Big Bang cosmology, as well as to produce the correct amount of metric fluctuations required to explain the observed temperature anisotropies of the CMB.

As a consequence of the factorization of the symmetry breaking parameter f_0^2 in the potential (17), the slow-roll parameters (11)–(13) do not depend on f_0. This further simplifies our analysis. For a given value of Λ/M it is easy to find the value of ϕ_e at the end of inflation and from there compute the value ϕ_{60} corresponding to $N = 60$ e-folds from the end of inflation.

Quantum fluctuations of the scalar condensate, $\delta\phi$, will create perturbations in the metric, $\mathcal{R} = H\delta\phi/\dot\phi$, which cross the Hubble scale during inflation and later re-enter during the matter era. Those fluctuations corresponding to the scale of the present horizon left 60 e-folds before the end of inflation, and are responsible via the Sachs-Wolfe effect for the observed temperature anisotropies in the microwave background [7]. From the amplitude and spectral tilt of these temperature fluctuations we can constrain the values of the parameters Λ and f_0.

Present observations of the power spectrum of temperature anisotropies on various scales, from COBE DMR to Saskatoon and CAT experiments, impose the following constraints on the amplitude of the tenth multipole and the tilt of the spectrum [8],

$$Q_{10} = 17.5 \pm 1.1\ \mu\mathrm{K}, \tag{18}$$

$$n = 0.91 \pm 0.10\,. \tag{19}$$

Assuming that the dominant contribution to the CMB anisotropies comes form the scalar metric perturbations, we can write [8] $A_S = 5\times10^{-5}\,(Q_{10}/17.6\ \mu\mathrm{K})$, where [6]

$$A_S^2 = \frac{1}{M^2}\Big(\frac{H}{2\pi}\Big)^2\frac{1}{2\epsilon}, \qquad n = 1 + 2\eta - 6\epsilon, \tag{20}$$

in the slow-roll approximation.

Since during inflation at large values of ϕ, corresponding to $N = 60$, the rate of expansion is dominated by the vacuum energy density (15), we can write, to very good approximation, $H^2 = f_0^2\Lambda^2/3\pi^2M^2$, and thus the amplitude of scalar metric perturbations is

$$A_S^2 = \frac{1}{24\pi^4}\frac{f_0^2\Lambda^2}{M^4}\frac{1}{\epsilon_{60}}\,. \tag{21}$$

Let us consider, for example, a model with $\Lambda = 0.1M$. In that case the end of inflation occurs at $\phi_e \simeq 1.5\Lambda$, still in the Higgs region, and 60 e-folds correspond to a relatively large value, $\phi_{60} = 14\Lambda$, deep in the weak coupling region. The corresponding values of the slow roll parameters are $\epsilon_{60} = 2\times10^{-5}$ and $\eta_{60} = -0.04$, which gives $A_S^2 = 5f_0^2/24\pi^4\Lambda^2$ and $n = 0.91$. In order to satisfy the constraint on the amplitude of perturbations (18), we require $f_0 = 10^{-3}\Lambda$, which is a very natural value from the point of view of the consistency of the theory. In particular, these parameters correspond to a vacuum mass scale of order $V_0^{1/4} \simeq 4 \times 10^{15}$ GeV, very close to the GUT scale. For other values of Λ we find numerically the relation $\log_{10}(f_0/\Lambda) = -4.5 + 1.54 \log_{10}(M/\Lambda)$, which is a very good fit in the range $1 \leq M/\Lambda \leq 10^3$. For $M > 800\Lambda$, the soft breaking parameter f_0 becomes greater than Λ, where our approximations breaks down, and we can no longer trust our exact solution. Meanwhile the spectral tilt is essentially invariant, $n = 0.913 - 0.003 \log_{10}(M/\Lambda)$, in the whole range of Λ. It is therefore a concrete prediction of the model. Surprisingly enough it precisely corresponds to the observed value (19). This might change however when future satellite missions will determine the spectral index n with better than 1% accuracy [9].

There are also tensor (gravitational waves) metric perturbations in this model, with amplitude $A_T^2 = 2H^2/\pi^2 M^2 = 2f_0^2\Lambda^2/3\pi^4 M^4$ and tilt $n_T = -2\epsilon$, see Ref. [6]. The relative contribution of tensor to scalar perturbations in the microwave background on large scales can be parametrized by $T/S \simeq 12.4\epsilon$. A very good fit to the ratio T/S in this model is given by $\log_{10}(T/S) = -2.6 - 0.91 \log_{10}(M/\Lambda)$, in the same range as above. Since $M \geq \Lambda$, we can be sure that no significant contribution to the CMB temperature anisotropies will arise from gravitational waves.

We still have to make sure that motion along the transverse direction (for $\mathrm{Im}\,\phi \neq 0$) does not lead to inflationary trajectories, since otherwise our simple one-dimensional analysis would break down and there could exist isocurvature as well as adiabatic perturbations. For that purpose we computed the corresponding epsilon parameter $(-\dot{H}/H^2)$ along the imaginary axis and confirmed numerically, for various values of Λ/M and the whole range, $\phi_e < \phi < \phi_{60}$, that $\epsilon(\mathrm{Im}\,\phi) > 1$, and thus inflation does not occur there. A similar result can be found for the epsilon parameter along the negative real axis. This ensures that whatever initial condition one may have, eventually the field will end in the inflationary positive real axis.

We have therefore found a new model of inflation, based on *exact* expressions for the scalar potential of a softly broken N=2 supersymmetric SU(2) theory, to all orders in perturbations and with all non-perturbative effects included. Inflation occurs along the weak coupling Higgs region where the potential is essentially flat, and ends when the gauge invariant scalar condensate enters the strong coupling confining phase, where the monopole acquires a VEV, and starts to oscillate around the minimum of the potential, reheating the universe. A simple argument suggests that during reheating explosive production of particles could occur in this model. The evolution equation of a generic scalar (or vector) particle has the form of a Mathieu equation and presents parametric resonance for certain values of the parameters, see Ref. [10]. Explosive production corresponds to large values of the ratio $q = g^2\Phi^2/4m^2$, where g is the coupling between ϕ and the corresponding scalar field, Φ is the amplitude of oscillations of ϕ, and m is its mass. As the inflaton field oscillates around $\phi_{\rm min}$ it couples strongly, $g \sim 1$, to the other particles in the supermultiplet since the minimum is in the strong coupling region. The amplitude of oscillations is of order the dynamical scale, $\Phi \sim \Lambda$, while the masses of all particles (scalars, fermions and vectors) are of order the supersymmetry breaking scale, $m \sim f_0 \ll \Lambda$, see Ref. [11]. This means that the q-parameter is large, thus inducing strong parametric resonance and explosive particle production [10]. These particles will eventually decay into ordinary particles, reheating the universe. This simple picture however requires a detailed investigation, see Ref. [4].

We have shown that for very natural values of the parameters it is possible to obtain the correct amplitude and tilt of scalar metric perturbations responsible for the observed anisotropies in the microwave background. Furthermore, in this model, the contribution of gravitational waves to the CMB anisotropies is very small. Since inflation occurs only along the positive real axis of moduli space, we have an effectively one-dimensional problem, which ensures that perturbations are adiabatic. But perhaps the most interesting aspect of the model is the fact that the inflaton field is not a fundamental scalar field, but a condensate, with an exact non-perturbative effective potential. The most obvious advantage with respect to other inflationary models based on supersymmetry is that one has control of the strong coupling reheating phase, thanks to the exact knowledge of the inflaton potential.

We are assuming throughout that we can embed this inflationary scenario in a more general theory that contains two sectors, the inflaton sector, which describes the soft breaking of N=2 supersymmetric SU(2) and is responsible for the observed flatness and homogeneity of our universe, and a matter sector with the particle content of the standard model, at a scale much below the inflaton sector. The construction of more realistic scenarios remains to be explored, in which the two sectors communicate via some messenger sector. For example, one could consider this SU(2) as a subgroup of the hidden E_8 of the heterotic string and the visible sector as a subgroup of the other E_8. No-scale supergravity could then be used as mediator of supersymmetry breaking from the strong coupling inflaton sector to the weakly coupled visible sector. For the scales of susy breaking considered above, $f_0 \sim 10^{-5} M_{\rm P}$, we can obtain a phenomenologically reasonable gravitino mass, $m_{3/2} \sim f_0^3/M_{\rm P}^2 \sim 10$ TeV [12]. This gravity mediated supersymmetry breaking scenario needs further study, but suggests that it is possible in principle to do phenomenology with this novel inflationary model.

ACKNOWLEDGEMENTS

I would like to thank Luis Alvarez-Gaumé, Costas Kounnas and Frederic Zamora for very stimulating discussions.

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