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# Extracting Supersymmetry-Breaking Effects from Wave-Function Renormalization

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**Abstract**

We show that in theories in which supersymmetry breaking is communicated by renormalizable perturbative interactions, it is possible to extract the soft terms for the observable fields from wave-function renormalization. Therefore all the information about soft terms can be obtained from anomalous dimensions and $\beta$ functions, with no need to further compute any Feynman diagram. This method greatly simplifies calculations which are rather involved if performed in terms of component fields. For illustrative purposes we reproduce known results of theories with gauge-mediated supersymmetry breaking. We then use our method to obtain new results of phenomenological importance. We calculate the next-to-leading correction to the Higgs mass parameters, the two-loop soft terms induced by messenger-matter superpotential couplings, and the soft terms generated by messengers belonging to vector supermultiplets.

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# 1 Introduction

Theories in which supersymmetry breaking is mediated by renormalizable perturbative interactions have an interesting advantage over the gravity-mediated scenario. This is because in these theories the soft terms are in priciple calculable quantities, very much like $g-2$ in QED. Gauge-mediated models [1, 2], in particular, have also a strong motivation as they elegantly solve the supersymmetric flavour problem. In the simplest version of these theories, the gaugino and sfermion masses arise respectively from one- and two-loop finite diagrams. Their evaluation, while conceptually straightforward, takes in practice some effort. With the increasing number of non-minimal examples it would certainly be very useful to have a quick and reliable method to evaluate soft masses. When scanning through models it is important to know right away if the predicted masses are phenomenologically consistent and whether there are any interesting predictions. It is the purpose of this paper to develop one such technique and to apply it to a number of interesting cases. These include theories in which gauge as well as Yukawa interactions mediate soft masses and also theories in which the messengers are massive gauge supermultiplets.

We will be interested in the case in which the breaking of supersymmetry and the scale of its mediation to the observable sector are completely determined by the scalar and auxiliary component vacuum expectation values (VEV) of a chiral superfield $\langle X\rangle = M+\theta^2 F$. In the case of conventional gauge mediation $X$ gives mass to the messengers via the superpotential coupling $X\Phi\bar{\Phi}$. Moreover we are interested in the regime $F \ll M^2$ for which, in the effective field theory below the messenger scale $M$, supersymmetry is broken *only* by soft effects. This allows us to use a manifestly supersymmetric formalism to keep track of supersymmetry-breaking effects as well. Due to the non-renormalization of the superpotential, the gaugino and sfermion masses arise just from $X$-dependent renormalizations of respectively the gauge and matter kinetic terms, *i.e.* from the gauge charge $g^2$ and matter wave-function renormalizations $Z_i$. In the presence of only one spurion $X$, the kinetic functions can be obtained by calculating first $g^2$ and $Z_i$ in the supersymmetric limit in which $X = M$ is just a c-number mass, and by substituting later $M$ with the superfield $X$. This is the crucial point of our paper. The $g^2(M,\mu)$ and $Z_i(M,\mu)$ renormalized at a scale $\mu \ll M$ are simply evaluated by solving the Renormalization Group (RG) equations. At the end, the substitution $M \to X$ in $g^2$ is made since this quantity, at least at one-loop, has to be holomorphic. On the other hand, the only substitution in $Z_i$ consistent with the chiral reparametrization $X \to e^{i\phi}X$ is given by $M \to \sqrt{XX^\dagger}$. As it will be shown below, a very interesting advantage of this method is that the two-loop sfermion masses of gauge mediation are determined only by the one-loop RG equations.

This paper is organized as follows. Section 2 contains the essence of our paper. There we describe our method, its regime of validity, and give the general formulae relating the supersymmetry-breaking terms to $X$-derivatives of wave-function renormalizations. For illustration, we reproduce the known soft terms of gauge-mediated theories. Later sections are devoted to some applications of phenomenological interest. In sect. 3 we compute the next-to-leading corrections to the Higgs mass parameters, while soft terms induced by superpotential couplings between messengers and matter are studied in sect. 4. The case of messengers belonging to vector supermultiplet is discussed in sect. 5, and sect. 6 contains a summary of our results.

## 2 The Method and its Application to Gauge Mediation

In this section we will derive the general formulae which relate the soft terms to $X$-derivatives of wave-function renormalizations. In our formulation $X$ is a background superfield which has both a scalar and an auxiliary VEV, $\langle X\rangle = M + \theta^2 F$. Here $M$ determines the mass scale of the messenger fields which communicate supersymmetry-breaking to the observable fields. For instance, in gauge-mediated theories, the messenger sector consists of $N$ pairs of chiral superfields $\Phi$, $\bar{\Phi}$ getting mass from the superpotential interaction to $X$

$$\int d^2\theta X\Phi\bar{\Phi} \ . \tag{1}$$

The gaugino mass renormalized at a scale $\mu$ is determined by the $X$-dependendent wave function $S$ of the gauge multiplet,

$$\mathcal{L} = \int d^2\theta S W^{a\alpha} W^a_\alpha + \text{h.c.} \tag{2}$$

Here $a$ and $\alpha$ are gauge and spinorial indices, respectively. At the one-loop order, to which we are now interested, $S$ is a holomorphic function $S(X,\mu)$. Then at the minimum $\langle X\rangle = M+\theta^2 F$, we obtain that the supersymmetry-breaking gaugino mass is given by

$$\tilde{M}_g(\mu) = -\frac{1}{2}\frac{\partial \ln S(X,\mu)}{\partial \ln X}\bigg|_{X=M} \frac{F}{M} \ . \tag{3}$$

The $S$ functional dependence on the Goldstino superfield $X$ can be obtained by observing that the scalar component of $S$ is

$$S(M,\mu) = \frac{\alpha(M,\mu)^{-1}}{16\pi} - \frac{i\Theta}{32\pi^2} \ , \tag{4}$$

where $\alpha$ is the gauge-coupling strength and $\Theta$ the topological vacuum angle. Now, the function $\alpha(M,\mu)$ is determined by integrating the RG evolution from an ultraviolet scale $\Lambda_{UV}$ down to $\mu$ across the messenger threshold $M$. The one-loop RG equation is

$$\frac{d}{dt}\alpha^{-1} = \frac{b}{2\pi} \ , \tag{5}$$

where $t = \ln\mu$ and $b$ is the $\beta$-function coefficient, given by $b = 3N_c - N_f$ in an $SU(N_c)$ gauge theories with $N_f$ flavours. Above the scale $M$, the $N$ messenger superfields transforming as $N_c + \bar{N}_c$ under $SU(N_c)$ contribute to the gauge running and the $\beta$-function coefficient is

$$b' = b - N \ , \tag{6}$$

if $b$ is the coefficient below $M$. By using tree level matching and one-loop running one gets

$$\text{Re}\ S(M,\mu) = \frac{\alpha^{-1}(M,\mu)}{16\pi} = \frac{\alpha^{-1}(\Lambda_{UV})}{16\pi} + \frac{b'}{32\pi^2}\ln\frac{|M|}{\Lambda_{UV}} + \frac{b}{32\pi^2}\ln\frac{\mu}{|M|} \ . \tag{7}$$

This expression and the requirement of holomorphy fix $S(X,\mu)$ to have the form

$$S(X,\mu) = S(\Lambda_{UV}) + \frac{b'}{32\pi^2}\ln\frac{X}{\Lambda_{UV}} + \frac{b}{32\pi^2}\ln\frac{\mu}{X} \ . \tag{8}$$

Notice that the above equation reproduces also the renormalization $\Theta \to \Theta + (b-b')\arg(X)$ as dictated by the chiral anomaly. By differentiating eq. (8) with respect to $\ln X$ keeping $S(\Lambda_{UV})$ fixed, we can express the gaugino mass in eq. (3) as

$$\tilde{M}_g(\mu) = \frac{\alpha(\mu)}{4\pi}\ N\ \frac{F}{M}. \tag{9}$$

This is the familiar expression obtained by explicit loop integration. Notice that it already contains its one-loop RG evolution.

We now turn to discuss the supersymmetry-breaking terms for the matter fields. Let $Z_Q$ be the wave-function renormalization of the chiral superfield $Q$

$$\mathcal{L} = \int d^4\theta Z_Q(X,X^\dagger)\ Q^\dagger Q\ . \tag{10}$$

In contrast to the case of the gauge multiplet, here we are dealing with the renormalization of a $D$-term and therefore $Z_Q$ is a real function of *both* $X$ and $X^\dagger$. Replacing the superfield-$X$ VEV in eq. (10), we find

$$\mathcal{L} = \int d^4\theta \left( Z_Q + \frac{\partial Z_Q}{\partial X} F\theta^2 + \frac{\partial Z_Q}{\partial X^\dagger} F^\dagger\bar\theta^2 + \frac{\partial^2 Z_Q}{\partial X \partial X^\dagger} FF^\dagger\theta^2\bar\theta^2 \right)\Bigg|_{X=M} Q^\dagger Q\ . \tag{11}$$

It is useful to define a new variable $Q'$ with canonically normalized kinetic terms,

$$Q' \equiv Z_Q^{\frac{1}{2}} \left(1 + \frac{\partial \ln Z_Q}{\partial X} F\theta^2\right)\Bigg|_{X=M} Q\ \equiv \mathcal{Z} Q. \tag{12}$$

Expressing eq. (11) in terms of $Q'$, we find that linear terms in $F$ disappear, and we are left with a quadratic term corresponding to a supersymmetry-breaking mass for the scalar component of the chiral multiplet

$$\tilde{m}^2_Q(\mu) = -\frac{\partial^2 \ln Z_Q(X,X^\dagger,\mu)}{\partial \ln X\ \partial \ln X^\dagger}\Bigg|_{X=M} \frac{FF^\dagger}{MM^\dagger}\ . \tag{13}$$

The derivatives in eq. (13) are computed keeping the couplings at $\Lambda_{UV}$ fixed. Again we have made explicit the dependence on the renormalization scale $\mu$. The redefinition of the chiral superfield $Q$ in eq. (12) however does not leave the superpotential invariant, and in particular it gives rise to $A$-type supersymmetry-breaking terms proportional to $F$. Considering a superpotential $W(Q)$ with the fields $Q_i$ redefined by $Q'_i = \mathcal{Z}_i Q_i$, we obtain an $A$-type contribution to the scalar potential

$$V = \sum_i A_i Q_i \partial_{Q_i} W(Q) + \text{h.c.} \tag{14}$$

involving the scalar components of the corresponding superfields, where

$$A_i(\mu) = \left.\frac{\partial \ln Z_{Q_i}(X, X^\dagger, \mu)}{\partial \ln X}\right|_{X=M} \frac{F}{M} \ . \tag{15}$$

Notice that eq. (15) corresponds to the coefficients $A_i$ defined in eq. (14) when $W(Q)$ is expressed in terms of renormalized fields and running coupling constants at the scale $\mu$.

Let us now obtain $Z_Q(X, X^\dagger)$ from RG evolution. The crucial remark is that $Z_Q$ must be a function of the product $XX^\dagger$. This is due to invariance under the chiral symmetry $X \to e^{i\varphi}X$, $\bar\Phi\Phi \to e^{-i\varphi}\bar\Phi\Phi$. This symmetry is anomalous, but this has no effect in perturbation theory as the anomaly only affects the $\Theta$ angle, as seen in eq. (8). It is then straightforward to solve the RG evolution for the c-number $Z_Q(M, \mu)$ and substitute $M \to \sqrt{XX^\dagger}$ at the end of the calculation. Now, $Z_Q$ will just be a power series in $L_\Lambda = \ln(\mu^2/\Lambda_{UV}^2)$ and $L_X = \ln(\mu^2/XX^\dagger)$. This property has a very important consequence that greatly simplifies the calculation of soft masses. The contribution to the wave-function renormalization at the loop order $\ell$ can be written as

$$\ln Z_Q(X, X^\dagger, \mu) = \alpha_{UV}^{\ell-1} P_\ell(\alpha_{UV} L_X, \alpha_{UV} L_\Lambda) \ , \tag{16}$$

where $\alpha_{UV} = \alpha(\Lambda_{UV})$ and $P_\ell$ ia s function computed by integrating the RG equations. Using eq. (13), we see that the corresponding contribution to the soft scalar mass is

$$\tilde m_Q^2(\mu) = \alpha(\mu)^{\ell+1} \tilde P_\ell \left(\alpha(\mu) L_X\right) \ , \tag{17}$$

where the dependence on $\Lambda_{UV}$ disappears because of the invariance under the RG, and $\tilde P_\ell$ is a function related to the second derivative of $P_\ell$. Therefore it is sufficient to use the RG at the order $\ell = 1$ to obtain the $\alpha^2$ contribution to soft masses. We conclude that, while sfermion masses arise from *finite* two loop diagrams, they can be extracted just by using the one-loop anomalous dimensions. Soft masses are generated from loop momenta of the order of the threshold $M$, but can be reconstructed from the behaviour of wave-function renormalization far away from threshold (RG evolution).

Let us now derive the well-known results for gauge-mediated soft terms in the scalar sector by using our technique. The one-loop RG equation for the wave-function renormalization of a chiral superfield is

$$\frac{d}{dt} \ln Z_Q = \frac{c}{\pi} \alpha \ . \tag{18}$$

Here $c$ is the quadratic Casimir of the $Q$ gauge representation ($c = (N^2-1)/(2N)$ for an $SU(N)$ fundamental). The function $Z_Q(M, \mu)$ is determined by integrating eq. (18) from the ultraviolet scale $\Lambda_{UV}$ down to $\mu$ with tree-level matching at the intermediate threshold $M$. Substituting $M \to \sqrt{XX^\dagger}$ we get

$$Z_Q(X, X^\dagger, \mu) = Z_Q(\Lambda_{UV}) \left[\frac{\alpha(\Lambda_{UV})}{\alpha(X)}\right]^{\frac{2c}{b'}} \left[\frac{\alpha(X)}{\alpha(\mu)}\right]^{\frac{2c}{b}} \ , \tag{19}$$

where the $X$ dependence of $\alpha(\mu)$ and $\alpha(X)$ is given by

$$\alpha^{-1}(\mu) = 16\pi \ \mathrm{Re}\ S(\mu) = \alpha^{-1}(X) + \frac{b}{4\pi} \ln \frac{\mu^2}{XX^\dagger} \ , \tag{20}$$

$$\alpha^{-1}(X) = 16\pi \text{ Re } S(X) = \alpha^{-1}(\Lambda_{UV}) + \frac{b'}{4\pi} \ln \frac{XX^\dagger}{\Lambda_{UV}^2} \ . \tag{21}$$

Notice that $S(X,\mu)$ is chiral, as required by supersymmetry, while $\alpha$, proportional to the real part of $S(X,\mu)$, is not. Using eq. (13) and performing the derivatives in $X$ while keeping the ultraviolet parameters $Z_Q(\Lambda_{UV})$ and $\alpha(\Lambda_{UV})$ fixed, it is easy to obtain the supersymmetry-breaking masses and trilinear terms,

$$\tilde{m}_Q^2(\mu) = 2c \ \frac{\alpha^2(\mu)}{(4\pi)^2} \ N \left[\xi^2 + \frac{N}{b}(1-\xi^2)\right] \ \left(\frac{F}{M}\right)^2 \ , \tag{22}$$

$$A_i(\mu) = \frac{2c_i}{b} \ \frac{\alpha(\mu)}{4\pi} \ N(\xi - 1) \ \frac{F}{M} \ , \tag{23}$$

$$\xi \equiv \frac{\alpha(M)}{\alpha(\mu)} = \left[1 + \frac{b}{2\pi}\alpha(\mu) \ln \frac{M}{\mu}\right]^{-1} \ . \tag{24}$$

If the superfield $Q$ is charged under different gauge groups, eqs. (22) and (23) are generalized by summing over the different gauge coupling constants. We have recovered the familiar formulae for the soft terms in theories with gauge-mediated supersymmetry breaking, including the leading-log effect in the renormalization from the messenger scale $M$ to the low energy scale $\mu$. In particular, all $A_i$ vanish at $\mu = M$ ($\xi = 1$), but at low-energies acquire a renormalization proportional to the gaugino mass.

A few comments are in order at this point. First of all, our results represent the lowest order contribution to soft terms in an expansion in powers of $y = F/M^2$. The higher-order terms correspond to higher-derivative interactions in the gauge and matter effective kinetic terms. Indeed the expansion parameter is given by $y = D^2(1/X)$ where $D^2$ is the superspace covariant derivative squared. These higher-derivative interactions are genuinely finite effects and cannot be reconstructed with our RG technique. This is not surprising as our method crucially relies on the approximate supersymmetry of the low-energy effective Lagrangian and, for $F \sim M^2$, the messenger spectrum badly violates supersymmetry. Our technique is however very useful as in a vast class of interesting models one has $y \ll 1$. Indeed, as shown by the explicit calculations of refs. [3, 4], the asymptotic formulae derived at $y = 0$ are reached very quickly, so that already for $F/M^2 \sim 0.3$ our approximation is extremely good. This is true also because the expansion parameter is actually $y^2$.

Our method as described above crucially relies on the possibility of substituting unambiguously $M$ with $X$ or $\sqrt{XX^\dagger}$. For one-loop RG evolution this surely poses no problem. The question however arises when considering the evolution of $S$ beyond one-loop. This is because in any practical regularization scheme like DRED, $S$ ceases being holomorphic in $X$ as soon as two-loop evolution is considered. For instance, at two-loops, in the expansion for $\alpha^{-1}(M,\mu)$, there appear $\ln^n(M/\mu)$ terms with $n > 1$. Thus the analytic continuation $M \to X$ is no longer unambiguous. These higher-order effects are absent in the Wilsonian $\beta$ function, for which holomorphy is explicitly manifest to all orders in perturbation theory [5]. On the other hand, for practical purposes, one needs the full gauge propagator and holomorphy has to be abandoned. The correct prescription, as discussed in ref. [6], is motivated by the all-order formula for $\alpha^{-1}(\mu)$. For instance in an abelian gauge theory this is given by [7]

$$\alpha^{-1}(M,\mu) = \alpha^{-1}(\Lambda_{UV}) + \frac{b}{2\pi}\ln\frac{\mu}{M} + \frac{b'}{2\pi}\ln\frac{M}{\Lambda_{UV}} - \frac{1}{2\pi}\sum_i T_i \ln Z_i(M,\mu)\ , \tag{25}$$

where $i$ runs over the light matter fields and $T_i$ are the Dynkin indices. The $\ln Z_i$ can be interpreted as originating from a non-holomorphic rescaling of the matter fields via the Konishi anomaly [8]. The rescaling, albeit non holomorphic, must be supersymmetric: a chiral superfield must be rescaled with a chiral background wave function. This is precisely what eq. (12) does. The right prescription to promote $\alpha$ to a superfield is then to substitute $Z$ with $\mathcal{Z}^\dagger\mathcal{Z}$, so that the complex version of eq. (25) becomes

$$S(X,\mu) = S(\Lambda_{UV}) + \frac{b}{32\pi^2}\ln\frac{\mu}{X} + \frac{b'}{32\pi^2}\ln\frac{X}{\Lambda_{UV}} - \frac{1}{32\pi^2}\sum_i T_i \ln \mathcal{Z}_i(M,\mu). \tag{26}$$

In the course of this paper we will only be concerned with quantities for which the one-loop gauge $\beta$ function suffices, so that we will not get involved into these subtleties. The only quantities which we study to the next-to-leading accuracy are the Higgs mass parameters for which the two-loop RG evolution occurs only in the matter wave-function.

# 3 Higgs Mass Parameters at the Next-to-Leading Order

In the previous section we have computed the supersymmetry-breaking terms induced by gauge couplings. Neglecting the Yukawa couplings is usually a good approximation in all practical cases, aside from the Higgs mass parameters. The large top-quark Yukawa coupling induces a three-loop contribution to the hypercharge +1 Higgs mass parameter $m_H^2 \sim \alpha_t\alpha_s^2/\pi^3 \times F^2/M^2$, which is of the same order of magnitude of the two-loop weak contribution $m_H^2 \sim \alpha_W^2/\pi^2 \times F^2/M^2$. Moreover, this top-quark Yukawa contribution plays a key rôle in the phenomenology of gauge mediation, since it drives $m_H^2$ negative, triggering electroweak-symmetry breaking. This contribution is known in the leading-log approximation and we will reproduce it here using our method. However, in realistic gauge-mediation models, the logarithm of the ratio between the messenger and squark masses $\ln(M/\tilde{m}_Q) \gtrsim \ln(2\pi/\alpha_s)$ can be as small as 4, and therefore the subleading constant term is not necessarily insignificant. The computation of this term requires to go beyond the leading-log approximation, and it has not been done in previous literature; we will present it in this section. This example will illustrate how our method can be simply used to perform computations which are exceedingly involved if viewed in terms of Feynman diagrams of component fields.

Let us start with the leading-log calculation. For simplicity we will consider only top-quark Yukawa and QCD effects, but the inclusion of electroweak effects is straightforward and gives rise to well-known results. At the one-loop level, the relevant RG equation for the hypercharge +1 Higgs wave-function renormalization is

$$\frac{d}{dt}\ln Z_H = -\frac{3}{2\pi}\alpha_t\ . \tag{27}$$

The RG equation for the top-quark Yukawa coupling, $\alpha_t = h_{top}^2/(4\pi)$, is

$$\frac{d}{dt}\alpha_t = \frac{\alpha_t}{\pi}\left(3\alpha_t - \frac{8}{3}\alpha_s\right) , \tag{28}$$

while the RG equation for the QCD gauge couplings $\alpha_s$ is given in eq. (5), with $b = 3$ below the messenger scale $M$ and $b' = 3 - N$ above $M$. Notice that we are actually calculating the evolution of wave functions and that, at any energy scale $\mu$, $\alpha_t = \alpha_t(\Lambda_{UV}) Z_{Q_L}^{-1} Z_{\bar{U}_R}^{-1} Z_H^{-1}$. The use of $\alpha_t$ is a convenient way to describe the evolution of the product of wave functions $Z_{Q_L} Z_{\bar{U}_R} Z_H$. The solution of eq. (27) is

$$Z_H(X, X^\dagger, \mu) = Z_H(\Lambda_{UV}) \left[\frac{\alpha_t(\Lambda_{UV})}{\alpha_t(\mu)}\right]^{\frac{1}{2}} \left[\frac{\alpha_s(\Lambda_{UV})}{\alpha_s(X)}\right]^{-\frac{8}{3b'}} \left[\frac{\alpha_s(X)}{\alpha_s(\mu)}\right]^{-\frac{8}{3b}} , \tag{29}$$

where the expressions for $\alpha_s(\mu)$ and $\alpha_s(X)$ are given in eqs. (20)–(21), and

$$\alpha_t(\mu) = \frac{\alpha_t(X)E}{1 - \frac{3}{\pi}\alpha_t(X)F} , \tag{30}$$

$$\alpha_t(X) = \frac{\alpha_t(\Lambda_{UV})E'}{1 - \frac{3}{\pi}\alpha_t(\Lambda_{UV})F'} , \tag{31}$$

$$E \equiv \left[\frac{\alpha_s(\mu)}{\alpha_s(X)}\right]^{\frac{16}{3b}} , \qquad F \equiv \frac{2\pi}{\frac{16}{3} - b}\left[\alpha_s^{-1}(X) - \alpha_s^{-1}(\mu)E\right] , \tag{32}$$

$$E' \equiv \left[\frac{\alpha_s(X)}{\alpha_s(\Lambda_{UV})}\right]^{\frac{16}{3b'}} , \qquad F' \equiv \frac{2\pi}{\frac{16}{3} - b'}\left[\alpha_s^{-1}(\Lambda_{UV}) - \alpha_s^{-1}(X)E'\right] . \tag{33}$$

Using the general formula in eq. (13), we can express the top-quark Yukawa contribution to $m_H^2$ as

$$m_H^2(\mu) = \frac{\alpha_t(\mu)}{\pi^2} N\,(1-\xi)\left\{N(1-\xi)\left[\frac{\alpha_t(\mu)}{8}(1-I)^2 - \frac{\alpha_s(\mu)}{9}\right] - \frac{\alpha_s(\mu)}{3}\xi I\right\}\left(\frac{F}{M}\right)^2 , \tag{34}$$

$$\xi \equiv \frac{\alpha_s(M)}{\alpha_s(\mu)} = \left[1 + \frac{3}{2\pi}\alpha_s(\mu)\ln\frac{M}{\mu}\right]^{-1} , \tag{35}$$

$$I \equiv \frac{9}{7}\,\frac{(1-\xi^{\frac{7}{9}})\xi}{(1-\xi)} . \tag{36}$$

If the logarithm $\ln(M/\mu)$ is not too large, it is convenient to expand eq. (34) and obtain the three-loop contribution in the leading-log approximation:

$$m_H^2(\mu) = -\frac{\alpha_t(\mu)\alpha_s^2(\mu)}{2\pi^3} N\left(\ln\frac{M}{\mu}\right)\left(\frac{F}{M}\right)^2 . \tag{37}$$

This is the known negative contribution to the Higgs mass squared parameter which leads to electroweak-symmetry breaking. In the following we will compute the three-loop $\alpha_t\alpha_s^2$ term with no logs.

Before proceeding to the next-to-leading order calculation, we remark that, for gauge-mediated models in which the Higgs mixing mass $\mu_H$ is a hard parameter[2] (*i.e.* $\mu_H$ is not generated by loops involving messenger fields), then eq. (29) also provides the expression of the bilinear parameter $B$ at low energies. From eqs. (14) and (15), we find

$$B(\mu) = \frac{\alpha_t(\mu)}{4\pi} \; N \; (1-\xi)(1-I) \; \frac{F}{M} \; , \tag{38}$$

which, expanded to the two-loop level, becomes

$$B(\mu) = \frac{\alpha_t(\mu)\alpha_s^2(\mu)}{2\pi^3} \; N \; \left(\ln^2 \frac{M}{\mu}\right) \; \frac{F}{M} \; . \tag{39}$$

If the mechanism responsible for generating the $\mu_H$ term also gives a contribution to $B$, this should be added to eqs. (38) and (39).

Let us now turn to the next-to-leading order calculation. We are interested in the subleading contribution to the term $\mathcal{O}(\alpha_t\alpha_s^2)$, *i.e.* the term suppressed by $1/\ln(M/\mu)$ with respect to eqs. (37) and (39). For this purpose it is sufficient to retain only the $\mathcal{O}(\alpha_t\alpha_s)$ term in the RG equation for $Z_H$, the $\mathcal{O}(\alpha_t\alpha_s^2)$ term in the equation for $\alpha_t$, and use the RG equation for $\alpha_s$ at the leading order, eq. (5). In the $\overline{DR}$ scheme, the relevant RG equations are [9]

$$\frac{d}{dt} \ln Z_H = -\frac{3}{2\pi}\alpha_t - \frac{2}{\pi^2}\alpha_t\alpha_s \; , \tag{40}$$

$$\frac{d}{dt}\alpha_t = \frac{\alpha_t}{\pi} \; \left(3\alpha_t - \frac{8}{3}\alpha_s\right) - \frac{2d}{\pi^2}\alpha_t\alpha_s^2 \; , \tag{41}$$

In eq. (41), $d = 1/9$ below the messenger scale $M$ and $d = (1-3N)/9$ above $M$. Expanding at the three-loop level the solution to the differential equation (40), we obtain

$$\ln \frac{Z_H(X, X^\dagger, \mu)}{Z_H(\Lambda_{UV})} = \frac{\alpha_t(\mu)\alpha_s^2(\mu)}{(2\pi)^3} \; N \; \left[\frac{1}{3}\ln^3\left(\frac{XX^\dagger}{\mu^2}\right) + 2\ln^2\left(\frac{XX^\dagger}{\mu^2}\right)\right] + f[\ln(\Lambda_{UV}/\mu)] \; . \tag{42}$$

Here $f$ is a function of $\ln(\Lambda_{UV}/\mu)$ and it is independent of $X$. The $\log^2$ term in eq. (42) represents the subleading correction. Calculation of the linear-log term (which contributes to $B$, but not to $m_H^2$) would require knowledge of the next order in perturbation theory for the $\beta$ functions and the anomalous dimensions. From eqs. (13) and (15), we obtain

$$m_H^2(\mu) = -\frac{\alpha_t(\mu)\alpha_s^2(\mu)}{2\pi^3} \; N \; \left(\ln\frac{M}{\mu} + 1\right) \; \left(\frac{F}{M}\right)^2 \; . \tag{43}$$

$$B(\mu) = \frac{\alpha_t(\mu)\alpha_s^2(\mu)}{2\pi^3} \; N \; \left(\ln^2\frac{M}{\mu} + 2\ln\frac{M}{\mu}\right) \; \frac{F}{M} \; . \tag{44}$$

To complete the next-to-leading order calculation we have to include the matching conditions at the ultraviolet scale $M$ and the infrared scale $\mu$. Notice that even though we only need to

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[2] In order not to generate confusion between the renormalization scale and the Higgs mixing mass parameter, we have chosen to denote the latter by $\mu_H$, instead of the usual symbol $\mu$.

evolve $\alpha_s$ at one-loop, it is crucial to correctly match $\alpha_s$ at the next-to-leading order in the chosen scheme. In the $\overline{DR}$ scheme, the matching of the running gauge coupling constant for thresholds of heavy scalars and fermions has to be done at a scale equal to the mass of the heavy particles [10]. Therefore in eqs. (43)–(44) we identify $M$ with the physical mass of the messenger fields. To determine the infrared matching, one has to include the one-loop effective potential which cancels the $\mu$ dependence at the appropriate order in perturbation theory. This can be seen explicitly by expanding the effective potential in powers of ratios between the Higgs fields $H$, $\bar{H}$ and the supersymmetry-breaking stop mass $\tilde{m}_t$, see eq. (22). This is a good approximation since, in realistic models, the stop turns out to be rather heavy. In the $\overline{DR}$ scheme, we find

$$\begin{aligned} V_{1-loop} &= \frac{1}{64\pi^2}\mathrm{STr}\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{\mu^2}-\frac{3}{2}\right)\simeq \\ &\simeq \frac{3\alpha_t(\mu)}{2\pi}\left[|A_t(\mu)H-\mu_H\bar{H}^\dagger|^2\ln\frac{\tilde{m}_t(\mu)}{\mu}+|H|^2\tilde{m}_t(\mu)^2\left(2\ln\frac{\tilde{m}_t(\mu)}{\mu}-1\right)\right] . \end{aligned} \tag{45}$$

Here $A_t$ is the coefficient of the supersymmetry-breaking trilinear interaction for the stop, and $\mu_H$ is the supersymmetric Higgs mixing mass. Therefore $V_{1-loop}$ gives an effective contribution to the soft-breaking parameters

$$\begin{aligned} \delta m_H^2 &= \frac{3\alpha_t(\mu)}{2\pi}\left[A_t^2(\mu)\ln\frac{\tilde{m}_t(\mu)}{\mu}+\tilde{m}_t^2(\mu)\left(2\ln\frac{\tilde{m}_t(\mu)}{\mu}-1\right)\right]\simeq \\ &\simeq \frac{\alpha_t(\mu)\alpha_s^2(\mu)}{2\pi^3}\ N\ \left(\ln\frac{\tilde{m}_t(\mu)}{\mu}-\frac{1}{2}\right)\ \left(\frac{F}{M}\right)^2 , \end{aligned} \tag{46}$$

$$\delta B = -\frac{3\alpha_t(\mu)}{2\pi}A_t(\mu)\ln\frac{\tilde{m}_t(\mu)}{\mu}\simeq\frac{\alpha_t(\mu)\alpha_s^2(\mu)}{\pi^3}\ N\ \ln\frac{\tilde{m}_t(\mu)}{\mu}\ln\frac{M}{\mu}\ \frac{F}{M} . \tag{47}$$

The right-hand sides of eqs. (46)–(47) are obtained by retaining only the genuine three-loop effects. These contributions can be reabsorbed in the definition of effective supersymmetry-breaking parameters. By adding eqs. (46)–(47) to eqs. (43)–(44) we obtain that the effective values of the supersymmetry-breaking parameters are

$$m^2_{H\mathrm{eff}} = -\frac{\alpha_t(\tilde{m}_t)\alpha_s^2(\tilde{m}_t)}{2\pi^3}\ N\ \left(\ln\frac{M}{\tilde{m}_t}+\frac{3}{2}\right)\ \left(\frac{F}{M}\right)^2 . \tag{48}$$

$$B_{\mathrm{eff}} = \frac{\alpha_t(\tilde{m}_t)\alpha_s^2(\tilde{m}_t)}{2\pi^3}\ N\ \left(\ln^2\frac{M}{\tilde{m}_t}+2\ln\frac{M}{\tilde{m}_t}\right)\ \frac{F}{M} . \tag{49}$$

The result in eq. (49) agrees with the explicit two-loop calculation of the component-field Feynman diagrams presented in eqs. (27) and (29) of ref. [11]. The next-to-leading correction to $m_H^2$, which has been calculated here for the first time, can be as large as 30%. This modifies by the same amount the extraction of the $\mu_H^2$ parameter from the electroweak-breaking condition and consequently the physical mass spectrum of charginos and neutralinos.

# 4 Superpotential Couplings between Messengers and Matter

Models with gauge-mediated supersymmetry breaking usually assume that messengers and ordinary matter do not couple directly in the superpotential. This is to avoid any new source of flavour breaking, which is, after all, the primary motivation for these models. However, $SU(3) \times SU(2) \times U(1)$ gauge invariance alone does not forbid such couplings. Actually superpotential interactions involving messenger and Higgs superfields were considered in ref. [12], while interactions between messengers and quarks or leptons were studied in ref. [13]. It was shown [12, 13] that these couplings do not induce at the leading order in $F/M$ one-loop supersymmetry-breaking masses for the scalar components of the matter superfields. This property was explained in ref. [14] using superfield language. In this section we will give the general expression for supersymmetry-breaking terms induced by messenger-matter couplings, including the two-loop soft masses for the scalar components of chiral superfields.

We will consider a coupling in the superpotential between one matter and two messenger superfields, $\lambda Q \Phi_1 \Phi_2$, or between two matter and one messenger superfields, $\lambda Q_1 Q_2 \Phi$. The formulae we present are the same in both cases. For simplicity we assume that all superfields are charged under a simple gauge group. The generalization to gauge groups made of direct products of simple groups is straightforward. The RG equations for the wave-function renormalization of the matter superfield $Q_i$ is

$$\frac{d}{dt} \ln Z_{Q_i} = \frac{1}{\pi} \left( c_i \alpha - \frac{d_i}{2} \alpha_\lambda \right) , \tag{50}$$

with $\alpha_\lambda \equiv \lambda^2/(4\pi)$. Here $c_i$ is the quadratic Casimir of the $Q_i$ gauge representation and $d_i$ is the number of fields circulating in the Yukawa loop. For instance, if the $\lambda$ interaction involves a fundamental, an anti-fundamental, and a singlet of $SU(N)$, then $d_i = 1$ or $N$ if $Q_i$ is a fundamental or a singlet, respectively. The RG equation for the gauge coupling constant is given in eq. (5) and the one for $\alpha_\lambda$ is

$$\frac{d}{dt} \alpha_\lambda = \frac{\alpha_\lambda}{\pi} \left( \frac{D}{2} \alpha_\lambda - C\alpha \right) . \tag{51}$$

Here $C = \sum_i c_i$, $D = \sum_i d_i$ with the sum extended to all superfields participating to the $\lambda$ interaction. The solution to eq. (50) is

$$Z_{Q_i}(X, X^\dagger, \mu) = Z_{Q_i}(\Lambda_{UV}) Z_{GM}(X, X^\dagger, \mu) Z_\lambda(X, X^\dagger, \mu) \tag{52}$$

$$Z_{GM}(X, X^\dagger, \mu) \equiv \left[ \frac{\alpha(\Lambda_{UV})}{\alpha(X)} \right]^{\frac{2c_i}{b'}} \left[ \frac{\alpha(X)}{\alpha(\mu)} \right]^{\frac{2c_i}{b}} , \tag{53}$$

$$Z_\lambda(X, X^\dagger, \mu) \equiv \left[ \frac{\alpha_\lambda(\Lambda_{UV})}{\alpha_\lambda(X)} \right]^{\frac{d_i}{D}} \left[ \frac{\alpha(\Lambda_{UV})}{\alpha(X)} \right]^{-\frac{2Cd_i}{Db'}} , \tag{54}$$

$$\alpha_\lambda(X) = \frac{\alpha_\lambda(\Lambda_{UV}) E'}{1 - \frac{D}{2\pi} \alpha_\lambda(\Lambda_{UV}) F'} , \tag{55}$$

$$E' \equiv \left[\frac{\alpha(X)}{\alpha(\Lambda_{UV})}\right]^{\frac{2C}{b'}} , \qquad F' \equiv \frac{2\pi}{2C-b'}\left[\alpha^{-1}(\Lambda_{UV}) - \alpha^{-1}(X)E'\right] . \tag{56}$$

The factor $Z_{GM}$ is the usual contribution from gauge mediation, while $Z_\lambda$ is the new contribution from the $\lambda$ interaction. Notice that $Z_\lambda$ is independent of $\mu$ (for $\mu < M$) since the $\lambda$ interaction is not present in the effective theory with messenger fields integrated out. The soft terms (which originate from differentiating the logarithm of $Z$) are given by the sum of the gauge-mediated contribution, given in eqs. (22)–(23), and a genuine new contribution given by

$$\delta\tilde{m}^2_{Q_i}(\mu) = \frac{d_i}{8\pi^2}\alpha_\lambda(M)\left[\frac{D}{2}\alpha_\lambda(M) - C\alpha(M)\right]\left(\frac{F}{M}\right)^2 , \tag{57}$$

$$\delta A_i(\mu) = -\frac{d_i}{4\pi}\alpha_\lambda(M)\frac{F}{M} . \tag{58}$$

As expected, supersymmetry-breaking masses vanish at one loop at the leading order in $F/M^2$. The one-loop contribution proportional to $(\alpha_\lambda/4\pi)(F^4/M^6)$ computed in ref. [13] is smaller than the two-loop expression shown in eq. (57), as long as $F/M^2 \lesssim \sqrt{\alpha_s/(4\pi)}$. In contrast to the case of gauge mediation, $A$ terms are generated at one loop, see eq. (58). Therefore, if superpotential matter-messenger interactions exist, their contribution to supersymmetry-breaking parameters can substantially modify the physical spectrum of new particles.

The superpotential $\lambda$ interactions can find an interesting application in the generation of the Higgs-mixing $\mu_H$ parameter. It is known [12] that gauge-mediation models have difficulties in generating dynamically the $\mu_H$ term. In particular, the introduction of a singlet superfield $N$ with superpotential couplings $W = NH\bar{H} + N^3$ does not easily solve the problem because, at the messenger scale $M$, all supersymmetry-breaking terms involving $N$ vanish at the leading order. A negative mass squared for the $N$ scalar component and trilinear couplings are generated by the RG evolution to the weak scale, but these parameters turn out to be too small to insure a correct symmetry-breaking pattern and an acceptable mass spectrum. A possible solution invoked in ref. [2] is to introduce new light particles which increase the renormalization effects.

The results shown in this section open a different possibility. Suppose $N$ interacts with the messenger fields $\Phi$, $\bar{\Phi}$ with a superpotential coupling

$$W = \lambda N\bar{\Phi}\Phi . \tag{59}$$

Now the scalar component of $N$ gets a two-loop negative mass squared proportional to $\alpha_\lambda\alpha_s$, see eq. (57), where $\alpha_s$ is the QCD coupling constant. Moreover supersymmetry-breaking trilinear terms involving the scalar field $N$ are generated at one loop. The electroweak-breaking conditions can now be much more easily satisfied. Notice however that the superfield $N$ has the same quantum numbers as the Goldstino superfield $X$, and therefore the wave-function renormalization mixes these two superfields generating a term $\int d^4\theta X^\dagger N$. After supersymmetry breaking this term leads to a tadpole for $F_N$ larger than the electroweak scale. This can be avoided by extending the messenger sector in such a way that $N$ and $X$ carry different quantum numbers under some new symmetry. A simple example is the case of two messenger flavours with superpotential

$$W = X(\bar{\Phi}_1\Phi_1 + \bar{\Phi}_2\Phi_2) + N\bar{\Phi}_1\Phi_2 . \tag{60}$$

The theory has a discrete $Z_3$ symmetry under which all chiral supefields (including Higgs and matter) have charge 1/3, but for $\Phi_1$ and $\bar{\Phi}_2$ which have charges -1/3 and for $X$ which is neutral. This symmetry, broken only at the weak scale, distinguishes between $X$ and $N$. Finally we just remark that a different possibility is using an interaction of the form $\lambda N H \bar{\Phi}$ instead of eq. (59). We have not investigated if this can lead to a successful mechanism of electroweak breaking.

# 5 Gauge Messengers

So far we have considered the most familiar case in which the messenger particles belong to chiral superfields. However it is also possible that gauge supermultiplets behave as messengers. We are envisaging a situation in which the supersymmetry-breaking VEV is also responsible for spontaneous breaking of some gauge symmetry containing the Standard Model (SM) as a subgroup. The vector bosons corresponding to the broken generators, together with their supersymmetric partners, receive masses proportional to $M$. However supersymmetry-breaking effects proportional to $F$ split the gauge supermultiplets at tree level and consequently soft terms for observable fields are generated by quantum effects. In this section we will compute these terms. We will show that our method enormously simplifies a calculation which is complicated if performed by evaluating component-field Feynman diagrams. But this is not merely an exercise to show the power of our method. We will find that the contribution from gauge messengers, which have never been calculated before, are of the same order as those from chiral messengers and therefore can significantly modify the phenomenology of certain classes of models. For instance, this happens in models based on Witten's inverse hierarchy [15] or in more recent proposals that combine the inverse hierarchy with dynamical supersymmetry breaking [16, 14].

Let us first consider the case in which $\langle X \rangle$ spontaneously breaks the gauge group $G \to H$, in such a way that the gauge coupling constant is continuous at the threshold. The calculation of the soft term generated by gauge interactions is completely analogous to the one presented in sect. 2 for chiral messengers. Equations (9), (22), and (23) are still valid, but now $N$ measures the total effect of chiral and gauge messengers:

$$N = b - b' = N_f - 2(C_G - C_H) \ . \tag{61}$$

$N$ is the difference between the low-energy ($H$) and high-energy ($G$) $\beta$-function coefficients. For chiral messengers, this is just the number of flavours of heavy multiplets $N_f$. In the case of gauge messengers, one has to add the contribution of heavy gauge bosons, equal to $-3(C_G - C_H)$, and the contribution of the would-be Goldstone bosons, equal to $(C_G - C_H)$. Here $C_G$ is the quadratic Casimir of the adjoint representation of $G$, equal to $N$ for an $SU(N)$ group. Therefore, the pure gauge-messenger effect is to reduce the value of $N$, and allow also negative values of the total $N$. Their contribution to the soft scalar masses at the scale $M$ is negative. Notice however that the renormalization to the low-energy scale gives a contribution to scalar masses proportional to $N^2$, which is always positive, see eq. (22).

Before discussing phenomenological applications, we believe it is useful to understand our results in terms of component fields. To follow the same procedure used in the case of chiral messengers, we first have to determine the tree-level mass splittings inside the gauge messenger supermultiplets, and then study how this is tranferred to the observable sector at the quantum level. The messenger mass spectrum can be derived from the kinetic terms

$$\mathcal{L} = \int d^4\theta X^\dagger e^V X + \left( \int d^2\theta \frac{1}{4g^2} W^{a\alpha} W^a_\alpha + \text{h.c.} \right) . \tag{62}$$

Instead of the usual Wess-Zumino gauge, it is more convenient to use a unitary gauge for the vector superfield, which eliminates the would-be Goldstone boson chiral multiplets from the superfield $X$. In this gauge, the vector superfield $V$ contains a vector boson $v_\mu$, two Weyl fermions $\lambda$ and $\chi$, a real scalar $C$, one real and one complex auxiliary fields $D$ and $N$ (we follow the conventions of ref. [17] and suppress the gauge index $a$). In order to have canonical kinetic terms and canonical field dimensions, we do the rescaling $(\lambda, v_\mu, D) \to g(\lambda, v_\mu, D)$ and $(C, \chi, N) \to M(C, \chi, N)$, where $g$ is the gauge coupling constant and $M$ is the VEV of the scalar component of the Goldstino superfield. Expanding eq. (62) in field bilinears, we find the usual kinetic terms for all propagating fields and the mass terms

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}(gM)^2 v^\mu v_\mu + \frac{1}{2}D^2 + \frac{1}{2}N^\dagger N + gMCD - gM(\chi\lambda + \text{h.c.}) + \\ &+ \frac{F}{M}\left( iCN - \frac{1}{2}\chi\chi + \text{h.c.} \right) + \frac{F^2}{M^2}C^2 . \end{aligned} \tag{63}$$

Eliminating the auxiliary fields $D$ and $N$, we obtain

$$\mathcal{L} = \frac{1}{2}(gM)^2 v^\mu v_\mu - \frac{1}{2}\left( g^2M^2 + 2\frac{F^2}{M^2} \right) C^2 - \frac{1}{2}\left( \psi^T \mathcal{M}_\psi \psi + \text{h.c.} \right) , \tag{64}$$

$$\psi \equiv \begin{pmatrix} \lambda \\ \chi \end{pmatrix} \qquad \mathcal{M}_\psi \equiv \begin{pmatrix} 0 & gM \\ gM & \frac{F}{M} \end{pmatrix} . \tag{65}$$

Therefore the gauge multiplet contains a vector boson with mass $gM$, two Weyl fermion with masses $[\sqrt{4g^2M^2 + (F/M)^2} \pm (F/M)]/2$, and one real scalar with mass squared $g^2M^2 + 2(F/M)^2$. Notice that the supersymmetry-breaking VEV $F$ spoils the usual superHiggs mass relation, but preserves the condition of vanishing supertrace.

Our result for the gauge-messenger contribution to gaugino masses, see eq. (61), can also be extracted from the result of an explicit component calculation of the GUT threshold corrections to gaugino masses [18]. For illustrative purposes, we want to rederive the same result from a component calculation in a simple model. Let us consider an $SU(2)$ gauge theory which is broken, in the supersymmetric limit, to $U(1)$ by the VEV of a triplet chiral superfield $X^i$ ($i = 1, 2, 3$), along the direction $\langle X_{1,2} \rangle = 0$, $\langle X_3 \rangle \neq 0$. We include the effect of supersymmetry breaking by introducing a superpotential interaction of $X$ with an external source $F$

$$W = F\sqrt{\vec{X}^2} = F\langle X_3 \rangle \left( 1 + \frac{X_+ X_-}{\langle X_3 \rangle^2} + ... \right) . \tag{66}$$

The interaction in eq. (66) describes a gauge-invariant implementation of the condition $\langle F_{X_3} \rangle = F$, $\langle F_{X_{1,2}} \rangle = 0$. Since we are interested in the mass of the light $U(1)$ gaugino (the "photino") at one loop, we just need to know the interaction Lagrangian for the charged fields. For this reason we have expanded eq. (66) around the X VEV, and retained only the leading terms involving

$X_\pm = (X_1 \pm iX_2)/\sqrt{2}$. In the Feynman-'t Hooft gauge and working at the first order in $F$, the scalar components of $X_+$ and $X_-$ are mass degenerate and therefore there is no physical mixing angle between the two states. On the other hand, the supersymmetry-breaking mass for the fermionic components of $X_+$ and $X_-$, see eq. (66), flips $X_+$ into $X_-$. Thus, at the first order in $F$ (and in the Feynman-'t Hooft gauge), there is no contribution to the "photino" mass from loops involving scalar and fermionic components of the "higgsinos" $X_\pm$. All we need to do is to compute a loop diagram with charged gauge bosons and gauginos. This gives a "photino mass"

$$\tilde{M} = -\frac{\alpha}{\pi}\,\frac{F}{\langle X_3\rangle}\ . \tag{67}$$

This result agrees with eq. (61). The high-energy $SU(2)$ theory with the chiral triplet $X$ has $b' = 6-2=4$, while $b=0$ since there are no light charged chiral superfields. The two-loop calculation of the supersymmetry-breaking scalar masses from gauge messengers is of course much more involved.

We want to apply now our results to the scenarios proposed in ref. [16, 14]. This requires to study the case in which $\langle X\rangle$ breaks the gauge group $G\times H \to H'$, with the coupling constant $\alpha$ of the low-energy gauge group $H'$ related to the coupling constants of the high-energy groups $G$ and $H$ by $\alpha^{-1} = \alpha_G^{-1} + \alpha_H^{-1}$. The case of an arbitrary mixing angle between gauge coupling constant is a trivial generalization of the formulae we present below. We also assume that ordinary matter is charged under $H$ and $H'$, but not under $G$.

The wave-function renormalizations for the gauge and matter superfields are, respectively

$$S(X,\mu) = S(\Lambda_{UV}) + \frac{b_H+b_G}{32\pi^2}\ln\frac{X}{\Lambda_{UV}} + \frac{b}{32\pi^2}\ln\frac{\mu}{X}\ , \tag{68}$$

$$Z_Q(X,X^\dagger,\mu) = Z_Q(\Lambda_{UV})\ \left[\frac{\alpha_H(\Lambda_{UV})}{\alpha_H(X)}\right]^{\frac{2c}{b_H}}\ \left[\frac{\alpha(X)}{\alpha(\mu)}\right]^{\frac{2c}{b}}\ , \tag{69}$$

$$\alpha^{-1}(\mu) = \alpha^{-1}(X) + \frac{b}{4\pi}\ln\frac{\mu^2}{XX^\dagger}\ , \tag{70}$$

$$\alpha^{-1}(X) = \alpha_G^{-1}(\Lambda_{UV}) + \alpha_H^{-1}(\Lambda_{UV}) + \frac{(b_H+b_G)}{4\pi}\ln\frac{XX^\dagger}{\Lambda_{UV}^2}\ , \tag{71}$$

$$\alpha_H^{-1}(X) = \alpha_H^{-1}(\Lambda_{UV}) + \frac{b_H}{4\pi}\ln\frac{XX^\dagger}{\Lambda_{UV}^2}\ . \tag{72}$$

Here $b_G$, $b_H$, and $b$ are the $\beta$-function coefficients of the high-energy groups $G, H$ and the low-energy group $H'$, respectively. From the general equations (3), (13), and (15) we obtain the expressions for the supersymmetry-breaking gaugino and scalar masses

$$\tilde{M}_g(\mu) = \frac{\alpha(\mu)}{4\pi}\ (b-b_H-b_G)\ \frac{F}{M}\ , \tag{73}$$

$$\tilde{m}_Q^2 = 2c\,\frac{\alpha^2(\mu)}{(4\pi)^2}\ \left\{\left[b+(R^2-2)b_H-2b_G\right]\xi^2 + \frac{(b-b_H-b_G)^2}{b}(1-\xi^2)\right\}\ \left(\frac{F}{M}\right)^2\ . \tag{74}$$

$$A_i(\mu) = \frac{2c_i}{b}\,\frac{\alpha(\mu)}{4\pi}\,\left[(bR - b_H - b_G)\xi - (b - b_H - b_G)\right]\,\frac{F}{M}\,, \tag{75}$$

$$\xi \equiv \frac{\alpha(M)}{\alpha(\mu)} = \left[1 - \frac{b}{2\pi}\alpha(\mu)\ln\frac{M}{\mu}\right]^{-1}\,, \tag{76}$$

$$R \equiv \frac{\alpha_H(M)}{\alpha(M)} = 1 + \frac{\alpha_H(M)}{\alpha_G(M)}\,. \tag{77}$$

Gauge messengers give a new contribution to scalar masses at $\mu = M$ ($\xi = 1$) proportional to $-2b_G$, see eq. (74). For an asymptotically-free group $G$, these contributions are negative and can destabilize squarks and sleptons. The RG evolution proportional to the gaugino mass squared helps to restore positivity, but there are strong constraints on the group $G$, as we will show below. It is also interesting to notice that the trilinear $A$ terms are generated even without running ($\xi = 1$), since $R > 1$, see eq. (75).

These formulae can now be directly applied to one of the examples discussed in ref. [14]: the "$SU(5)^3$ model". We have to identify $G$ with $SU(5)$, $H = H'$ with $SU(3)\times SU(2)\times U(1)$, $b$ with the SM coefficients, and $b_H = b - 5$, $b_G = 10$. It is easy to see from eq. (74) that the right-handed sleptons turn out to have negative squared masses. This disease could be cured by adding matter charged under $G$ in order to reduce $b_G$, source of the negative contribution. Making the group $G$ less asymptotically free can be dangerous, since this can destabilize the minimum of $X$ along the flat direction found in ref. [14]. This is a general difficulty of models with gauge messengers. Nevertheless, the other models presented in ref. [14] are viable, since $\langle X\rangle$ does not spontaneously break a gauge group which contains the SM.

Another example of a model with gauge messengers has been proposed by Murayama [16]. Although the Goldstino resides in a single field ($\Sigma$ in the notation of ref. [16]), the model has three different mass thresholds, because some fields ($S$ and $\phi$) acquire their masses from Planck-suppressed higher-dimensional operators. The generalization of the previous equations to this case is straightforward. Identifying $H = H'$ with the SM group and $G$ with $SU(5)$, we obtain the following expressions for gaugino and scalar masses:

$$\tilde{M}_g(\mu) = \frac{\alpha(\mu)}{4\pi}\,(4b - b_\phi - 2b_S - b_H - b_G)\,\frac{F}{M}\,, \tag{78}$$

$$\begin{aligned}
\tilde{m}^2_Q &= 2c\,\frac{\alpha^2(\mu)}{(4\pi)^2}\,\left[R^2 b_H\frac{\alpha^2(M_Q)}{\alpha^2(\mu)} - \frac{(b_H+b_G)^2}{b_S}\frac{\alpha^2(M_Q)}{\alpha^2(\mu)} + \right.\\
&+ \left(\frac{1}{b_S} - \frac{1}{b_\phi}\right)(b_H + b_G + 2b_S)^2\frac{\alpha^2(M_S)}{\alpha^2(\mu)} + \left(\frac{1}{b_\phi} - \frac{1}{b}\right)(b_H + b_G + 2b_S + b_\phi)^2\frac{\alpha^2(M_\phi)}{\alpha^2(\mu)} + \\
&+ \left.\frac{1}{b}(b_H + b_G + 2b_S + b_\phi - 4b)^2\right]\left(\frac{F}{M}\right)^2\,,
\end{aligned} \tag{79}$$

$$R \equiv \frac{\alpha_H(M_Q)}{\alpha(M_Q)} = 1 + \frac{\alpha_H(M_Q)}{\alpha_G(M_Q)}\,. \tag{80}$$

The three mass thresholds are given by $M_Q = \lambda v/\sqrt{5}$, $M_S = (v/\sqrt{5})^3/M^2_{Pl}$, $M_\phi = (v/\sqrt{5})^4/M^3_{Pl}$, where $M_{Pl}$ is the reduced Planck mass, and $v$ is the symmetry-breaking VEV lying somewhere between $3 \times 10^{14}$ GeV and the GUT scale. The $\beta$-function coefficients $b$ are those of the SM, and $b_\phi = b - 2$, $b_S = b - 5$, $b_H = b - 9$, $b_G = 6$.

The coefficient $(4b - b_\phi - 2b_S - b_H - b_G)$ of the gaugino mass, see eq. (78) is equal to 15. Had we ignored the gauge messenger contribution, this would be equal to 25. The gauge messenger effect is also important for the scalar masses, see eq. (79). We find that at least the right-handed slepton has negative mass squared, unless $v$ coincides with the GUT scale and $\alpha_G$ is large ($R$ close to 1). Of course in this case one is sensitive to GUT physics, and one should investigate specific unified models.

The model of ref. [16] is less problematic than the "SU(5) model" of ref. [14]. This is because of the larger matter content and because the higher-dimensional couplings between messengers and Goldstino superfield enhance the effective supersymmetry-breaking scale $F/M$. However a complete study of the minimization conditions is still lacking. At any rate, even in the model of ref. [16], right handed sleptons have negative squared masses, unless $v$ becomes equal to the GUT scale.

In conclusion, gauge messengers drastically change the usual mass relations obtained with chiral messengers. However, in the models presented in the literature, the new contributions to scalar mass squared from gauge messengers turn out to be negative and spoil the viability of the models. It would be interesting to construct realistic theories with gauge messengers.

# 6 Conclusions

We have described a new simple method to derive the supersymmetry-breaking terms for the observable sector mediated by renormalizable perturbative interactions. We are dealing with theories in which the relevant mass scales are set by the VEV of the Goldstino chiral superfield $X$, $\langle X \rangle = M + \theta^2 F$. Some fields, called the messengers, receive masses of order $M$ from their tree-level couplings to $X$. Assuming $F \ll M^2$, we can use the non-renormalization theorems of supersymmetry, and obtain the $M$ functional dependence of the wave-function renormalizations from the RG evolution through the threshold $M$. Then we perform the appropriate replacement of $M$ with the chiral superfield $X$, dictated either by holomorphy or by chiral reparametrization. Finally, by replacing $X$ with its VEV $\langle X \rangle = M + \theta^2 F$, we obtain all supersymmetry-breaking effects, at the leading order in $F/M^2$.

Operatively, the method is extremely simple. The supersymmetry-breaking gaugino masses, scalar masses, and coefficients of the $A$-type terms have the expressions, derived in sect. 2,

$$\tilde{M}_g(\mu) = -\frac{1}{2}\frac{\partial \ln S(X,\mu)}{\partial \ln X}\bigg|_{X=M} \frac{F}{M} , \tag{81}$$

$$\tilde{m}^2_Q(\mu) = -\frac{\partial^2 \ln Z_Q(X,X^\dagger,\mu)}{\partial \ln X \, \partial \ln X^\dagger}\bigg|_{X=M} \frac{FF^\dagger}{MM^\dagger} , \tag{82}$$

$$A_i(\mu) = \frac{\partial \ln Z_{Q_i}(X,X^\dagger,\mu)}{\partial \ln X}\bigg|_{X=M} \frac{F}{X} . \tag{83}$$

The expressions for the gauge and chiral wave-function renormalizations $S$ and $Z_Q$ are obtained by integrating the well-known RG differential equations. There is no need to evaluate any

Feynman diagram. The method allows one to derive with no effort results which before required laborious calculations. In particular, notice that the gauge-mediated two-loop sfermion masses are obtained by integrating the one-loop RG equation.

We have applied our method to derive several new results of phenomenological interest. We have computed the sub-leading $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections to the Higgs mass parameters in gauge-mediated models, suppressed by a factor $1/\ln(M/\tilde{m}_t)$ with respect to the known leading result. This correction modifies by at most 30 % the value of the Higgs mixing mass $\mu_H^2$ extracted from the electroweak symmetry-breaking condition.

We have computed the supersymmetry-breaking terms induced by superpotential couplings between matter and messenger chiral superfields. These include $A$-type soft terms at one loop. We have shown that an interaction of this kind involving a Higgs singlet $N$ offers a new possibility for a solution to the $\mu$ problem in gauge-mediated theories. Indeed, the superpotential coupling between messengers and $N$, generates a negative mass squared and trilinear couplings for the scalar component of $N$, which can help to achieve the correct electroweak breaking.

Finally, we have investigated the possibility that the messengers belong to gauge supermultiplets instead of chiral supermultiplets, and calculated the resulting soft terms. We have shown that, if the gauge coupling constant is continuous at the symmetry-breaking threshold, then the soft terms have the same form as in the chiral messenger case, with the index $N$ replaced by a function of the gauge group Casimir invariants, see eq. (61). In particular, $N$ can become negative. We have also generalized the formulae to the case in which the gauge coupling constant has a discontinuity at the threshold. Now one-loop $A$-type soft terms are generated at the messenger scale $M$. Finally, we have shown that some recently-proposed models of dynamical supersymmetry breaking have contributions from gauge messengers which drive negative slepton mass squared. The presence of negative contributions to sfermion masses seems a generic problem of models with gauge messengers. Since these models are otherwise compelling, we believe it is interesting to search for realistic examples.

Since two-loop $\beta$ functions and anomalous dimensions in supersymmetric theories are know, our method seems well suited for next-to-leading order calculations. However, we wish to warn the reader that to extend our method beyond the leading order, one has to deal with some subtleties related to the analytic continuation of the gauge wave-function [19].

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# References

[1] M. Dine, W. Fischler, and M. Srednicki, *Nucl. Phys.* **B189** (1981) 575;
S. Dimopoulos and S. Raby, *Nucl. Phys.* **B192** (1981) 353;
M. Dine and W. Fischler, *Phys. Lett.* **B110** (1982) 227;
M. Dine and M. Srednicki, *Nucl. Phys.* **B202** (1982) 238;
M. Dine and W. Fischler, *Nucl. Phys.* **B204** (1982) 346;
L. Alvarez-Gaumé, M. Claudson, and M. Wise, *Nucl. Phys.* **B207** (1982) 96;
C.R. Nappi and B.A. Ovrut, *Phys. Lett.* **B113** (1982) 175;
S. Dimopoulos and S. Raby, *Nucl. Phys.* **B219** (1983) 479.

[2] M. Dine and A.E. Nelson, *Phys. Rev.* **D48** (1993) 1277;
M. Dine, A.E. Nelson, and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362;
M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, *Phys. Rev.* **D53** (1996) 2658.

[3] S. Dimopoulos, G.F. Giudice, and A. Pomarol, *Phys. Lett.* **B389** (1996) 37.

[4] S.P. Martin, *Phys. Rev.* **D55** (1997) 3177.

[5] M. Shifman and A. Vainshtein, *Nucl. Phys.* **B277** (1986) 456; *ibid.* **B359** (1991) 571.

[6] J. Hisano and M. Shifman, preprint TPI-MINN-97/14-T, hep-ph/9705417.

[7] M. Shifman, A. Vainshtein and V. Zakharov, *JETP Lett.* **42** (1985) 224; *Phys. Lett.* **B166** (1986) 334.

[8] K. Konishi, *Phys. Lett.* **B135** (1984) 439;
K. Konishi and K. Shizuya, *Nuovo Cim.* **A90** (1985) 111.

[9] J.E. Björkman and D.R.T. Jones, *Nucl. Phys.* **B259** (1985) 533.

[10] S. Weinberg, *Phys. Lett.* **B91** (1980) 51;
L.J. Hall, *Nucl. Phys.* **B178** (1981) 75.

[11] R. Rattazzi and U. Sarid, preprint CERN-TH/96-349, hep-ph/9612464.

[12] G. Dvali, G.F. Giudice, and A. Pomarol, *Nucl. Phys.* **B478** (1996) 31.

[13] M. Dine, Y. Nir, and Y. Shirman, *Phys. Rev.* **D55** (1997) 1501.

[14] S. Dimopoulos, G. Dvali, R. Rattazzi, and G.F. Giudice, preprint CERN-TH/97-98, hep-ph/9705307.

[15] E. Witten, *Phys. Lett.* **B105** (1981) 267.

[16] H. Murayama, preprint LBNL-40286, hep-ph/9705271.

[17] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press.

[18] J. Hisano, H. Murayama, and T. Goto, *Phys. Rev.* **D49** (1994) 1446.

[19] G.F. Giudice and R. Rattazzi, in preparation.