

Approximations for W-Pair Production at Linear-Collider Energies[†]

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Abstract

We determine the accuracy of various approximations to the $\mathcal{O}(\alpha)$ corrections for on-shell W-pair production. While an approximation based on the universal corrections arising from initial-state radiation, from the running of α , and from corrections proportional to m_t^2 fails in the Linear-Collider energy range, a high-energy approximation improved by the exact universal corrections is sufficiently good above about 500 GeV. These results indicate that in Monte Carlo event generators for off-shell W-pair production the incorporation of the universal corrections is not sufficient and more corrections should be included.

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1 Introduction

One of the most important processes for testing the Minimal Standard Model (MSM) in the future is the production of W pairs [1–3]. It allows a direct study of the non-Abelian triple gauge couplings in the clean environment of e^+e^- collisions. The sensitivity of this process to anomalous gauge-boson couplings grows strongly with energy owing to the fact that these couplings in general spoil the unitarity cancellations that are present in the MSM for longitudinal gauge bosons. Consequently, more stringent limits on non-standard couplings can be obtained at higher energies.

In order to test for anomalous couplings, the cross-section for W-pair production has to be known with an accuracy of 1% or better. At this level, the inclusion of radiative corrections is mandatory. However, so far the complete electroweak $\mathcal{O}(\alpha)$ corrections to off-shell W-pair production are not available and the present Monte Carlo event generators include only the known leading universal corrections.

In order to assess the theoretical uncertainty inherent in these event generators, on-shell W-pair production can be used as guideline. The corresponding cross-section in the MSM is known, including the complete set of electroweak $\mathcal{O}(\alpha)$ corrections [4]. The size of the non-leading $\mathcal{O}(\alpha)$ corrections in the on-shell case should provide a reasonable estimate for the corresponding left-out non-leading corrections in the off-shell case.

In this short article we discuss the quality of an improved Born approximation (IBA) for on-shell W-pair production that is based on the known leading universal corrections. We concentrate on the Linear-Collider (LC) energy range and emphasize the differences to the LEP2 case studied in Ref. [5]. In addition, we compare the full one-loop results with a form-factor approximation (FFA), which corresponds to the best possible improved Born approximation, and with a consistent high-energy approximation (HEA). We also include some remarks on the radiative corrections to W-pair productions in photon–photon collisions.

2 Form-factor approximation

Whereas in the lowest-order matrix element for $e^+e^- \rightarrow W^+W^-$ only three different tensor structures occur, at $\mathcal{O}(\alpha)$ twelve independent tensor structures are required, each

of which is associated with an independent invariant function. The dominant radiative corrections, such as those that are related to UV, IR or mass singularities, in general have factorization properties and are at $\mathcal{O}(\alpha)$ restricted to those invariant functions that appear already at lowest order. Therefore, the contributions of the other invariant functions should be relatively small. Indeed, a numerical analysis reveals that in a suitably chosen representation for the basic set of independent matrix elements only the three Born-like invariant functions plus one extra right-handed piece are relevant for a sufficiently good approximation [6,7]. This suggests an approximation for the matrix element of the form

$$\mathcal{M}_{\text{app}}^\kappa = \mathcal{M}_I^\kappa F_I^\kappa + \mathcal{M}_Q^\kappa F_Q^\kappa, \quad (1)$$

where—following the notation of Ref. [1]— \mathcal{M}_I and \mathcal{M}_Q denote the tensor structures associated with the charged-current coupling and the electromagnetic coupling, respectively, in the lowest-order matrix element, and κ is the electron helicity. The term involving F_I^+ is only needed for right-handed electrons in the LEP2 energy region. Neglecting the other invariant functions in the basis of Ref. [6] defines the FFA. The form factors F_I^κ and F_Q^κ are independent of the W polarizations and include all corrections that are related to the Born structure. However, they are both energy- and angular-dependent and their evaluation is as complicated as the exact one-loop calculation.

3 Improved Born approximation

In order to construct an IBA one has to specify simple expressions that reproduce the invariant functions $F_{I,Q}^\kappa$ with sufficient accuracy. In the LEP2 energy region the following expressions provide a reasonable ansatz [6]

$$\begin{aligned} F_{I,\text{IBA}}^\kappa &= \left[2\sqrt{2}G_\mu M_W^2 + \frac{4\pi\alpha}{2s_W^2} \frac{\pi\alpha}{4\beta} (1 - \beta^2)^2 \right] \delta_{\kappa-}, \\ F_{Q,\text{IBA}}^\kappa &= \left[4\pi\alpha(s) + 4\pi\alpha \frac{\pi\alpha}{4\beta} (1 - \beta^2)^2 \right]. \end{aligned} \quad (2)$$

As usual, s , t , and u denote the Mandelstam variables, and $\beta = \sqrt{1 - 4M_W^2/s}$ is the W-boson velocity in the centre-of-mass system.

The terms containing G_μ and $\alpha(s)$ incorporate all leading universal corrections associated with the running of α and the corrections $\propto \alpha m_t^2/M_W^2$ associated with the ρ parameter. As these are linked to the renormalization of the electric charge at zero momentum transfer and of the weak mixing angle, they only contribute to the structures already present at lowest order. The $1/\beta$ term describes the effect of the Coulomb singularity, which is only relevant close to threshold and can be omitted at high energies. The factor $(1 - \beta^2)^2$ is introduced by hand to restrict the $1/\beta$ contribution to the threshold region. The ansatz (2) includes all leading logarithms originating from light fermions and all corrections $\propto \alpha m_t^2/M_W^2$. As has been studied in Ref. [6], contributions $\propto \alpha \log(m_t/M_W)$ and $\propto \alpha \log(M_H/M_W)$ that occur when the masses are heavy and are not covered by (2) have a negligible numerical impact.

In addition to the contributions described so far, one has to include the leading logarithmic QED corrections. These can be calculated using the structure-function method

[8]. They comprise all contributions $\propto \alpha \log(m_e/M_W)$. Since we are interested in approximations for the virtual corrections, we restrict ourselves to the inclusion of real photonic corrections in the soft-photon approximation. In this approximation the leading logarithmic QED corrections to the cross-section are given by the factor

$$\delta_\gamma = 2\frac{\alpha}{\pi} \left[\ln \frac{s}{m_e^2} \ln \frac{2\Delta E}{\sqrt{s}} + \frac{3}{4} \ln \frac{Q^2}{m_e^2} - \ln \frac{2\Delta E}{\sqrt{s}} \left(2 - 2 \ln \frac{M_W^2 - t}{M_W^2 - u} - \frac{s - 2M_W^2}{s\beta} \ln \frac{1 + \beta}{1 - \beta} \right) \right], \quad (3)$$

where ΔE is the soft-photon cutoff energy, and Q^2 is a scale that is not determined in the leading-log approximation and set to $Q^2 = s$ for the numerics.

We note that all present event generators for off-shell W-pair production include only those radiative corrections that are included in our IBA.

4 High-energy approximation

At high energies the full $\mathcal{O}(\alpha)$ corrections can be approximated by exploiting the fact that $s, |t|, |u| \gg M_{W,Z}^2 \gg m_{e,\mu,\tau,d,u,s,c,b}$. Such an approximation has been constructed for the process $e^+e^- \rightarrow W^+W^-$ by a systematic expansion of the exact one-loop corrections for arbitrary m_t and M_H [9,10]. For intermediate energies (500 GeV to 2 TeV) the high-energy approximation is improved [9] by exactly taking into account the leading universal corrections of the IBA, defined in Section 3, apart from the Coulomb singularity. As the HEA has already been discussed in Ref. [11], we summarize here only its most important features.

The dominant corrections at high energies result from terms proportional to the high-energy leading logarithms $\alpha \log(q_1^2/M_W^2) \log(q_2^2/M_W^2)$ with $q_i^2 = s, |t|, |u|$. These corrections are non-universal and arise from vertex and box diagrams. For longitudinal gauge bosons, additional sizeable corrections appear, depending on the values of the top-quark and Higgs-boson masses. In the limit $s \gg m_t^2, M_H^2 \gg M_W^2$, terms containing $\alpha m_t^2/M_W^2 \log(m_t/M_W)$ or $\alpha M_H^2/M_W^2$ arise as a consequence of incomplete screening. In the limit $m_t^2, M_H^2 \gg s \gg M_W^2$, corrections of the form $\alpha s/M_W^2 \log(m_t/M_W)$ and $\alpha s/M_W^2 \log(M_H/M_W)$ result from delayed unitarity effects.

5 Numerical results

For the numerical evaluation we used the parameters given in Ref. [5]. The corresponding values for the W-boson, Higgs-boson, and top-quark masses are $M_W = 80.26$ GeV, $M_H = 300$ GeV, and $m_t = 165.3$ GeV.

The unpolarized lowest-order cross-sections integrated over the angular region $10^\circ \leq \theta \leq 170^\circ$ and the differential cross section for three angles are listed in Table 1 at various energies. The quality Δ_{app} is defined as

$$\Delta_{\text{app}} = \left| \frac{\delta\sigma_{\text{app}} - \delta\sigma_{\text{exact}}}{\sigma_{\text{exact}}^{\text{Born}}} \right|, \quad (4)$$

where $\delta\sigma_{\text{exact}}$ is the full virtual and soft-photonic $\mathcal{O}(\alpha)$ correction, $\delta\sigma_{\text{app}}$ the corresponding approximation and $\sigma_{\text{exact}}^{\text{Born}}$ the exact lowest-order cross-section. When calculating the

\sqrt{s}/GeV	θ	$\sigma_{\text{B}}/\text{fb}$	$\Delta_{\text{IBA}}/\%$	$\Delta_{\text{FFA}}/\%$	$\Delta_{\text{HEA}}/\%$
161	(10°, 170°)	3753.2	1.5	0.00	37
	10°	367.0	1.6	0.00	36
	90°	300.7	1.4	0.00	37
	170°	250.0	1.3	0.00	37
175	(10°, 170°)	15591	1.3	0.03	12
	10°	3380	1.7	0.00	10
	90°	1001	1.0	0.05	12
	170°	439	0.59	0.00	12
200	(10°, 170°)	17107	1.5	0.01	3.7
	10°	6463	1.8	0.00	2.3
	90°	812	1.4	0.02	4.7
	170°	255	1.3	0.00	3.8
500	(10°, 170°)	4413.1	4.7	-0.06	-0.85
	10°	11604.4	1.9	0.00	-0.67
	90°	75.4	10	-0.29	-0.05
	170°	6.5	14	-0.19	3.5
1000	(10°, 170°)	1084.3	11	0.06	0.21
	10°	3292.3	3.9	0.00	1.1
	90°	16.7	23	0.08	0.54
	170°	0.6	28	-0.77	6.4
2000	(10°, 170°)	267.57	22	0.12	0.17
	10°	823.35	9.7	0.02	0.64
	90°	4.03	39	-0.16	0.34
	170°	0.09	46	-2.3	5.4

Table 1: Quality of the approximations for various energies

difference $\delta\sigma_{\text{app}} - \delta\sigma_{\text{exact}}$, care has to be taken that the leading higher-order corrections are treated in the same way in the two expressions.

The FFA is excellent; the corresponding error is well below the per-cent level, whenever the cross-section is sizeable. At high energies and large scattering angles, where the cross section is extremely small, larger differences appear. This signals a strong dominance of the Born structure and demonstrates that improved Born approximations are possible. Above LEP2 energies the IBA is reasonable only for extremely small scattering angles, and deviates for most angles by several per cent easily. This shows that approximations based on the leading universal corrections are certainly not sufficient for a comparison with experiment at high energies. The HEA on the other hand is good at energies above

about 500 GeV for sizeable cross-sections, i.e. its deviation from the full $\mathcal{O}(\alpha)$ corrections is below 1%.

6 Remarks on $\gamma\gamma \rightarrow W^+W^-$

Another important process for the study of anomalous gauge-boson couplings is W-pair production in photon–photon collisions. The corresponding cross-section approaches 80 pb at high energies and is thus much larger than the one for W-pair production in e^+e^- collisions. The corrections to the corresponding on-shell process have been studied in Refs. [12] and briefly summarized in Ref. [13]. Almost all leading universal corrections are absent. There are no effects from the running of α , no corrections proportional to m_t^2 , and no enhanced logarithms arising from soft or collinear photons. Not even corrections $\propto \alpha \log(M_H/M_W)$ arise in the limit of a large Higgs-boson mass. The Coulomb singularity is unimportant at LC energies. Thus, an IBA that is equivalent to the one discussed for $e^+e^- \rightarrow W^+W^-$ at high energies becomes trivial, i.e. equal to the Born approximation. All dominating corrections are of non-universal origin and their size, which reaches several per cent at 500 GeV and increases with energy, indicates that they cannot be neglected. Unlike for $e^+e^- \rightarrow W^+W^-$, no form-factor and high-energy approximations have been worked out for $\gamma\gamma \rightarrow W^+W^-$ so far.

7 Conclusions

At high energies the $\mathcal{O}(\alpha)$ corrections to on-shell W-pair production in e^+e^- collisions are large [$\mathcal{O}(10\text{--}40\%)$] and cannot be neglected. They are not dominated by universal corrections from initial-state radiation, from the running of α , and from corrections proportional to m_t^2 , but by contributions like $\alpha \log^2(s/M_W^2)$, which originate from the bosonic (vertex and box) corrections. For on-shell W-pair production they are well approximated by a consistent high-energy expansion that is improved by the exact leading universal corrections in order to yield a sufficient accuracy at intermediate energies.

Present event generators for off-shell W-pair production include only the universal corrections. The theoretical uncertainty from neglecting non-universal corrections strongly increases with energy. While this uncertainty is only 1–2% at LEP2 energies it amounts to several tens of per cent in the TeV range. Thus, the non-universal corrections are required for adequate theoretical predictions. This applies in the same way to $\gamma\gamma \rightarrow W^+W^-$ where essentially all non-universal corrections are absent.

A first step towards the inclusion of non-universal corrections into Monte Carlo event generators for $e^+e^- \rightarrow 4$ fermions is provided by the fermion-loop scheme [14], where gauge-boson widths are consistently introduced by the resummation of the fermion-loop corrections. The incorporation of the bosonic corrections, which dominate the non-universal corrections at high energies, is more complicated owing to their complexity and problems with gauge invariance. The most promising approach consists in an expansion according to the degree of resonance. In this approach the corrections are to a large extent given by the known corrections for on-shell W bosons or, at LC energies, by the corresponding high-energy approximation. For a genuine on-shell approach to W-pair production and the subsequent W decay, the $\mathcal{O}(\alpha)$ corrections have already been included in an event generator (see e.g. Ref. [15]). The combination of off-shell effects and $\mathcal{O}(\alpha)$ corrections is still under investigation.

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