Branes, central charges and U-duality invariant BPS conditions

Sergio Ferrara[1] and Juan Maldacena[2]

[1] *CERN Theoretical Division, CH 1211 Geneva 23, Switzerland*

[2] *Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855, USA*

Abstract

In extended supergravity theories there are p-brane solutions preserving different numbers of supersymmetries, depending on the charges, the spacetime dimension and the number of original supersymmetries (8, 16 or 32). We find U-duality invariant conditions on the quantized charges which specify the number of supersymmetries preserved with a particular charge configuration. These conditions relate U-duality invariants to the picture of intersecting branes. The analysis is carried out for all extended supergravities with 16 or 32 supersymmetries in various dimensions.

[1] FERRARAS@vxcern.cern.ch

[2] malda@physics.rutgers.edu

1. Introduction

Extended supergravity theories contain BPS black hole solutions which preserve some supersymmetries. Given a generic charge configuration we can find an extremal black hole solution, extremal in the sense of the cosmic censorship bound, i.e. the black hole solution with a mass that saturates the bound coming from demanding that there be no naked singularities. In some cases the extremal black hole is also BPS [1] and in some others it is not BPS. Even in the case when the extremal solution is supersymmetric it can preserve different numbers of supersymmetries. The charges transform under a group G characteristic of the supergravity theory. We show how different cases are separated by G-invariant conditions on the charges.

Maximally extended supergravities in d spacetime dimensions are the low energy limit of type II (A or B) string theory compactified on a 10-d dimensional torus. The classical supergravity theories are formulated in terms of an underlying non-compact group G [2][3] which is $G = E_{11-d}$, i.e. E_7 in four dimensions, E_6 in five dimensions, $E_5 = S0(5,5)$ in six dimensions, $E_4 = SL(5,R)$ in seven dimensions, $E_3 = SL(3,R) \times SL(2,R)$ in eight dimensions and $E_2 = SL(2,R) \times O(1,1)$ in nine dimensions. These theories contain some Abelian $p+1$-form potentials which, for each p, arrange themselves into multiplets of the group G. They also have a large number of scalar fields which live in the coset space G/H, where H is a maximal compact subgroup of the non-compact group G. Even though the symmetry of the theory is only H the charges transform under the group G. In the full quantum theory, charges are quantized and the duality symmetry is broken to a discrete subgroup $G(Z)$, which is the U-duality group [4]. Classical supergravity solutions correspond to the limit of large values of integer quantized charges. In this case the action of $G(Z)$ becomes almost continuous, so we will consider the action of the continuous group. We normalize the charges so that they become integers in the quantum theory. The various $p+1$-forms couple to p-dimensional objects and we always consider $p < d-3$ so that fields decay fast enough at infinity, enabling the action of the various (super)symmetry generators on the configuration to be well defined. If this condition is not satisfied the geometry will not be asymptotically Minkowski in the presence of a brane. Configurations with p and $d-p-4$ are related by Dirac electric-magnetic duality. The corresponding charges transform in the gradient and contragradient representations of the group G, except for the case $p = \frac{d-4}{2}$ where the same representation includes both electric and magnetic objects. The commutator of two supersymmetries contains several "central

charge matrices". These central charge matrices depend on the charges and on the moduli of the theory, which are the values of the scalar fields at infinity. By using a transformation in the group H it is possible to reduce the central charges into a normal form where they are "diagonal" [5]. BPS states with enhanced supersymmetry restrict the eigenvalues of the central charge matrices, giving constraints on the charges. The central charges in the normal frame still depend on the moduli. These are residual G/H transformations that keep the diagonal structure of the matrix in the normal frame. These tranformations are just some number of $O(1,1)$ rotations. We can view this choice of the normal frame as choosing a particular background of a square torus where the charges are aligned in a simple way on the torus. The residual transformations are related to the possibility of changing the size of the torus, etc. [6].

In the case of extremal BPS black hole solutions it is possible to write the entropy in terms of the charges in a U-dual fashion. This makes use of particular quartic [7] and cubic [8][9] invariants in 4 and 5 dimensions respectively. This is the situation when the charges are generic. There are however some charge configurations for which the area of the black hole horizon is zero, and also some configurations which preserve a larger number of supersymmetries. There are also configurations that do not preserve any supersymmetries. In different dimensions the number of possible preserved supersymmetries can be calculated as follows. A localized particle-like configuration breaks the Lorentz group into the little group $SO(d-1)$. The preserved supersymmetry has to be in a representation of $SO(d-1)$. For $d = 4,5$ the spinor representation has 4 real components and for $d = 6,7,8,9$ the spinor representation has 8 real components. So depending on the number of original supersymmetries (which is 32 in the maximal case) we have different numbers of possible preserved supersymmetries: 1/2, 1/4, 1/8 depending on the dimension. For example, in $d \geq 6$ we can have only 1/2 or 1/4 BPS solutions.

We write down U-duality invariant expressions which separate the various cases. We also argue that one can choose a "basis" for the charges in which each element by itself breaks 1/2 of the supersymmetry and that, taken together, they break more supersymmetries. This "basis" has a representation in terms of intersecting branes.

We can decompactify the d-dimensional theory into a $d-1$ dimensional one by letting one of the radii of the torus go to infinity. The duality group decomposes as $E_{11-d} \rightarrow E_{10-d} \times O(1,1)$ where the $O(1,1)$ is related to the T-duality symmetry that is lost when

a circle becomes infinite[3]. It will be useful to analyze the behaviour of the representations under this decomposition.

It is also instructive to decompose E_{11-d} under S, T duality. The decomposition reads $E_{11-d} \to O(1,1) \times O(10-d, 10-d)$ for $d \geq 5$ and $E_7 \to SL(2, R) \times O(6,6)$ for $d = 4$ [10]. This decomposition separates NS and R charges in string theory.

This paper is organized as follows: In section 2 we analyze maximal supergravities (32 supersymmetries) in various dimensions and for different extended objects. The conditions for a state to preserve different numbers of supersymmetries are presented. In section 3 the analysis is extended to theories with 16 supersymmetries in $d = 4, 5$, such as heterotic on T^6, T^5 or the dual type II on $K3 \times T^2$, etc.

2. Maximal supergravity in various dimensions, $4 \leq d \leq 9$

2.1. Five dimensions

We start with the five-dimensional case. We have 27 Abelian gauge fields which transform in the fundamental representation of E_6. The electrically charged objects are point-like and the magnetic duals are one-dimensional, or string-like. The first invariant of E_6 is the cubic invariant $I_3 = T_{ijk}q^i q^j q^k$. In fact, the entropy of a black hole with charges q^i is proportional to $\sqrt{I_3}$ [8][9]. We will see that a configuration with $I_3 \neq 0$ preserves 1/8 of the supersymmetries. If $I_3 = 0$ and $\frac{\partial I_3}{\partial q^i} \neq 0$ then it preserves 1/4 of the supersymmetries, and finally if $\frac{\partial I_3}{\partial q^i} = 0$ (and the charge vector q^i is non-zero), the configuration preserves 1/2 of the supersymmetries. We will show this by choosing a particular basis for the charges, the general result following by U-duality.

In five dimensions the compact group H is $USp(8)$.[4] In the commutator of the supersymmetry generators we have a central charge matrix Z_{ab} which can be brought to a normal form by a $USp(8)$ transformation. In the normal form the central charge matrix can be written as

$$
e_{ab} = \begin{pmatrix} s_1 + s_2 - s_3 & 0 & 0 & 0 \\ 0 & s_1 + s_3 - s_2 & 0 & 0 \\ 0 & 0 & s_2 + s_3 - s_1 & 0 \\ 0 & 0 & 0 & -(s_1 + s_2 + s_3) \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (2.1)
$$

---

[3] In the quantum theory $O(1,1) \to Z_2$.

[4] We choose our conventions so that $USp(2) = SU(2)$.

4

we can order s_i so that $s_1 \geq s_2 \geq |s_3|$. The cubic invariant, in this basis, becomes [9]

$$I_3 = T_{ijk}q^i q^j q^k = s_1 s_2 s_3 \ . \tag{2.2}$$

Even though the eigenvalues s_i might depend on the moduli, the invariant (2.2) only depends on the quantized values of the charges. We can write a generic charge configuration as UeU^t, where e is the normal frame as above, and the invariant will then be (2.2). There are three distinct possibilities

$$
\begin{array}{lll}
I_3 \neq 0 & s_1, \ s_2, \ s_3 \neq 0 & \\
I_3 = 0, \quad \dfrac{\partial I_3}{\partial q^i} \neq 0 & s_1, \ s_2 \neq 0, & s_3 = 0 \\
I_3 = 0, \quad \dfrac{\partial I_3}{\partial q^i} = 0 & s_1 \neq 0, & s_2, \ s_3 = 0
\end{array}
\tag{2.3}
$$

Taking the case of type II on T^5 we can choose the rotation in such a way that, for example, s_1 corresponds to solitonic five-brane charge, s_2 to fundamental string winding charge along some direction and s_3 to Kaluza-Klein momentum along the same direction. We can see that in this specific example the three possibilities in (2.3) break 1/8, 1/4 and 1/2 supersymmetries. This also shows that one can generically choose a basis for the charges so that all others are related by U-duality. The basis chosen here is the S-dual of the D-brane basis usually chosen for describing black holes in type II B on T^5 [11][12]. All others are related by U-duality to this particular choice. The sign of the invariant (2.2) is not important since it changes under a CPT transformation.

In five dimensions there are also string-like configurations which are the magnetic duals of the configurations considered here. They transform in the contragradient 27 representation and the solutions preserving 1/2, 1/4, 1/8 supersymmetries are characterized in an analogous way. We could also have configurations where we have both point-like and string-like charges. If the point-like charge is uniformly distributed along the string, it is more natural to consider this configuration as a point-like object in $d = 4$ by dimensional reduction.

It is useful to decompose the U-duality group into the T-duality group and the S-duality group [10]. The decomposition reads $E_6 \rightarrow O(5,5) \times O(1,1)$, leading to

$$\mathbf{27} \rightarrow \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4 \ . \tag{2.4}$$

The last term in (2.4) corresponds to the NS five-brane charge. The $\mathbf{16}$ correspond to the D-brane charges and the $\mathbf{10}$ correspond to the 5 directions of KK momentum and the 5

directions of fundamental string winding, which are the charges that explicitly appear in string perturbation theory. The cubic invariant has the decomposition

$$(\mathbf{27})^3 \rightarrow \mathbf{10}_{-2} \ \mathbf{10}_{-2} \ \mathbf{1}_4 + \mathbf{16}_1 \ \mathbf{16}_1 \ \mathbf{10}_{-2} \ . \tag{2.5}$$

This is saying that in order to have a non-zero area black hole we must have three NS charges (more precisely some "perturbative" charges and a solitonic five-brane); or we can have two D-brane charges and one NS charge. In particular, it is not possible to have a black hole with a non-zero horizon area with purely D-brane charges.

Notice that the non-compact nature of the groups is crucial in this classification.

2.2. Four dimensions

In four dimensions the duality group is E_7 and the charges transform in the 56 representation of E_7. In this case electric and magnetic charges are all point-like and are included in the same representation of the duality group. The invariant is quartic $I_4 = T_{ijkl}q^i q^j q^k q^l$ [7][13][14] and it can also be expressed in terms of the central charge matrix $Z_{AB}(q, \phi)$. Of course, the dependence on the scalar fields drops out from the expression for I_4. Again, by performing an $SU(8)$ transformation one can choose the charges in the normal frame form

$$Z_{ab} = \begin{pmatrix} z_1 & 0 & 0 & 0 \\ 0 & z_2 & 0 & 0 \\ 0 & 0 & z_3 & 0 \\ 0 & 0 & 0 & z_4 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \ , \tag{2.6}$$

where $z_i = \rho_i e^{i\varphi_i}$ are complex. Actually the relative phases of z_i can be changed, but the overall phase $\varphi = \sum \varphi_i$ cannot be removed by an $SU(8)$ transformation; it is related to an extra parameter in the class of black hole solutions [15]. In this basis the quartic invariant takes the form [7]

$$
\begin{aligned}
I_4 &= \sum_i |z_i|^4 - 2\sum_{i<j} |z_i|^2 |z_j|^2 + 4(z_1 z_2 z_3 z_4 + \bar{z}_1 \bar{z}_2 \bar{z}_3 \bar{z}_4) \\
&= (\rho_1 + \rho_2 + \rho_3 + \rho_4)(\rho_1 + \rho_2 - \rho_3 - \rho_4)(\rho_1 - \rho_2 + \rho_3 - \rho_4)(\rho_1 - \rho_2 - \rho_3 + \rho_4) + \\
&\quad + 8\rho_1 \rho_2 \rho_3 \rho_4 (\cos\varphi - 1) \ .
\end{aligned}
\tag{2.7}
$$

It was shown in [9][16] that for a 1/8 supersymmetric solution

$$M_{BPS}^4(\phi_h, q) = I_4 \ , \qquad z_1(\phi_h, q) \neq 0, \qquad z_i(\phi_h, q) = 0, \quad i = 2, 3, 4 \ , \tag{2.8}$$

6

where ϕ_h are the values of the moduli at the horizon, given by ratios of the quantized charges [17][18]. This implies that $I_4 \geq 0$ for a BPS solution and if $I_4 < 0$ then the extremal solution is not BPS (as opposed to $d = 5$ where the sign of I_3 was not important). We also see that at the horizon we can choose $\varphi_h = 0$.

The condition for a 1/4 BPS state is [9]

$$|z_1(\phi, q)| = |z_2(\phi, q)| , \qquad |z_3(\phi, q)| = |z_4(\phi, q)|. \qquad (2.9)$$

This happens when $\frac{\partial I_4}{\partial z_i} = 0$, in the normal frame this implies, in particular, $\phi = 0$ and also $\rho_1 = \rho_2$, $\rho_3 = \rho_4$. We then see that there is no extra phase if the configuration preserves at least 1/4 supersymmetry.

If the state preserves 1/2 of the supersymmetries the condition on the central charges is $\phi = 0$, $\rho_1 = \rho_2 = \rho_3 = \rho_4$ [9]. This translates into the condition that the second derivatives of I_4 projected on the adjoint representation vanish,

$$\left. \frac{\partial^2 I_4}{\partial q^i \partial q^j} \right|_{Adj.} \sim T_{ijkl} q^k q^l |_{Adj} = 0 . \qquad (2.10)$$

There is no constraint on the **1463** representation of E_7 present in the above symmetric polynomial [19]. Note that under $SU(8)$: **1463** $= \mathbf{1} + \cdots$, where the singlet is the 1/2 BPS mass which can be extracted from I_4 as follows

$$M_{BPS}^2 = -\frac{1}{8} \sum_i \frac{\partial^2 I_4}{\partial z_i \partial \bar{z}_i} = \rho^2 \qquad (2.11)$$

If the phase vanishes, I_4 becomes

$$I_4 = T_{ijkl} q^i q^j q^k q^l = s_1 s_2 s_3 s_4 , \qquad (2.12)$$

where we have defined s_i by

$$\begin{aligned} s_1 &= \rho_1 + \rho_2 + \rho_3 + \rho_4 \\ s_2 &= \rho_1 + \rho_2 - \rho_3 - \rho_4 \\ s_3 &= \rho_1 - \rho_2 + \rho_3 - \rho_4 \\ s_4 &= \rho_1 - \rho_2 - \rho_3 + \rho_4 . \end{aligned} \qquad (2.13)$$

and we order the s_i so that $s_1 \geq s_2 \geq s_3 \geq |s_4|$. Now the distinct possibilities are

$$
\begin{aligned}
& I_4 \neq 0 && s_1, s_2, s_3, s_4 \neq 0 \\
& I_4 = 0, \qquad \frac{\partial I_4}{\partial q^i} \neq 0 && s_1, s_2, s_3 \neq 0, \qquad s_4 = 0 \\
\frac{\partial I_4}{\partial q^i} = 0, \qquad & \left.\frac{\partial^2 I_4}{\partial q^i \partial q^j}\right|_{Adj\,E_7} \neq 0 && s_1, s_2 \neq 0, \qquad s_3, s_4 = 0 \\
& \left.\frac{\partial^2 I_4}{\partial q^i \partial q^j}\right|_{Adj\,E_7} = 0 && s_1 \neq 0, \qquad s_2, s_3, s_4 = 0
\end{aligned}
\tag{2.14}
$$

We can choose a basis of four charges $(q_i)_I$, $I = 1, 2, 3, 4$ such that for each element only $s_I \neq 0$ and $s_i = 0$, $i \neq I$. Any charge vector is then related by an E_7 rotation to a configuration with only these four charges if the phase vanishes. An example of this would be a set of four D-three-branes oriented along 456, 678, 894, 579 (where the order of the three numbers indicates the orientation of the brane). Note that in choosing the basis the sign of the D-3-brane charges is important; here they are chosen such that taken together with positive coefficients they form a BPS object. The first two possibilities in (2.14) preserve 1/8 of the supersymmetries, the second 1/4 and the last 1/2. It is interesting that there are two types of 1/8 BPS solutions. In the supergravity description, the difference between them is that the first in (2.14) has non-zero horizon area. If $I_4 < 0$ the solution is not BPS. This case corresponds, for example, to changing the sign of one of the three-brane charges discussed above. By U-duality transformations we can relate this to configurations of branes at angles such as in [20].

Going from four to five dimensions it is natural to decompose the $E_7 \to E_6 \times O(1,1)$ where E_6 is the duality group in five dimensions and $O(1,1)$ is the extra T duality that appears when we compactify from five to four dimensions. According to this decomposition the representation breaks as $\mathbf{56} \to \mathbf{27}_1 + \mathbf{1}_{-3} + \mathbf{27'}_{-1} + \mathbf{1}_3$ and the quartic invariant becomes

$$
\mathbf{56}^4 = (\mathbf{27}_1)^3 \mathbf{1}_{-3} + (\mathbf{27'}_{-1})^3 \mathbf{1}_3 + \mathbf{1}_3 \mathbf{1}_3 \mathbf{1}_{-3} \mathbf{1}_{-3} + \mathbf{27}_1 \mathbf{27}_1 \mathbf{27'}_{-1} \mathbf{27'}_{-1} + \mathbf{27}_1 \mathbf{27'}_{-1} \mathbf{1}_3 \mathbf{1}_{-3}
\tag{2.15}
$$

The $\mathbf{27}$ comes from point-like charges in five dimensions an the $\mathbf{27'}$ comes from string-like charges.

Decomposing the U-duality group into T- and S-duality groups, $E_7 \to SL(2, R) \times O(6,6)$ we find $\mathbf{56} \to (\mathbf{2}, \mathbf{12}) + (\mathbf{1}, \mathbf{32})$ where the first term corresponds to NS charges and the second term to D-brane charges. Under this decomposition the quartic invariant (2.12) becomes $\mathbf{56}^4 \to \mathbf{32}^4 + (\mathbf{12}.\mathbf{12'})^2 + \mathbf{32}^2.\mathbf{12}.\mathbf{12'}$. This means that we can have configurations

with a non-zero area that carry only D-brane charges, or only NS charges or both D-brane and NS charges. We can then express the charges as $e_{AB} = (v_i^\alpha, S^a)$, where $\alpha = 1, 2$, $i = 1, ..., 12$ is the vector index and $a = 1, 32$ is the spinor index. In order to gain some light on the conditions in (2.14) involving the projections on the adjoint we decompose them according to the S-T-duality groups. The adjoint representation of E_7 decomposes as $\mathbf{133} \rightarrow (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{66}) + (\mathbf{2}, \mathbf{32})$. The last condition in (2.14) becomes, with this decomposition,

$$\frac{\partial^2 I_4}{\partial S^a \partial S^b}(\gamma^{ij})_{ab} + \frac{\partial^2 I_4}{\partial e_i^\alpha \partial e_j^\beta}\epsilon_{\alpha\beta} = 0 \ ,$$

$$\frac{\partial^2 I_4}{\partial S^a \partial e_i^\alpha}(\gamma_i)_{ab} = 0 \ , \qquad \frac{\partial^2 I_4}{\partial e_i^\alpha \partial e_j^\beta}\eta_{ij} = 0 \ .$$

(2.16)

where γ^i are the $O(6, 6)$ gamma-matrices.

An interesting case where I_4 is negative corresponds to a configuration carrying electric and magnetic charges under the same gauge group, for example a 0-brane plus 6-brane configuration which is U-dual to a KK-monopole and plus KK-momentum [21][22]. This case corresponds to $z_i = \rho e^{i\varphi/4}$ and the phase is $\tan\varphi/4 = e/g$ where e is the electric charge and g is the magnetic charge. Using (2.7) we find that $I_4 < 0$ unless the solution is purely electric or purely magnetic. In [23] it was suggested that $0 + 6$ does not form a supersymmetric state. Actually it was shown in [24] that a 0+6 configuration can be T-dualized into a non-BPS configuration of four intersecting D-3-branes. Of course, I_4 is negative for both configurations. Notice that even though these two charges are Dirac dual (and U-dual) they are not S-dual in the sense of filling out an $SL(2, Z)$ multiplet. In fact, the KK-monopole forms an $SL(2, Z)$ multiplet with a fundamental string winding charge under S-duality [25].

2.3. Six dimensions

In this case the duality group is O(5,5) and we have vector fields and two form field potentials. The vector fields couple to point-like configurations and their magnetic duals to two-dimensional configurations. The two-form potentials and their magnetic duals both couple to one dimensional string-like objects. All point-like charges belong to the $\mathbf{16}$ representation (spinor of O(5,5)) while one-brane charges belong to the $\mathbf{10}$ (vector of O(5,5)).

Going from five to six by decompactifying one dimension leads to the decomposition $E_6 \to O(5,5) \times O(1,1)$ and the representations decompose as

$$\mathbf{27} \to \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4$$
$$\mathbf{27}' \to \mathbf{16}'_{-1} + \mathbf{10}_2 + \mathbf{1}_{-4} \ . \tag{2.17}$$

The $\mathbf{1}_4$ corresponds to KK momentum and the $\mathbf{1}_{-4}$ to KK monopole charge. From group theory, this is the same decomposition as in (2.4) but the interpretation is different. The cubic invariant has the decomposition

$$(\mathbf{27})^3 \to \mathbf{10}_{-2} \ \mathbf{10}_{-2} \ \mathbf{1}_4 + \mathbf{16}_1 \ \mathbf{16}_1 \ \mathbf{10}_{-2} \ . \tag{2.18}$$

Solutions carrying one-brane charge can preserve $1/2$ or $1/4$ of the supersymmetries according to whether the vector $\mathbf{10}$ is null or not, respectively. Similarly a point-like solution, characterized by the spinor S^a, can preserve $1/2$ or $1/4$ according to whether $S^a \gamma^\mu_{ab} S^b$ is zero (as a vector) or not, respectively. We see that both conditions are U-duality invariant.

A one-dimensional solution can also carry "zero-dimensional" charge; this charge can be spread uniformly along the string. These configurations can break more supersymmetries, leaving only $1/8$, when the invariant $\mathbf{16}_1 \ \mathbf{16}_1 \ \mathbf{10}_{-2}$ is non-zero, and they have a natural interpretation as black holes in $d = 5$.

2.4. Seven dimensions

The duality group is $SL(5)$. We have again vector potentials and two-form potentials. So we have point-like configurations whose magnetic duals are three-branes and stringlike configurations whose magnetic duals are two-branes. In going from six to seven dimensions the duality group breaks as $O(5,5) \to SL(5) \times O(1,1)$ and the representations $\mathbf{10} \to \mathbf{5}_2 + \tilde{\mathbf{5}}'_{-2}$, $\mathbf{16} \to \mathbf{10}_{-1} + \mathbf{5}'_3 + \mathbf{1}_{-5}$. The point-like charges belong to the antisymmetric tensor $\mathbf{10}$ and the string-like solutions to the $\mathbf{5}'$ (vector), and the three-branes and two-branes to $\mathbf{10}'$ and $\mathbf{5}$, respectively. Going to six dimensions the $\tilde{\mathbf{5}}'$ and $\mathbf{5}'$ correspond to leaving the string unwrapped or to wrapping it in the extra circle, respectively. In the type IIA the point-like charges would be the 3 directions of KK momentum, D0-brane charge, 3 directions of fundamental string winding, and 3 possible D2-brane wrapping modes. The string like-charges are 1 D4-brane, 3 D2-branes and one fundamental string. The two- and three-brane charges are the magnetic duals of these.

10

The invariants break as

$$\mathbf{10}^2 \rightarrow \mathbf{5}_2 \ \tilde{\mathbf{5}}'_{-2}$$

$$\mathbf{16} \ \mathbf{16} \ \mathbf{10} \rightarrow \mathbf{10}_{-1} \ \mathbf{5}'_3 \ \tilde{\mathbf{5}}'_{-2} + \mathbf{5}_2 \ \mathbf{5}'_3 \ \mathbf{1}_{-5} + \mathbf{10}_{-1} \ \mathbf{10}_{-1} \ \mathbf{5}_2 \ .$$

(2.19)

We see that there is no quadratic condition we can impose on a $\mathbf{5}$; this is related to the fact that all one-dimensional configurations with only string charges break $1/2$ of the supersymmetries (a fundamental string ending on a D-brane would preserve $1/4$ but the configuration would have to extend along two different directions in space). On the other hand we can have point-like solutions preserving $1/2$ or $1/4$ of the supersymmetries according to whether the $\epsilon^{ijklm} T_{ij} T_{kl}$ is zero or not respectively (T_{ij} is the $\mathbf{10}$ representation).

The M(atrix) theory description involves the (0,2) non-trivial fixed point theory describing the world-volume degrees of freedom of coincident 5-branes in M-theory. M-theory on T^4 is defined by compactifying this (0,2) theory on T^5 [26]. The SL(5,Z) duality symmetry is just the modular group of a five-torus [26]. The (0,2) theory contains a two-form potential with a self dual three-form field strength. The poin-tlike charges correspond to fluxes along three of the spatial dimensions of the fivetorus, they are naturally in the $\mathbf{10}$ of SL(5). The string-like solutions correspond to momentum modes along the torus. The five possible directions give the five possible string-like charges. They represent strings along the longitudinal direction of the M(atrix) description [27].

2.5. Eight dimensions

In eight dimensions the duality group is SL(3,Z)×SL(2,Z). We have point-like configurations and their magnetic 4-brane duals, string-like configurations and their magnetic 3-brane duals, and finally two-brane configurations. The point-like configurations $w_{\alpha b}$ are in $\mathbf{3} \times \mathbf{2}$, the string-like configurations in $\mathbf{3}' \times \mathbf{1}$ and the dyonic two-brane in $\mathbf{1} \times \mathbf{2}$.

In M(atrix) theory, the description is based on a 3+1 YM theory on a torus. SL(3,Z) comes from the symmetries of the torus while SL(2,Z) comes from the S-duality of YM [28]. The six point-like charges correspond to fluxes in the YM theory, the string-like charges correspond to momentum modes along the three-torus.

It is clear that there are no invariant conditions that select $1/4$ or $1/2$ in the case of solutions with purely string charges or purely two-brane charges. This fits in with the fact that those configurations can only be $1/2$ BPS.

The point-like solutions could be $1/2$ or $1/4$ BPS according to whether $\epsilon^{ab} w_{\alpha a} w_{\beta b}$ is zero or not, where $w_{\alpha a}$ are the point-like charges.

2.6. Nine dimensions

In nine dimensions the duality group is $SL(2, Z) \times Z_2$. The point-like charges belong to **2** (v_α) and **1** (v). In the IIB case they would correspond to the two wrapped strings, the fundamental string and the D-string, and the KK momentum mode. The string-like charges are in **2** (the fundamental string and the D-string), the two-brane charge is a singlet **1** (from the ten-dimensional D3-brane) and the rest are the magnetic duals of the above.

With point-like charges we can preserve 1/2 of the supersymmetries if $v = 0$ or $v_\alpha = 0$ (i,e $vv_\alpha = 0$) and 1/4 if both are non-zero.

3. Supergravities with 16 supersymmetries

Now we turn to the discussion of supergravity theories with 16 supersymmetries like $N = 4$ in $d = 4$. We will analyze the $d = 4, 5$ cases. If we take a supergravity theory with n matter multiplets the duality groups are $SL(2, R) \times O(6, n)$ and $O(1, 1) \times O(5, n)$ respectively. If we think of heterotic strings on T^6 then $n = 22$ and $SL(2, Z)$ in the S-duality symmetry of $N = 4$ four dimensional heterotic strings.

3.1. Five dimensions

The charges form a vector Q_i under $O(5, n)$ and a singlet Q_H. There are two invariants Q_H and Q^2. In order for a state to be BPS we need $Q^2 \geq 0$. If either $Q_H = 0$ or $Q_i = 0$ (as a vector) 1/2 of the supersymmetries are preserved and only 1/4 are preserved if both are non-zero. In addition, when $Q_H Q^2$ is non-zero, the corresponding configuration gives rise to a black hole with non-zero entropy. Strominger and Vafa computed the microscopic entropy of these black holes using D-branes for a general configuration [11]. They did the computation for the type II theory on $K3 \times S_1$. The charge Q_H corresponds to KK momentum along S_1 and the charges Q_i correspond to $D1$-, $D3$- and $D5$-branes wrapping along S_1 and a 0-cycle, a 2-cycle, and a 4-cycle on $K3$ respectively.

3.2. Four dimensions

In four dimensions we have electric Q_i and magnetic P_i charges, which are vectors of $O(6, n)$ and together form a doublet of $SL(2, Z)$. It is sometimes convenient to write the charges as $V_{\alpha i} = (Q_i, P_i)$ where $\alpha = 1, 2$ is the $SL(2, Z)$ index. We can form the symmetric matrix $M_{\alpha\beta} = V_{\alpha i} V_{\beta j} \eta^{ij}$ where η^{ij} is the $O(6, n)$ metric. The black hole entropy is proportional to [15][6][29]

$$S = \sqrt{det(M)} = \sqrt{Q^2 P^2 - (Q.P)^2} \tag{3.1}$$

which is clearly invariant.

The condition for having a BPS solution is that $M_{\alpha\beta}$ is semi-definite positive which means that $detM \geq 0$ and $M_{11} \geq 0$, which implies, in particular, that $Q^2 \geq 0$ and $P^2 \geq 0$.

The condition for having a 1/2 BPS solution is that $\epsilon^{\alpha\beta} V_{\alpha i} V_{\beta j} = 0$ which means that Q and P are parallel vectors, so that by means of an $SL(2, Z)$ transformation the configuration can be dualized into one with only electric (or only magnetic) charges. We can present this statement, in analogy to (2.10), by saying that the projection of the second derivatives of the invariant $detM$ projected on the adjoint of $O(6, n)$ vanishes. As in $N = 8$, there is generically a phase that cannot be removed. This is a phase between the central and matter charges, reflecting the fact that five parameters are necessary to obtain the general solution [15]. Again this phase automatically vanishes if we have a 1/2 BPS state.

References

[1] R. Kallosh, A. Linde, T. Ortin, A. Peet, A. Van Proeyen, Phys. Rev. D46 (992) 5278, hep-th/9205027.

[2] E. Cremmer and B. Julia, Nucl. Phys. B159 (1979) 141.

[3] E. Cremmer in "*Supergravity*", ed. by S. Ferrara and J. Taylor, p. 313; B. Julia in "*Superspace & Supergravity*", ed. by S. Hawking and M. Rocek, Cambridge (1981) p. 331.

[4]   C. Hull and P. Townsend, Nucl. Phys. B438 (1995) 109, hep-th/9410167.

[5]   S. Ferrara, C. Savoy, B. Zumino, Phys. Lett. 100B (1981) 393.

[6]   A. Sen, Nucl. Phys. B440 (1995) 421, hep-th/9411187.

[7]   R. Kallosh and B. Kol, Phys. Rev. D53 (1996) 5344, hep-th/9602014.

[8]   J. Polchinski, private communication.

[9]   S. Ferrara and R. Kallosh, Phys. Rev. D54 (1996) 1525, hep-th/9603090.

[10] L. Andrianopoli, R. D'Auria, S. Ferrara, P. Fré, M. Trigiante, "*R-R Scalars, U-Duality and Solvable Lie Algebras*", hep-th/9611014.

[11] A. Strominger and C. Vafa, Phys. Lett. B379 (1996) 99, hep-th/9601029.

[12] C. Callan and J. Maldacena, Nucl. Phys. B475 (1996) 645, hep-th/9602043.

[13] M. Cvetic and C. Hull, Nucl. Phys. B480 (1996) 296, hep-th/9606193.

[14] M. Cvetic and I. Gaida, hep-th/9703134.

[15] M. Cvetic and D. Youm, hep-th/9512127; M. Cvetic and A. Tseytlin, Phys. Rev. D53 (1996) 5619, hep-th/9512031.

[16] L. Andrianopoli, R. D'Auria, S. Ferrara, "*U-Invariants, Black-Hole Entropy and Fixed Scalars*", hep-th/9703156.

[17] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. D52 (1995) 5412, hep-th/9508072.

[18] S. Ferrara and R. Kallosh, Phys. Rev. D54 (1996) 1514, hep-th/9602136.

[19] R. Slansky, Phys. Rep. 79 (1981) 1 (see pages 108, 112, 113).

[20] V. Balasubramanian, F. Larsen and R. Leigh, hep-th/9704143.

[21] R. Kuhri and T. Ortin, Phys. Lett. B373 (1996) 56, hep-th/9512178.

[22] H. Sheinblatt, "*Statistical Entropy of an Extremal Black Hole with 0- and 6-Brane Charges*", hep-th/9705054.

[23] J. Polchinski, "*TASI lectures on D-branes*", hep-th/9611050.

[24] W. Taylor, "*Adhering 0-Branes to 6-Branes and 8-Branes*", hep-th/9705116.

[25] A. Sen, "*Kaluza-Klein Dyons in String Theory*", hep-th/9705212.

[26] M. Rozali, "*Matrix theory and U-duality in seven dimensions*", hep-th/9702136; M. Berkooz, M. Rozali and N. Seiberg, "*Matrix Theory Description of M-theory on T^4 and T^5*", hep-th/9704089.

[27] T. Banks, W. Fishler, S. Shenker and L. Susskind, "*M theory as a Matrix Model: A Conjecture*", hep-th/9610048.

[28] L. Susskind, "*T duality in M(atrix) theory and S duality in field theory*", hep-th/9611202; O. Ganor, S. Ramgoolam and W. Taylor, "*Branes, Fluxes and Duality in M(atrix) Theory*", hep-th/9611202.

[29] M. Duff, J. T. Liu and J. Rahmfeld, Nucl. Phys. B459 (1996) 125, hep-th/9508094.