European Laboratory for Particle Physics

Large Hadron Collider Project

LHC Project Report 114

Estimates for Long–Term Stability for the LHC

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Abstract

Since about 10 years survival plots have been used to evaluate single–particle long–term stability. In a recent paper (M. Giovannozzi et al.) this concept has been reviewed, using a dynamic aperture (Dyn.Aper.) definition based on the average over different ratios of emittances. It has been shown that the survival times evaluated according to this procedure decay with the inverse of the logarithm of the number of turns in several different systems. In this paper the validity of this conjecture is tested in the case of the latest LHC lattice which has been studied extensively.

The inverse log conjecture also predicts a non–zero Dyn.Aper. at infinite times called D_∞. The tracking data are analysed for the LHC lattice to determine the relation between D_∞ and the onset of chaos determined through Lyapunov exponents. Two different methods to automate the prediction of the Lyapunov exponent are tested and are compared with D_∞.

This research was supported in part by the National Science Foundation under Grant No. PHY94-07194.

Contribution to the workshop on “New Ideas for Particle Accelerators”, Santa Barbara, November 1996 and presented, in part, at the Particle Accelerator Conference, Vancouver, May 1997.

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Geneva, 12 May 1997

1 Introduction

In Ref. [1] (see also Ref. [2]) it has been shown that for several dynamical systems the evolution of the Dyn.Aper. $D(N)$ as a function of turn number N is well described by the following equation here called the *Inverse Log Conjecture*:

$$D(N) = D_\infty \left(1 + \frac{b}{log_{10}(N)} \right). \tag{1}$$

The D_∞ can be interpreted as the Dyn.Aper. after an infinite number of turns while the b appears to be a measure of the range of amplitudes where particle loss will take place, e.g. a value $b = 3$ means that after 1'000 turns the Dyn.Aper. is still a factor of two larger than D_∞. For this relation to work a precondition is to average the Dyn.Aper. over the four dimensional phase space as described in Ref. [3]:

$$D(N) = \left(\int_0^{\pi/2} [D_\alpha(N)]^4 \sin(2\alpha) d\alpha \right)^{1/4}, \tag{2}$$

where α is related to emittance ratio ϵ_{II}/ϵ_I by:

$$\alpha = atan\sqrt{\epsilon_{II}/\epsilon_I}, \tag{3}$$

e.g. $(\alpha = 45°)$ corresponds to a emittance ratio of $(\epsilon_{II}/\epsilon_I = 1)$. As the tracking for the LHC is usually done in the full six dimensional phase space one could argue that an average over the six dimensions is needed. This is not done for the following reasons: firstly the nonlinear coupling between longitudinal and transverse planes is small which allows the separate treatment of the longitudinal plane, secondly for the LHC tracking the initial conditions in the longitudinal phase space are not varied but fixed to one set of pessimistic (large) values and lastly the tracking effort would have to be increased by another factor of ten. One aim of this report is to check the conjecture for the LHC version 4 which has been extensively studied (see Ref.[4]). Another aim is the understanding of the relation between D_∞ and the onset of chaos.

2 Fitting Technique

One can rewrite Eq. 1 as follows:

$$D(N) \cdot \log_{10}(N) = D_\infty \cdot \log_{10}(N) + D_\infty \cdot b, \tag{4}$$

where $\log_{10}(N)$ is treated as an independent variable on the right–hand side. Thus D_∞ denotes the slope and $D_\infty \cdot b$ the offset of a linear function which describes $D(N) \cdot \log_{10}(N)$. A linear regression yields both quantities with a certain error Δ. The error of $D(N)$ is calculated to be:

$$\Delta(D(N)) = \Delta(D_\infty) + \Delta(D_\infty \cdot b)\frac{1}{log_{10}(N)} \tag{5}$$

It should be noted that the multiplication of $D(N)$ with $\log_{10}(N)$ in Eq. 4 puts a stronger weight on loss boundaries where they are most relevant, i.e. at larger turn numbers N.

3 Conjecture Test

Figure 1 summarises the tracking data and the fitting result for one realization of the imperfect LHC: the tracking has been performed for 17 emittance ratios up to 10^6 turns. For the emittance ratio of one ($\alpha = 45°$) the tracking has been prolonged to 10^7 turns. A linear regression fit according to Eq. 4 is performed up to 10^5 and 10^6 turns. The fits are extrapolated to 10^7 turns and quoted with their errors. The data for $\alpha = 45^\circ$ which deviate from the phase space averaged data at small turn numbers are consistent with both fits beyond 10^6 turns within their errors. Moreover, reducing the number of angles to 9 changes the predicted D_∞ by a mere 1.1%.

A bit worrying is the fact (see Figure 2) that both the fitted D_∞ and the error of the fit are increasing between 10^5 and 10^6 turns. The figure shows that this is due to a monotonically increasing sliding fit of D_∞ after a few thousand of turns, i.e. the Dyn.Aper. decreases less rapidly than the linear fit does imply. Using the conjecture fit for 60 machine realization (Figure 3) reveals a small anti correlation between D_∞ and b which could mean that the linear relation of Eq. 4 is based on a too simple assumption. On the other hand the figure also shows that the fit constants and the Dyn.Aper. scaled from 10^5 to 10^6, using the inverse log conjecture, have small errors. Even though the significance of the fit parameters remains unclear the fit with two parameters may still be useful to extrapolate the Dyn.Aper. to larger turn numbers.

To check this assumption emittance ratio scans have been extended up to 10^6 turns for 5 different realizations of the random errors (see Figure 4). The fit involving data up to 10^5 turns and the tracking data for 10^6 turns agree within the error bars of the extrapolation.

4 Chaos and D_∞

Since many years the chaotic boundary has been used to estimate the long–term Dyn.Aper. (see Ref. [5]). D_∞ determined from the conjecture fit should agree with the onset of chaos because both quantities describe the stability boundary in phase space. Agreement of the two independent methods would give D_∞ a physical meaning at least in a heuristic manner.

It is well known that there cannot be a rigorous non–zero loss boundary over infinite number of turns in a system with more than two degrees of freedom due to the loss of particles in the Arnold web (see Ref.[6]). However tracking studies for various systems have clearly shown that there always seems to be a hard core of stability in the amplitude space which is equivalent to a non–zero D_∞.

Two models have been tested: the four dimensional Hénon model and the LHC case for which the conjecture fit is shown in Figure 1. Due to its simplicity the first model can be tracked for a large number of angles and turn numbers (40 and 10^7 respectively). The LHC has been tracked for only 17 angles and 10^6 turns which has required two weeks of CPU time of a powerful 10 processor workstation cluster [7].

The two models are meant to be independent: the LHC is tracked at its nominal set of tunes (Q_x=63.28, Q_y=63.31) while the tunes of the Hénon model are chosen so as to maximise the chaotic regime (Q_x=0.168, Q_y=0.201) [8]. The *Top Left* and *Bottom Left* part of figure 5 depict the Dyn.Aper. versus emittance ratio of the Hénon and LHC model respectively, each curve corresponding to a different number of turns. The variations of the Dyn.Aper. can be large and depend on the choice of the tunes and on the realization of the random errors. For the phase space averaged Dyn.Aper. the conjecture fit agrees well with the tracking data in both cases (see *Top Right* and *Bottom Right* of Figure 5).

Chaos is detected by tracing the path of two initially close–by particles. This method is preferred over the original one introduced by Benettin et al. [9] as in this context the most sensitive measure is more relevant than the precise knowledge of the Lyapunov exponent.

Owing to the fact that the automatic detection of the onset of chaos is much more difficult than the reliable but time consuming inspection by eye a new approach has been attempted. Two different values can be automatically extracted from the tracking data: in the first method the distance in phase space must exceed a threshold which is larger than the final separation of any two regular (initially close–by) particles at the end of the tracking, whereas in the second method the motion is deemed chaotic when the slope, calculated from the evolution of the distance in phase space in a double logarithmic scale, is outside a certain interval of slope values (for regular motion the slope is one). In the following these techniques are called the **distance** and the **slope** method respectively. For simplicity the threshold and the slope interval are kept constant but it may be advantageous to vary them as a function of turn number.

The distance method is certainly safe due to its definition. However, it is an optimistic estimate because weakly chaotic particles may not have enough time to separate beyond the chosen threshold. The slope method is less precisely defined: it may be optimistic in case the motion is so weakly chaotic that the slope is not affected but it could also be pessimistic because it can pick up large oscillations of particles which are close to some resonance but which are nevertheless regular. The slope method is chosen as the preferred indicator as it is usually more pessimistic and more consistent with the inspection by eye. It should be mentioned that both methods can be improved by using frequency analysis [10] which allows to eliminate most of the regular oscillations from the evolution of the distance in phase space.

In the case of the Hénon model the slope method is pessimistic (Top right in Figure 5) and very close to the D_∞ fit. From the above discussion it is not surprising that D_∞ itself varies widely as a function of turns. As expected the distance method is optimistic at low turn numbers. At 10^7 turns, however, all three curves converge to almost the same point. It should be noted that this behaviour has been reproduced at two other tune working points. Tab. 1 summarises the results for all three cases.

Table 1: Results for the Hénon Model

Turn Number	Q_x	Q_y	D_∞	Chaos (Slope Method)	Chaos (Distance Method)
10^5	0.168	0.201	114.6	118.1	122.3
10^7			119.7	121.8	120.0
10^5	0.201	0.168	–	123.7	127.3
10^7			125.3	125.1	125.1
10^5	0.201	0.112	–	69.8	73.6
10^7			72.7	72.9	71.7

For the LHC the distance and the slope method are both optimistic (Bottom right in Figure 5). The fact that the slope method is optimistic means that the Dyn.Aper. of the LHC is determined by very weak chaotic motion. Still, at large turn numbers the chaotic boundary determined by the latter method agrees quite well with D_∞.

In both models the fit of D_∞ appears to be pessimistic in an intermediate turn number regime. In fact, in all studied LHC cases D_∞ is a too pessimistic estimate of long–term stability.

5 Conclusion

The inverse log conjecture has been thoroughly tested for the LHC version 4. Although doubts remain about the physical meaning of D_∞ and b the fit can be used to extrapolate the Dyn.Aper. from 10^5 to 10^6 turns. There are indications that this extrapolation can be further extended to 10^7 turns.

The chaos and D_∞ border seem to converge for large turn numbers for both the Hénon and the LHC model.

6 Acknowledgements

We would like to thank E. Todesco for discussing the conjecture fit for the Hénon model with one of the authors at the Santa Barbara workshop.

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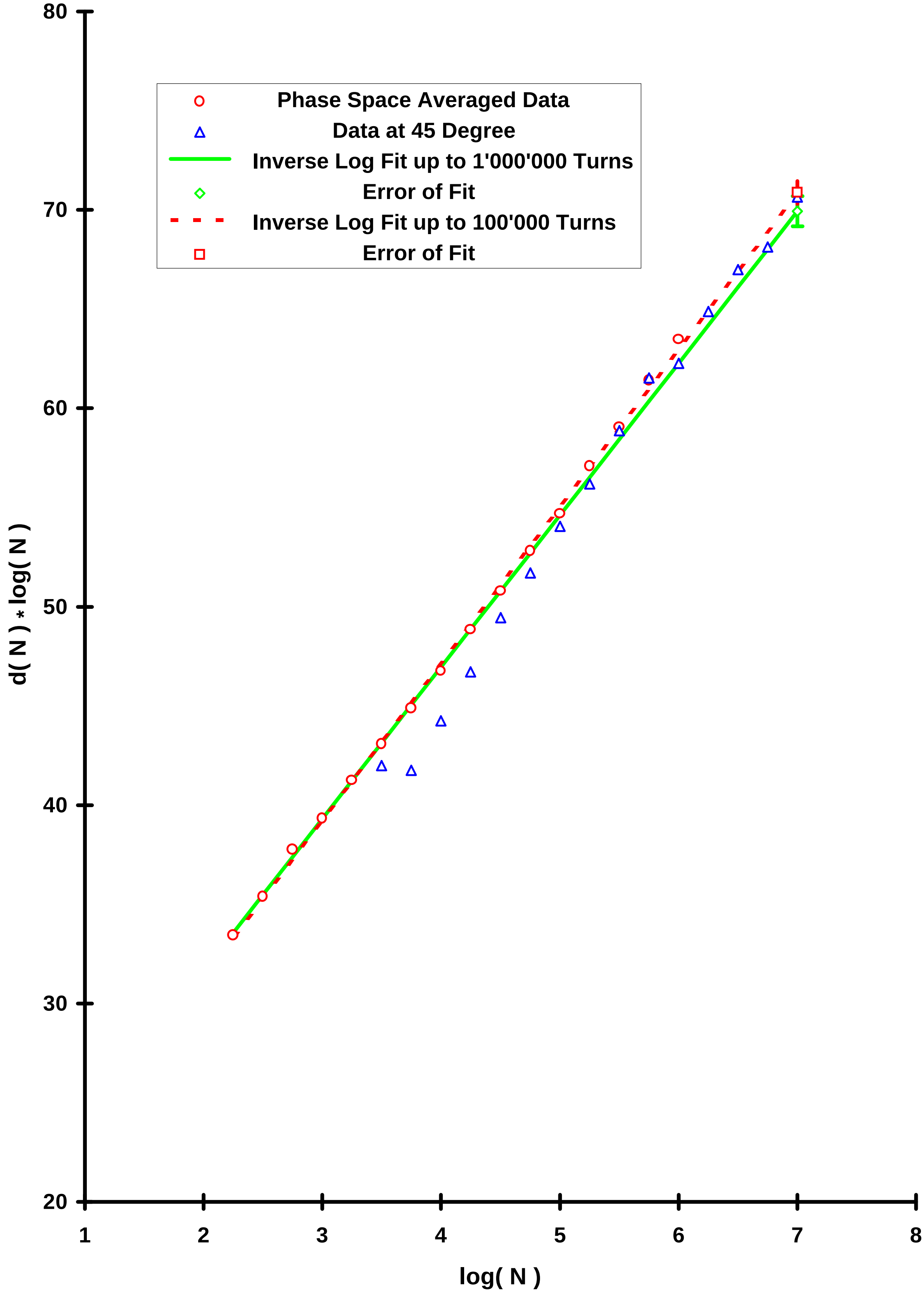

Figure 1: *Fits of Eq. 4 from* 10^2 *to* 10^5 *and* 10^6 *as well as the extrapolation to* 10^7 *turns for one realization of the imperfect LHC*

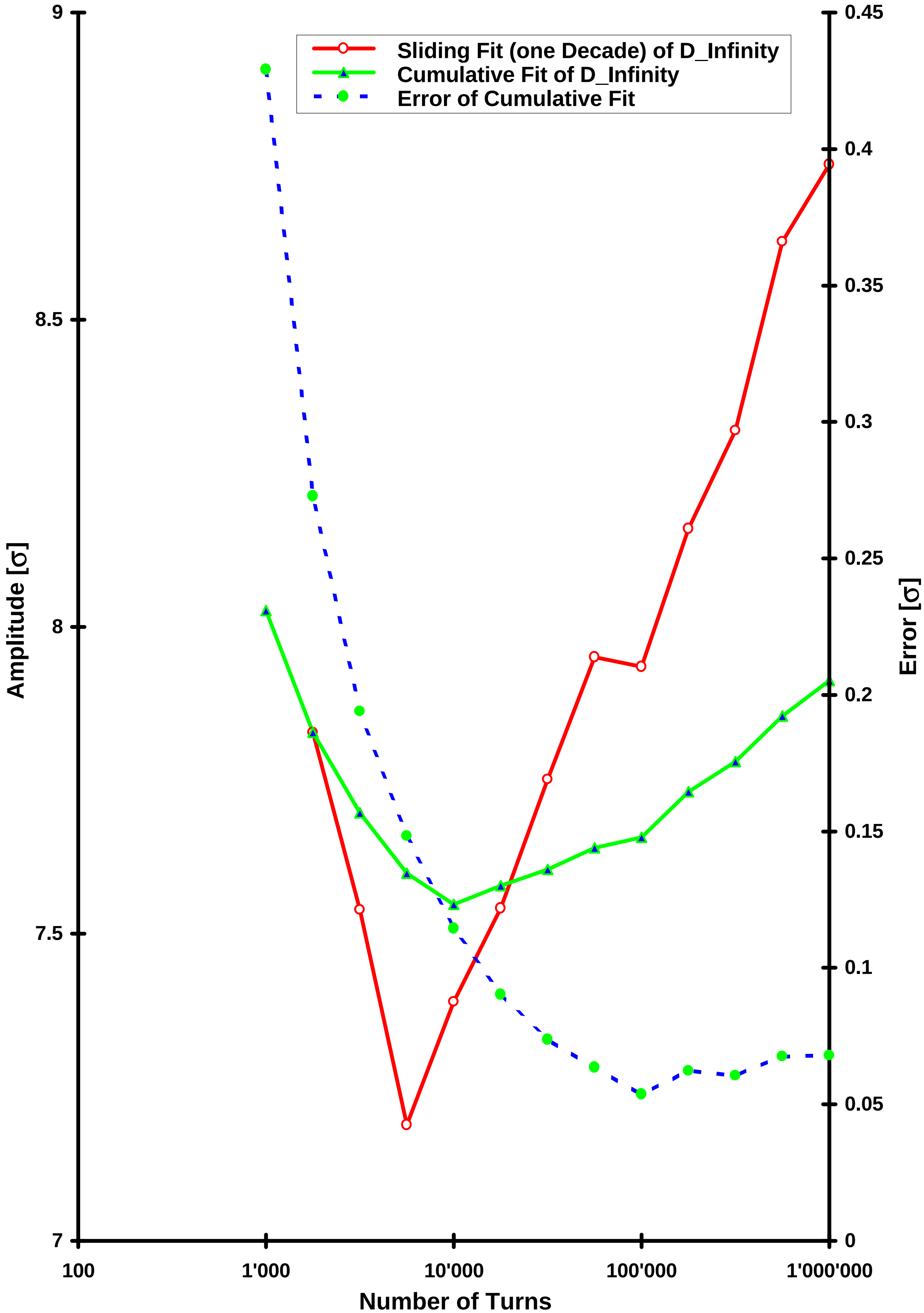

Figure 2: D_∞ *determined from a cumulative fit and a sliding fit*

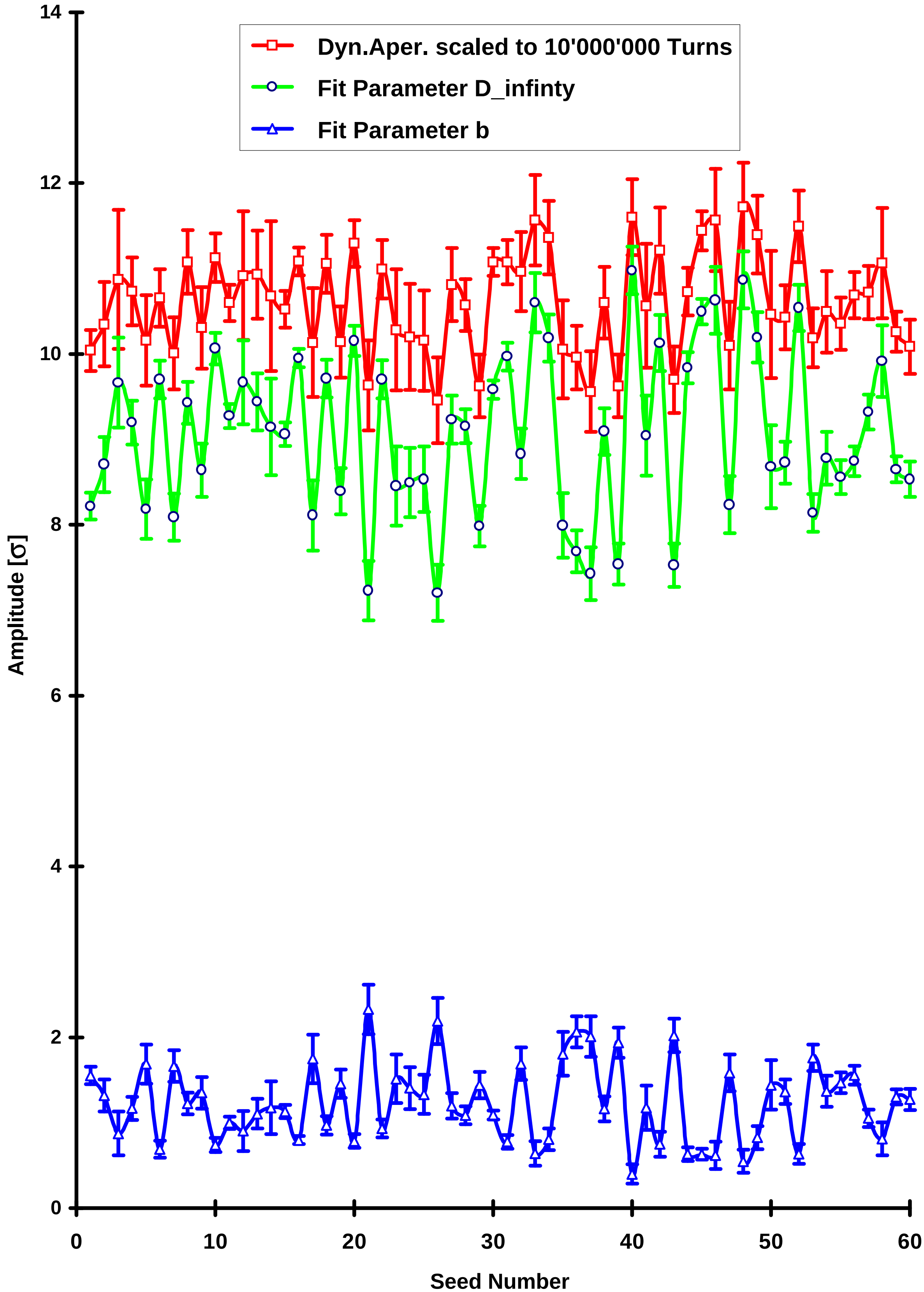

Figure 3: *Scaled Dyn.Aper. and the conjecture fit parameters* D_∞ *and* b

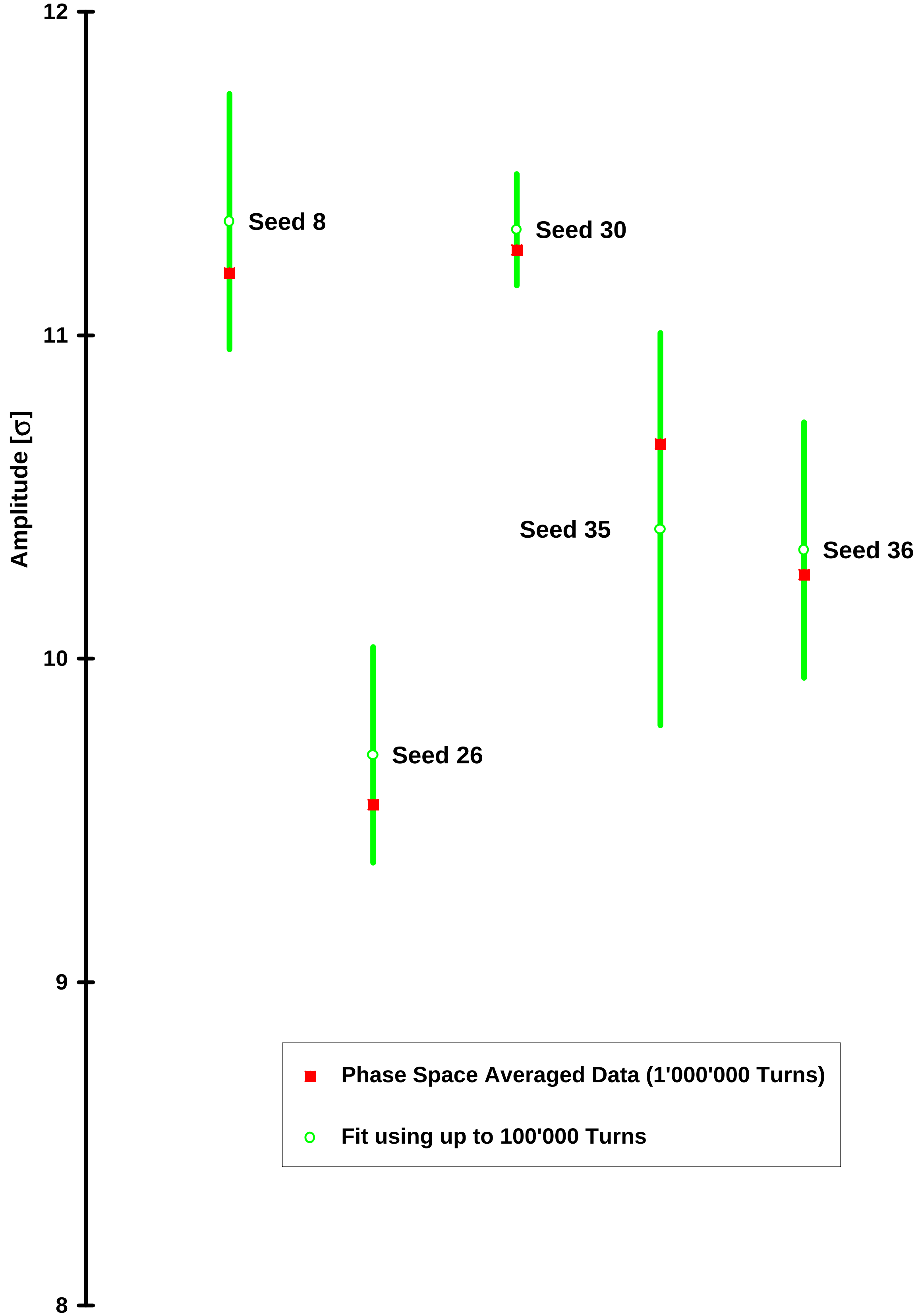

Figure 4: *Comparison of tracked and scaled Dyn.Aper.*

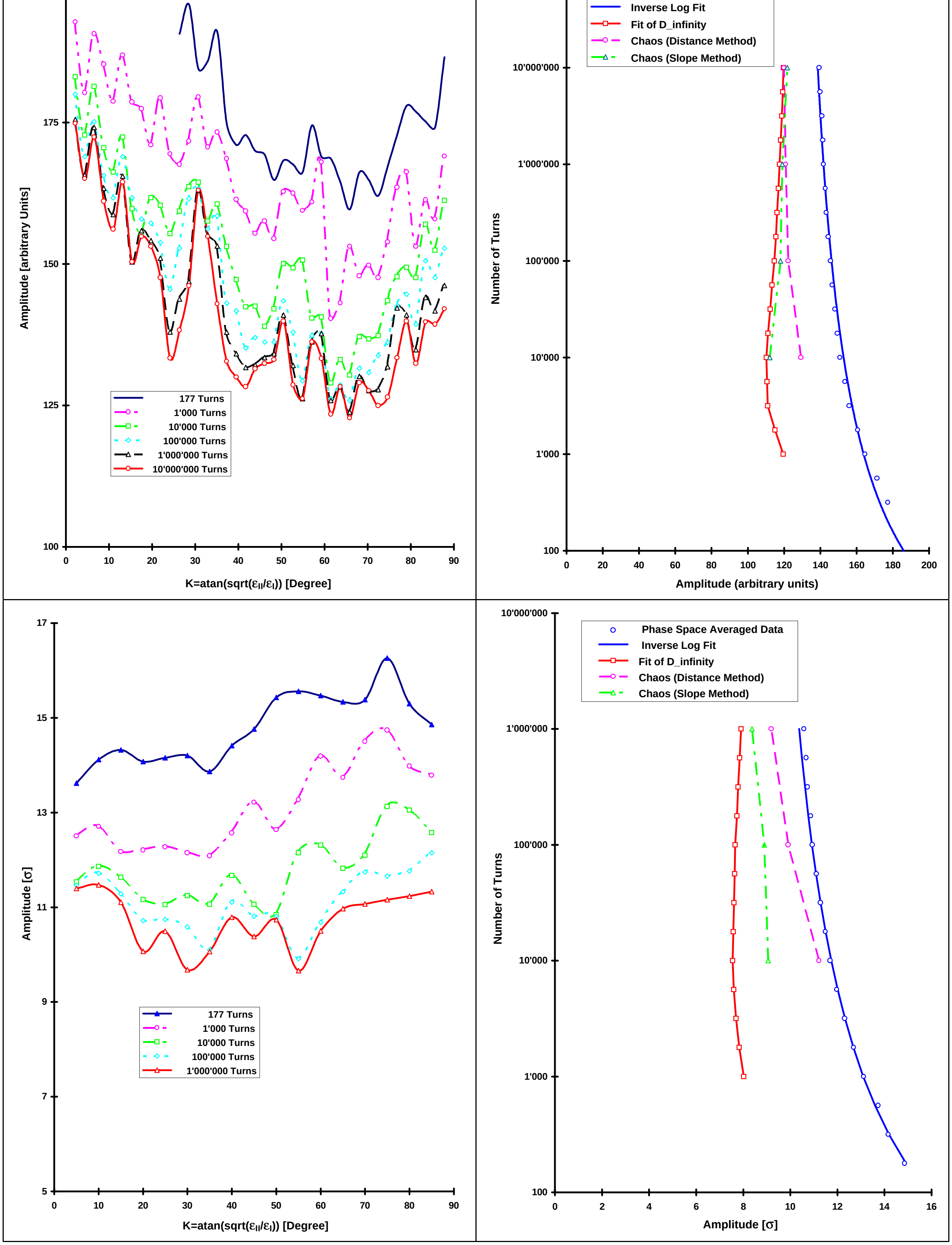

Figure 5: *The Hénon model – Top Left: Stable amplitude versus emittance ratio between* 10^2 *and* 10^7 *turns, Top Right: Survival plot, conjecture fit and chaos boundaries,*
The LHC model – Bottom Left: Stable amplitude versus emittance ratio between 10^2 *and* 10^6 *turns, Bottom Right: Survival plot, conjecture fit and chaos boundaries*