

String and Particle with Two Times

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An action for a string and a particle with two timelike dimensions is proposed and analyzed. Due to new gauge symmetries and associated constraints, the motion of each system in the background of the other is equivalent to effective motion with a single timelike dimension. The quantum constraints are consistent only in critical dimensions. For the bosonic system in flat spacetime the critical dimension is 27 or 28, with signature (25,2) or (26,2), depending on whether the particle is massive or massless respectively. For the supersymmetric case the critical dimensions are 11 or 12, with signature (9,2) or (10,2), under the same circumstances. Generalizations to multi particles, strings and p-branes are outlined.

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I. CLASSICAL PARTICLES AND STRINGS

The idea that the fundamental theory may be formulated in twelve or more dimensions has been receiving increased attention [1] - [19]. It has become apparent that some of the extra dimensions are timelike, and thus the issues associated with more than one timelike dimensions must be addressed seriously. As a first step toward theories with two or more timelike dimensions, in which the traditional problems are overcome, we have proposed a set of gauge symmetries and associated constraints, as well as a cosmological scenario [13]. As an example we formulated an action principle for two particles which move freely except for a global constraint on each other's momenta. In this paper we generalize this type of action principle by discussing the example of a string and a particle in detail, and then showing how to apply the same methods to more particles, strings and p-branes. We discuss the quantum constraints, the emergence of critical dimensions, and the quantum consistent sectors. The supersymmetric generalization is outlined.

A. Reformulation of the two particles

Consider two particles described by their worldlines $x_1^\mu(\tau), x_2^\mu(\tau)$. In our previous work [13] we presented an action with appropriate gauge invariances that produced the following constraints for the momenta of the two particles

$$p_1^2 + m_1^2 = 0, \quad p_2^2 + m_2^2 = 0, \quad p_1 \cdot p_2 = 0. \quad (1)$$

The two particles move freely, except for the mutual constraint $p_1 \cdot p_2 = 0$. Two orthogonal timelike momenta cannot exist in a space with a single timelike dimension. The extra constraint was the key for two timelike dimensions and their interpretation given in [13]. In this section we

would like to give another formulation of the action that leads to the same results. The new formulation is better adapted to generalizations to strings and p-branes.

We consider the following action for two particles

$$S = S_1(x_1, A_1, e_1, \lambda_2) + S_2(x_2, A_2, e_2, \lambda_1) + \lambda_1^\mu \lambda_2^\nu \eta_{\mu\nu}$$

$$S_1 = \frac{1}{2} \int_0^T d\tau \left[e_1^{-1} (\partial_\tau x_1^\mu - \lambda_2^\mu A_1)^2 - e_1 m_1^2 \right] \quad (2)$$

and similarly for S_2 , where $x_i^\mu(\tau), A_i(\tau), e_i(\tau)$ are functions of τ while λ_i^μ are independent of τ . Note that λ_2^μ appears in S_1 and λ_1^μ appears in S_2 . As we will see, λ_2^μ is determined in terms of canonical variables that belong to particle #2, so that the presence of particle #2 influences particle #1, and vice versa. In the path integral we integrate over the λ_i^μ as well as the other fields. Therefore, in the classical theory we minimize the action with respect to the λ_i^μ as well as the other fields.

The τ reparametrization invariance of $S_i(x_i, A_i, e_i, \lambda_i)$ is independent for each i (we denote $1' = 2$ and $2' = 1$), hence there are two reparametrizations that eventually allow the two gauge choices $e_i(\tau) = 1$. The equations of motion for the e_i lead to two constraints

$$p_i^2 + m_i^2 = 0, \quad (3)$$

where

$$p_i^\mu = e_i^{-1} (\partial_\tau x_i^\mu - \lambda_{i'}^\mu A_i), \quad \partial_\tau p_i^\mu = 0 \quad (4)$$

are the canonical momenta, which are conserved according to the equations of motion for x_i^μ .

The action is gauge invariant under the following two gauge transformations with parameters $\Lambda_i(\tau)$

$$\delta x_i^\mu(\tau) = \lambda_{i'}^\mu \Lambda_i(\tau), \quad \delta A_i(\tau) = \partial_\tau \Lambda_i(\tau). \quad (5)$$

The covariant derivatives $\partial_\tau x_i^\mu - \lambda_{i'}^\mu A_i$ may be seen as arising from the gauging of an Abelian subgroup in the

spirit of gauged Wess-Zumino-Witten (WZW) models. Because of these gauge invariances, the equations of motion for A_i lead to two other constraints that help remove degrees of freedom

$$\lambda_{i'} \cdot p_i = 0. \quad (6)$$

Finally we come to the role of the coupling term $\lambda_1 \cdot \lambda_2$ in the action. If it were not for this coupling term there would be two independent Lorentz symmetries, one for each particle. However, because of this coupling there is a single Lorentz symmetry, in a d -dimensional space with a metric $\eta_{\mu\nu}$ whose signature will be determined by the solution of all the constraints. The equations of motion for the λ_i give

$$\lambda_1^\mu = \int_0^T d\tau A_1 p_1^\mu, \quad \lambda_2^\mu = \int_0^T d\tau A_2 p_2^\mu. \quad (7)$$

Since the momenta are conserved one finds

$$\lambda_1^\mu \sim p_1^\mu, \quad \lambda_2^\mu \sim p_2^\mu. \quad (8)$$

Therefore, the two constraints in (6) reduce to the single constraint

$$p_1 \cdot p_2 = 0. \quad (9)$$

We have demonstrated that the new action in (2) reproduces the same system of constraints in (1) given by our old action. Hence, we need to introduce two time-like dimensions and interpret them as in our previous work. This reformulation is more elegant and permits generalizations to strings and p-branes, as discussed below.

B. A string and a particle

Consider a string and a particle described by the a worldsheet $x_1^\mu(\tau, \sigma)$ and a worldline $x_2^\mu(\tau)$ respectively, and introduce the action

$$\begin{aligned} S &= S_1(x_1, A_{1m}, g_{mn}, \lambda_2) + S_2(x_2, A_2, e_2, \lambda_1) + \lambda_1 \cdot \lambda_2 \\ S_1 &= \frac{1}{2} \int_0^T d\tau \int d\sigma \sqrt{-g} g^{mn} (\partial_m x_1^\mu - \lambda_2^\mu A_{1m}) \\ &\quad \times (\partial_n x_1^\nu - \lambda_2^\nu A_{1n}) \eta_{\mu\nu} \quad (10) \\ S_2 &= \frac{1}{2} \int_0^T d\tau \left[e_2^{-1} (\partial_\tau x_2^\mu - \lambda_1^\mu A_2)^2 - e_2 m_2^2 \right] \end{aligned}$$

The two actions $S_{1,2}$ are invariant under independent reparametrizations, hence one can choose the usual conformal gauge for the string $\sqrt{-g}g^{mn} = \eta^{mn}$, and $e_2 = 1$ for the particle, and obtain the following constraints from the equations of motion of g^{mn}, e_2 respectively

$$(D_\pm x_1^\mu)^2 = 0, \quad p_2^2 + m_2^2 = 0, \quad (11)$$

where

$$\begin{aligned} D_\pm x_1^\mu &= (\partial_\pm x_1^\mu - \lambda_2^\mu A_{1\pm}), \quad (12) \\ p_2^\mu &= e_2^{-1} (\partial_\tau x_2^\mu - \lambda_1^\mu A_2), \end{aligned}$$

and the ∂_\pm derivatives are with respect to the light-cone variables $\sigma^\pm \equiv \tau \pm \sigma$. The equations of motion for $x_1^\mu(\tau, \sigma), x_2^\mu(\tau)$, in the gauges we have chosen, are

$$\partial_+ (D_- x_1^\mu) + \partial_- (D_+ x_1^\mu) = 0, \quad \partial_\tau p_2^\mu = 0. \quad (13)$$

As in the two particle case, there are additional gauge invariances, which may be understood in the spirit of gauged WZW models

$$\begin{aligned} \delta_1 x_1^\mu &= \lambda_2^\mu \Lambda_1(\tau, \sigma), \quad \delta_1 A_{1m} = \partial_m \Lambda_1(\tau, \sigma), \quad (14) \\ \delta_2 x_2^\mu &= \lambda_1^\mu \Lambda_2(\tau), \quad \delta_2 A_2 = \partial_\tau \Lambda_2(\tau), \end{aligned}$$

and these explain the structure of the covariant derivatives ($g_{mn}, e_2, \lambda_{1,2}^\mu$ are invariant under $\delta_{1,2}$). The equations of motion for $A_{1\pm}, A_2$ give the constraints associated with these gauge invariances

$$\lambda_2 \cdot D_\pm x_1 = 0, \quad \lambda_1 \cdot p_2 = 0. \quad (15)$$

Finally, the equations of motion for $\lambda_{1,2}^\mu$ give

$$\begin{aligned} \lambda_1^\mu &= \int_0^T d\tau \int d\sigma (D_+ x_1^\mu A_{1-} + D_- x_1^\mu A_{1+}), \quad (16) \\ \lambda_2^\mu &= \int_0^T d\tau A_2 p_2^\mu, \end{aligned}$$

Using the constraints (15) we deduce

$$\lambda_1 \cdot \lambda_2 = 0. \quad (17)$$

Since p_2^μ is conserved, one finds $\lambda_2^\mu \sim p_2^\mu$.

From the first equation in (15) we deduce $A_{1\pm} = \frac{1}{\lambda_2^\pm} \lambda_2 \cdot \partial_\pm x_1$ which allows us to write

$$D_\pm x_1^\mu = \partial_\pm x_1^\mu - \frac{1}{p_2^2} p_2^\mu p_2 \cdot \partial_\pm x_1, \quad (18)$$

which solves the constraint $\lambda_2 \cdot D_\pm x_1 = p_2 \cdot D_\pm x_1 = 0$ explicitly, provided $p_2^2 \neq 0$. From this form one sees that the component of x_1^μ that is parallel to p_2^μ drops out of the string system when $p_2^2 \neq 0$. Using the $\Lambda_1(\tau, \sigma)$ gauge invariance one may choose the gauge

$$p_2 \cdot x_1(\tau, \sigma) = 2c_1 p_2^2 \tau = c_1 p_2^2 (\sigma^+ + \sigma^-), \quad (19)$$

where c_1 is a constant. In this gauge one has

$$D_\pm x_1^\mu = \partial_\pm x_1^\mu - c_1 p_2^\mu, \quad A_{1\pm}(\tau, \sigma) = c_1 \sqrt{p_2^2 / \lambda_2^2}, \quad (20)$$

which leads to the simplification

$$\lambda_1^\mu = c_1 \sqrt{p_2^2 / \lambda_2^2} \int_0^T d\tau \int d\sigma (\partial_\tau x_1^\mu - 2c_1 p_2^\mu). \quad (21)$$

The canonical momentum density for the string is $D_\tau x_1^\mu = \partial_\tau x_1^\mu - A_{1\tau} \lambda_2^\mu = \partial_\tau x_1^\mu - 2c_1 p_2^\mu$. Inserting this

in the equation above, one finds that λ_1^μ is proportional to the *total conserved* momentum of the string # 1

$$p_1^\mu = \int d\sigma (\partial_\tau x_1^\mu - 2c_1 p_2^\mu), \quad (22)$$

and that it is orthogonal to the momentum of particle # 2. Since both λ_1^μ and p_1^μ are gauge independent quantities, their relation which was derived in a specific gauge, is also gauge invariant. Hence we have deduced that

$$\lambda_1^\mu \sim p_1^\mu, \quad \lambda_2^\mu \sim p_2^\mu, \quad p_1 \cdot p_2 = 0, \quad (23)$$

just as in the case of two particles of the previous section.

In the lightlike case $\lambda_2^2 = 0 = p_2^2$ (which is consistent only if $m_2 = 0$), the action S_1 has no $A_{1+}A_{1-}$ term, and acquires an additional gauge symmetry beyond (14)

$$\delta_3 x_1^\mu(\tau, \sigma) = 0, \quad \delta_3 A_{1\pm} = \pm \partial_\pm \Lambda_3(\tau, \sigma). \quad (24)$$

The constraint $p_2 \cdot D_\pm x_1 = p_2 \cdot \partial_\pm x_1 = 0$ eliminates a component of $x_1^\mu(\tau, \sigma)$ not parallel to p_2^μ (this is consistent with the gauge choice (19) although in this case it follows from the constraint). The extra $\Lambda_3(\tau, \sigma)$ gauge symmetry can be used to gauge fix $A_{1\pm}$ to $A_{1\pm} = \partial_\pm \gamma(\tau, \sigma)$ and then use the gauge symmetry (14) to fix $\gamma(\tau, \sigma)$ so that $D_\pm x_1$ and $A_{1\pm}$ take the form in (20). In this way all the results above, including (23), apply in the massless particle case as well, and this may be understood as the limit in which p_2^2/λ_2^2 remains finite. However, in the background of the massless particle, two string components, rather than only one, are eliminated by the gauge invariances: $\partial_\pm x_1^\mu - c_1 p_2^\mu$ has no components along the lightlike p_2^μ , and $p_2 \cdot \partial_\pm x_1 = 0$, whereas for the massive particle these two conditions correspond to one and the same component. This kind of phenomenon happened also in the massless limit of the two particle case, as explained in [13].

The equation of motion for the string is easily solved since it has the free string form

$$\partial_+ \partial_- x_1^\mu = 0. \quad (25)$$

As usual, the general solution is given in terms of left and right movers

$$x_{1\mu} = x_{1\mu}^{(+)}(\sigma^+) + x_{1\mu}^{(-)}(\sigma^-) + c_1 p_{2\mu} \tau$$

$$x_{1\mu}^{(\pm)}(\sigma^\pm) = \frac{1}{2} \left(q_{1\mu} + \frac{\sigma^\pm}{2\pi} p_{1\mu} \right) - i \sum_{n \neq 0} \frac{1}{n} \alpha_{n\mu}^{(\pm)} e^{in\sigma^\pm}$$

The term proportional to $c_1 p_2^\mu \tau$ is added to be consistent with the definition of the total string momentum p_1^μ that followed from the canonical formalism. The canonical pair is $(q_{1\mu}, p_{1\mu})$, while the $\alpha_{n\mu}^{(\pm)}$ have the usual string oscillator commutation rules.

The equations of motion for particle #2 are also solved easily since p_2^μ is conserved. The constraint in eq.(15), together with the definition of p_2^μ in terms of the velocity $\partial_\tau x_2^\mu$ give $A_2(\tau) = \frac{1}{\lambda_1} \lambda_1 \cdot \partial_\tau x_2$, so that

$$p_2^\mu = \partial_\tau x_2^\mu - p_1^\mu \frac{1}{p_1^2} p_1 \cdot \partial_\tau x_2. \quad (26)$$

Using the $\Lambda_2(\tau)$ gauge invariance one can choose the gauge

$$p_1 \cdot \partial_\tau x_2(\tau) = c_2 p_1^2 \quad (27)$$

where c_2 is a constant. In this gauge the conserved momentum of particle #2 and A_2 become

$$p_2^\mu = \partial_\tau x_2^\mu - c_2 p_1^\mu, \quad A_2 = c_2 \sqrt{p_1^2/\lambda_1^2} \quad (28)$$

This form is valid for the massive as well as massless string states (i.e. p_1^2/λ_1^2 finite as $p_1^2 \rightarrow 0$, as above). The solution of the particle equation is

$$x_2^\mu(\tau) = (p_2^\mu + c_2 p_1^\mu) \tau + q_2^\mu, \quad (29)$$

showing that it moves as a free particle, except for the orthogonality constraint $p_1 \cdot p_2 = 0$. The canonical pair is (q_2^μ, p_2^μ) .

By reexamining the equations for $\lambda_{1,2}^\mu$ one finds that

$$\lambda_1^\mu = T a_1 p_1^\mu, \quad \lambda_2^\mu = T a_2 p_2^\mu, \quad (30)$$

where $a_{1,2}$ are constant zero modes of the gauge fields that have survived in the general solution

$$A_{1\pm}(\tau, \sigma) \equiv a_1, \quad A_2(\tau) = a_2, \quad (31)$$

and that the two constants $c_{1,2}$ are equal and given by $c_1 = c_2 = T a_1 a_2$.

II. QUANTIZATION AND CRITICAL DIMENSIONS

In the previous section it was shown that the particle and string systems move as free systems except for a set of constraints. The canonical degrees of freedom (q_2^μ, p_2^μ) , (q_1^μ, p_1^μ) , $\alpha_{n\mu}^\pm$ satisfy the first class constraints that follow from eqs.(11,15,23)

$$m_2 \neq 0 : \Phi \equiv p_2^2 + m_2^2 = 0, \quad L_n^\pm = 0, \quad J_0^\pm \equiv p_2 \cdot p_1 = 0,$$

$$m_2 = 0 : \Phi \equiv p_2^2 = 0, \quad L_n^\pm = 0, \quad J_n^\pm \equiv p_2 \cdot \alpha_n^\pm = 0, \quad (32)$$

where the Virasoro operators are

$$m_2 \neq 0 : L_n^\pm = \frac{1}{2} \sum_{m=-\infty}^{\infty} \alpha_{n-m}^{(\pm)\mu} \cdot \alpha_m^{(\pm)\nu} \left(\eta_{\mu\nu} - \frac{p_{2\mu} p_{2\nu}}{p_2^2} \right)$$

$$m_2 = 0 : L_n^\pm = \frac{1}{2} \sum_{m=-\infty}^{\infty} \alpha_{n-m}^{(\pm)\mu} \cdot \alpha_m^{(\pm)\nu} \eta_{\mu\nu} \quad (33)$$

We have used $\alpha_{0\mu}^\pm = p_{1\mu}$, therefore the constraint $J_0^\pm \equiv p_1 \cdot p_2 = 0$ is included above for $m_2 = 0$ as well. The second term in L_n^\pm for $m_2 \neq 0$ could be dropped since $p_2 \cdot \alpha_n^\pm = 0$ was taken as a gauge choice, which may be

added as an additional constraint $J_n^\pm \equiv p_2 \cdot \alpha_n^\pm = 0$ for the massive case as well. This may be done classically as well as in a lightcone quantization, as we will see below. However, this gauge choice becomes a second class constraint in the massive case $m_2 \neq 0$, that is harder to deal with in a covariant quantization (since it has an anomaly proportional to m_2^2)

$$[J_n^\pm, J_m^\pm] = n \Phi \delta_{n+m} - n m_2^2 \delta_{n+m}. \quad (34)$$

As we will see, this will play a role in the covariant quantization of the system and in the determination of the critical dimension.

It is evident from the analogous two particle problem that there is no solution (either classical or quantum) unless there are two or more timelike dimensions. Assuming two timelike dimensions the signature of the space is given by $\eta_{\mu\nu} = \text{diag}(-1, -1, 1, \dots, 1)$. Hence there is a $\text{SO}(d-2, 2)$ covariance in d dimensions. d will be determined in the quantum theory. It will be fixed to a critical dimension $d_{crit} = 27$ for $m_2 \neq 0$, or $d_{crit} = 28$ for $m_2 = 0$, to eliminate quantum anomalies.

We will first solve the constraints classically and hence we will add $J_n^\pm = 0$ as a gauge choice to the massive case as well. This solution will be used in a lightcone quantization. Using the $\text{SO}(d-2, 2)$ symmetry one can boost p_2^μ to a timelike rest frame if the mass $m_2 \neq 0$, or to a lightlike frame if the mass $m_2 = 0$,

$$\hat{p}_2^\mu = (m_2, 0; \vec{0}, 0) \quad \text{or} \quad \hat{p}_2^\mu = (|p_2|, 0; \vec{0}, p_2), \quad (35)$$

where the first two entries are timelike and the rest are spacelike. This allows the solution of three constraints in the form

$$\begin{aligned} m_2 \neq 0: \quad \hat{p}_{1\mu} &= (0, \hat{p}_{1I}), \quad \hat{\alpha}_{\mu n}^\pm = (0, \hat{\alpha}_{In}^\pm), \\ m_2 = 0: \quad \hat{p}_1^\mu &= (0, \hat{p}_{1I}, 0), \quad \hat{\alpha}_{\mu n}^\pm = (0, \hat{\alpha}_{In}^\pm, 0), \end{aligned} \quad (36)$$

leaving one constraint to be solved

$$L_n^\pm = \frac{1}{2} \sum_{m=-\infty}^{\infty} \hat{\alpha}_{n-m}^\pm \cdot \hat{\alpha}_m^\pm = 0, \quad (37)$$

where $\hat{\alpha}_{In}^\pm$ are the string variables in the rest frame or lightcone frame of particle #2, and the index I labels a subspace of signature $(-1, 1, \dots, 1)$ whose dimension is $(d-1)$ for $m_2 \neq 0$ and $(d-2)$ for $m_2 = 0$. Therefore there remains a Lorentz symmetry $\text{SO}(d-2, 1)$ for $m_2 \neq 0$ and $\text{SO}(d-3, 1)$ for $m_2 = 0$. Note that for $m_2 = 0$, the solutions $\hat{\alpha}_{\mu n}^\pm, \hat{p}_1^\mu$ have no components along the lightlike \hat{p}_2^μ . This is due to the gauge symmetry (24) as explained above.

A. Covariant quantization

To quantize covariantly and implement the constraints (32) on the states one may use the BRST formalism.

The BRST procedure is valid for first class constraints. We will pretend as if it applies to both massless and massive cases with the constraints Φ, L_n^\pm, J_n^\pm in order to illustrate the problem with the anomaly in (34), and will find out that this set of constraints work only for $m_2 = 0$. Then we will discuss separately the massive case by dealing directly only with the first class constraints (without the J_n^\pm).

Corresponding to the constraints Φ, L_n^\pm, J_n^\pm one introduces the ghosts $(B, C), (b_n^\pm, c_n^\pm)$ and $(\beta_n^\pm, \gamma_n^\pm)$. The algebra of the constraints and their anomalies are

$$\begin{aligned} [L_n^\pm, L_m^\pm] &= (n-m) L_{n+m}^\pm + \frac{d}{12} (n^3 - n) \delta_{n+m, 0} \\ [L_n^\pm, J_m^\pm] &= -m J_m^\pm \\ [J_n^\pm, J_m^\pm] &= n \Phi \delta_{n+m} - n m_2^2 \delta_{n+m} \end{aligned} \quad (38)$$

The BRST operator is

$$\begin{aligned} Q &= C \Phi + \sum_{\pm} \sum_n [c_{-n}^\pm L_n^\pm + \gamma_{-n}^\pm J_n^\pm] \\ &\quad - \frac{1}{2} \sum_{\pm} \sum_{n, m} (n-m) c_{-n}^\pm c_{-m}^\pm b_{n+m}^\pm \\ &\quad - \frac{1}{2} \sum_{\pm} \sum_m m \gamma_{-m}^\pm \gamma_m^\pm B \\ &\quad - \sum_{\pm} \sum_{n, m} (-m) c_{-n}^\pm \gamma_{-m}^\pm \beta_{n+m}^\pm - \sum_{\pm} c_0^\pm a^\pm - CA \end{aligned} \quad (39)$$

where a^\pm, A are anomaly constants to be determined by requiring $Q^2 = 0$. By anticommuting Q with the antighosts B, b_n, β_n one gets the total gauge generators of the BRST quantized theory

$$\begin{aligned} \Phi^{tot} &= \Phi - A \\ L_n^{(\pm)tot} &= L_n^\pm + \sum_m (n-m) b_{n+m}^\pm c_{-m}^\pm \\ &\quad + \sum_m (-m) \beta_{n+m}^\pm \gamma_{-m}^\pm - a^\pm \delta_{n0} \\ J_n^{(\pm)tot} &= J_n^\pm + n \left(\gamma_n^\pm B + \sum_m \beta_{n+m}^\pm c_{-m}^\pm \right) \end{aligned} \quad (40)$$

Requiring $Q^2 = 0$ is equivalent to requiring the total generators to close without anomalies. For a ghost system of dimension h the Virasoro generator is $L_n^{(h)} = \sum_m (n(h-1) - m) b_{n+m}^{(h)} c_{-m}^{(1-h)}$ and the anomaly in its algebra is

$$\begin{aligned} \text{anom}(h) &= \delta_{n+m} \frac{1}{12} [n^3 c_h + 2n] \\ c_h &= -12h^2 + 12h - 2. \end{aligned} \quad (41)$$

Therefore the total anomaly in the algebra of $L_n^{(\pm)tot}$ is

$$\frac{n^3}{12} (d + c_{h=2} + c_{h=1}) + \frac{n}{12} (-d + 24a^\pm + 2 + 2) \quad (42)$$

with the total central charge

$$c_{tot} = d + c_{h=2} + c_{h=1} = d - 28. \quad (43)$$

The $J_n^{(\pm)tot}$ algebra is

$$\begin{aligned} [J_n^{(\pm)tot}, J_m^{(\pm)tot}] &= n \Phi^{tot} \delta_{n+m} \\ &+ n (A - m_2^2) \delta_{n+m}, \end{aligned} \quad (44)$$

and

$$[L_n^{(\pm)tot}, J_m^{(\pm)tot}] = -m J_{n+m}^{(\pm)tot}. \quad (45)$$

So the total anomaly cancels if

$$d = 28, \quad a^\pm = 1, \quad A = m_2^2. \quad (46)$$

Under these conditions one finds

$$\Phi^{tot} = p_2^2 + m_2^2 - A = p_2^2 = 0. \quad (47)$$

So, the quantum particle must be *massless* $p_2^2 = 0$, and the critical dimension for the total system is 28. The global Lorentz symmetry of this quantum system is $SO(26, 2)$. Therefore, the analysis applies correctly only to the $m_2 = 0$ case, as anticipated above.

For $m_2 \neq 0$ the correct treatment of the second class constraint $J_n^{(\pm)} = 0$ needs more care. Since the present model is in the class of gauged WZW models one may imitate the BRST procedure advocated in [20] to show that the final result is equivalent to the standard coset construction. In the coset language our case corresponds to G/H with $G = R^d$ and $H = R$, and our Virasoro generator in (33) is indeed the coset construction for the conformal field theory for R^d/R . The central charge of this Virasoro generator is $d - 1$, and for the quantum consistency of the conformal field theory it must be set equal to 26. Hence

$$m_2 \neq 0: \quad d_{crit} = 27, \quad (48)$$

and the global symmetry of the system with a massive particle #2 is $SO(25, 2)$.

This result may be obtained more directly without appealing to the formalism of [20]. Namely, one can avoid the $J_n^{(\pm)} = 0$ gauge fixing and work directly with the oscillators

$$\tilde{\alpha}_{n\mu}^{(\pm)} = \alpha_{n\mu}^{(\pm)} - \frac{1}{p_2^2} p_{2\mu} p_2 \cdot \alpha_n^{(\pm)} \quad (49)$$

since these $\tilde{\alpha}_{n\mu}^{(\pm)}$ solve explicitly the original constraints $p_2 \cdot D_\pm x_1 = 0$. The Virasoro constraints in (33) are written directly in terms of these oscillators

$$m_2 = 0: \quad L_n^\pm = \frac{1}{2} \sum_{m=-\infty}^{\infty} \tilde{\alpha}_{n-m}^{(\pm)\mu} \cdot \tilde{\alpha}_m^{(\pm)\nu} \eta_{\mu\nu}, \quad (50)$$

and their commutation rules are

$$[\tilde{\alpha}_n^{(\pm)\nu}, \tilde{\alpha}_m^{(\pm)\nu}] = n \delta_{n+m} \left(\eta^{\mu\nu} - \frac{p_2^\mu p_2^\nu}{p_2^2} \right). \quad (51)$$

The only constraints that need to be considered are the first class constraints $L_n^\pm = 0$, $\Phi = p_2^2 + m_2^2 = 0$, $J_0 = p_1 \cdot p_2 = 0$. In the algebra of these constraints all commutators are zero, except for

$$\begin{aligned} [L_n^\pm, L_m^\pm] &= (n - m) L_{n+m}^\pm \\ &+ \frac{d-1}{12} (n^3 - n) \delta_{n+m,0} \end{aligned} \quad (52)$$

where the $d - 1$ anomaly comes from

$$\left(\eta^{\mu\nu} - \frac{p_2^\mu p_2^\nu}{p_2^2} \right) \left(\eta_{\mu\nu} - \frac{p_{2\mu} p_{2\nu}}{p_2^2} \right) = d - 1. \quad (53)$$

The BRST operator is then

$$\begin{aligned} Q &= \sum_{\pm} \sum_n c_{-n}^\pm L_n^\pm - \frac{1}{2} \sum_{\pm} \sum_{n,m} (n - m) c_{-n}^\pm c_{-m}^\pm b_{n+m}^\pm \\ &+ C\Phi + \gamma_0 J_0 - \sum_{\pm} c_0^\pm a^\pm \end{aligned} \quad (54)$$

and the standard procedure gives the critical dimension $d - 1 = 26$.

B. Lightcone quantization

The remaining constraints in the rest frame or lightcone frame of particle #2 are $(\partial_\pm \hat{x}_{1\pm}^I)^2 = 0$ or $L_n^\pm = 0$ of eq.(37). These are the familiar Virasoro constraints of string theory, which can be solved explicitly by taking advantage of the conformal invariance of the string system and choosing the lightcone gauge $\hat{x}_1^+ = \hat{p}_1^+ \tau$

$$\hat{\alpha}_n^{(\pm)-} = \frac{1}{2\hat{p}_1^+} \sum_{m=-\infty}^{\infty} \sum_I \hat{\alpha}_{n-m}^{(\pm)i} \hat{\alpha}_m^{(\pm)i} - \frac{\tilde{a}}{\hat{p}_1^+} \delta_{n,0}, \quad (55)$$

$$\hat{\alpha}_n^{(\pm)+} = \hat{p}_1^+ \delta_{n,0},$$

The unconstrained degrees of freedom are the transverse string oscillators $\tilde{\alpha}_n^{(\pm)i}$ that describe the left/right moving string excitations and the center of mass canonical degrees of freedom for the string (\hat{q}_1, \hat{p}_1) and particle (\hat{q}_2, \hat{p}_2) , all of which are expressed in the rest frame or lightcone frame of particle #2.

The Lorentz symmetry $SO(d - 2, 1)$ (or $SO(d - 3, 1)$) which was manifest in the special frame (35-37) is hidden in the lightcone gauge (55) for the string. As is well known, the quantum algebra for the *normal ordered* generators $\hat{M}^{IJ} = \left(\hat{M}^{-+}, \hat{M}^{i+}, \hat{M}^{i-}, \hat{M}^{ij} \right)$

$$\begin{aligned}
\hat{M}^{-+} &= \hat{q}_1^- \hat{p}_1^+, \quad \hat{M}^{i+} = \hat{q}_1^i \hat{p}_1^+, \\
\hat{M}^{i-} &= \hat{q}_1^i \hat{p}_1^- - \hat{q}_1^- \hat{p}_1^i - i \sum_{\pm} \sum_{n \neq 0} \frac{1}{n} : \hat{\alpha}_{-n}^{(\pm)i} \hat{\alpha}_n^{(\pm)-} :, \quad (56) \\
\hat{M}^{ij} &= \hat{q}_1^i \hat{p}_1^j - \hat{q}_1^j \hat{p}_1^i - i \sum_{\pm} \sum_{n \neq 0} \frac{1}{n} : \hat{\alpha}_{-n}^{(\pm)i} \hat{\alpha}_n^{(\pm)j} :,
\end{aligned}$$

of this symmetry closes correctly only if the number of transverse dimensions labeled by i is 24 and $\tilde{a} = 1$ (see e.g. [21]). The number of transverse dimensions is $(d-3) = 24$ for $m_2 \neq 0$, and $(d-4) = 24$ for $m_2 = 0$. Therefore the particle and string system has a critical dimension

$$\begin{aligned}
m_2 \neq 0 : \quad d_{crit} &= 27, \\
m_2 = 0 : \quad d_{crit} &= 28,
\end{aligned} \quad (57)$$

so the generators \hat{M}^{IJ} represent correctly $\text{SO}(25,1)$ at the quantum level.

The original action was invariant under the full classical rotation invariance $\text{SO}(d-2,2)$. For the critical dimension this classical symmetry is $\text{SO}(25,2)$ if $m_2 \neq 0$ and $\text{SO}(26,2)$ if $m_2 = 0$. We now need to show that the quantum theory has the higher symmetry by verifying that the Lorentz algebra closes. To construct the remaining generators of $\text{SO}(25,2)$ (or $\text{SO}(26,2)$) we need to boost back to the general frame of particle #2 and include the canonical degrees of freedom of particle #2.

For the massive particle $m_2 \neq 0$ in the general frame the mass shell constraint is solved by the $\text{SO}(d-2,2)$ covariant vector

$$p_2^\mu = (E_2', p_2^I), \quad E_2' = \sqrt{p_2 \cdot p_2 + m_2^2} \quad (58)$$

where p_2^I is a $\text{SO}(25,1)$ vector. The boost of any vector $\hat{v}^\mu = (\hat{v}^{0'}, \hat{v}^I)$ defined in the particle rest frame (denoted with the hats $\hat{}$) to the vector $v^\mu = (v^{0'}, v^I)$ defined in the particle general frame (no $\hat{}$) is given by

$$\begin{aligned}
v^{0'} &= \frac{1}{m_2} \left(E_2' \hat{v}^{0'} + p_2 \cdot \hat{v} \right), \\
v^I &= \hat{v}^I + \frac{p_2^I}{m_2} \left(\frac{p_2 \cdot \hat{v}}{E_2' + m_2} + \hat{v}^{0'} \right)
\end{aligned} \quad (59)$$

where the sum over I in the dot products is $\text{SO}(25,1)$ covariant in 26 dimensions. Of course, the transformation is such that dot products are $\text{SO}(25,2)$ invariant in the full 27 dimensions $\hat{v}^2 = v^2$. The string and particle can now be described in the general frame by boosting the rest frame solution in (35-37). Taking into account $\hat{\alpha}_n^{(\pm)0'} = 0$ one obtains

$$\alpha_n^{(\pm)0'} = \frac{p_2 \cdot \hat{\alpha}_n^\pm}{m_2}, \quad \alpha_n^{(\pm)I} = \hat{\alpha}_n^{(\pm)I} + \frac{p_2^I}{m_2} \frac{p_2 \cdot \hat{\alpha}_n^\pm}{E_2' + m_2}. \quad (60)$$

These are expected to form covariant $\text{SO}(25,2)$ vectors $\alpha_n^{(\pm)\mu} = \left(\alpha_n^{(\pm)0'}, \alpha_n^{(\pm)I} \right)$, as will be verified below. Furthermore, because of the $\text{SO}(25,2)$ invariance of dot

products, the fully $\text{SO}(25,2)$ covariant Virasoro constraints in (33) are equal to (37) for any $p_2^\mu = (E_2', p_2^I)$. Therefore, the explicit solution of these constraints is given in terms of only the 24 transverse oscillators in (55). Thus, the 27 components $\alpha_n^{(\pm)\mu}$ given in (60) are also expressed in terms of the 24 oscillators in (55) which are the ones that solve all the constraints in the general frame of the massive particle. So, for example the $0'$ component is

$$\alpha_n^{(\pm)0'} = \frac{1}{m_2} \left(-p_2^+ \cdot \hat{\alpha}_n^{(\pm)-} - p_2^- p_1^+ \delta_{n,0} + p_2^i \alpha_n^{(\pm)i} \right) \quad (61)$$

where $\hat{\alpha}_n^{(\pm)-}$ is quadratic in the 24 transverse oscillators as given in (55).

We are now ready to construct the generators $M^{\mu\nu} = (M^{0'I}, M^{IJ})$ of $\text{SO}(25,2)$ in the general frame of the massive particle #2. They are given by

$$\begin{aligned}
M^{0'I} &= \frac{1}{2} (q_2^I E_2' + E_2' q_2^I) + \frac{p_{2J} \hat{M}^{JI}}{E_2' + m_2} \\
M^{IJ} &= q_2^I p_2^J - q_2^J p_2^I + \hat{M}^{IJ}
\end{aligned} \quad (62)$$

where \hat{M}^{IJ} , which satisfy the $\text{SO}(25,1)$ Lie algebra, are given in terms of the 24 transverse string oscillators in eq.(56). It can be checked that these $M^{\mu\nu}$ satisfy the $\text{SO}(25,2)$ Lie algebra without any anomalies, and furthermore, that they rotate the $\alpha_n^{(\pm)\mu}$ of (60) as vectors

$$\left[M^{\mu\nu}, \alpha_n^{(\pm)\lambda} \right] = i \eta^{\mu\lambda} \alpha_n^{(\pm)\nu} - i \eta^{\nu\lambda} \alpha_n^{(\pm)\mu}. \quad (63)$$

This last property is trivial for the $M^{IJ}, \alpha_n^{(\pm)K}$ since it is the same as the usual 26 dimensional string in the light-cone gauge. The new feature is the structure of $M^{0'I}$. It can be checked that this structure automatically closes into the higher algebra $\text{SO}(25,2)$ provided the \hat{M}^{IJ} form the $\text{SO}(25,1)$ Lie algebra.

The form of $M^{0'I}$ follows from rather general properties of cosets. The $\alpha_n^{(\pm)\mu}$ are given by boosting the $\hat{\alpha}_n^{(\pm)\mu}$ with a p_2^μ dependent boost

$$\alpha_n^{(\pm)\mu} = T^\mu{}_\nu(p_2) \hat{\alpha}_n^{(\pm)\nu}. \quad (64)$$

where $T^\mu{}_\nu(p_2)$ is in the coset $\text{SO}(25,2)/\text{SO}(25,1)$. When a general $\text{SO}(25,2)$ transformation is applied, it can be rewritten as follows

$$\begin{aligned}
\alpha_n^{(\pm)\mu} &\rightarrow \Lambda^\mu{}_\nu \alpha_n^{(\pm)\nu} = (\Lambda T(p_2))^\mu{}_\nu \hat{\alpha}_n^{(\pm)\nu} \\
&= (T(p_2') H)^\mu{}_\nu \hat{\alpha}_n^{(\pm)\nu}
\end{aligned} \quad (65)$$

where $p_2'^\mu = \Lambda^\mu{}_\nu p_2^\nu$, and $H(p_2, \Lambda)$ is an element in the subgroup $\text{SO}(25,1)$ but its parameters depend on a function of both p_2^μ and $\Lambda^\mu{}_\nu$. When Λ is an element of $\text{SO}(25,1)$ one has $H = \Lambda$, therefore the subgroup is implemented on the 26 $\hat{\alpha}_n^{(\pm)I}$ and p_2^I by the *total* particle

and string generators M^{IJ} . Of course, the $\hat{\alpha}_n^{(\pm)0'}$ is invariant under this combined transformation since it is a dot product. The remaining $\text{SO}(25, 2)/\text{SO}(25, 1)$ coset transformations have generators that are precisely the $M^{0'I}$ given above, and they automatically take into account the complicated nature of $H(p_2, \Lambda)$.

The outcome of the lightcone quantization for $m_2 \neq 0$ is a critical dimension $d = 27$ with signature $(25, 2)$, in agreement with the covariant quantization.

The lightcone quantization for the $m_2 = 0$ case can be done in a similar way. One needs to boost back from the lightcone frame of particle #2 to the general frame. It is a straightforward exercise and there are no oscillator ordering problems, just as in the massive case. Therefore the critical dimension is $d = 28$ with signature $(26, 2)$, in agreement with the BRST quantization.

III. MULTI PARTICLES, STRINGS, P-BRANES

The type of action discussed in this paper can be generalized to other systems. For example for three particles

$$S = S_1 + S_2 + S_3 + \lambda_1 \cdot \lambda_2 + \lambda_2 \cdot \lambda_3 + \lambda_3 \cdot \lambda_1 \quad (66)$$

where $S_1(x_1^\mu, A_{12}, A_{13}, e_1, \lambda_2^\mu, \lambda_3^\mu)$ is the action for particle #1 in the background of particles #2,3, constructed in terms of gauge covariant derivatives

$$D_\tau x_1^\mu = \partial_\tau x_1^\mu - A_{12} \lambda_2^\mu - A_{13} \lambda_3^\mu \quad (67)$$

in the spirit of gauged WZW models. So, the coset is R^d/R^2 . Similarly for the actions $S_{2,3}$ which are obtained by a cyclic permutation of the indices 1,2,3. Because of the gauge invariances one finds constraints, and going through a similar analysis as the two particle case one determines $\lambda_i^\mu \sim p_i^\mu$ and the constraints

$$p_i \cdot p_j + m_i^2 \delta_{ij} = 0. \quad (68)$$

The solution of this system of constraints requires 3 time-like coordinates. In the bosonic case there seems to be no limit on the number of particles and corresponding new timelike dimensions, but with supersymmetry there are hints for both sufficient and necessary reasons to have a minimum as well as a maximum of three timelike dimensions in a setting that is $\text{SO}(11, 3)$ covariant [14]. The structures of [14] were found to be necessary and sufficient to unify type-A and type-B supersymmetries. This unification is possible with a minimum of three timelike dimensions and extending the general structure beyond 14 dimensions is not required by any known phenomena. Furthermore, there seems to be an obstruction to Yang-Mills supersymmetric systems beyond 14 dimensions [16], thus providing a hint for a maximum of three time like dimensions.

The same approach can be applied to a string and two particles, with results that can be guessed from the previously discussed cases, of a string & one particle, and

three particles. The general result is that when a massive particle is added one needs to add $(0, 1)$ dimensions, i.e. one timelike dimension but no spacelike dimensions, and when a massless particle is added one must add $(1, 1)$, i.e. one time plus one space dimensions. Similarly one may substitute a membrane for a string, and so on for other p-branes, in the case of classical p-branes. Of course, one does not know how to solve the quantum theory that includes membranes or p-branes, and therefore there is no reliable statement on the number of dimensions for which the quantum theory is consistent. However, there are partial hints that $d_{crit} = 11$ for supermembranes [22], therefore for the combined supermembrane & superparticle system, one may extrapolate these hints to $d = 12$ with signature $(10, 2)$ if the superparticle is massive or $d = 13$ with signature $(11, 2)$ if the superparticle is massless. The supersymmetry of such a system is not standard, as discussed in [23] and the next section.

Next we consider two strings, with an action $S = S_1 + S_2 + \lambda_1 \cdot \lambda_2$, where both $S_{1,2}$ are string actions of the form (10). The equations of motion and constraints for each string can be solved, both classically and quantum mechanically, following the same steps as sections 3.4. But now we find new features in the quantum consistency of the combined system. Recall that in consistent sectors the masses of each string are given by $-p_i^2 = N_i - 1$, where N_i are the oscillator excitation levels. Perhaps the simplest way to arrive at the critical dimension is to note that a quantum consistent string must have 26 dimensions after putting the other string in one of its massive or massless states. If the second string is in a massless state, its effect on the first string is the same as a massless particle. Then the total number of dimensions for string #1 has to be 28 with signature $(26, 2)$, as we have shown in sections 3.4. On the other hand, if the second string is in a massive state, its effect on the first string is the same as a massive particle, and the number of dimensions must be 27, with signature $(25, 2)$. There is also a third case when the mass of the second string is tachyonic. Then the number of dimensions is also 27, but with signature $(26, 1)$. The roles of the two strings may be reversed and similar statements would be made for the critical dimension of string #2. These statements cannot be all simultaneously right in the same theory, since the classical theory is defined with a fixed number of dimensions for both strings. Hence, as the consistent quantum sectors, we must select only the mass sectors that are simultaneously consistent for both strings for a fixed number of dimensions. The sectors are defined by whether the masses $(-p_1^2, -p_2^2)$ are simultaneously zero, positive or negative, and evidently the only consistent sectors are

$$\begin{aligned} (26, 2) : & \quad (-p_1^2, -p_2^2) = (0, 0) \\ (25, 2) : & \quad (-p_1^2, -p_2^2) = (+, +) \\ (26, 1) : & \quad (-p_1^2, -p_2^2) = (-, -) \end{aligned} \quad (69)$$

The $(0, 0)$ sector which is possible in 28 dimensions has

only one state, similarly the $(-, -)$ sector in 27 dimensions has only one state, while the $(+, +)$ sector in 27 dimensions has an infinite number of massive states from each string. Here we have assumed that the conformal field theory for each string has no spectator sectors, that is that all degrees of freedom of both strings are coupled via the coupling $\lambda_1 \cdot \lambda_2$. Of course, this assumption can be modified by changing the model.

Consider a model that has one dimension for string #1 which remains uncoupled, while all other $(d_1 - 1)$ dimensions of string #1 are coupled to the d_2 dimensions of string #2 via $\lambda_1 \cdot \lambda_2 \sim p_1 \cdot p_2$, so that $d_2 = (d_1 - 1)$. So, string #1 has one extra dimension. We need to consider again the values of $(-p_1^2, -p_2^2)$. The mass shell conditions in consistent sectors are now $-p_1^2 + \tilde{p}^2 = N_1 - 1$ and $-p_2^2 = N_2 - 1$, where \tilde{p}^2 is the zero mode of the extra dimension of string #1. If this dimension is compact there would be contributions from the winding sectors as well. In this model the sector $(-p_1^2, -p_2^2) = (+, 0)$ is consistent for $d_1 = 28$ with signature $(26, 2)$ and $d_2 = 27$ with signature $(25, 2)$. There are an infinite number states from string #1 and only one state from string #2. Evidently, one can construct various consistent models provided the sectors are selected as above.

However, it is not clear that such sectors are self consistent by themselves under interactions. It is not yet clear what interactions should be considered. If all interactions defined through vertex operator products of both strings are included, then the sectors identified above do not seem to remain isolated from others. Perhaps one can make sense of interactions that mix sectors of different dimensions and signatures. More study is required to understand such issues. These questions did not arise for the string & particle or string & two particle systems.

IV. SUPERSYMMETRY

In a separate publication the supersymmetric version of the superstring and a massive or massless superparticle will be discussed in detail [23]. This involves a construction of an action for the massive superparticle and a more general superstring action that is invariant under a generalized supersymmetry. Here we wish to mention some generalities and ideas for future applications and improvements. By extrapolating from the results of the present paper to the supersymmetric case, one expects critical dimensions $d = 12$ for $m_2 = 0$ and $d = 11$ for $m_2 \neq 0$ with a Lorentz symmetry $SO(10, 2)$ and $SO(9, 2)$ respectively. Furthermore, as expected quite generally from [5], and from discussions in [12,13,16], the generalized superalgebra has to be

$$\{Q_\alpha, Q_\beta\} = \gamma_{\alpha\beta}^{\mu\nu} p_{1\mu} p_{2\nu}, \quad (70)$$

where Q_α is the Majorana-Weyl spinor of $SO(10, 2)$ or $SO(9, 2)$ with 32 real components. In the lightcone frame of the massless particle #2 (as well as in the rest frame

of the massive particle #2), the remaining Lorentz symmetry is $SO(9, 1)$ and the supersymmetry reduces to the standard form of the ten dimensional type IIA. Furthermore, the superstring reduces to the usual 10 dimensional type IIA string. By contrast, in the general frame, or in the action, there is full covariance under $SO(10, 2)$ or $SO(9, 2)$.

We believe that the massless system $m_2 = 0$, underlies a supergravity theory in 12 dimensions, with bilocal fields $\Phi(x_1, x_2)$ describing the string-particle system, along the lines first suggested in [5]. This supergravity theory has been partially realized in the special lightcone frame in [17], but in a single Kaluza-Klein mode of the bilocal fields $\Phi(x_1, x_2)$ (that is particle #2 has been frozen to be at a fixed momentum p_2^μ). Similar considerations for two particles (rather than string and particle) underlie the Yang-Mills theories in 12 dimensions, which have also been only partially realized in a similar Kaluza-Klein mode [7].

As has been argued in [14] the unification of type-A and type-B supersymmetries point to a unifying supersymmetric structure in 14 dimensions with signature $(11, 3)$. Some sectors of such a structure can be constructed by considering three superparticles, or a superstring with two superparticles, etc.. It is expected from [14], and it has been confirmed in [16], that the three superparticle system underlies a super Yang-Mills theory in 14 dimensions. This theory has been partially constructed [15] in a Kaluza-Klein sector in the same sense as the two particle case (i.e. the sector in which the momenta of two particles out of three are frozen). A superstring and two superparticles probably underlie a supergravity in 14 dimensions that would generalize [17] to 14 dimensions. A more general approach that includes all Kaluza-Klein modes has been illustrated in [12] for free fields. This approach needs to be further developed to include interactions by figuring out the calculus of representations of the new supersymmetry (70). It could then be applied to the construction of the full 12 or 14 dimensional supergravity and super Yang-Mills theories in all Kaluza-Klein sectors.

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- [1] M. Duff and M.P. Blencowe, Nucl. Phys. **B310** (1988) 387.
 - [2] I. Bars, Phys. Rev. **D54**, 5203 (1996) ; “Duality and hidden dimensions”, hep-th/9604200, to be published in the proceedings of the conference *Frontiers in Quantum Field Theory*, Toyonaka, Japan, Dec. 1995.
 - [3] C. Vafa, Nucl.Phys. **B469**, 403 (1996) ; D.R. Morrison and C. Vafa, Nucl.Phys. **B473**, 74 (1996); E. Witten, Nucl. Phys. **B471**, 195 (1996); A. Kumar and C. Vafa, “U-manifolds”, hep-th/9611007.
 - [4] D. Kutasov and E. Martinec, Nucl. Phys. **B477**, 652 (1996); Nucl. Phys. **B477**, 675 (1996); hep-th/9612102.
 - [5] I. Bars, “S-Theory”, Phys. Rev. **D55**, 2373 (1997); I. Bars, “Algebraic Structures in S-Theory”, hep-th/9608061, to appear in Proc. of Second Sakharov conf. 1996, and Strings-96 conf.
 - [6] E. Martinec, “Geometrical structures of M-theory”, hep-th/9608017.
 - [7] H. Nishino and E. Sezgin, Phys. Lett. **B388**, 569 (1996).
 - [8] V. Periwal, Phys. Rev. **D55**, 1711 (1997).
 - [9] S. Hewson and M. Perry, hep-th/9612008.
 - [10] A. A. Tseytlin, “Type IIB instanton as a wave in twelve dimensions”, hep-th/9612164.
 - [11] I. Bars, “Black hole entropy reveals a twelfth dimension”, hep-th/9610074, Phys. Rev. **D55**, 3633 (1997) .
 - [12] I. Bars and C. Kounnas, “A new supersymmetry”, hep-th/9612119.
 - [13] I. Bars and C. Kounnas, “Theories with two times”, hep-th/9703060, to appear in Phys. Lett.
 - [14] I. Bars, “A case for 14 dimensions”, hep-th/9704054, to appear in Phys. Lett.
 - [15] I. Rudychev and E. Sezgin - “Superparticles in $D > 11$ ”, hep-th/9704057.
 - [16] E. Sezgin, “Super Yang-Mills in (11,3) dimensions”, hep-th/9703123.
 - [17] H. Nishino, “Supergravity in 10+2 dimensions ...”, hep-th/9703214.
 - [18] J. de Boer and K. Skenderis, “Self-Dual Supergravity from N=2 Strings”, hep-th/9704040.
 - [19] P. K. Townsend, “Membrane tension and manifest IIB S-duality”, hep-th/9705160.
 - [20] D. Karabali and H. Schnitzer, Nucl. Phys. **B329** (1990) 649.
 - [21] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory*, Vol. I, Cambridge Univ. Press (1987).
 - [22] I. Bars, C. N. Pope and E. Sezgin, Phys. Lett. **198B**, 455 (1987); I. Bars, Nucl. Phys. **B308**, 462 (1988); I. Bars and C. N. Pope, Class. and Quant. Grav. **5**, 1157 (1988); Gen. Rel. and Grav. **21**, 545 (1989).
 - [23] I. Bars, “Superstring and superparticle in (10, 2) and (9, 2) dimensions”, in preparation.