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# Dynamical Inflation and Unification Scale on Quantum Moduli Spaces

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**Abstract**

We show that simple strongly coupled supersymmetric gauge theories with quantum moduli spaces can naturally lead to hybrid inflation. These theories contain no input dimensionful or small parameters. The effective superpotential is linear in the inflaton field; this ensures that supergravity corrections do not spoil the slow roll conditions for inflation. We construct a simple theory in which the classical moduli space exhibits neither GUT-symmetry-breaking nor inflation whereas its quantum modification exhibits both. As a result, the dynamical origin and scales of inflation and grand unification coincide.

The inflationary solution [1] to the standard big bang problems assumes that the Universe has gone through a period of de Sitter phase with an exponentiallly growing scale factor. The temporarily nonzero cosmological "constant" is usually associated to the potential energy of a slowly rolling scalar field, the inflaton. In order to have successful inflation the slow-roll conditions must be satisfied. These conditions imply that the inflaton potential is very flat on mass scales of the order of the Hubble parameter. Inflation ends whenever this is not the case. Most inflationary scenarios exhibit two aesthetic problems: *i*) the desired potential is arranged by invoking an extremely small coupling constants and/or an input "hard" mass scale; *ii*) the sector that drives inflation is not usually motivated by particle physics considerations. In this paper we want to propose a class of theories where both problems are addressed. We will attempt to naturally reconcile the breaking of the grand unified gauge symmetry with a successful inflationary scenario, within a single dynamical mechanism, with no input mass parameters and/or very small coupling constants. Both the value of the inflationary potential and the GUT scale is set by a single dynamically generated mass parameter, which COBE measurements indicate to be around the scale of supersymmetric GUTs. Our inflationary scenario can be viewed as a natural realization of "hybrid" inflation [2]. In order to give a general idea, let us consider the "idealized" case of an exactly flat potential for the inflaton field $S$

$$V(S) = \Lambda^4 \tag{1}$$

In the (globally) supersymmetric context this simply corresponds to a linear superpotential

$$W(S) = S\Lambda^2 \tag{2}$$

This exhibits a continuous $R$-symmetry and breaks supersymmetry through the expectation value $F_S = \partial_S W = \Lambda^2$. Of course, in order to have a realistic situation one needs in addition: *i*) a force that drives the inflaton field; *ii*) a trigger which sooner or later switches off the cosmological constant $\Lambda^4$ thus ending inflation. Both these conditions are automatically satisfied provided there are particles that get masses by coupling to $S$. The simplest way is to have[3],[4]

$$W_{tree} = \Lambda^2 S + \frac{g}{2} S Q^2. \tag{3}$$

Here $Q$ is another chiral superfield and $g$ is a coupling constant. The perturbative dynamics of this system is not difficult to understand. For large values

$$|S| >> S_c = \frac{\Lambda}{\sqrt{g}} \tag{4}$$

the $Q$-particle has a zero vacuum expectation value (VEV), a large mass $\sim gS$ and it decouples. By integrating it out we get an effective superpotential given by eq. (2). Moreover the one-loop corrected Kähler potential has the form

$$K = SS^+ \left(1 - \frac{g^2}{16\pi^2} \ln \frac{SS^+}{M_P^2}\right) \tag{5}$$

where our ultraviolet cut-off is the reduced Planck mass $M_P$. This simply translates into the following effective potential [4]

$$V(S) = \Lambda^4 \left(1 + \frac{g^2}{16\pi^2} ln \frac{SS^+}{M_P^2}\right) \tag{6}$$

which drives inflation. When $S$ drops below $S_c$, $Q$ picks up a nonzero VEV and cancels the cosmological constant. Note that the end of inflation does not necessarily coincide with this transition, it generically happens earlier, when the slow-roll conditions break down. The scenario based on the above potential was already considered in [4]. The main drawback of this scenario is that the scale $\Lambda$ is an input of the theory. The purpose of this letter is to show that: (1) inflation with a linear superpotential can be induced dynamically; (2) the dynamically generated scale, as suggested by $\frac{\delta\rho}{\rho}$ can be identified with the scale of grand unification. We construct a simple example which achieves these results.

Let us consider an $SU(2)$ gauge theory with matter consisting of four doublet chiral superfields $Q_I, \bar{Q}^J$ ($I, J = 1, 2$ are flavour indices)[1]. We also add a singlet superfield $S$ and assume the following classical (bare) superpotential

$$W_{cl} = gS(Q_1\bar{Q}_1 + Q_2\bar{Q}_2) \tag{7}$$

where $g$ is a Yukawa coupling constant. A similar model was considered in Refs. [5],[6] in the context of dynamical supersymmetry breaking. At the classical level, in the absence of this superpotential ($g = 0$), the space of vacua ($D$-flat directions) is parametrized by a set of complex fields consisting of $S$ plus the following 6 $SU(2)$ invariants (mesons and baryons)

$$M_I^J = Q_I\bar{Q}^J, \quad B = \epsilon^{IJ}Q_IQ_J, \quad \bar{B} = \epsilon_{IJ}\bar{Q}^I\bar{Q}^J. \tag{8}$$

The invariants are however subject to the constraint

$$\mathrm{Det}M - \bar{B}B = 0 \tag{9}$$

so that in the end the space of vacua at $g = 0$ has complex dimension 6. In the presence of the superpotential, the classical moduli space has two branches described as follows:

1) $S \neq 0$, with $M_I^J = B = \bar{B} = 0$. On this branch the quarks get a mass $\sim gS$ from the superpotential and the gauge symmetry is unbroken. This branch has (complex) dimension 1.

2) $S = 0$, with non-zero mesons and baryons satisfying two constraints. One is eq. (9) while the other is $F_S = \mathrm{Tr}M = 0$. Here the gauge group is broken. The dimension of this branch is 4.

This moduli space is further reduced by quantum effects. In particular a non-zero vacuum energy is generated along the $S \neq 0$ branch. This energy provides the cosmological constant

[1] We write two of the doublets as anti-doublets and distinguish between meson and baryon bilinears in eq. (8). Our notation allows for a straightforward generalization to $SU(N)$ gauge theory with $N$ flavors.

that drives inflation and $S$ plays the role of the inflaton. This is easily established by considering the effective theory far away along $S \neq 0$. Here the quark fields get masses of order $S$ and decouple. The effective theory consists of the (free) singlet $S$ plus a pure $SU(2)$ gauge sector. The effective scale $\Lambda_L$ of the low-energy $SU(2)$ along this trajectory is given to all orders by the 1-loop matching of the gauge couplings at the quarks' mass $gS$ and reads[7]-[10]

$$\Lambda_L^6 = g^2 S^2 \Lambda^4 \tag{10}$$

where $\Lambda$ is the scale of the original theory with massless quarks. In the pure $SU(2)$ gauge theory gauginos condense [8, 9] and an effective superpotential $\sim \Lambda_L^3$ is generated

$$W_{eff} = gS\Lambda^2. \tag{11}$$

Thus

$$F_S = g\Lambda^2 \tag{12}$$

and supersymmetry is broken, with a vacuum energy density $F_S^2$ which is independent of $S$: this is a perfect condition for inflation[2]. We will later discuss the effects that generate a small curvature on this flat potential. Notice that this result for $W_{eff}$ can also be obtained by considering the confined superpotential for mesons and baryons. Quantum corrections[13] modify eq. (9) to

$$\mathrm{Det}M - \bar{B}B - \Lambda^4 = 0 \tag{13}$$

Then, with the aid of a Lagrange multiplier field $A$, we can write the full quantum superpotential for the confined mesons and baryons as [3]

$$W = A(\mathrm{Det}M - \bar{B}B - \Lambda^4) + gS\mathrm{Tr}M. \tag{14}$$

This equation shows that at $S \neq 0$ F-flatness cannot be satisfied, so that branch 1) is lifted. It also shows that branch 2) is modified by eq. (13) but unlifted. Again at $gS \gg \Lambda$ one can integrate out the composites and obtain the effective result eq. (11).

One could also recover this result by an explicit analysis of the $F$-terms behavior for large values of $S$. These F-terms are

$$\begin{gathered} F_A = \det M - B\bar{B} - \Lambda^4, \quad F_{M_I^J} = A\epsilon_{IK}\epsilon^{JL}M_L^K + g\delta_I^J S, \\ F_S = g\mathrm{TrM} = 0, \quad F_B = A\bar{B}, \quad F_{\bar{B}} = AB \end{gathered} \tag{15}$$

For $S \to \infty$, we want to identify the energetically most favorable path, the one along which the tree-level scalar potential assumes an (asymptotically) constant value. Such trajectories can be classified by two possible asymptotics for $F_S$ as $S \to \infty$: either $F_S = 0$ or $F_S = const \neq 0$. First, it is easy to understand that $F_S = 0$ gives that either $F_{M_1^1}$ or $F_{M_2^2}$ should grow with $S$

[2]Different scenarios based on the idea of dynamical inflation were considered in Refs. [11, 12].

[3]See also [14] where a somewhat similar effective $W$ was studied for constant $S$ field.

as $S \to \infty$. Thus we are lead to a solution for constant, but nonzero $F_S$. There exists such a solution with all other $F$-term vanishing

$$\bar{B} = B = 0, \quad M_I^J = \delta_I^J \Lambda^2, \quad A\Lambda^2/g = -S, \quad F_S = g\Lambda^2 \tag{16}$$

This matches the result of the gaugino condensation argument. This is energetically the most attractive trajectory in field space subject to $S \to \infty$.

Again the one-loop corrections to the Kähler potential

$$K = SS^+ \left(1 - \frac{g^2}{4\pi^2} \ln(SS^+/M_P^2)\right) \tag{17}$$

provide an effective potential via eq. (11)

$$V(S) = \frac{|\partial_S W|^2}{\partial^2_{SS^+} K} \simeq g^2 \Lambda^4 \left(1 + \frac{g^2}{4\pi^2} \ln(SS^+/M_P^2)\right). \tag{18}$$

This logarithmic slope pushes $S$ towards the origin and, for sufficiently large initial $S >> S_c$, leads to inflation. For large values of $S$ the potential is very flat and gradually becomes steeper as the inflaton rolls towards the origin. The slow roll regime is characterized by two conditions

$$\epsilon = 2\left|\frac{M_P V'}{V}\right|^2 << 1, \quad |\eta| = \left|\frac{M_P^2 V''}{V}\right| << 1 \tag{19}$$

where $V$ is the scalar potential, the primes denote derivatives with respect to the inflaton field and again $M_P$ is the reduced Planck mass. This fixes a lower bound on the value of $S$ at the end of inflation

$$S = \frac{g}{2\pi} M_P \tag{20}$$

as a result, for $g < 1$, the last 60 e-foldings can take place for values of $S$ well below the Planck scale. This fact provides a big advantage for avoiding a large gravity-mediated curvature of the inflaton potential, which is a grave difficulty for most inflationary scenarios in supergravity. We will come back on this point later on. To be precise, neglecting supergravity corrections, the value of $S$ at $N$ e-foldings before the end of inflation is given by [4]

$$S_N = \frac{g}{2\pi}\sqrt{2N} M_P. \tag{21}$$

The predicted cosmological parameters such as the spectral index $n = 1 + 2\eta$, density perturbations and relative contribution of gravitational waves $R = 12\epsilon$ has to be evaluated at the value of $S$ for which the scale of interest crossed out of the de Sitter horizon. Because of the logarithmic shape of the potential, $n$ is independent on either $g$ or $\Lambda$. Indeed $n$ is also independent on the gauge group. We have [4]

$$n \simeq 1 - \frac{1}{N}, \quad R \simeq \frac{6g^2}{N\pi^2}, \quad \frac{\delta T}{T} \simeq \frac{1}{2}\sqrt{\frac{N}{45}} \left(\frac{\Lambda^2}{M_P^2}\right) \tag{22}$$

The COBE normalization fixes the scale $\Lambda \sim 10^{16} - 10^{15}$ GeV, which is very close to the GUT scale. This remarkable coincidence suggests the intriguing possibility that, in a suitably modified scenario, $\Lambda$ may be the dynamically generated grand unification scale[4]. The most straightforward way to implement this is to add a coupling to matter in the adjoint of the GUT group. Let us consider, for definiteness, $SU(5)$ with an adjoint $\Sigma$ and with superpotential

$$W_{tree} = gS(Q_1\bar{Q}_1 + Q_2\bar{Q}_2) + \frac{g'}{2}S\mathrm{Tr}\Sigma^2 + \frac{h}{3}\mathrm{Tr}\Sigma^3. \tag{23}$$

Remarkably, this theory has, at the classical level, the same moduli space of the original one (without $\Sigma$). This was characterized in the points 1) and 2) above. It is easily checked. Along $S \neq 0$, the $F$-flatness conditions for the $Q$'s and $\bar{Q}$'s saturated to form invariants give $SM_I^J = SB = S\bar{B} = 0 \rightarrow M = B = \bar{B} = 0$. Then $F_S = \mathrm{Tr}\Sigma^2 = 0$ implies $\Sigma = 0$, so that the branch $S \neq 0$ is the same as before. At $S = 0$, we have $F_\Sigma = 0 \rightarrow h\Sigma^2 = 0$ so that also on this branch $SU(5)$ is unbroken and the moduli space is described by 2) above. In conclusion, $SU(5)$ is unbroken all over the classical moduli space, and along the $S \neq 0$ branch all other chiral superfields are massive $m_{Q\bar{Q}} = gS$ and $m_\Sigma \sim g'S$. Then, as long as $SU(5)$ is weak or in a phase where it cannot generate a superpotential, the quantum dynamics far away along $S$ is the same as before: $W_{eff} = gS\Lambda^2$ and Kähler logs drive inflation. Now there is also a contribution $\delta K \sim g'^2 \ln SS^+$, but it does not change the qualitative conclusions and it is even quantitatively negligible for $g' < g$ (which we will later argue to be favored by other considerations). Thus also the modified theory in eq. (23) naturally produces inflation. The interesting differences with respect to the previous case arise when considering the full quantum moduli space described by

$$W_{eff} = A\left(\mathrm{Det}M - B\bar{B} - \Lambda^4\right) + S\left(g\mathrm{Tr}M + \frac{g'}{2}\mathrm{Tr}\Sigma^2\right) + \frac{h}{3}\mathrm{Tr}\Sigma^3. \tag{24}$$

The $S = 0$ branch goes through unchanged, as here $\Sigma = 0$. At $S \neq 0$ there however appear two isolated points where $SU(5)$ is Higgsed respectively to $SU(4) \times U(1)$ and to $SU(3) \times SU(2) \times U(1)$. The one of physical interest is given by

$$\langle\Sigma\rangle = \Lambda\sqrt{\frac{-2g}{15g'}}(2, 2, 2, -3, -3), \quad M_I^J = \delta_I^J\Lambda^2, \quad B = \bar{B} = 0,$$
$$S = \Lambda h\sqrt{\frac{-2g}{15}}g'^{-\frac{3}{2}}, \quad A = \Lambda^{-1}h\sqrt{\frac{2}{15}}\left(\frac{-g}{g'}\right)^{\frac{3}{2}} \tag{25}$$

It is interesting that in this theory $SU(5)$ is unbroken on the classical moduli space. Quantum effects introduce a deformation that $a)$ smoothes one branch at $S = 0$ and $b)$ creates $SU(5)$ breaking isolated vacua near the origin. At the end of inflation the system will relax into one of its vacua. At this stage it is difficult to decide whether the vacuum with $SU(3) \times SU(2) \times U(1)$ residual group will be preferred over the one with $SU(4) \times U(1)$. It is however possible to

[4]See also Ref. [15], which discusses, from a different perspective, a model with a dynamical GUT scale.

argue that the points with broken $SU(5)$ are preferred over the continuum where $\Sigma = 0$. As we have seen, inflation starts out with large and slowly decreasing $S$ and $A$ (and with "frozen" mesons) satisfying eq. (16). Notice that in the supersymmetric isolated minima with broken $SU(5)$, the fields $B, \bar{B}, M, S, A$ satisfy a special case of eq. (16). In other words the points with broken $SU(5)$ lie on the inflationary branch. This suggests that at the end of inflation the system will very likely first relax to one of these vacua. Moreover, at least for $g' < g$, there is further indication that $SU(5)$, and not $SU(2)$, ends up being broken. This is beacuse in this limit $\Sigma$ becomes tachionic before the other fields, and drives $F_S$ to zero. It is clear that there still remains the question of whether the system may "overshoot" into the other branch of vacua at $S = 0$. This question, as the one concerning 3-2-1 versus 4-1 GUT breaking, will not be discussed here. It involves control over the behaviour of the Kähler potential in the strong coupling region (small $S$). This is beyond the purposes of our paper. Our model is a simple prototype illustrating the general idea. Nonetheless, we find that the feature we can control, the occurrence of an inflationary plateau ending in GUT breaking isolated minima, is interesting *per se*.

In supergravity the Planck scale suppressed corrections to the Kähler potential usually generate a large curvature to the inflaton potential and are incompatible with the slow-roll assumption. In our case the situation looks much more promising due to the fact that: 1) The inflaton superpotential is linear and 2) inflation (at least for the last 60 e-foldings) can take place for values of $S$ below $M_P$. For generic Kähler metric, our inflationary potential can be expanded as

$$V = g^2\Lambda^4 \left[1 + \frac{g^2}{4\pi} ln \frac{SS^+}{M_P^2} + \alpha \frac{SS^+}{M_P^2} + \text{higher powers}\right] \tag{26}$$

Here the bilinear comes from the non-minimal term $\alpha(SS^+)^2/M_P^2$ in the Kähler potential. The fact that there is no contribution to the $\alpha$-term from the canonical part $SS^+$ of the Kähler potential is a result of the effective linear superpotential (13). The same cancellation was observed in [3] in the context of a non-dynamical model. However the inflaton potential obtained by those authors was very different since the radiative corrections were not considered. Unless $g$ is tuned to a very small value these corrections are very important and lead to rather different conclusions. The contribution to the inflaton mass from the canonical Kähler metric does not vanish for other type of superpotentials. This is why the generic potentials, which lead to inflation in the global SUSY limit, in general have a problem (already in minimal) supergravity. (The case of zero effective inflationary superpotential, with $D$-term driven inflation is exceptional and avoids this problem[5][16]). To satisfy the slow roll conditions all we need is that $\alpha$ is somewhat smaller than one. Depending on the balance between the parameters one can face different regimes. For $g \sim 0.1 - 1$, the end of inflation is practically always triggered by the log-term whenever inflaton drops to (20), irrespective of the value of $\alpha$ (provided the slow roll conditions are not broken by this term at the beginning). The earlier stages are more sensitive to the precise values of $g$ and $\alpha$. For $g \lesssim 0.1$ the beginning of last 60 e-foldings can be dominated by gravity corrections if $\alpha$ is large enough[18] (crudely, the

[5] An alternative possibility is the inflaton being a pseudo-Goldstone mode[17].

evolution above $S_N$ will be dominated by the $\alpha$-term provided $\alpha > 1/2N$). For $g \gtrsim 1$ higher supergravity terms, like $\Lambda^4(SS^+)^2/M_P^4$ in $V(S)$, will dominate the early stage of inflation, even for arbitrarily small $\alpha$ [19]. Notice that for $gS \sim \Lambda$ the nonperturbative corrections to the Kähler potential from gaugino condensation become $\mathcal{O}(1)$. These corrections switch-off as $\sim \Lambda^4/(g^2S^4)$ [20] for large values of $S$ and therefore can dominate the potential at the later stages of inflation if $g$ is small. In conclusion the predictions on the smaller scales (small $N$ region in eqs. (21-22)) can be affected by gaugino condensation when $g$ is $\lesssim 0.1$, whereas those at large scale can be modified by supergravity corrections for $g \gtrsim 1$. The predictions for intermediate $N$ are less sensitive to the variation of $g$. The values of $g$ for which gravity and strong dynamics effects on eq. (22) can be ignored is somewhere between 0.1 and 1.

In conclusion we have constructed a scenario with dynamically generated inflaton potential, and without a slow-roll problem. The scenario is quite restrictive and, under reasonable assumptions, the predicted cosmological observables depend on a single parameter, the dynamically generated scale, which is also of the order of the GUT scale. This lead us to think that both inflation in the early Universe and the spontaneous breaking of the GUT symmetry in the present vacuum may have a common dynamical origin. We have shown that within the given inflationary scenario this connection can be naturally established.

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