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Non-Supersymmetric (but) Extreme Black Holes, Scalar Hair and Other Open Problems

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Abstract

We give a brief overview of black-hole solutions in four-dimensional supergravity theories and their extremal and supersymmetric limits. We also address problems like cosmic censorship and no-hair theorems in supergravity theories. While supergravity by itself seems not to be enough to enforce cosmic censorship and absence of primary scalar hair, superstring theory may be.

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1 Unbroken Supersymmetry in Supergravity Theories

1.1 Bogomol'nyi Bounds and Supersymmetric configurations

Some of the basic material in this section can also be found Ref. [1]. In general, the solutions of a supergravity (SUGRA) theory are not invariant under (local) supersymmetry (SUSY) transformations which can be written schematically in the form

$$\begin{cases} \delta_\epsilon B \sim \epsilon F, \\ \delta_\epsilon F \sim \partial\epsilon + \epsilon B, \end{cases} \quad (1)$$

for bosons (B) and fermions (F). Purely bosonic ($F = 0$) configurations³ which are invariant under *some* SUSY transformations generated by the SUSY parameters $\epsilon_{\text{Killing}}(x)$ are said to be supersymmetric or to have unbroken supersymmetries. By definition $\epsilon_{\text{Killing}}(x)$ satisfies the *Killing spinor equation*

$$\delta_{\epsilon_{\text{Killing}}} F \sim \partial\epsilon_{\text{Killing}} + \epsilon_{\text{Killing}} B = 0, \quad (2)$$

and is called *Killing spinor*. The Killing spinor has to be a generator of the global SUSY algebra at infinity (i.e. a constant spinor in asymptotically flat spaces). A solution of the Killing spinor equation that goes to zero at infinity simply generates SUSY gauge transformation and is irrelevant.

Classical solutions of SUGRA theories with unbroken supersymmetries enjoy special properties:

1. Classical supersymmetric solutions are simpler and depend on a smaller number of functions. Their simplicity is also due to the fact that:
2. In general a configuration admitting Killing spinors also admits Killing vectors which can be formed with the bilinear

$$k_{\text{Killing}}^\mu \sim \bar{\epsilon}_{\text{Killing}} \gamma^\mu \epsilon_{\text{Killing}}. \quad (3)$$

k_{Killing}^μ is timelike or null on $d = 4$, null in $N = 1, d = 10$ SUGRA etc.⁴ The null Killing vector is associated to massless representations of the global SUSY algebra with vanishing values of the central charges and

³These are the configurations that correspond to classical solutions.

⁴A exception seems to be $N = 1, d = 11$ SUGRA.

the associated solutions are gravitational waves. The timelike Killing vectors are in general associated to massive representations of the global SUSY algebra with or without central charges and the corresponding solutions are objects like black holes (BHs) and (M-, D-) p -branes.

- Supersymmetric configurations saturate Bogomol'nyi (B) bounds when they are asymptotically flat and massive⁵. These bounds can be obtained via a Nester construction [3] (see also [4]) or by associating the classical solution to a state in the quantum SUGRA theory. In the first case, certain assumptions concerning boundary and energy conditions have to be made. In the second case, one first has to make sure that such a state really does exist. In that case, given the action of the N -extended global SUSY algebra on the state one can derive the inequalities [5]

$$M - |Z_i| \geq 0, \quad i = 1, \dots, [N/2], \quad (4)$$

where the Z_i 's are the central charge matrix's complex skew eigenvalues and are complicated combinations of the electric and magnetic charges of the vector fields in the supergravity multiplet (*graviphotons*) and the constant values of the scalars at infinity (*moduli*). Since a SUSY transformation on states is of the form

$$\delta_\epsilon \sim \bar{\epsilon} Q | \rangle, \quad (5)$$

the existence of Killing spinors in the associated classical configuration is related to the state's annihilation by $\bar{\epsilon}_{\text{Killing}}^\infty Q$ which implies that at least one of the above B bounds (say $i = 1$) is saturated

$$M = |Z_1| \geq |Z_i|, \quad i \neq 1. \quad (6)$$

The number of bounds saturated depends on the number of independent Killing spinors (i.e. the number of independent SUSY charges $\bar{\epsilon}_{\text{Killing}}^\infty Q$ that annihilate the state. In $N = 8, d = 4$ SUGRA there are four Z_i 's and three possibilities (up to permutations, which are related to duality transformations):

⁵Other bounds could be found for different asymptotics. For instance, for asymptotically Taub-NUT spaces [2]

$$\left\{ \begin{array}{l} M = |Z_1| \geq |Z_2|, |Z_3|, |Z_4| \Rightarrow 1/8 \\ M = |Z_1| = |Z_2| \geq |Z_3|, |Z_4| \Rightarrow 1/4 \\ M = |Z_1| = \dots = |Z_4| \Rightarrow 1/2 \end{array} \right. \quad (7)$$

where the fraction of the total $N = 8$ supersymmetries which are unbroken is on the right.

States that belong to the third case (which is the only in which the saturated bound is duality-invariant and can be written as a expression linear on the electric and magnetic charges) have maximal unbroken SUSY and minimal mass for the given charge sector which ensures maximal classical and quantum stability⁶. They are called *BPS states*. *Vacuum states* have all supersymmetries unbroken.

4. Supersymmetric configurations obey *no-force conditions* and multi-pole solutions describing several supersymmetric objects in equilibrium can be found. For instance, if we take two objects (which in a SUGRA theory would be extreme RN BHs) satisfying $M_i = \pm Q_i$ (which is the $N = 2, d = 4$ SUGRA B bound), wherever they are placed, the Newtonian and Coulombian forces cancel each other:

$$F_{ij} = -\frac{M_i M_j}{r_{ij}^2} + \frac{Q_i Q_j}{r_{ij}^2}, \quad (8)$$

and it is reasonable to expect that there are *static* classical solutions describing them in equilibrium⁷. These solutions are the Majumdar-Papapetrou solutions [6] and describe arbitrary number of BHs satisfying $M_i = \pm Q_i$ [7].

1.2 Supersymmetric Embeddings

Given a solution of GR coupled to matter, we would like to know whether it is supersymmetric or not. For that we have to be able to identify it with a solution of a SUGRA theory, i.e. we have to find a solution of a SUGRA theory with the same metric⁸, or a *supersymmetric embedding* of the original

⁶In the ungauged SUGRA theories that we are considering here there are no particles carrying the graviphotons' electric and magnetic charges which are both, therefore, of topological nature.

⁷If they do not satisfy the B bound, there are also solutions describing them in motion, but they are much more complicated, although of great interest in astrophysics.

⁸In general it will have different matter fields.

solution whether or not it has unbroken supersymmetries. If there is one, then there are many possible embeddings for a given GR solution, due to the dualities in the SUGRA theories. These dualities in general respect unbroken supersymmetries and, thus, all duality-related embeddings are equally good from that point of view. Apart from the duality-related embeddings there are some more or less exceptional embeddings which are not duality-related to the others and have a different number of unbroken supersymmetries.

Example 1 The electric RN BH solution is a solution of the Einstein Maxwell system

$$S = \int d^4x \sqrt{|g|} [R - F^2] . \quad (9)$$

Given that the bosonic part of the action of $N = 2, d = 4$ SUGRA coincides with the Einstein-Maxwell action, an embedding of the electric RN BH solution consists in identifying the metric and the vector field of both theories. There is a second embedding: one can identify the metrics and the vector field of the Einstein-maxwell system with the dual vector field of the SUGRA theory. Both embeddings are related by electric-magnetic duality and both have the same number of unbroken supersymmetries (1/2 of the $N = 2$ in the extreme limit.).

Example 2 If we want to embed the same solution into $N = 4, d = 4$ SUGRA, whose action is

$$S = \int d^4x \sqrt{|g|} \left[R + 2(\partial\phi)^2 + e^{4\phi}(\partial a)^2 - e^{-2\phi} \sum_{i=1}^6 (F^i)^2 - a \sum_{i=1}^6 F^{i*} F^i \right] , \quad (10)$$

we also have to satisfy the scalar equations of motion. In particular, the dilaton ϕ equation of motion

$$\nabla^2 \phi \sim e^{-2\phi} \sum_{i=1}^6 (F^i)^2 , \quad (11)$$

implies that to have no dilaton (which would change the RN metric) we must have $\sum_{i=1}^6 (F^i)^2 = 0$. For this it is necessary to have two non-trivial vector fields one with electric charge q_1 and the other with magnetic charge $p_3 = \pm q_1$. Different choices of non-trivial vector fields are related by T duality transformations. Another embedding would

consist of two vector fields with electric and magnetic charge $q_1 = \pm p_1$, $q_3 = \mp p_3$ ⁹.

Example 3 To embed the RN BH into $N = 4, d = 4$ SUGRA coupled to six vector multiplets (the theory that one obtains when one compactifies $N = 1, d = 10$ SUGRA on T^6) we can simply embed $N = 4$ into $N = 4 + 6V$ and so the vector field of the Einstein-Maxwell system is still identified with a graviphoton. There is another possibility: to identify this vector field with a matter vector field. This embedding breaks all supersymmetries and it is not related to the previous one by duality [8, 9]. From the point of view of the string effective action one needs four vector fields with charges $|q_1| = |q_2| = |p_3| = |p_4| = |q|$. The two possible sets of embeddings correspond to the two possible relative signs $q_1 = \pm q_2$, $p_3 = \pm p_4$.

Example 4 To embed the RN BH in $N = 8, d = 4$ SUGRA (the theory that one obtains when one compactifies $N = 2A, N = 2B, d = 10$ SUGRA on T^6 , the effective field theories of the type II string theories) one can simply embed again $N = 4 + 6V$ into $N = 8$ using the embedding of $N = 1, d = 10$ into the $N = 2, d = 10$ theories (which consists in the identification of the NS-NS fields). However, now, the two kinds of embeddings described before turn out to be supersymmetric in $N = 8$ (the matter vector fields of $N = 4$ are graviphotons of $N = 8$). There are now more possible embeddings too: from the string point of view, one needs four non-trivial vector fields to embed the RN solution into $N = 8, d = 4$ SUGRA. All that is required is that the four charges have the same absolute value: $|q_1| = |q_2| = |p_3| = |p_4| = |q|$. There are now eight different relative sign choices (embeddings up to T dualities). In the extremal limit only a half (i.e. four) of them are supersymmetric.

The non-supersymmetric embeddings (that is, supersymmetric embeddings without unbroken SUSY) share many properties with the supersymmetric ones (after all, they have the same metric): Bogomol'nyi-like identities are satisfied [10] and the solutions seem to have the same stability properties [11]. This may suggest that there is a theory with more SUSY ($N = 16?$), of which $N = 8$ is a consistent truncation, in which all of these embeddings are supersymmetric [12, 10]. This theory may have a 12-dimensional origin. Similar ideas have been suggested in Refs. [13].

⁹Two are necessary to satisfy the axion a equation with vanishing axion.

There seems to be a difference between embeddings in globally and locally supersymmetric theories. The 't Hooft-Polyakov monopole in the BPS limit can be embedded both in $N = 2$ and $N = 4$ SYM and in both cases it has $1/2$ of the supersymmetries unbroken, i.e. one and two respectively. However, if we restrict ourselves to supersymmetric embeddings with unbroken SUSY the extreme RN has $1/2$ of $N = 2$ $1/4$ of $N = 4$ and $1/8$ of $N = 8$ unbroken SUSYs, i.e. always only one.

1.3 BH Thermodynamics and SUSY

1.3.1 $N = 1, d = 4$ SUGRA

The bosonic part of this theory is GR. The B bound is equivalent to the positivity of energy [3] $M \geq 0$. The only static BH solution is Schwarzschild's which depends only on M and becomes Minkowski space in the $M = 0$ (extreme and supersymmetric) limit.¹⁰ If we associate a quantum state to Schwarzschild's solution¹¹ (or assume some energy conditions) M will always be positive and the solution with $M < 0$ which has naked singularities will be excluded from the theory. SUSY seems to act here as a cosmic censor [8].

The temperature and entropy are given by

$$\begin{aligned} T &= \frac{1}{8\pi M} \xrightarrow{M \rightarrow 0} \infty, \\ S &= 4\pi M^2 \xrightarrow{M \rightarrow 0} 0. \end{aligned} \tag{12}$$

Observe that T does not go to zero in the extreme limit while it is reasonable to say that Minkowski's temperature is zero. This is one of many examples in which a family of metrics depending on continuous parameters (in this case the mass M) has physical properties which are not continuous functions of those parameters. The obvious reason is that the family of metrics itself is not a continuous function (in the space of metrics) of those parameters. Here, any metric with $M \neq 0$, no matter how small, has a horizon and is different from Minkowski's. The lesson to be learned in this simple example is that the properties of Minkowski's space cannot be calculated by naively taking the $M \rightarrow 0$ limit in Schwarzschild's.

¹⁰Minkowski's space has all (one) supersymmetries unbroken. It is a vacuum of the theory. In all other cases this will also be true and we will not mention this fact any more.

¹¹It is not clear that this can be done.

1.3.2 $N = 2, d = 4$ SUGRA

The bosonic part of this theory is the Einstein-Maxwell system Eq. (9). The B bound is $M \geq |Q + iP| = |Z|$. The only static BH solution is RN's which depends on the ADM mass M , the electric charge Q and magnetic charge P . When the B bound is saturated and the SUSY limit is reached the extreme limit in which the two RN horizons coincide is also reached (extreme RN (ERN) solution). The B bound ensures that no naked singularities will arise from trespassing the extreme limit and, in this sense, SUSY acts again as a cosmic censor. The ERN solution has one unbroken SUSY.

The temperature and entropy are given by

$$T = \frac{1}{2\pi} \frac{\sqrt{M^2 - |Q + iP|^2}}{M + \sqrt{M + \sqrt{M^2 - |Q + iP|^2}}} \xrightarrow{M \rightarrow |Q + iP|} 0, \quad (13)$$

$$S = \pi \left[M + \sqrt{M^2 - |Q + iP|^2} \right]^2 \xrightarrow{M \rightarrow |Q + iP|} \pi M^2. \quad (14)$$

Our experience in the $N = 1$ case should prevent us from trusting the above limits because the ERN geometry (with a single horizon) is completely different from arbitrary close to extremality RN geometry. In fact, the Hawking evaporation of a non-extreme RN BH takes an infinite time to produce an ERN BH. It is, though, reasonable to expect the temperature to be zero since the ERN has the lowest possible mass in the quantum theory and should not Hawking-radiate. With respect to the entropy, if we assume that the identification between horizon area and entropy holds even at zero temperature, then the above limit is also correct because the ERN BH has non-zero horizon area. However, Euclidean semiclassical calculations gave as result $S = 0$ for the ERN BH [14], in accordance with the conventional content of the third law of thermodynamics [15] and in open contrast with string state-counting results [16]. In Ref. [17] and argument due to Sen is given explaining the discrepancy and a discussion from the point of view of the third law of thermodynamics can be found in Ref. [18]. On the other hand one may always take the point of view that the geometry near the horizon is very different in the string picture [19] and the semiclassical approach breaks down there. In any case, the SUGRA result is $S = 0$, the string theory prediction is $S = \pi M^2$ and we should regard this difference as a test (even if in *gedanken* experiments) for both theories.

What about rotating BHs? Let us go back to $N = 1$. The extreme limit of the Kerr BH is reached when $M = J$, before the supersymmetric limit $M = 0$ is reached (the angular momentum does not appear in the B bound but it does in the extremality bound), but the supersymmetric limit implies the extreme because $M = 0$ implies $J = 0$. The extreme Kerr BH has finite horizon area and usually this is identified with non-vanishing entropy, although it is not clear whether from the Euclidean semiclassical point of view this would be so. The temperature is zero.

In $N = 2$ the extreme and supersymmetric limits in presence of angular momentum are even more different. The situation is essentially the same but now $M \rightarrow |Q + iP|$ (we stress that J does never appear in the B bound) is always beyond the extremality limit and produces naked singularities.

It seems that SUSY only acts as a cosmic censor in absence of angular momentum.

There are more supersymmetric solutions in pure $N = 2, d = 4$ SUGRA (apart from pp -waves) [20]. They all belong to the IWP class [21] and include objects with angular momentum and Taub-NUT charge. The only ones with regular horizons (i.e. BHs) are the ERN (Majumdar-Papapetrou) ones [7].

1.3.3 $N = 4, d = 4$ SUGRA

The bosonic part of this theory is described by the action (10). There are two B bounds: $M^2 - |Z_{1,2}|^2 \geq 0$. Neither of them is in general duality-invariant separately and so we take their product and divide by M^2 to get the duality-invariant *generalized B bound*

$$M^2 + \frac{|Z_1 Z_2|^2}{M^2} - |Z_1|^2 - |Z_2|^2 \geq 0, \quad (15)$$

which is the one that should appear in the metric because the metric is duality-invariant. The term $|Z_1 Z_2|^2 M^{-2}$ can be identified with the charges of the scalars in regular BH solutions.

There are now two ways approaching the supersymmetric limit: $M \rightarrow |Z_{1,2}| \neq |Z_{2,1}|$ (1/4 of the supersymmetries unbroken) and $M \rightarrow |Z_1| = |Z_2|$ (1/2 of the supersymmetries unbroken). To study them, it is useful to have handy the most general supersymmetric solutions of this theory, the *SWIP* solutions [22, 23, 24]. They can be built by following the recipe

1. Choose any two complex harmonic functions $\mathcal{H}_1, \mathcal{H}_2$

$$\partial_{\bar{i}} \partial_{\bar{i}} \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{pmatrix} (\vec{x}) = 0, \quad (16)$$

which constitute an $SL(2, \mathbb{R})$ doublet associated to S duality.

2. Choose a set of complex constants $k^{(i)}$ satisfying

$$\sum_{i=1}^6 (k^{(i)})^2 = 0, \quad \sum_{i=1}^6 |k^{(i)}|^2 = 1/2. \quad (17)$$

These constants constitute an $SO(6)$ vector associated to T duality.

3. Define the functions U and $\omega_{\underline{i}}$

$$\begin{cases} e^{-2U} = 2\Im(\mathcal{H}_1 \bar{\mathcal{H}}_2), \\ \partial_{[\underline{i}} \omega_{\underline{j}]} = \epsilon_{ijk} \Re(\mathcal{H}_1 \partial_{\underline{k}} \bar{\mathcal{H}}_2 - \bar{\mathcal{H}}_2 \partial_{\underline{k}} \mathcal{H}_1). \end{cases} \quad (18)$$

4. In terms of these objects they can be constructed as follows:

$$\begin{cases} ds^2 = e^{2U} (dt + \omega_{\underline{i}} dx^{\underline{i}})^2 - e^{-2U} d\vec{x}^2, \\ a + ie^{-2\phi} = \mathcal{H}_1 / \mathcal{H}_2, \\ A_t^i = 2e^{2U} \Re(k^{(i)} \mathcal{H}_2), \\ \tilde{A}_t^i = -2e^{2U} \Re(k^{(i)} \mathcal{H}_1), \end{cases} \quad (19)$$

where $\tilde{A}_\mu^{(i)}$ is the dual vector potential.

When the harmonic functions are chosen properly the SWIP solutions describe isolated (i.e. asymptotically flat) charged, point-like objects¹² that always satisfy the identity

$$M^2 + |\Upsilon|^2 - 4 \sum_{i=1}^6 |Q^i + iP^i|^2 = 0, \quad (20)$$

where Υ is a complex combination of the scalar charges. This identity is the explicit form of the generalized B bound (15). These solutions represent extreme BHs and so the supersymmetric limit is also the extremal limit.

The temperature is always zero. The area is zero when there are 1/2 of the supersymmetries unbroken and finite when there are only 1/4. From the

¹²Angular momentum and NUT charge can also be included.

semiclassical Euclidean SUGRA point of view the entropy is zero in the 1/4 case (for the same reasons as in $N = 2$ and in contradiction with the stringy calculation). In the 1/2 case the entropy is zero in both schemes.

When matter is added to $N = 4$ the rule that 1/4 of the supersymmetries unbroken implies regular horizon of finite area breaks down (for instance, in the $a = 1/\sqrt{3}$ dilaton BH) although from the string theory point of view the entropy should remain finite. We have also discussed that the extreme limit does no longer imply the supersymmetric and the extremality limit in general.

In $N = 4$ SUGRA SUSY acts as a cosmic censor only in absence of angular momentum [25].

1.3.4 $N = 8, d = 4$ SUGRA

The most general solution is unknown. We have already discussed that there are extremal but non-supersymmetric BH solutions in this theory. In any case we expect that all $N = 8$ SUGRA extreme BH solutions satisfy a generalized duality-invariant B identity of the form

$$M^{-6} \prod_{i=1}^4 (M^2 - |Z_i|^2) = 0. \quad (21)$$

Many kinds of scalar charges do appear in this identity, and, as in the $N = 4$ case they are all of secondary type, i.e. they are completely determined by the graviphotons' electric and magnetic charges. As we are going to argue now in the next section, primary charges should in some cases, and under certain assumptions, be included in B bounds [26].

2 Scalar Charges versus SUSY and Duality

Scalar charges, not being protected by a gauge symmetry, are not conserved charges. For minimally-coupled scalars the standard no-hair theorems apply and any non-vanishing value implies the presence of naked singularities. The prototype of this kind of singular solution with non-trivial scalar hair (called *primary hair*) is the one given in Refs. [27] for the theory with a massless scalar φ and action

$$S = \int d^4x \sqrt{|g|} \left[R + \frac{1}{2} (\partial\varphi)^2 \right]. \quad (22)$$

The solutions take the form

$$\begin{cases} ds^2 &= W^{\frac{M}{r_0}-1} W dt^2 - W^{1-\frac{M}{r_0}} [W^{-1} dr^2 + r^2 d\Omega^2], \\ \varphi &= \varphi_0 - \frac{\mathcal{Q}_d}{r_0} \ln W, \end{cases} \quad (23)$$

where

$$\begin{cases} W &= 1 - 2r_0/r, \\ r_0^2 &= M^2 + \mathcal{Q}_d^2. \end{cases} \quad (24)$$

The solution is determined by three independent parameters: the mass M , the scalar charge \mathcal{Q}_d and the value of the scalar at infinity φ_0 . Only when $\mathcal{Q}_d = 0$ one has a regular solution (Schwarzschild). In all other cases there is a singularity at $r = r_0$. It can be embedded in $N = 4$ SUGRA identifying $\varphi = 2\phi$ in Eq. (10). Observe that the above family of solutions includes a non-trivial *massless* solution. Setting $M = 0$ above we find

$$\begin{cases} ds^2 &= dt^2 - dr^2 - W r^2 d\Omega^2, \\ \varphi &= \varphi_0 - \ln W, \end{cases} \quad (25)$$

with

$$W = 1 - \frac{2\mathcal{Q}_d}{r}. \quad (26)$$

For non-minimally coupled scalars regular BH solutions with *secondary scalar hair* do exist (just see above). In those solutions, the scalar (dilaton) charge is identical to a certain fixed combinations of the other, conserved, charges (for simplicity we only consider one $U(1)$ field):

$$\mathcal{Q}_d \sim \frac{P^2 - Q^2}{2M}. \quad (27)$$

In $N = 4$ we found that the axidilaton charge in regular BH solutions was equal to

$$|\Upsilon| = M^{-1} |Z_1 Z_2|. \quad (28)$$

The existence of secondary hair does not preclude the existence of primary hair. In fact, the solutions above can be interpreted in the framework of string theory with primary but no secondary scalar hair and there are solutions which have both kinds of hair at the same time [28].

Primary scalar hair always seems to imply the presence of naked singularities, and the no-hair theorem should maybe be called *no-primary hair theorem*.

In the standard derivations of the different B bound formulae only conserved electric and magnetic charges appear and only when all the scalar hair is secondary and given by Eq. (27,28) one can derive the generalized B bounds of the previous section in which the scalar charges appear.

We are going to argue, however, that *primary scalar hair* should be incorporated into the generalized B bounds.

Let us consider a simple example: Schwarzschild's solution (given above just by setting $\mathcal{Q}_d = 0$). This solution has no unbroken supersymmetries, which can be understood in terms of non-saturation of the B bound ($M \geq 0$). A Buscher T duality transformation in the time direction preserves the SUSY properties and asymptotic behavior of the solution giving new asymptotically flat solution with no unbroken supersymmetries: precisely the massless solution with only primary scalar hair written above in Eqs. (25,26). It is easy to check that this solution admits no $N = 4$ Killing spinors and so it has no unbroken supersymmetries¹³. However, the fact that this solution has no unbroken supersymmetries would not have been clear from the B bound point of view, had we used the once-standard form in which primary hair should not added to it, since its mass and all the other conserved charges are zero, meaning that the bound would be trivially saturated.

All that happened in this transformation is that the mass M , which does appear in the B bound has completely transformed in primary dilaton charge \mathcal{Q}_d which in principle does not.

It is clear that to reconcile these two results one has to admit that the generalized B bound formula Eq. (20) does apply to all kinds of scalar charge and not only to the secondary-type one. Only in this way becomes consistent the invariance of the B bound with the covariance of the Killing spinor equations under T duality.

Although our reasoning is completely clear when we look on specific solutions one should be able to derive B bounds including primary scalar charges using a Nester construction based on the SUSY transformation laws of the fermions of the supergravity theory under consideration. To be able to do this one has to be able to manage more general boundary conditions including the seemingly unavoidable naked singularities that primary hair implies.

Although we have kept this discussion strictly four-dimensional it is easy to generalize these arguments to higher dimensions. In fact, solutions gen-

¹³The dilatino SUSY transformation rule would be equal to $\delta_\epsilon \lambda^I \sim \not{\partial} \hat{\phi} \epsilon^I$ which only vanishes for $\epsilon^I = 0$. (I is an $SU(4)$ index here).

eralizing the one above to higher (d) dimensions can be straightforwardly found

$$\begin{cases} ds^2 &= W^{\frac{M}{r_0}-1} W dt^2 - W^{\frac{1}{d-3} \left(1 - \frac{M}{r_0}\right)} \left[W^{-1} d\rho^2 + \rho^2 d\Omega_{(d-2)}^2 \right], \\ \phi &= \phi_0 + \frac{\mathcal{Q}_d}{r_0} \ln W. \end{cases} \quad (29)$$

where

$$W = 1 - \frac{2r_0}{\rho^{d-3}}, \quad (30)$$

and now

$$r_0^2 = M^2 + 2 \left(\frac{d-3}{d-2} \right) \mathcal{Q}_d. \quad (31)$$

For $\mathcal{Q}_d = 0$ we recover the d -dimensional Schwarzschild solution. In all other cases we have metrics with naked singularities either at $\rho = 0$ or $\rho^{d-3} = 2r_0$.

Further examples can be found in Ref. [26]. It seems that most massless BHs found in string theory are T dual to extremal BHs with vanishing dilaton.

It is not clear whether it should be possible to find supersymmetric solutions which saturate the generalized B bound with primary scalar charges. It seems that, although the B *bound* should include the primary scalar hair, the saturation of the bound is always reached with secondary scalar hair only, but there is not proof of this fact.

3 Cosmic Censorship, No-Hair Theorems and String Theory

We have seen that (classical) SUSY seems to act as a cosmic censor in general only in static situations. Solutions that saturate the B bound and have angular momentum have naked singularities but are allowed by SUSY. In Ref. [29] it was observed that SUSY allows for massless solutions that can be interpreted as being made of constituents with positive and negative mass. SUSY does not forbid the presence of negative mass as long as the total mass is not negative.

Something similar happens with primary scalar hair: SUSY does not constrain its existence.

It is precisely here where a good quantum gravity theory should make an improvement. In particular, once identified the elementary degrees of freedom and rules under which they can be combined, such a theory should predict that objects with naked singularities cannot be built within its framework. With respect to the angular momentum problem, at least, this seems to be a success of string theory. In SUGRA, if we start with an ERN BH, the rules seem to allow us to add angular momentum while keeping the B bound saturated (by diminishing the mass). This is not possible in string theory [30] where it seems that we can only add angular momentum by adding fermions increasing at the same time the mass so the B bound is no longer saturated but the extremality bound is exactly saturated and never trespassed.

With primary scalar hair the situation is not so clear. In fact, one could argue that there are no sources within string theory for primary scalar hair and, as such, it should be impossible to build string theory solutions with primary scalar hair. However, it also looks difficult in string theory to identify the source of the mass when the B bound is not saturated and the mass is not entirely determined by the electric and magnetic charges (just like the secondary hair). It can be argued that the duality transformation that takes us from Schwarzschild's BH (for which a string theory model is still lacking) to the massless purely scalar solution, and which we have used heuristically, is not a good string symmetry, but that would not change the fact that the massless purely scalar solution is a solution of the low-energy effective action at least as good as Schwarzschild's. Clearly more work on this area is needed to clarify these crucial issues.

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