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INVESTIGATION OF A BEAM OF CHARGED PARTICLES BY MEANS OF PICK-UP ELECTRODES

by

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Abstract

A theoretical investigation has been carried out on the problem of the measurement of the different characteristic values of a beam of charged particles with the help of pick-up electrodes. Recommendations are given on the choice of the form of the electrodes.

For the observation of beams of charged particles in accelerators usually pick-up electrodes are being used. In the general case these electrodes are a system of conducting bodies isolated from ground and from each other. Under the influence of the electrical field of the beam potential differences appear between the electrodes. It is of some interest to clarify which are the characteristic values of a beam one can study with the help of pick-up electrodes and which form one should give to the electrodes. In agreement with the usually applied technique, we assume that the directly measured values are the potential differences between two pick-up electrodes.

It is convenient to start the calculations from the following formula, the deduction of which is given in the appendix.

$$\varphi_A - \varphi_B = \int \varphi\ (x,y,z)\ \rho\ (x,y,z)\ dx\ dy\ dz \qquad (1)$$

where φ_A and φ_B are the potentials which appear on the conductors A and B if charges are brought in which are distributed in space with the density $\rho\ (x,y,z)$ and $\varphi\ (x,y,z)$ is the potential of the point (x,y,z), appearing if a positive unit charge is brought onto the conductor A and the negative unit charge on the conductor B.

1. Two-dimensional field

Let us consider first the case of a two-dimensional field where the charged distribution and the geometry of the problem do not depend on z. The triple integral in eq. (1) goes then over into a double one, ρ is then the two-dimensional charge density and on the electrodes A and B one must bring unit charges per centimetre length. If charges are brought onto the electrodes A and B, the field distribution in any region which does not envelope the electrodes (1), is described by a function $\varphi(x,y)$ which fulfils in this region the Laplace equation. Any such function can be written in the form of a series:

$$\varphi(r,\alpha) = \sum_m (a_m r^m \cos m\alpha + b_m r^m \sin m\alpha) . \qquad (2)$$

Substituting eq. (2) into eq. (1) we find:

$$\varphi_A - \varphi_B = \sum_{m=0}^{\infty} a_m I_{1,m} + \sum_{m=1}^{\infty} b_m I_{2,m} , \qquad (3)$$

where:

$$\left.\begin{aligned} I_{1,m} &= \iint r^{m+1} \rho(r,\alpha) \cos m\alpha dr d\alpha = \mathrm{Re} \int W^m \rho(W) dS \\ I_{2,m} &= \iint r_m + 1_\rho (r,\alpha) \text{ sic } m\alpha dr d\alpha = \mathrm{Im} \int W^m \rho(W) dS \end{aligned}\right\}, \qquad (4)$$

$W = x + iy$; dS the surface element in the plane x,y.
The values of the coefficients a_m and b_m depend on the form of electrodes. If one wants, one can arrange that all coefficients except one are zero (see the end of this section). Consequently, each of the integrals $I_{1,m}$ and $I_{2,m}$ can be measured independently.

However, certain parameters of the beam can not be measured by no matter what system of pick-up electrodes. Let us consider for instance a beam in the cross-section of which the particles are homogeneously distributed on a circle of the radius R with its centre in the origin of the coordinate system. In this case, as it is easy to see, all integrals $I_{1,m}$, $I_{2,m}$ are equal to zero, with the exception of the integral $I_{1,0} = \pi R^2 \rho = Q$. The full charge is therefore the only parameter of the beam

(1) Here, and in what follows, is assumed that the electrodes are outside the region occupied by the beam.

that can be measured with the help of pick-up electrodes; the other important parameter the radius of the beam R is impossible to be measured.

Some Systems of Pick-Up Electrodes

a) The measurement of the beam charge.

Let us consider electrodes for which all the coefficients a_i, b_i, are zero, with the exception of a_o. As can be seen from eqs. (3) and (4), such electrodes measure $I_{1,0}$ that is the charge of the beam. Esq. (2) shows that in the region that is not occupied by the beam the following equation must hold:

$$\varphi(r,a) = a_o = \text{const.} \tag{5}$$

This condition is fulfilled in the region which is inside of a conductor. The second conductor must envelope the first one. The constant in eq. (5) is equal to the potential of the inner electrodes which appears if a unit charge is brought onto it, namely:

$$a_o = 1/C,$$

where C is the specific capacity of the inner electrode with respect to the outer one.

By this way:

$$\varphi_A - \varphi_B = Q/C \tag{6}$$

b) Measurement of the beam displacement (first momenta)

If a_1 is different from zero the single electrodes measure:

$$I_{1,1} = \mathrm{Re} \int W \, \rho dS = \int x \, \rho dS = \langle x \rangle Q \tag{7}$$

The potential distribution is described by the formula:

$$\varphi \, (r,a) = a_1 x \tag{8}$$

Such field is created by an endless flat condensor, the plates of which are vertically positioned. However, one should not make the condensor plates higher than necessary. In fact if one increases the height of the plates towards infinity, their specific capacity with respect to each other (as well as their capacity with respect to the shield) goes to infinity too, and, consequently, the sensitivity of the system approaches zero. With the help of eq. (1) it is not difficult to find the relationship between the potential distance of the electrodes and the capacity C(Fig. 1):

$$\varphi_A - \varphi_B = Q \langle x \rangle / d \, . \tag{9}$$

Under capacity we understand here (and everywhere) a value which is the inverse of the potential difference appearing between the electrodes if unit specific charges are brought onto them in the real geometry of the experiment, that is in presence of a shield, of amplifier tubes to which the electrodes are connected and so on.

The height of the electrodes is determined by the accuracy with which one wants to suppress the contribution of terms of higher order.

The case in which not a_1 is different from zero, but b_1 is different from the considered one by a rotation by $\pi/2$. In this case the electrodes measure the displacements of the beam with respect to the y-axis.

c) The measurement of quadrupole moments.

If a_2 is different from zero, the single electrodes measure:

$$I_{1,2} = \mathrm{Re} \int W^2 \rho dS = \int (x^2 - y^2) \rho ds = Q \left[\langle x^2 \rangle - \langle y^2 \rangle \right]. \quad (10)$$

The single electrodes must be connected in pairs and must have a hyperbolical shape (Fig. 2):

$$x^2 - y^2 = \pm d^2/4 \quad (11)$$

The potential difference between the electrodes is determined by the formula:

$$\varphi_A - \varphi_B = \frac{2}{C} \frac{\langle x^2 \rangle - \langle y^2 \rangle}{d^2} Q , \quad (12)$$

where C is the capacity of the system, and is determined in the same way as in the foregoing case. By choosing the points where the hyperbolae breaks off one must again keep in mind that an increase in the electrode size (for a given d) not only the contribution of higher moment is being suppressed, but that also the sensitivity of the system goes down.

The case where not a_2 but b_2 is different from zero is different from the considered case by a rotation by the angle $\pi/4$.

Consequently, the simplest (after the beam displacement) characteristic value of the beam which can be measured with the help of single electrodes is the average value $x^2 - y^2$. This value characterizes the asymmetry of the beam distribution. It is zero for rotationally symmetrical beam and has a maximum for a ribbon-shaped beam with one of the two diameters much smaller than the other one. In the case of a beam of negligibly small vertical dimension the electrodes measure $\langle x^2 \rangle = \langle x \rangle^2 + \langle (\Delta x)^2 \rangle$. In the absence of the beam displacement this value describes the horizontal root mean square beam width. In the general case for the determination of $\langle (\Delta x)^2 \rangle$ one must measure as well $\langle x \rangle^2$ as $\langle x \rangle$. If the beam has a small horizontal width, the same electrodes measure its average vertical extension (and give a signal of the opposite sign). If one tries to measure the ribbon-shaped beam, which is inclined by an angle of 45° with respect to the x-axis, the electrodes under consideration don't show anything, and it is necessary then to use electrodes which are rotated by an angle of $\pi/4$ (the case $b_2 \neq$ zero). The electrodes can in this way observe the loss of beam stability in one dimension, if one of the beam dimensions is blowing up, e.g. due to a parameter instability of the betatron oscillation.

The Form of the Electrodes

Let us consider the problem of finding the form of electrodes creating a field for which all the coefficients a_i, b_i are zero, except a_n (or b_n). The electrical field of this type is described by one term of the formula (2), e.g.:

$$\varphi\ (r,\ a) = a_n r^n \cos n\alpha. \qquad (13)$$

The electrodes are equipotential planes of the field and must follow a curve $\varphi(r,\alpha) =$ constant, e.g. fulfil the equations:

$$r_1^n \cos n\alpha_1 = c, \qquad (14')$$

$$r_2^n \cos n\alpha_2 = d, \qquad (14'')$$

where c and d are arbitrary constants.

The equations (14') and (14'') have in general several branches (Fig. 2). The electrodes placed on all the branches of one equation must be electrically connected. The constants c and d can be chosen arbitrarily. If $c = -d$, the electrodes are placed symmetrically with respect to the origin of the coordinate system.

In some cases it can be useful to add to the surfaces of the single electrodes according to (14') and (14'') a surface:

$$r_3^n \cos n\alpha_3 = 0 \;, \qquad (14''')$$

which must be connected to the shield. In the case considered on Fig. 2, such a surface is the line kk. The system of single electrodes has in this case not two, but only one hyperbolical electrodes of either sign, and the screen includes one half of the cylindrical surface shown on Fig. 2 and the surface kk.

2. Three-Dimensional Fields

When trying to calculate the potential districution in a three-dimensional case, one encounters serious mathematical difficulties. Usually it is simpler to choose the form of the electrodes by an empirical method. We limit ourselves therefore to some remarks of a general character:

a) Equation (1) shows that the sensitivity of the system is proportional to $\varphi(x,y,z)$, e.g. inversely proportional to the capacity of the electrodes (capacity must here be understood in the same sense as explained in section 1.b);

b) The stray fields on the edges of the pick-up electrodes have not the same distribution as the field between the electrodes. The contribution of the undesirable component depends therefore strongly on the contribution of the stray fields to the integral of eq. (1). It is better to work with electrodes the length of which is considerabl bigger than the transverse dimensions;

c) In most of the practical problems the dependence of the field of the electrodes on z is much stronger than the dependence of ρ on z ("long" beam). Assuming that in the region in which $\varphi \neq 0$, ρ is independent of z, we find from eq. (1):

$$\varphi_A - \varphi_B = \int dx\, dy\, \rho(x,y) \int \varphi(x,y,z)\, dz, \qquad (15)$$

where the integration has to be carried out over the region in which φ is essentially different from zero.

The integral field distribution:

$$\Phi(x,y) = \int \varphi(x,y,z)\, dz \qquad (16)$$

fulfils the two-dimensional Laplace equation and can be expressed in the form of a series according to eq. (2). The experimentally measurable value $\varphi_A - \varphi_B$ is defined by this series expression. One obtains the result leading to important practical conclusions. Let us consider, for instance, the measurement of the beam displacement. As it has already been said (see section 1.b) in the region which is occupied by the beam the field of the electrodes should be expressed by the formula $\varphi = ax$.

The necessary properties has a field which is enclosed in a square, as is shown on Fig. 3, if the potential on the dotted border line varies from - to + according to a linear law. In the case of two flat electrodes, this condition is fulfilled only by the field of an infinitely large flat condensor. A field with the required properties can be created by the electrodes as shown on Fig. 4. Here a field of the necessary configuration can be obtained with considerably smaller capacity of the system. An analogous method can be applied for the improvement of the electrodes of Fig. 2.

So far we discussed the problem from a pure electrostatic point of view. It is easy to see that the motion of the beam does not change our formula as long as the difference between the length of the beam and the length of the electrodes exceeds considerably the transverse dimensions of the system. For short beams the relativistic field compression has to be taken into account. This, however, is hardly of any practical interest.

The author is indebted to E.K. Tarasov and P.R. Zenkyevitch for the discussion of the results.

APPENDIX

We want to prove the theory mentioned in the introduction. We make use of Green's equation [1] which is valid for any finite and contir. ous functions φ_1 and φ_2 :

$$\int (\varphi_2 \Delta \varphi_1 - \varphi_1 \Delta \varphi_2) dv = \oint (\varphi_2 \frac{\partial \varphi_1}{n} - \varphi_1 \frac{\partial \varphi_2}{n}) \, dS \, . \qquad (A.1)$$

Let φ_1 and φ_2 be the potentials for two different charge distributions. Then is:

$$\Delta\varphi_1 = - 4\pi\rho_1 \; ; \qquad \partial\varphi_1 / \partial n = - E_{1n}$$
$$\Delta\varphi_2 = - 4\pi\rho_2 \; ; \qquad \partial\varphi_2 / \partial n = - E_{2n} \qquad (A.2)$$

Therefore:

$$\int (\varphi_2\rho_1 - \varphi_1\rho_2) \, dv = \frac{1}{4\pi} \oint (\varphi_2 E_{1n} - \varphi_1 E_{2n}) \, dS, \quad (A.3)$$

where ρ_1 and ρ_2 are the charge distributions, and E_{1n} and E_{2n} the normal components of the electrical field strength vector. The integral on the right-hand side of eq. (A.3) is carried out over the surfaces of the electrodes, the screen (if the system has one) or an infinitely far remoted sphere. The integral over the infinitely far remoted sphere equals zero, because φ and E go both simultaneously to zero. The integral over grounded screen is also zero, because $\varphi = 0$. On the surface of conducting bodies, the potential does not depend on the coordinates and:

$$\oint E_n \, dS = - 4 \pi Q \, , \qquad (A.4)$$

where Q is the charge of the body.

We obtain therefore from eq. (A.3):

$$\int(\varphi_2\rho_1 - \varphi_1\rho_2)dv + \sum_i (\varphi_{2i}Q_{1i} - \varphi_{1i}Q_{2i}) = 0 \quad , \qquad (A.5)$$

where the sum has to be taken over all conductors. Let us assume that in the first distribution a charge q is located on the point x,y,z and in the second distribution the charges $+1$ and -1 are located on the bodies A and B (the pick-up electrodes). We find then from eq. (A.5):

$$\varphi\,(x,\,y,\,z)\;q = \varphi_A - \varphi_B \qquad (A.6)$$

Equation (A.6) can be generalized for the case of a condinuous charge distribution:

$$\varphi_A - \varphi_B = \varphi(x,y,z)\;\rho\;(x,y,z)\;dv \;, \qquad (A.7)$$

where φ_A and φ_B are the potentials of the bodies A and B, which appear for a charge density distribution $\rho(x,y,z)$ and $\varphi(x,y,z)$ is the potential which appears on the point (x,y,z) if one brings a unit positive charge onto the body A and the negative unit charge onto the body B.

<u>Reference:</u>

(1) I.E. TAMM, Fundamentals of the Theory of Electricity, 1946, Gosteichisdat, Moscow.

Figure Captions:

Fig. 1 Electrodes measuring the beam displacement.

Fig. 2 Electrodes measuring $\langle x^2 \rangle - \langle y^2 \rangle$

Fig. 3 -----

Fig. 4 Electrodes measuring the beam displacement.

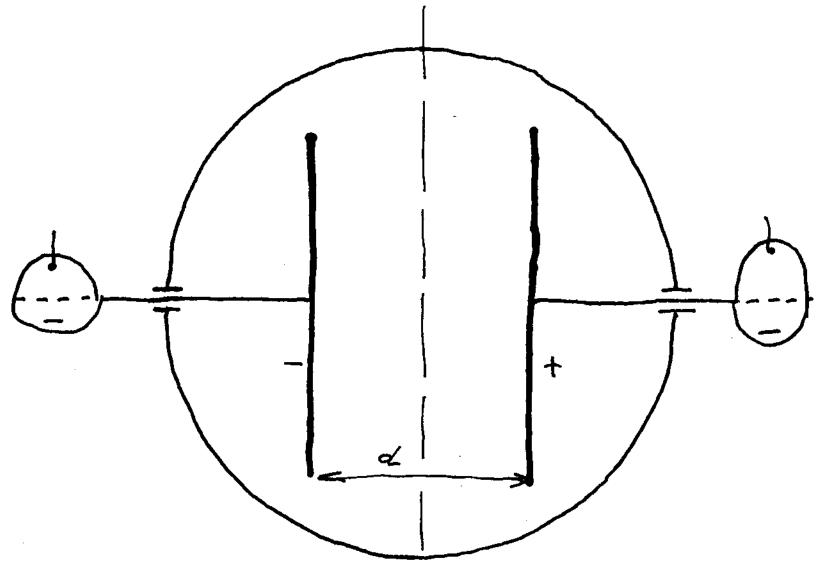

Fig. 1

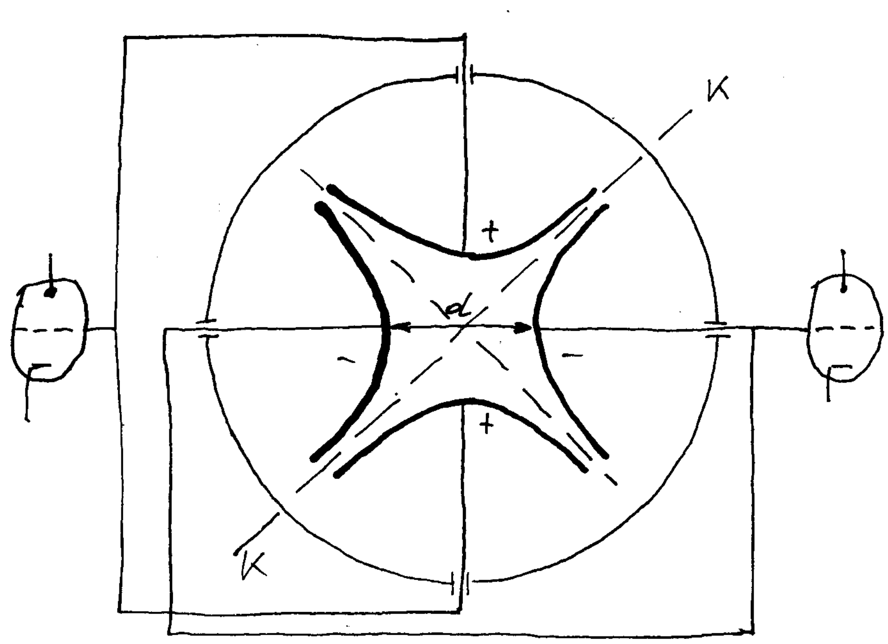

Fig. 2

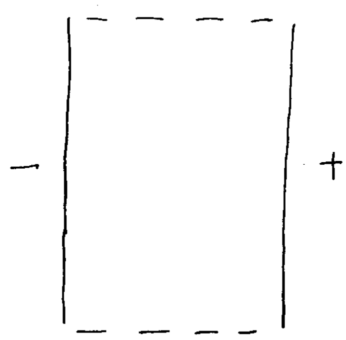

Fig. 3

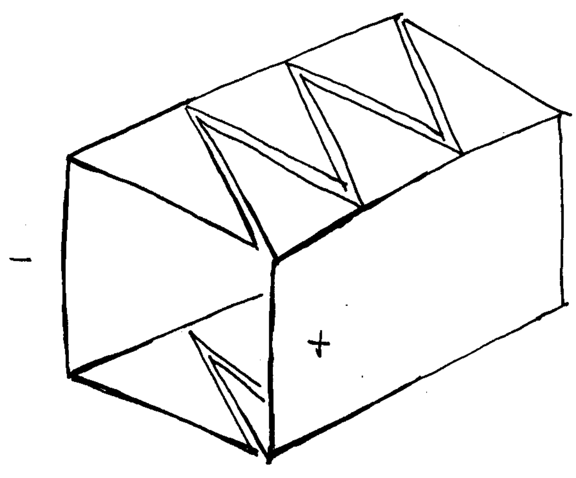

Fig. 4