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Non-Leptonic Weak Decays of B Mesons

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Abstract

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1   Introduction

The weak decays of hadrons containing a heavy quark offer the most direct way to determine the weak mixing angles of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and to explore the physics of CP violation. At the same time, they are of great help in studying strong-interaction physics related with the confinement of quarks and gluons into hadrons. Indeed, both tasks complement each other: an understanding of the connection between quark and hadron properties is a necessary prerequisite for a precise determination of the CKM matrix and CP-violating phases.

The simplest processes are those involving a minimum number of hadrons, i.e. a single hadron in the final state of a semileptonic decay, or two hadrons in the final state of a non-leptonic decay. In recent years, much progress has been achieved in understanding these processes. Simple bound-state models are able to describe, in a semiquantitative way, the current matrix elements occurring in semileptonic decay amplitudes [1–11]. A factorization prescription for reducing the hadronic matrix elements of four-quark operators to products of current matrix elements shed light onto the dynamics of non-leptonic processes [12,13], where even drastic effects had been lacking an explanation before.

More recently, the discovery of heavy-quark symmetry [14–19] and the establishment of the heavy-quark effective theory [20–31] have provided a solid theoretical framework to calculate exclusive semileptonic transitions between two hadrons containing heavy quarks, such as the decays $\bar{B} \to D^{(*)} \ell \bar{\nu}$. Moreover,

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the heavy-quark expansion has been applied to the calculation of inclusive semileptonic, non-leptonic and rare decay rates [32–40] and of the lifetimes of charm and bottom hadrons [41–47]. These developments are discussed in detail in the article of one of us (M. Neubert) in this volume. In particular, they lead to a precise determination of the CKM matrix element governing the strength of $b \to c$ transitions:

$$|V_{cb}| = 0.039 \pm 0.002 \,. \tag{1}$$

In this article, we shall discuss non-leptonic two-body decays of B mesons. The dynamics of non-leptonic decays, in which only hadrons appear in the final state, is strongly influenced by the confining colour forces among the quarks. Whereas in semileptonic transitions the long-distance QCD effects are described by few hadronic form factors parametrizing the hadronic matrix elements of quark currents, non-leptonic processes are complicated by the phenomenon of quark rearrangement due to the exchange of soft and hard gluons. The theoretical description involves matrix elements of local four-quark operators, which are much harder to deal with than current operators. These strong-interaction effects prevented for a long time a coherent understanding of non-leptonic decays. The $\Delta I = \frac{1}{2}$ rule in strange particle decays is a prominent example. Although this selection rule had been known for almost four decades, only recently successful theoretical approaches have been developed to explain it in a semiquantitative way [48–51]. The strong colour force between two quarks in a colour-antitriplet state has been identified as the dominant mechanism responsible for the dramatic enhancement of $\Delta I = \frac{1}{2}$ processes.

The discovery of the heavy charm and bottom quarks opened up the possibility to study a great variety of new decay channels. By now, there is an impressive amount of experimental data available on many exclusive and inclusive decay modes. In many respects, non-leptonic decays of heavy mesons are an ideal instrument for exploring the most interesting aspect of QCD, i.e. its non-perturbative, long-range character. Since the initial state consists of an isolated heavy particle and the weak transition operator exhibits a well-known and simple structure, a detailed analysis of decays into particles with different spin and flavour quantum numbers provides valuable information about the nature of the long-range forces influencing these processes, the same forces that determine the internal structure of all hadrons.

In energetic two-body transitions, hadronization of the decay products does not occur until they have traveled some distance away from each other [52]. The reason is that once the quarks have grouped into colour-singlet pairs, soft gluons are ineffective in rearranging them. The decay amplitudes are then expected to factorize into products of hadronic matrix elements of colour-singlet quark currents. The factorization approximation has been applied to many

two-body decays of B and D mesons[13,16,53−62]. It relates the complicated non-leptonic decay amplitudes to products of meson decay constants and hadronic matrix elements of current operators, which are similar to those encountered in semileptonic decays. The decay constants are fundamental hadronic parameters providing a measure of the strength of the quark-antiquark attraction inside a hadronic state. As some of them are not directly accessible in leptonic or electromagnetic processes, their extraction from non-leptonic transitions may provide important information.

In Section 2, we discuss the effective Hamiltonian relevant for decays of the bottom quark. In Section 3, we provide a prescription for the calculation of non-leptonic decay amplitudes in the factorization approximation. We discuss the problems connected with factorization, such as the choice of a suitable factorization scale and its possible dependence on the energy released in a decay process. In Section 4, we then discuss in more detail the structure of non-factorizable corrections. Using the $1/N_c$ expansion, we argue that in energetic two-body decays of B mesons a generalized factorization prescription holds, which involves two parameters $a_1^{\text{eff}} \approx c_1 + \zeta c_2$ and $a_2^{\text{eff}} \approx c_2 + \zeta c_1$ depending on a hadronic parameter $\zeta = O(1/N_c)$. We estimate that the process dependence of ζ is very mild, suppressed as $\Delta E / m_b$, where ΔE is the difference in the energy release in different two-body decay channels. Some basic aspects of final-state interactions are briefly covered in Section 5. The remainder of this article is devoted to a phenomenological description of non-leptonic two-body decays of B mesons. In the factorization approximation, the ingredients for such an analysis are meson decay constants and transition form factors. In Section 6, we collect the current experimental information on meson decay constants. In Section 7, we describe two simple models that provide a global description of heavy-to-heavy and heavy-to-light weak decay form factors, embedding the known constraints imposed by heavy-quark symmetry [18,19,63]. With only a few parameters, these models reproduce within errors the known properties of decay form factors and predict those form factors which are yet unknown. Our approach is meant to provide global predictions for a large set of non-leptonic decay amplitudes. We do not attempt here a more field-theoretical calculation of weak amplitudes, nor do we perform a dedicated investigation of individual decay channels.[a] In Sections 8 and 9, we compare our predictions with the experimental data on two-body non-leptonic decays of B mesons, present tests of the factorization assumption, and extract the decay constants of D_s and D_s^* mesons. In Section 10, we briefly address the interesting possibility of B-meson decays into baryons. A summary and conclusions are given in Section 11.

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[a]Attempts of a more rigorous calculation of some particular decay modes are discussed in the article by R. Rückl in this volume.

2   Effective Hamiltonian

At tree level, non-leptonic weak decays are described in the Standard Model by a single W-exchange diagram. Strong interactions affect this simple picture in a two-fold way. Hard-gluon corrections can be accounted for by perturbative methods and renormalization-group techniques. They give rise to new effective weak vertices. Long-distance confinement forces are responsible for the binding of quarks inside the asymptotic hadron states. The basic tool in the calculation of non-leptonic amplitudes is to separate the two regimes by means of the operator product expansion (OPE)[64], incorporating all long-range QCD effects in the hadronic matrix elements of local four-quark operators. This treatment appears well justified due to the vastly different time and energy scales involved in the weak decay and in the subsequent formation of the final hadrons.

Integrating out the heavy W-boson and top-quark fields, one derives the effective Hamiltonian for $b \to c, u$ transitions [65,66]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} \left[c_1(\mu) Q_1^{cb} + c_2(\mu) Q_2^{cb} \right] \right.$$

$$\left. + V_{ub} \left[c_1(\mu) Q_1^{ub} + c_2(\mu) Q_2^{ub} \right] + \text{h.c.} \right\}$$

$$+ \text{penguin operators.} \qquad (2)$$

It consists of products of local four-quark operators renormalized at the scale μ and scale-dependent Wilson coefficients $c_i(\mu)$. V_{cb} and V_{ub} are elements of the quark mixing matrix. The operators Q_1 and Q_2, written as products of colour-singlet currents, are given by

$$Q_1^{cb} = \left[(\bar{d}'u)_{V-A} + (\bar{s}'c)_{V-A} \right] (\bar{c}b)_{V-A},$$

$$Q_2^{cb} = (\bar{c}u)_{V-A} (\bar{d}'b)_{V-A} + (\bar{c}c)_{V-A} (\bar{s}'b)_{V-A},$$

$$Q_1^{ub} = \left[(\bar{d}'u)_{V-A} + (\bar{s}'c)_{V-A} \right] (\bar{u}b)_{V-A},$$

$$Q_2^{ub} = (\bar{u}u)_{V-A} (\bar{d}'b)_{V-A} + (\bar{u}c)_{V-A} (\bar{s}'b)_{V-A}, \qquad (3)$$

where d' and s' denote weak eigenstates of the down and strange quarks, respectively, and $(\bar{c}b)_{V-A} = \bar{c}\gamma_\mu(1-\gamma_5)b$ etc. Without strong-interaction effects we would have $c_1 = 1$ and $c_2 = 0$. This simple result is modified, however, by gluon exchange: the original weak vertices get renormalized, and new types of interactions (such as the operators Q_2) are induced. Not explicitly shown in (2) are the so-called penguin operators [66]. Since their Wilson coefficients are
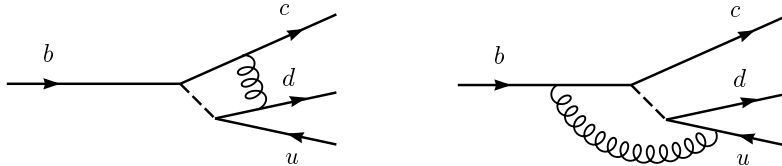
Figure 1: Hard-gluon corrections giving rise to the Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$ in the effective weak Hamiltonian.

very small, the corresponding contributions to weak decay amplitudes only become relevant in rare decays, where the tree-level contribution is either strongly CKM-suppressed, as in $\bar{B} \to \bar{K}^{(*)}\pi$, or where matrix elements of the Q_1 and Q_2 operators do not contribute at all, as in $\bar{B} \to \bar{K}^*\gamma$ and $\bar{B}^0 \to \bar{K}^0\phi$.

The Wilson coefficients $c_i(\mu)$ take into account the short-distance corrections arising from the exchange of gluons with virtualities between m_W and some hadronic scale μ, chosen large enough for perturbation theory to be applicable. The coefficients $c_1(\mu)$ and $c_2(\mu)$, for example, arise from the hard-gluon exchanges shown in Figure 1. The effects of soft gluons (with virtualities below the scale μ) remain in the hadronic matrix elements of the local four-quark operators Q_i. In the course of the evolution from m_W down to the scale μ there arise large logarithms of the type $[\alpha_s \ln(m_W/\mu)]^n$, which must be summed to all orders in perturbation theory. This is achieved by means of the renormalization-group equation (RGE). The combinations $c_\pm(\mu) = c_1(\mu) \pm c_2(\mu)$ of the Wilson coefficients have a multiplicative evolution under change of the renormalization scale. They can be obtained from the solution of the RGE

$$\left(\mu \frac{\mathrm{d}}{\mathrm{d}\mu} - \Gamma_\pm \right) c_\pm(\mu) = 0 \,, \tag{4}$$

with the initial condition $c_\pm(m_W) = 1$, corresponding to $c_1(m_W) = 1$ and $c_2(m_W) = 0$. The quantities $\Gamma_\pm$ in (4) are the anomalous dimensions of the operators $(Q_1 \pm Q_2)$. At one-loop order, they are given by [67,68]

$$\Gamma_\pm = \gamma_\pm \frac{\alpha_s}{4\pi} \,; \qquad \gamma_\pm = 6 \left(\pm 1 - \frac{1}{N_c} \right) \,, \tag{5}$$

where $N_c = 3$ is the number of colours. To leading logarithmic order (LO), the solution of the RGE is

$$c_\pm(\mu) = \left(\frac{\alpha_s(m_W)}{\alpha_s(\mu)} \right)^{\gamma_\pm/2\beta_0} \,; \qquad \beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f \,, \tag{6}$$

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where β_0 is the first coefficient of the β function, and n_f the number of active flavours (in the region between m_W and μ). The physical origin of the enhancement of $c_-(\mu)$ can be traced back to the attractive force between two quarks in the colour-antitriplet channel of the scattering process $b + u \to c + d$. Similarly, in the colour-sextet channel the force is repulsive, leading to $c_+(\mu) < 1$.

The leading logarithmic approximation can be improved by including the next-to-leading (NLO) corrections of order $\alpha_s[\alpha_s \ln(m_W/\mu)]^n$. The result is

$$c_\pm(\mu) = \left(\frac{\alpha_s(m_W)}{\alpha_s(\mu)} \right)^{\gamma_\pm/2\beta_0} \left\{ 1 + R_\pm \frac{\alpha_s(\mu) - \alpha_s(m_W)}{4\pi} \right\} , \qquad (7)$$

where the coefficients $R_\pm$ are given by [69,70] (β_1 is the two-loop coefficient of the β function)

$$R_\pm = \frac{N_c \mp 1}{2N_c} \left\{ \pm \frac{6\beta_1}{\beta_0^2} + \frac{1}{2\beta_0} \left(21 \mp \frac{57}{N_c} \pm \frac{19}{3} N_c \mp \frac{4}{3} n_f \right) \mp 11 \right\} . \qquad (8)$$

In Table 1, we show the values of $c_1(m_b)$ and $c_2(m_b)$ obtained at leading and next-to-leading order. The evolution of the running coupling constant is done at two-loop order, using the normalization $\alpha_s(m_Z) = 0.118 \pm 0.003$. For comparison, we show in Table 2 the values obtained at a lower scale, which may be relevant to charm decays. The negative sign of $c_2(\mu)$ has important consequences for decays in which there is significant interference between c_1 and c_2 amplitudes.

In a typical hadronic decay process such as $\bar{B}^0 \to D^+ \varrho^-$, there are several mass scales involved: the hadron masses, the quark masses, the energy release, etc. Thus, there is an uncertainty in the choice of the "characteristic scale" of a process. In principle this is not a problem, since the products of the Wilson coefficients with the hadronic matrix elements of the local four-quark operators are scale independent. In practice, however, one often employs simple model estimates of the matrix elements, which usually do not yield an explicit scale dependence that could compensate for that of the Wilson coefficients. Instead, these model calculations are assumed to be valid on a particular scale. A related technical problem is that whereas the operator evolution from m_W down to m_b can be calculated in a straightforward way, a more complicated scaling behaviour is expected below the mass of the b quark, where all the other mass scales start to become relevant. There have been attempts to account for the scaling in the region $\mu < m_b$ by summing logarithms of the type $[\alpha_s \ln(m_b/\mu)]^n$ and $[\alpha_s \ln(E/\mu)]^n$, with $E = O(m_b)$ being the large energy of a light particle produced in a two-body decay of a B meson [71,72]. However, at present the treatment of such corrections is still associated with large uncertainties.

Table 1: Values of the Wilson coefficients at the scale $m_b = 4.8$ GeV, both at leading (LO) and next-to-leading (NLO) order

$\alpha_s(m_Z)$	$c_1^{\mathrm{LO}}(m_b)$	$c_2^{\mathrm{LO}}(m_b)$	$c_1^{\mathrm{NLO}}(m_b)$	$c_2^{\mathrm{NLO}}(m_b)$
0.115	1.102	-0.239	1.124	-0.273
0.118	1.108	-0.249	1.132	-0.286
0.121	1.113	-0.260	1.140	-0.301

Table 2: Values of the Wilson coefficients at the scale $m_c = 1.4$ GeV

$\alpha_s(m_Z)$	$c_1^{\mathrm{LO}}(m_c)$	$c_2^{\mathrm{LO}}(m_c)$	$c_1^{\mathrm{NLO}}(m_c)$	$c_2^{\mathrm{NLO}}(m_c)$
0.115	1.240	-0.476	1.313	-0.576
0.118	1.263	-0.513	1.351	-0.631
0.121	1.292	-0.556	1.397	-0.696

We shall thus stay with the conventional choice $\mu = O(m_b)$ adopted in most previous work on non-leptonic B decays.

3 Factorization

In weak interactions, a meson (or meson resonance) can be directly generated by a quark current carrying the appropriate parity and flavour quantum numbers. The corresponding contribution to a decay amplitude factorizes into the product of two current matrix elements [73,74]. As an example, consider the transition $\bar{B}^0 \to D^+ \pi^-$. The factorizable part of the amplitude is given by

$$A_{\mathrm{fact}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 \langle \pi^- | (\bar{d}u)_A | 0 \rangle \langle D^+ | (\bar{c}b)_V | \bar{B}^0 \rangle. \tag{9}$$

The coefficient a_1 will be discussed below. The $\bar{B}^0 \to D^+$ transition matrix element is of the same type as that encountered in the semileptonic decay $\bar{B}^0 \to D^+\ell^-\nu$. It can be determined using data on semileptonic decays together with theoretical arguments based on heavy-quark symmetry [19]. The amplitude for creating a pion from the vacuum via the axial current is parametrized by the decay constant f_π and is proportional to the momentum of the pion:

$$\langle \pi^-(p) | \bar{d}\gamma_\mu\gamma_5 u | 0 \rangle = i f_\pi p_\mu. \tag{10}$$

Thus it seems natural to assume that the amplitude for energetic weak decays, in which the directly generated meson carries a large momentum, is dominated by its factorizable part. This assumption can be substantiated by a more detailed analyses of the kinematic situation in the above decay process: a fast moving $(\bar{u}d)$ pair created in a point-like interaction, with both quarks leaving the interaction region in the same direction and with a velocity close to the speed of light, will hadronize only after a time given by its γ factor times a typical hadronization time $\tau_{\mathrm{had}} \sim 1$ fm/c. In the above example, this means that hadronization occurs about 20 fm away from the remaining quarks. Inside the interaction region, the $(\bar{u}d)$ pair behaves like a colourless and almost point-like particle. It does little interact with the remaining quarks. Because of this intuitive "colour transparency argument" [52], one expects that the factorizable part (9) of the decay amplitude does indeed give the dominant contribution to the full amplitude. A more formal investigation of this situation, using an effective theory for heavy quarks and fast-moving light quarks, has been presented in Ref. 72.

In the following applications, we shall adopt the factorization ansatz also for processes in which the γ factors of the outgoing particles are not necessarily large. Examples of such decays are $\bar{B} \to \bar{K} J/\psi$ and $\bar{B} \to D \bar{D}_s$. In these cases, the kinematic argument given above does no longer apply. Nevertheless, in a two-body process, the concentration of energy into colour-singlet states together with the fact that (axial) vector current matrix elements increase with the particle momentum favours the direct current-induced production of mesons over more complicated production mechanisms. Only comparison with experiment can tell whether factorization is a useful concept also for those processes. We will analyse the structure of non-factorizable corrections in Section 4, and discuss some tests of the factorization approximation in Section 9.

By factorizing matrix elements of the four-quark operators contained in the effective Hamiltonian (2), one can distinguish three classes of decays [12,13]. The first class contains those decays in which only a charged meson can be generated directly from a colour-singlet current, as in $\bar{B}^0 \to D^+ \pi^-$. For these processes, the relevant QCD coefficient is given by the combination

$$a_1 = c_1(\mu_f) + \zeta \, c_2(\mu_f), \qquad \text{(class I)} \qquad (11)$$

where $\zeta = 1/N_c$ (N_c being the number of quark colours), and $\mu_f = O(m_b)$ is the scale at which factorization is assumed to be relevant. The term proportional to $c_2(\mu_f)$ arises from the Fierz reordering of Q_2 operators and factorization of the product of colour-singlet currents contained in it. At this point, the colour-octet term resulting from the Fierz reordering is simply being discarded. In

order to remove this deficiency, one should treat ζ as a free parameter. This will be discussed in more detail in the next section.

A second class of transitions consists of those decays in which the meson generated directly from the current is neutral, like the J/ψ particle in the decay $\bar{B} \to \bar{K} J/\psi$. The corresponding decay amplitude,

$$A_{\text{fact}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \, a_2 \, \langle J/\psi | \, (\bar{c}c)_V \, | \, 0 \, \rangle \, \langle \bar{K} | \, (\bar{s}b)_V \, | \bar{B} \rangle \,, \tag{12}$$

is proportional to the QCD coefficient

$$a_2 = c_2(\mu_f) + \zeta \, c_1(\mu_f) \,. \qquad \text{(class II)} \tag{13}$$

Due to the different sign and magnitude of the Wilson coefficients (see Table 1), the combination a_2 is particularly sensitive to the value of the factorization scale and to any additional long-distance contributions (i.e., to the precise value of ζ). The QCD coefficient a_1, on the other hand, can be estimated quite reliably.

The third class of transitions covers decays in which the a_1 and a_2 amplitudes interfere, such as in $B^- \to D^0 \pi^-$. Their final state contains a charged as well as a neutral meson, both of which can be generated from a current of one of the operators of the effective Hamiltonian. The corresponding amplitudes involve a combination

$$a_1 + x \, a_2 \,, \qquad \text{(class III)} \tag{14}$$

where $x = 1$ in the formal limit of a flavour symmetry for the final-state mesons, as it is realized in the strongly CKM-suppressed decay $B^- \to \pi^0 \pi^-$.

In principle, there is another type of factorizable contribution to weak decay amplitudes, which is however significantly different from the ones covered so far: the so-called "weak annihilation contribution" [75,76], in which the decaying heavy meson is annihilated by a current of one of the operators of the effective Hamiltonian. For a charged meson, this contribution is proportional to a_1, while it is proportional to a_2 for a neutral meson. In the latter case, the weak annihilation is in fact the exchange of a W boson between the two constituent quarks. In a weak annihilation process, the second current in the four-quark operator produces all the recoiling final-state particles out of the vacuum, which implies a sizable form factor suppression. Therefore, annihilation amplitudes are expected to be small, and it is commonly assumed that they may be neglected for all except some rare processes. This approximation is not an essential part of the factorization scheme, however. It would be straightforward to include the annihilation contributions if the relevant form factors at large time-like values of q^2 were known.

Let us come back to our first example of a class I decay, collecting all the different pieces of the factorizable contribution to the decay amplitude. Inserting (10) and a suitable form factor decomposition of the hadronic matrix element (see Appendix) into (9), we obtain

$$A_{\text{fact}}(\bar{B}^0 \to D^+\pi^-) = -i\frac{G_F}{\sqrt{2}}\, V_{cb}\, V_{ud}^*\, a_1\, f_\pi\, (m_B^2 - m_D^2)\, F_0^{B \to D}(m_\pi^2)\,. \quad (15)$$

Note that it is the longitudinal form factor F_0 which enters this expression, whereas (in the limit of vanishing lepton mass) the semileptonic decay amplitude for $\bar{B}^0 \to D^+\ell^-\nu$ involves the transverse form factor F_1. Fortunately, heavy-quark symmetry provides relations between all the $\bar{B} \to D^{(*)}$ transition form factors [19,63]. They allow us to relate F_0 to form factors which are easily accessible experimentally.[b]

Note that the hadronic matrix elements of the vector and axial currents resulting from the factorization of the matrix elements of four-quark operators do not show any scale dependence that could compensate the scale dependence of the Wilson coefficients. Strictly speaking, therefore, factorization cannot be correct. What one may hope for is that it provides a useful approximation if the Wilson coefficients (or equivalently the QCD coefficients a_1 and a_2) are evaluated at a suitable scale μ_f, the factorization point. Because the factorized hadronic matrix elements can only account for the interaction between quarks remaining together in the same hadron, the Wilson coefficients in effective Hamiltonian should contain those gluon effects which redistribute the quarks. Thus, we have to evolve these coefficients down to a scale where the gluons are no longer effective in changing the particle momenta in a significant way. For very energetic decays, the "colour transparency argument" shows that gluons with virtualities much below the mass of the decaying particle are ineffective in rearranging the final-state quarks [52,72], and $\mu_f = O(m_b)$ seems a reasonable choice for the factorization scale. However, for processes in which the decay products carry only little kinetic energy, gluons with virtualities well below m_b can still lead to a redistribution of the quarks before hadronization sets in. This fact may be used as an argument in favour of a lower factorization scale in such processes. On a qualitative level, the connection between the factorization scale and the energy release in the final state can be seen from Figure 2, where we show the ratio a_2/a_1 as a function of $\alpha_s(\mu_f)$. As we will see in Section 8, the value preferred by $\bar{B} \to D\pi$ decays is positive and corresponds to a rather small coupling, indicating $\mu_f = O(m_b)$. On the other hand, D

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[b]In addition, in the present case the smallness of the pion mass allows us to use the kinematic constraint $F_0(0) = F_1(0)$ (see Appendix) to relate F_0 to a measurable semileptonic form factor.
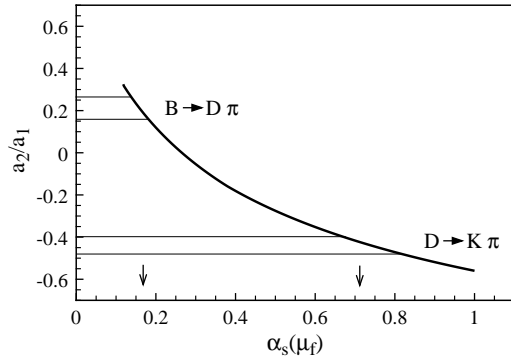
Figure 2: The ratio a_2/a_1 as a function of the running coupling constant evaluated at the factorization scale. The bands indicate the phenomenological values of a_2/a_1 extracted from $\bar{B} \to D\pi$ and $D \to K\pi$ decays.

decays indicate a negative value of a_2/a_1, corresponding to a significantly lower value of the factorization scale. This is in accordance with the fact that in these processes the energy released to the final-state particles is only about 1 GeV. Moreover, it reflects that for low-energetic processes hadronic weak decays exhibit a strong $\Delta I = \frac{1}{2}$ enhancement, which is significant in D decays and even spectacular in K decays. A strict $\Delta I = \frac{1}{2}$ selection rule would correspond to $a_2/a_1 = -1$. Because of these intuitive arguments, we are led to abandon the "naive" factorization prescription adopted in most previous work, where a fixed factorization scale $\mu_f = m_b$ was taken for all B decays, in favour of a more flexible factorization scheme, in which μ_f – or equivalently ζ in (11) and (13) – is treated as a process-dependent parameter.

We end this section with a remark on very low-energetic processes such as K decays. Factorization of hadronic matrix elements of four-quark operators into two matrix elements of colour-singlet currents implies that only those non-perturbative forces that act between quarks and antiquarks are taken into account. The remaining interactions including, in particular, the gluon exchange between two quarks or two antiquarks are treated perturbatively. However, in the decays of strange particles the long-distance attraction between two quarks in a colour-antitriplet state gives the dominant contribution to $\Delta I = \frac{1}{2}$ decay amplitudes. A detailed analysis of these "diquark effects" can be found in Ref. 51. As will be briefly discussed in Section 10, we expect similar effects to be important in B decays into baryon-antibaryon pairs. For energetic B decays into two mesons, on the other hand, the two quarks (or antiquarks) do

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not end up in the same final-state hadron, and a perturbative treatment of their mutual interaction seems justified.

4   Corrections to Factorization and the $1/N_c$ Expansion

Let us now illustrate in more detail the structure of the corrections to the factorization approximation (for a similar discussion, see Refs. 59, 60). For a given class I or class II two-body decay channel, the effective Hamiltonian (2) can be rewritten using Fierz identities in such a way that the quarks are paired according to the flavour quantum numbers of the final-state hadrons.[c] This introduces products of two colour-singlet or two colour-octet current operators. The hadronic matrix elements of the latter ones are being neglected in the naive factorization approximation.

Consider again the example of the decay $\bar{B}^0 \to D^+ \pi^-$. In this case, the appropriate form to write the effective Hamiltonian is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\, V_{cb}\, V_{ud}^* \left\{ \left(c_1(\mu) + \frac{c_2(\mu)}{N_c} \right)\, (\bar{d}u)_{V-A}\, (\bar{c}b)_{V-A} \right.$$
$$\left. + \frac{c_2(\mu)}{2}\, (\bar{d}t_a u)_{V-A}\, (\bar{c}t_a b)_{V-A} \right\} + \ldots, \qquad (16)$$

where $(\bar{d}\, t_a u)_{V-A} = \bar{d}\, \gamma_\mu (1 - \gamma_5) t_a u$ etc., and we have shown explicitly the dependence on the number of colours. For a given decay process, let us now define two hadronic parameters as follows:

$$\varepsilon_1^{(BD,\pi)}(\mu) \equiv \frac{\langle \pi^- D^+ | (\bar{d}u)_{V-A}\, (\bar{c}b)_{V-A} | \bar{B}^0 \rangle}{\langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle\, \langle D^+ | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle} - 1\,,$$

$$\varepsilon_8^{(BD,\pi)}(\mu) \equiv \frac{\langle \pi^- D^+ | (\bar{d}t_a u)_{V-A}\, (\bar{c}t_a b)_{V-A} | \bar{B}^0 \rangle}{2\langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle\, \langle D^+ | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle}\,. \qquad (17)$$

They parametrize the non-factorizable contributions to the hadronic matrix elements and are process dependent. Here the subscript refers to the colour structure of the operator, whereas the superscript indicates the particles involved in the process. The final-state particle with flavour quantum numbers of one of the currents is given last. This superscript is written only when necessary to avoid confusion. In terms of these new parameters the decay amplitude resumes the form given in (9), however, with a_1 replaced by the new coefficient

$$a_1^{\text{eff}} = \left(c_1(\mu) + \frac{c_2(\mu)}{N_c} \right) \left[1 + \varepsilon_1^{(BD,\pi)}(\mu) \right] + c_2(\mu)\, \varepsilon_8^{(BD,\pi)}(\mu)\,. \qquad (18)$$

[c]In all our applications, class III amplitudes are related to combinations of class I and II amplitudes by isospin relations.

Similarly, in the class II transition $\bar{B}^0 \to D^0 \pi^0$ we encounter the effective coefficient

$$a_2^{\text{eff}} = \left(c_2(\mu) + \frac{c_1(\mu)}{N_c} \right) \left[1 + \varepsilon_1^{(B\pi,D)}(\mu) \right] + c_1(\mu)\, \varepsilon_8^{(B\pi,D)}(\mu) \,. \qquad (19)$$

Since we have introduced these parameters without loss of generality, the effective coefficients a_i^{eff} take into account all contributions to the matrix elements and are thus μ independent. In other words, the hadronic parameters $\varepsilon_i(\mu)$ restore the correct μ dependence of the matrix elements, which is lost in the naive factorization approximation. Using the RGE for the coefficient functions, it is then straightforward to show that the μ dependence of the hadronic parameters is, in the leading logarithmic order, given by

$$\begin{aligned}
1 + \varepsilon_1(\mu) &= \frac{1}{2} \left[\left(1 + \frac{1}{N_c} \right) \left[1 + \varepsilon_1(\mu_0) \right] + \varepsilon_8(\mu_0) \right] \kappa_+ \\
&\quad + \frac{1}{2} \left[\left(1 - \frac{1}{N_c} \right) \left[1 + \varepsilon_1(\mu_0) \right] - \varepsilon_8(\mu_0) \right] \kappa_- \,, \\
\varepsilon_8(\mu) &= \frac{1}{2} \left[\left(1 - \frac{1}{N_c} \right) \varepsilon_8(\mu_0) + \left(1 - \frac{1}{N_c^2} \right) \left[1 + \varepsilon_1(\mu_0) \right] \right] \kappa_+ \\
&\quad + \frac{1}{2} \left[\left(1 + \frac{1}{N_c} \right) \varepsilon_8(\mu_0) - \left(1 - \frac{1}{N_c^2} \right) \left[1 + \varepsilon_1(\mu_0) \right] \right] \kappa_- \,,
\end{aligned}$$
$$(20)$$

where $\kappa_\pm = [\alpha_s(\mu)/\alpha_s(\mu_0)]^{\gamma_\pm / 2\beta_0}$, and μ_0 is an arbitrary normalization point. It is interesting to evaluate these expressions assuming that there exists a scale $\mu_0 = \mu_f$ where factorization holds, i.e. where $\varepsilon_i(\mu_f) = 0$. It then follows that

$$\begin{aligned}
\varepsilon_1(\mu) &= \frac{1}{2} \left(1 + \frac{1}{N_c} \right) \kappa_+ + \frac{1}{2} \left(1 - \frac{1}{N_c} \right) \kappa_- - 1 = O(1/N_c^2) \,, \\
\varepsilon_8(\mu) &= \frac{1}{2} \left(1 - \frac{1}{N_c^2} \right) (\kappa_+ - \kappa_-) = O(1/N_c) \,,
\end{aligned} \qquad (21)$$

and expanding in powers of α_s we obtain

$$\varepsilon_1(\mu) = O(\alpha_s^2) \,, \qquad \varepsilon_8(\mu) = -\frac{4\alpha_s}{3\pi} \ln \frac{\mu}{\mu_f} + O(\alpha_s^2) \,. \qquad (22)$$

To deduce the dependence on the number of colours, we have used the expressions for the anomalous dimension given in (5). Note that the large-N_c

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counting rules, $\varepsilon_1 = O(1/N_c^2)$ and $\varepsilon_8 = O(1/N_c)$, are independent of the assumption that factorization holds at the scale μ_f [77,78]. Hence, from first principles of QCD, we expect that $|\varepsilon_1| \ll 1$, whereas contributions from ε_8 can be more sizable.

Similar counting rules can be derived for the Wilson coefficients $c_i(\mu)$; however, they can be obscured by the presence of the large logarithm $L = \ln(m_W/\mu)$. In general, we have

$$c_1(\mu) = 1 + O(L/N_c^2), \qquad c_2(\mu) = O(L/N_c). \tag{23}$$

At the scale $\mu = m_b$, it is evident from Table 1 that one can consistently treat $L = O(1)$, and thus $c_1 = 1 + O(1/N_c^2)$ and $c_2 = O(1/N_c)$. For much lower scales, such as $\mu = m_c$, however, Table 2 shows that it is appropriate to take $L/N_c = O(1)$, and therefore $c_1 = 1 + O(1/N_c)$ and $c_2 = O(1)$.

Let us now discuss the relation of this general approach to the conventional factorization scheme, focusing first on the case of B decays. Naive factorization corresponds to setting $\varepsilon_i(m_b) = 0$, in which case a_1 and a_2 are universal, process-independent coefficients, which are simply linear combinations of the Wilson coefficients $c_1(m_b)$ and $c_2(m_b)$. In (18) and (19) we have shown how these relations are modified by the presence of non-factorizable corrections. From the above discussion it follows that non-factorizable corrections are expected to be small in class I transitions. In class II decays, on the other hand, the contribution proportional to ε_8 is enhanced by the large value of the ratio $|c_1/c_2|$. Hence, as a general rule, we expect sizable violations of the naive factorization approximation in class II decays only. Explicitly, evaluating the general expressions in the large-N_c limit, we find

$$a_1^{\text{eff}} = c_1(\mu) + c_2(\mu)\left(\frac{1}{N_c} + \varepsilon_8^{(BD,\pi)}(\mu)\right) + O(1/N_c^2) = 1 + O(L/N_c^2),$$

$$a_2^{\text{eff}} = c_2(\mu) + c_1(\mu)\left(\frac{1}{N_c} + \varepsilon_8^{(B\pi,D)}(\mu)\right) + O(L/N_c^3). \tag{24}$$

In B decays, with $L = O(1)$ and neglecting terms of order $1/N_c^2$, we can rewrite this as

$$a_1^{\text{eff}} \approx 1, \qquad a_2^{\text{eff}} \approx c_2(m_b) + \zeta\, c_1(m_b), \tag{25}$$

with

$$\zeta \equiv \frac{1}{N_c} + \varepsilon_8^{(B\pi,D)}(m_b). \tag{26}$$

Note that, with the same accuracy, the first relation in (25) may be replaced by $a_1^{\text{eff}} \approx c_1(m_b) + \zeta\, c_2(m_b)$. It is important to stress at this point that the

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naive choice $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$ does not correspond to any consistent limit of QCD [78]; in particular, this is not a prediction of the $1/N_c$ expansion. Since the parameter ε_8 is of order $1/N_c$, the two contributions to ζ are expected to be of similar magnitude, and hence ζ should be considered as an unknown dynamical parameter. In other words, the large-N_c counting rules of QCD predict that related class I and class II two-body decays, such as $\bar{B}^0 \to D^+\pi^-$ and $\bar{B}^0 \to D^0\pi^0$, or $\bar{B}^0 \to D^+\varrho^-$ and $\bar{B}^0 \to D^0\varrho^0$, can be described (up to corrections of order $1/N_c^2$) using factorization with a single phenomenological parameter $\zeta = O(1/N_c)$. Strictly speaking, however, this parameter will take different values for different class II decay channels. The picture emerging from this discussion is equivalent to the concept of using a process-dependent factorization scale μ_f, discussed in the previous section. This scale is defined such that $\varepsilon_8(\mu_f) \equiv 0$. Using (22) one may then calculate the corresponding value of $\varepsilon_8(m_b)$ and thus the ζ parameter for each process.

We can go a step further and estimate the expected size of the process dependence of the parameter ζ. From Figure 2, we know that for the particular case of $\bar{B} \to D\pi$ transitions factorization holds at a high scale, i.e. $\varepsilon_8^{(B\pi,D)}(\mu_f) = 0$ for $\mu_f \gtrsim m_b$ and thus $\zeta \approx 1/3$. If we now assume that in other two-body decays of B mesons the factorization scale is lower because there is less energy released to the final-state particles, we may use relation (22) to obtain for the change in ζ:

$$\Delta\zeta = \Delta\varepsilon_8(m_b) \approx \frac{4\alpha_s(m_b)}{3\pi}\frac{\Delta E}{m_b} \approx 0.02 \times \frac{\Delta E}{1\text{ GeV}}, \tag{27}$$

where ΔE is the difference in the energy release in different decay channels. In the two-body B decays of interest to us, ΔE is always smaller than 1 GeV, so that $\Delta\zeta$ becomes a parameter of order $1/m_b$. The corresponding variations of ζ are of order a few per cent, which is small compared with the value of ζ itself ($\zeta \approx 1/3$ for $\bar{B} \to D\pi$ decays).

To summarize this discussion, we repeat that in energetic two-body decays of B mesons the large-N_c counting rules of QCD predict that factorization with $a_1^{\text{eff}} \approx 1$ works well for class I transitions, whereas class II transitions can be described by a phenomenological coefficient $a_2^{\text{eff}} \approx c_2(m_b) + \zeta\,c_1(m_b)$ with $\zeta = O(1/N_c)$. Based on an intuitive argument, we have estimated that the process dependence of ζ is very mild, since $\Delta\zeta \sim \Delta E/m_b$. Thus, we have provided a theoretical basis for the phenomenological treatment of using a factorization prescription in which the phenomenological parameter ζ is taken to be the same for all energetic two-body decays of B mesons.

Let us briefly also discuss the case of charm decays. Here the large-N_c counting rules are different, because empirically $L/N_c = O(1)$. Then, neglect-

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ing terms of order $1/N_c^2$, we find instead of (25) the relations

$$a_1^{\text{eff}} \approx c_1(m_c) + \zeta' \, c_2(m_c) \,, \qquad a_2^{\text{eff}} \approx c_2(m_c) + \zeta \, c_1(m_c) \,, \qquad (28)$$

with

$$\zeta' \equiv \frac{1}{N_c} + \varepsilon_8^{(DK,\pi)}(m_c) \,, \qquad \zeta \equiv \frac{1}{N_c} + \varepsilon_8^{(D\pi,K)}(m_c) \,. \qquad (29)$$

In general, class I and II transitions can no longer be described by the same ζ parameter. However, we may again argue that the expected process dependence of the ζ parameters is a mild one. In (27) now appears the smaller charm-quark mass in the denominator, but on the other hand the energy difference ΔE in the numerator is smaller than in B decays. As a consequence, we still have $\Delta \zeta$ of order a few per cent; in particular, then, we expect $\zeta' \approx \zeta$. That ζ in (25) should be treated as a phenomenological parameter, rather than fixed to the value $\zeta = 1/3$ predicted by naive factorization, was the basis of the approach of Bauer et al.[13] (see also Ref. 53). It turns out that setting $\zeta \approx 0$ provides a successful description of many two-body decays of D mesons. To some extend, this phenomenological "rule of discarding $1/N_c$ terms" can be understood in the context of QCD sum rules[79], and using more formal considerations based on the heavy-quark expansion[80]. From the above argument, it follows that if $\zeta \approx 0$ for one D decay mode, it is expected to be a small parameter also for other channels.

As a final comment, we note that whereas the concept of introducing effective, process-dependent parameters a_i^{eff} works in most two-body decays of B and D mesons, it has to be modified in decays where the final state consists of two vector particles ($P \to VV$ decays). The polarization of the final-state particles in such processes is very sensitive to non-factorizable contributions and final-state interactions. The ratios between S-, P- and D-wave amplitudes predicted in the factorization approximation are affected since non-factorizable contributions to the amplitude will, in general, have a different structure for different partial waves[59]. In other words, the matrix elements describing the non-factorizable contributions to the decay amplitudes are spin-dependent, thus affecting the polarization of the final-state particles. Likewise, final-state interactions are different for different partial waves. They may change S- into D- waves even without changing the total decay rate. A case of particular interest is the polarization of the J/ψ particle in the decay $B \to \bar{K}^* J/\psi$. Most models predict a longitudinal polarization of around 40% (a model that we shall introduce in Section 7 predicts 48%), whereas the experimental world average is[81] (78 ± 7)%. A recent CLEO measurement, however, gives[82] $(52 \pm 7 \pm 4)$%. Although a clear picture has not yet emerged, it is possible that the first deviations from factorization predictions in B decays will be seen

in polarization data. Indeed, it has been argued that spin-dependent non-factorizable effects are necessary to understand the data[83] (see, however, also the discussion in Ref. 8).

5 Final-State Interactions

In the conventional factorization approximation, the scattering of the final-state particles off each other is neglected, and all amplitudes are real (apart from an arbitrary common phase). Watson's theorem, however, requires the amplitudes to have the same phases as the corresponding scattering amplitudes[84]. Moreover, in two-body decays, the final state must be of low angular momentum, and we know that S- and P-wave scattering amplitudes in the GeV region are, in general, inelastic.

Let us first consider a scattering eigenstate, i.e. an eigenstate of the S matrix. The phase of the amplitude for the decay into this state, which is in general a linear combination of physical final states, is identical to the corresponding scattering phase. But final-state interactions will, in general, also affect the magnitude of the scattering amplitudes. By definition, these interactions occur in a space-time region where the final-state particles have already been formed in their ground states, but are still strongly interacting while recoiling from each other. Accordingly, for a very energetic two-body decay with a pion in the final state, the "colour transparency argument" of the previous section already excludes the possibility of significant final-state interactions, since the light quark-antiquark pair will have left the region of strong interaction long before it hadronizes into a pion. In a weak decay involving small recoil energies, however, the product kR (particle momentum times the radius of the strong-interaction region) is of the same order as the scattering phase δ, which is $O(1)$. Therefore, in such a decay rescattering will indeed change also the magnitude of the decay amplitude. In $K \to 2\pi$ decays, for example, rescattering effects lead to an enhancement of the $\Delta I = \frac{1}{2}$ amplitude by $\approx 30\%$ and to a reduction of the $\Delta I = \frac{3}{2}$ amplitude by $\approx 10\%$[51]. Fortunately, in very energetic processes such as exclusive B decays, the phase δ is negligible compared to kR, and we may safely assume that the magnitude of the amplitude for decays into a scattering eigenstate remains unchanged by final-state interactions. As a consequence, the relation between the amplitudes A_i for decays into final states i (which are, in general, not scattering eigenstates) to the "bare" amplitudes A_i^0 is

$$A_i = \left(S^{1/2}\right)_{ij} A_j^0 \,, \tag{30}$$

where S denotes the strong-interaction S matrix.

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The S matrix can redistribute the amplitudes into different channels carrying the same quantum numbers, including those channels which were not originally coupled to the weak process. To illustrate the consequences of (30) we consider, as an extreme example, a two-channel S matrix with maximal absorption ($S_{11} = S_{22} = 0$):

$$ S = \begin{pmatrix} 0 & e^{i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix} . \tag{31} $$

The square root of this matrix is

$$ S^{1/2} = \frac{1}{\sqrt{2}} \exp\left\{ i \left(\frac{\varphi}{2} - \frac{\pi}{4} \right) \right\} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} . \tag{32} $$

For a vanishing "bare" amplitude for decays into the second channel ($A_2^0 = 0$), the amplitude A_1 for decays into the first channel is reduced by a factor $1/\sqrt{2}$, leading to a 50% reduction in the corresponding branching ratio. The second channel, in spite of not being directly accessible through the weak decay, obtains an amplitude of the same magnitude. Although this is an extreme example, one should be aware of the fact that different decay channels influence each other. In particular, decays with small branching ratios may have been modified or even caused by a "spill-over" from stronger modes.

Obviously, using the factorization approximation we can only attempt to calculate the "bare" decay amplitudes A_i^0. However, summing over all decay channels with the same conserved quantum numbers, the uncertainties connected with final-state interactions drop out. The reason is that, because of (30) and the unitarity of the S matrix, we have

$$ \sum_i |A_i|^2 = \sum_i |A_i^0|^2 , \tag{33} $$

i.e. this sum of decay rates remains unaffected by final-state interactions. By measuring the branching ratios of several related decay channels, one can extract the magnitudes of the corresponding isospin amplitudes as well as their relative phases [85]. The same isospin amplitudes can also be obtained from the "bare" decay amplitudes calculated in the factorization approximation. Neglecting inelastic rescattering, one can then extract the QCD coefficients a_1^{eff} and a_2^{eff} from a comparison of measured and calculated isospin amplitudes.

It is well-known that final-state interactions are important in D decays; they must be included in the comparison of theoretical predictions with data [57]. For B decays, on the other hand, we expect smaller final-state interaction effects. Because of the large energy released to the particles in the final state,

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the magnitude of the isospin amplitudes should not be modified significantly by final-state interactions. Moreover, there do not exist any charm resonances in the B meson mass region that could lead to strong final-state interactions. As an example, consider the decays $\bar{B} \to D\pi$. The four-quark operator $(\bar{d}u)(\bar{c}b)$, which is part of the effective weak Hamiltonian relevant for B decays, carries isospin quantum numbers $I = 1$ and $I_3 = +1$ and thus can transform a $\bar{B}^0$ meson into a state with $I = \frac{1}{2}$ or $I = \frac{3}{2}$, while the B^- can only decay into final states with $I = \frac{3}{2}$. Accordingly, we define the following (complex) isospin amplitudes:

$$A_{1/2} = \langle D\pi; I = \tfrac{1}{2} | H_{\text{eff}} | \bar{B}^0 \rangle$$

$$A_{3/2} = \langle D\pi; I = \tfrac{3}{2} | H_{\text{eff}} | \bar{B}^0 \rangle = \frac{1}{\sqrt{3}} \langle D\pi; I = \tfrac{3}{2} | H_{\text{eff}} | B^- \rangle. \qquad (34)$$

They are related with the physical decay amplitudes through

$$A(\bar{B}^0 \to D^+\pi^-) = \sqrt{\frac{1}{3}} A_{3/2} + \sqrt{\frac{2}{3}} A_{1/2}$$

$$A(\bar{B}^0 \to D^0\pi^0) = \sqrt{\frac{2}{3}} A_{3/2} - \sqrt{\frac{1}{3}} A_{1/2}$$

$$A(B^- \to D^0\pi^-) = \sqrt{3} A_{3/2}. \qquad (35)$$

Although the phases of the measured decay amplitudes are unknown, we can extract from (35) the isospin amplitudes as well as their relative phase:

$$|A_{1/2}|^2 = |A(\bar{B}^0 \to D^+\pi^-)|^2 + |A(\bar{B}^0 \to D^0\pi^0)|^2$$

$$- \frac{1}{3} |A(B^- \to D^0\pi^-)|^2$$

$$|A_{3/2}|^2 = \frac{1}{3} |A(B^- \to D^0\pi^-)|^2$$

$$\cos(\delta_{3/2} - \delta_{1/2}) = \frac{3|A(\bar{B}^0 \to D^+\pi^-)|^2 - 2|A_{1/2}|^2 - |A_{3/2}|^2}{2\sqrt{2}|A_{1/2}||A_{3/2}|}. \qquad (36)$$

Let us assume that the magnitudes of the three physical amplitudes had been measured. We could then use these relations to determine the magnitudes of the two isospin amplitudes as well as their relative phase. In order to compare the results with the amplitudes calculated in the factorization approximation, we must assume that the magnitude of the isospin amplitudes remains unaffected by final-state interactions. In other words, the contribution of inelastic

Table 3: Upper limits for the relative phase shifts between the two isospin amplitudes in some exclusive hadronic B decays

Ratio	CLEO (90% CL)	$\lvert\delta_{3/2} - \delta_{1/2}\rvert$
$\mathrm{B}(\bar{B}^0 \to D^0\pi^0)/\mathrm{B}(B^- \to D^0\pi^-)$	< 0.07	$< 36°$
$\mathrm{B}(\bar{B}^0 \to D^0\varrho^0)/\mathrm{B}(B^- \to D^0\varrho^-)$	< 0.05	$< 30°$
$\mathrm{B}(\bar{B}^0 \to D^{*0}\pi^0)/\mathrm{B}(B^- \to D^{*0}\pi^-)$	< 0.13	$< 53°$
$\mathrm{B}(\bar{B}^0 \to D^{*0}\varrho^0)/\mathrm{B}(B^- \to D^{*0}\varrho^-)$	< 0.07	$< 36°$

scattering into or from other channels has to be negligible. Then neglecting final-state interactions simply amounts to neglecting the relative phase shift between the different isospin amplitudes, and we can directly compare the theoretical predictions obtained for $\lvert A_{1/2}\rvert$ and $\lvert A_{3/2}\rvert$ with the experimental values for these quantities.

Unfortunately, in B decays complete measurements of the class of decay amplitudes relevant to the isospin analysis described above are not yet available. Still, we can use existing data to obtain upper bounds on the relative phase shifts in some interesting cases. In our example, it is the amplitude $A(\bar{B}^0 \to D^0\pi^0)$ that has not yet been measured. However, from the current upper limit for the corresponding branching ratio we can derive an upper limit for the relative phase shift $\lvert\delta_{3/2} - \delta_{1/2}\rvert$. To this end, we represent the second relation in (35) as a triangle in the complex plane. An elementary geometrical argument shows that the angle between $A_{1/2}$ and $A_{3/2}$ is maximal when there is a right angle between $A_{1/2}$ and $A(\bar{B}^0 \to D^0\pi^0)$. Hence

$$\sin^2(\delta_{3/2} - \delta_{1/2}) \leq \frac{9}{2} \frac{\tau(B^-)}{\tau(\bar{B}^0)} \frac{\mathrm{B}(\bar{B}^0 \to D^0\pi^0)}{\mathrm{B}(B^- \to D^0\pi^-)} \,. \tag{37}$$

Similar inequalities can be derived for the phase shifts in other decay channels. Using then experimental upper limits for colour-suppressed B decay modes obtained by the CLEO Collaboration [81,86], as well as $\tau(B^-)/\tau(\bar{B}^0) = 1.06 \pm 0.04$ for the lifetime ratio [87], we obtain the results shown in Table 3. The next generation of CLEO data is likely to produce still smaller upper bounds for the various phase shifts, but even the current limits are a strong indication that final-state interactions in hadronic two-body decays of B mesons are much less important than in the corresponding decays of D mesons. We will therefore neglect them in our analysis.

6 Decay Constants

The evaluation of factorized amplitudes requires the knowledge of meson decay constants and hadronic form factors of current matrix elements. We shall discuss first what is known about decay constants. For a pseudoscalar meson $P = (q_1\bar{q}_2)$, we define

$$\langle\, 0\,|\, \bar{q}_2\gamma_\mu\gamma_5 q_1\, |P(p)\rangle = if_P p_\mu\,. \tag{38}$$

The decay constant f_V of a vector meson $P = (q_1\bar{q}_2)$ is defined by [d]

$$\langle\, 0\,|\, \bar{q}_2\gamma_\mu q_1\, |V(\epsilon)\rangle = \epsilon_\mu m_V f_V\,. \tag{39}$$

The experimental values of the decay constants of the charged pion and kaon, as obtained from their leptonic decays $P^+ \to \ell^+\nu_\ell\,(\gamma)$, are [88]

$$f_\pi = (130.7 \pm 0.37)\ \mathrm{MeV}\,, \qquad f_K = (159.8 \pm 1.47)\ \mathrm{MeV}\,. \tag{40}$$

For charm mesons, the uncertainties in the experimental values of the leptonic decay rates are much larger. For the D^+ meson, only an upper bound can be deduced from the published data [89]:

$$f_D < 310\ \mathrm{MeV} \quad (90\%\ \mathrm{CL})\,. \tag{41}$$

Four groups (WA75[90], CLEO[91], BES[92], and E653[93]) have measured the $D_s^+ \to \mu^+\nu_\mu$ branching ratio, from which the decay constant f_{D_s} can be extracted. The average value is [87]

$$f_{D_s} = (241 \pm 37)\ \mathrm{MeV}\,. \tag{42}$$

Later, we shall compare this result with an independent determination of f_{D_s} from non-leptonic decays.

The decay constants of light charged mesons can also be obtained from the 1-prong hadronic decays of the τ lepton. Denoting the relevant CKM-matrix element by V_{ij}, and neglecting radiative corrections, we write the corresponding decay width as

$$\Gamma(\tau^- \to M^-\nu_\tau) = \frac{m_\tau^3}{16\pi}\, G_F^2\, |V_{ij}|^2\, f_M^2\, \left(1 - \frac{m_M^2}{m_\tau^2}\right)^2 \left(1 + b_M\frac{m_M^2}{m_\tau^2}\right)\,, \tag{43}$$

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[d]For the neutral mesons ϱ^0 and ω, we use the current $\bar{u}\gamma_\mu u$ on the left-hand side of (39) and divide the right-hand side by $\sqrt{2}$. With this definition and under the assumption of isospin symmetry, the decay constants of neutral and charged ϱ mesons have the same numerical value.

Table 4:   Decay constants of light charged mesons extracted from hadronic τ decays

M	π	K	ϱ	K^*	a_1
f_M [MeV]	134 ± 1	158 ± 3	208 ± 1	214 ± 7	229 ± 10

Table 5:   Decay constants of neutral vector mesons extracted from their electromagnetic decays

V	ϱ	ω	ϕ	J/ψ	$\psi(2\mathrm{S})$
c_V	$1/2$	$1/18$	$1/9$	$4/9$	$4/9$
f_V [MeV]	216 ± 5	195 ± 3	237 ± 4	405 ± 14	282 ± 14

with $b_P = 0$ for pseudoscalar and $b_V = 2$ for (axial) vector mesons. We use $m_\tau = 1777$ MeV for the mass of the τ lepton and $\tau_\tau = 291.0 \pm 1.5$ fs for its lifetime [88]. Moreover, we use the following branching ratios for $\tau^- \to M^- \nu_\tau$ [88,94]: $(11.31 \pm 0.15)\%$ (π), $(0.69 \pm 0.03)\%$ (K), $(24.94 \pm 0.16)\%$ (ϱ), $(1.25 \pm 0.08)\%$ (K^*), and $(17.65 \pm 0.32)\%$ (a_1). This leads to the values of the decay constants shown in Table 4. Only experimental errors are quoted. The results for f_π and f_K are in excellent agreement with those given in (40).

The decay constants of neutral vector mesons can be extracted from their electromagnetic decay width, using

$$\Gamma(V^0 \to e^+ e^-) = \frac{4\pi}{3} \frac{\alpha^2}{m_V} f_V^2 \, c_V \,, \tag{44}$$

where c_V are factors related to the electric charge of the quarks that make up the vector meson. From the measured widths [88], we obtain the results shown in Table 5. The errors reflect the uncertainty in the experimental data only. The value for f_ϱ obtained in this way is somewhat larger than that derived from τ decays. In our analysis below, we take $f_\varrho = 210$ MeV. In all other cases, we take the central values shown in the Tables.

We note for completeness that the decay constants of light neutral pseudoscalar mesons can be extracted from the two-photon decay $P^0 \to \gamma\gamma$. Averaging the results reported by the TPC/2γ [95] and CELLO [96] Collaborations, we find

$$f_\eta = (131 \pm 6) \text{ MeV}\,, \qquad f_{\eta'} = (118 \pm 5) \text{ MeV}\,. \tag{45}$$

Decay constants not yet known experimentally are left as free parameters in our expressions for the branching ratios. However, we will often need an estimate for the decay constants of charm mesons, such as D and D^*. In the heavy-quark limit ($m_c \to \infty$), spin symmetry predicts that $f_D = f_{D^*}$, and most theoretical predictions indicate that symmetry-breaking corrections enhance the ratio f_{D^*}/f_D by 10–20% [19]. Hence, we take as our central values

$$f_D = 200 \text{ MeV}, \qquad f_{D^*} = 230 \text{ MeV},$$
$$f_{D_s} = 240 \text{ MeV}, \qquad f_{D_s^*} = 275 \text{ MeV}. \tag{46}$$

The value for f_D lies in the ball park of most theoretical predictions, whereas that for f_{D_s} corresponds to the central experimental result quoted in (42).

7 Transition Form Factors

The most important ingredient of factorized decay amplitudes are the hadronic form factors defined in terms of the covariant decomposition of hadronic matrix elements of current operators. In particular, we need matrix elements of the type

$$\langle M | \bar{q} \gamma_\mu (1 - \gamma_5) b | B \rangle, \tag{47}$$

where M is a pseudoscalar or vector meson with mass m_M. There are two form factors (F_0, F_1) describing the transition to a pseudoscalar particle, and four form factors (V, A_0, A_1, A_2) for the transition to a vector particle. The definitions of these form factors are given in the Appendix. Conventionally, they are written as functions of the invariant momentum transfer q^2. To obtain reliable theoretical predictions for the form factors is the main obstacle in the analysis of hadronic weak decays, once the factorization hypothesis is accepted.

In the case of the heavy-to-heavy transitions $\bar{B} \to D$ and $\bar{B} \to D^*$, heavy-quark symmetry implies simple relations between the various transition form factors. In the heavy-quark limit, they read [63]

$$F_1(q^2) = V(q^2) = A_0(q^2) = A_2(q^2) = \frac{m_B + m_M}{2\sqrt{m_B m_M}} \xi(w),$$
$$F_0(q^2) = A_1(q^2) = \frac{2\sqrt{m_B m_M}}{m_B + m_M} \frac{w + 1}{2} \xi(w), \tag{48}$$

where

$$w = v_B \cdot v_M = \frac{m_B^2 + m_M^2 - q^2}{2 m_B m_M} \tag{49}$$

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is the product of the velocities of the two mesons, and the universal (mass-independent) function $\xi(w)$ is called the Isgur-Wise form factor [18,28]. This function is normalized to unity at the kinematic point $w = 1$, where the two mesons have a common rest frame. For the realistic case of finite heavy-quark masses, the above relations are modified by corrections that break heavy-quark symmetry. They can be analysed in a systematic way using the heavy-quark effective theory [19−31]. This is discussed in detail in the article by one of us (M. Neubert) in this volume. As a consequence, it has become possible to extract the $\bar{B} \to D^{(*)}$ form factors relevant for all class I transitions considered in this article from semileptonic decay data with good precision and in an essentially model-independent way. The results are compiled in Ref. 57. The use of heavy-quark symmetry constitutes a significant improvement compared with earlier estimates of non-leptonic amplitudes, which were based on model calculations of the relevant form factors.

Unfortunately, there has not been similar progress in the calculation of hadronic current matrix elements between heavy and light mesons. For these transitions, we still must rely on the results obtained using some phenomenological model. One such model (below referred to as the NRSX model) was introduced in Ref. 57, where we used the overlap integrals of the BSW model [1,13] to obtain the form factors at zero momentum transfer, and then proposed a specific ansatz for their q^2 dependence motivated by pole dominance and the relations in (48). Another model [7], which we shall not discuss in detail, has been used by Deandrea et al. [58] to perform a global analysis of non-leptonic decays comparable to ours.

In order to get an idea about the unavoidable amount of model dependence, which will affect our predictions for class II decays of B mesons, we will consider an alternative to the NRSX model. The main purpose here is not to provide a new approach that is more sophisticated; on the contrary, we will make several strong assumptions to obtain a model as simple as possible. Yet, we have checked that it agrees, within reasonable limits, with the available data on semileptonic decays, as well as with most theoretical predictions. We thus feel confident that this model may also be used advantageously to obtain rough estimates for other processes, such as rare B decays and decays of B_s mesons.

In the case of heavy-to-light transitions, the symmetry relations in (48) are no longer valid. We account for this by introducing, for each form factor, a function $\xi_i(w)$ replacing the Isgur-Wise function. There is, however, still a connection between various form factors provided by kinematic constraints at zero momentum transfer ($q^2 = 0$), corresponding to

$$w = w_{\max} = \frac{m_B^2 + m_M^2}{2 m_B m_M}\,. \tag{50}$$

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There, the form factors satisfy the relations

$$F_1(0) = F_0(0) \,,$$

$$A_0(0) = \frac{m_B + m_M}{2m_M} A_1(0) - \frac{m_B - m_M}{2m_M} A_2(0) \,. \tag{51}$$

The normalization condition for the Isgur-Wise function is replaced by the inequalities

$$\xi_{F_0}(1) \leq 1 \,, \qquad \xi_{A_1}(1) \leq 1 \,, \tag{52}$$

which follow from equal-time commutation relations for a heavy b quark. For an estimate of the functions $\xi_i(w)$ we shall adopt a simple pole model, which apart from the masses of some resonances has no tunable parameters. This model reproduces the relations (48) in the heavy-quark limit. It will serve for an immediate estimate of all form factors in the physical region of q^2. As a starting point, we write $\xi_i(w) = \sqrt{2/(w+1)}\, h_i(w)$, where $h_i(w)$ will be simple monopole form factors. In the heavy-quark limit, the prefactor $\sqrt{2/(w+1)}$ can be shown to yield an upper bound for the Isgur-Wise function [63]. For heavy-to-light form factors, on the other hand, this factor ensures that $\xi_i(w) \sim w^{-3/2}$ for large w, in accordance with the scaling rules obtained in Ref. 97. In the next step we make the strong assumption that the symmetry relations in (48) still hold for heavy-to-light form factors, but only at the point $q^2 = 0$. In other words, we assume that all the $h_i(w_{\max})$ are equal. This is a minimal ansatz that preserves the kinematic relations in (51). For the functions $h_i(w)$ with $i = F_1$, V, A_0 and A_2, we take

$$h_i(w) = N\, \frac{w_{\max} - w_i}{w - w_i} \,, \tag{53}$$

where N is a common normalization factor. Note that, according to its definition, the value of $w_{\max}$ in h_{F_1} is different from that in the other three cases. The locations of the poles are at

$$w_i = \frac{m_B^2 + m_M^2 - M_i^2}{2 m_B m_M} \,, \tag{54}$$

where M_i denotes the mass of the nearest resonance with the appropriate spin-parity quantum numbers. Most of these masses are known experimentally; some others (such as the masses of the B_c and B_c^* mesons) are taken from potential models. To obtain the masses of the 1^+ resonances, we simply add 400 MeV to the masses of the corresponding 1^- states, as suggested by the spectroscopy of the light and charm mesons, and use the same values for the A_1 and A_2 form factors.

It remains to find an ansatz for $h_{F_0}(w)$ and $h_{A_1}(w)$. For simplicity, we choose to normalize these functions to unity at $w = 1$, corresponding to a complete overlap of the initial and final states at the same velocity [cf. (52)]. Moreover, we have to satisfy the condition $h_{F_0}(w_{\max}) = h_{A_1}(w_{\max}) = N$. Thus, we take (for $j = F_0, A_1$)

$$ h_j(w) = \frac{1}{1 + r \dfrac{w - 1}{w_{\max} - 1}} \, , \tag{55} $$

where $N = 1/(1 + r)$ is still a free parameter. To fix it, we try to mimic the effect of the physical axial vector (1^+) pole on the form factor $A_1(w)$ near $w = 1$. In order to have full correspondence with the pole structure of the other form factors in (53), the factor $\frac{1}{2}(w + 1)$ in (48) has to be included. Therefore, we require that the derivative of the product $\frac{1}{2}(w + 1) \, h_{A_1}(w)$ at $w = 1$ be equal to the derivative of a single-pole form factor with the physical pole mass M_{A_1}. The result is

$$ r = \frac{(m_B - m_V)^2}{4 m_B m_V} \left(1 + \frac{4 m_B m_V}{M_{A_1}^2 - (m_B - m_V)^2} \right) , \tag{56} $$

where V is a vector meson. The same value of r is then taken for $h_{F_0}(w)$. With these simple assumptions, all form factors are determined.

A few tests of this model may be quoted here: The branching ratios for the semileptonic decays $\bar{B} \to D^* \ell \bar{\nu}$ and $\bar{B} \to D \ell \bar{\nu}$ are found to be 5.3% and 1.8%, respectively. The corresponding experimental values are [88] $(4.64 \pm 0.26)\%$ and $(1.8 \pm 0.4)\%$. In the semileptonic decay $\bar{B} \to \varrho \ell \bar{\nu}$, the values for the form factors at $q^2 = 0$ are $V = A_1 = A_2 = 0.26$, in accordance with the predictions $V = 0.35 \pm 0.07$, $A_1 = 0.27 \pm 0.05$ and $A_2 = 0.28 \pm 0.05$ obtained using QCD sum rules [98]. The expected branching ratio for this decay is $25.5|V_{ub}|^2$, which when combined with the experimental result [99] $B(\bar{B} \to \varrho \ell \bar{\nu}) = (2.5^{+0.8}_{-0.9}) \times 10^{-4}$ yields $|V_{ub}| = (3.1 \pm 0.5) \times 10^{-3}$. The calculated ϱ/π ratio in exclusive semileptonic B decays is 2.2, as compared with the experimental value of [99] 1.4 ± 0.6. Finally, comparing the calculated $\bar{B} \to \pi$ form factor $F_1(q^2)$ at threshold with the B^*-pole structure at this point [100,101] gives $g_{BB^*\pi} = 0.54 \times (0.2\,\text{GeV}/f_{B^*})$ for the $BB^*\pi$ coupling constant, which is of the expected order.

8 Theoretical Predictions, Comparison with Experiment and Determination of a_1^{eff} and a_2^{eff}

We first discuss the results obtained using the NRSX model [57]. To obtain predictions for the branching ratios from the factorized decay amplitudes, we use $\tau(\bar{B}^0) = 1.55 \pm 0.04$ ps and $\tau(B^-) = 1.65 \pm 0.04$ ps for the B-meson lifetimes [87], and $V_{cb} = 0.039 \pm 0.002$ from (1). The small errors in these quantities, as well as uncertainties in the decay constants of light mesons, are neglected. Our predictions for the branching ratios of the dominant non-leptonic two-body decays of B mesons are given in Tables 6 and 7. The QCD coefficients a_1^{eff} and a_2^{eff} (for simplicity called a_1 and a_2 in the tables), as well as the unknown decay constants of charm mesons, have been left as parameters in the expressions for the branching ratios. For comparison, we show the world average experimental results for the branching ratios, as recently compiled in the review article in Ref. 102. They are dominated by the CLEO II measurements. The quoted upper limits for the colour-suppressed $\bar{B}^0$ decay modes also include recent CLEO data reported in Ref. 86. In Tables 8–10, we show for comparison the predictions obtained using the simple form-factor model described in the previous section, including some new decay channels which had not been considered in Ref. 57. In the theoretical expression for the class III decay $B^- \to D^{*0}a_1^-$, the term proportional to a_2^{eff} requires the knowledge of the current matrix element between the B meson and the pseudovector meson a_1. We use the axial current as an interpolating field for the a_1 particle and employ chiral symmetry to relate the corresponding form factors to those of the $B^- \to \varrho^-$ matrix element.

Let us now compare in detail the theoretical predictions with the data. Due to the uncertainty in the values of the phenomenological parameters a_1^{eff} and a_2^{eff}, we first concentrate on ratios of branching fractions in which these coefficients cancel. The comparison of such predictions with data constitutes a test of the factorization hypotheses and of the quality of our form factors. From the class I transitions listed in Tables 6 and 8, we obtain the predictions

$$R_1 = \frac{\text{B}(\bar{B}^0 \to D^+\pi^-)}{\text{B}(\bar{B}^0 \to D^{*+}\pi^-)} \approx 1.04 \quad [1.07],$$

$$R_2 = \frac{\text{B}(\bar{B}^0 \to D^+\varrho^-)}{\text{B}(\bar{B}^0 \to D^{*+}\varrho^-)} \approx 0.88 \quad [0.89]. \tag{57}$$

Here and below we use the NRSX model as our nominal choice, and quote results obtained with the new model of Section 7 in parentheses. In the heavy-quark limit both ratios become equal to unity, because the spin symmetry relates D and D^*. Only part of the deviations from this limit is a form factor effect; the remainder is of kinematic origin. The experimental results for these

Table 6: Branching ratios of non-leptonic $\bar{B}^0$ decays (in %) obtained in the NRSX model[57]. In the third column, the factors containing not yet known decay constants have been suppressed. The last column shows the world average experimental results[102,86]. Upper limits are given at the 90% confidence level

$\bar{B}^0$ Modes	NRSX Model	$a_1^{\mathrm{eff}} = 1.08$ $a_2^{\mathrm{eff}} = 0.21$	Experimental Average
Class I			
$D^+\pi^-$	$0.257\ a_1^2$	0.30	$0.31 \pm 0.04 \pm 0.02$
D^+K^-	$0.020\ a_1^2$	0.02	
D^+D^-	$0.031\ a_1^2\ (f_D/200)^2$	0.04	
$D^+D_s^-$	$0.879\ a_1^2\ (f_{D_s}/240)^2$	1.03	$0.74 \pm 0.22 \pm 0.18$
$D^+\varrho^-$	$0.643\ a_1^2$	0.75	$0.84 \pm 0.16 \pm 0.07$
D^+K^{*-}	$0.035\ a_1^2$	0.04	
D^+D^{*-}	$0.030\ a_1^2\ (f_{D^*}/230)^2$	0.03	
$D^+D_s^{*-}$	$0.817\ a_1^2\ (f_{D_s^*}/275)^2$	0.95	$1.14 \pm 0.42 \pm 0.28$
$D^+a_1^-$	$0.719\ a_1^2$	0.84	
$D^{*+}\pi^-$	$0.247\ a_1^2$	0.29	$0.28 \pm 0.04 \pm 0.01$
$D^{*+}K^-$	$0.019\ a_1^2$	0.02	
$D^{*+}D^-$	$0.022\ a_1^2\ (f_D/200)^2$	0.03	
$D^{*+}D_s^-$	$0.597\ a_1^2\ (f_{D_s}/240)^2$	0.70	$0.94 \pm 0.24 \pm 0.23$
$D^{*+}\varrho^-$	$0.727\ a_1^2$	0.85	$0.73 \pm 0.15 \pm 0.03$
$D^{*+}K^{*-}$	$0.042\ a_1^2$	0.05	
$D^{*+}D^{*-}$	$0.072\ a_1^2\ (f_{D^*}/230)^2$	0.08	
$D^{*+}D_s^{*-}$	$2.097\ a_1^2\ (f_{D_s^*}/275)^2$	2.45	$2.00 \pm 0.54 \pm 0.49$
$D^{*+}a_1^-$	$1.037\ a_1^2$	1.21	$1.27 \pm 0.30 \pm 0.05$
Class II			
$\bar{K}^0 J/\psi$	$2.262\ a_2^2$	0.10	0.075 ± 0.021
$\bar{K}^0 \psi(2\mathrm{S})$	$1.051\ a_2^2$	0.05	< 0.08
$\bar{K}^{*0} J/\psi$	$3.645\ a_2^2$	0.16	0.153 ± 0.028
$\bar{K}^{*0} \psi(2\mathrm{S})$	$1.939\ a_2^2$	0.09	0.151 ± 0.091
$\pi^0 D^0$	$0.164\ a_2^2\ (f_D/200)^2$	0.007	< 0.033
$\pi^0 D^{*0}$	$0.230\ a_2^2\ (f_{D^*}/230)^2$	0.010	< 0.055
$\varrho^0 D^0$	$0.111\ a_2^2\ (f_D/200)^2$	0.005	< 0.055
$\varrho^0 D^{*0}$	$0.240\ a_2^2\ (f_{D^*}/230)^2$	0.011	< 0.117

Table 7: Branching ratios (in %) of non-leptonic B^- decays in the NRSX model

B^- Modes	NRSX Model	$a_1^{\text{eff}} = 1.08$ $a_2^{\text{eff}} = 0.21$	Experimental Average
Class I			
$D^0 D^-$	$0.033\ a_1^2\,(f_D/200)^2$	0.04	
$D^0 D_s^-$	$0.938\ a_1^2\,(f_{D_s}/240)^2$	1.09	$1.36 \pm 0.28 \pm 0.33$
$D^0 D^{*-}$	$0.032\ a_1^2\,(f_{D^*}/230)^2$	0.04	
$D^0 D_s^{*-}$	$0.873\ a_1^2\,(f_{D_s^*}/275)^2$	1.02	$0.94 \pm 0.31 \pm 0.23$
$D^{*0} D^-$	$0.023\ a_1^2\,(f_D/200)^2$	0.03	
$D^{*0} D_s^-$	$0.639\ a_1^2\,(f_{D_s}/240)^2$	0.75	$1.18 \pm 0.36 \pm 0.29$
$D^{*0} D^{*-}$	$0.077\ a_1^2\,(f_{D^*}/230)^2$	0.09	
$D^{*0} D_s^{*-}$	$2.235\ a_1^2\,(f_{D_s^*}/275)^2$	2.61	$2.70 \pm 0.81 \pm 0.66$
Class II			
$K^- J/\psi$	$2.411\ a_2^2$	0.11	0.102 ± 0.014
$K^- \psi(2S)$	$1.122\ a_2^2$	0.05	0.070 ± 0.024
$K^{*-} J/\psi$	$3.886\ a_2^2$	0.17	0.174 ± 0.047
$K^{*-} \psi(2S)$	$2.070\ a_2^2$	0.09	< 0.30
Class III			
$D^0 \pi^-$	$0.274\,[a_1 + 1.127\,a_2\,(f_D/200)]^2$	0.48	$0.50 \pm 0.05 \pm 0.02$
$D^0 \varrho^-$	$0.686\,[a_1 + 0.587\,a_2\,(f_D/200)]^2$	0.99	$1.37 \pm 0.18 \pm 0.05$
$D^{*0} \pi^-$	$0.264\,[a_1 + 1.361\,a_2\,(f_{D^*}/230)]^2$	0.49	$0.52 \pm 0.08 \pm 0.02$
$D^{*0} \varrho^-$	$0.775\,[a_1^2 + 0.661\,a_2^2\,(f_{D^*}/230)^2$ $+\ 1.518\,a_1 a_2\,(f_{D^*}/230)]$	1.19	$1.51 \pm 0.30 \pm 0.02$

Table 8:   Branching ratios (in %) of class I non-leptonic $\bar{B}^0$ decays in the new model described in Section 7

$\bar{B}^0$ Modes	New Model	$a_1^{\text{eff}} = 0.98$ $a_2^{\text{eff}} = 0.29$	Experimental Average
Class I			
$D^+\pi^-$	$0.318\ a_1^2$	0.30	$0.31 \pm 0.04 \pm 0.02$
D^+K^-	$0.025\ a_1^2$	0.02	
D^+D^-	$0.037\ a_1^2\,(f_D/200)^2$	0.03	
$D^+D_s^-$	$1.004\ a_1^2\,(f_{D_s}/240)^2$	0.96	$0.74 \pm 0.22 \pm 0.18$
$D^+\varrho^-$	$0.778\ a_1^2$	0.75	$0.84 \pm 0.16 \pm 0.07$
D^+K^{*-}	$0.041\ a_1^2$	0.04	
D^+D^{*-}	$0.032\ a_1^2\,(f_{D^*}/230)^2$	0.03	
$D^+D_s^{*-}$	$0.830\ a_1^2\,(f_{D_s^*}/275)^2$	0.80	$1.14 \pm 0.42 \pm 0.28$
$D^+a_1^-$	$0.844\ a_1^2$	0.81	
$D^{*+}\pi^-$	$0.296\ a_1^2$	0.28	$0.28 \pm 0.04 \pm 0.01$
$D^{*+}K^-$	$0.022\ a_1^2$	0.02	
$D^{*+}D^-$	$0.023\ a_1^2\,(f_D/200)^2$	0.02	
$D^{*+}D_s^-$	$0.603\ a_1^2\,(f_{D_s}/240)^2$	0.58	$0.94 \pm 0.24 \pm 0.23$
$D^{*+}\varrho^-$	$0.870\ a_1^2$	0.84	$0.73 \pm 0.15 \pm 0.03$
$D^{*+}K^{*-}$	$0.049\ a_1^2$	0.05	
$D^{*+}D^{*-}$	$0.085\ a_1^2\,(f_{D^*}/230)^2$	0.08	
$D^{*+}D_s^{*-}$	$2.414\ a_1^2\,(f_{D_s^*}/275)^2$	2.32	$2.00 \pm 0.54 \pm 0.49$
$D^{*+}a_1^-$	$1.217\ a_1^2$	1.16	$1.27 \pm 0.30 \pm 0.05$
$\pi^+\pi^-$	$50.0\ a_1^2\,|V_{ub}|^2$		
$\pi^+\varrho^- + \varrho^+\pi^-$	$176.9\ a_1^2\,|V_{ub}|^2$		

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Table 9: Branching ratios (in %) of class II non-leptonic $\bar{B}^0$ decays in the new model. We take $\theta = 20°$ for the η–η' mixing angle

$\bar{B}^0$ Modes	New Model	$a_1^{\text{eff}} = 0.98$ $a_2^{\text{eff}} = 0.29$	Experimental Average
Class II			
$\bar{K}^0 J/\psi$	$0.800\ a_2^2$	0.07	0.075 ± 0.021
$\bar{K}^0 \psi(2S)$	$0.326\ a_2^2$	0.03	< 0.08
$\bar{K}^{*0} J/\psi$	$2.518\ a_2^2$	0.21	0.153 ± 0.028
$\bar{K}^{*0} \psi(2S)$	$1.424\ a_2^2$	0.12	0.151 ± 0.091
$\pi^0 J/\psi$	$0.018\ a_2^2$	0.002	< 0.006
$\varrho^0 J/\psi$	$0.050\ a_2^2$	0.004	< 0.025
$\pi^0 D^0$	$0.084\ a_2^2\ (f_D/200)^2$	0.007	< 0.033
$\pi^0 D^{*0}$	$0.116\ a_2^2\ (f_{D^*}/230)^2$	0.010	< 0.055
$\varrho^0 D^0$	$0.078\ a_2^2\ (f_D/200)^2$	0.007	< 0.055
$\varrho^0 D^{*0}$	$0.199\ a_2^2\ (f_{D^*}/230)^2$	0.017	< 0.117
$\omega\, D^0$	$0.081\ a_2^2\ (f_D/200)^2$	0.007	< 0.057
$\omega\, D^{*0}$	$0.203\ a_2^2\ (f_{D^*}/230)^2$	0.017	< 0.120
$\eta\, D^0$	$0.058\ a_2^2\ (f_D/200)^2$	0.005	< 0.033
$\eta\, D^{*0}$	$0.073\ a_2^2\ (f_{D^*}/230)^2$	0.006	< 0.050

Table 10: Branching ratios (in %) of non-leptonic B^- decays in the new model

B^- Modes	New Model	$a_1^{\text{eff}} = 0.98$ $a_2^{\text{eff}} = 0.29$	Experimental Average
Class I			
$D^0 D^-$	$0.039\, a_1^2\, (f_D/200)^2$	0.04	
$D^0 D_s^-$	$1.069\, a_1^2\, (f_{D_s}/240)^2$	1.03	$1.36 \pm 0.28 \pm 0.33$
$D^0 D^{*-}$	$0.034\, a_1^2\, (f_{D^*}/230)^2$	0.03	
$D^0 D_s^{*-}$	$0.883\, a_1^2\, (f_{D_s^*}/275)^2$	0.85	$0.94 \pm 0.31 \pm 0.23$
$D^{*0} D^-$	$0.025\, a_1^2\, (f_D/200)^2$	0.02	
$D^{*0} D_s^-$	$0.642\, a_1^2\, (f_{D_s}/240)^2$	0.62	$1.18 \pm 0.36 \pm 0.29$
$D^{*0} D^{*-}$	$0.091\, a_1^2\, (f_{D^*}/230)^2$	0.09	
$D^{*0} D_s^{*-}$	$2.570\, a_1^2\, (f_{D_s^*}/275)^2$	2.47	$2.70 \pm 0.81 \pm 0.66$
Class II			
$K^- J/\psi$	$0.852\, a_2^2$	0.07	0.102 ± 0.014
$K^- \psi(2S)$	$0.347\, a_2^2$	0.03	0.070 ± 0.024
$K^{*-} J/\psi$	$2.680\, a_2^2$	0.23	0.174 ± 0.047
$K^{*-} \psi(2S)$	$1.516\, a_2^2$	0.13	< 0.30
$\pi^- J/\psi$	$0.038\, a_2^2$	0.003	0.0057 ± 0.0026
$\varrho^- J/\psi$	$0.107\, a_2^2$	0.009	< 0.077
Class III			
$D^0 \pi^-$	$0.338\, [a_1 + 0.729\, a_2\, (f_D/200)]^2$	0.48	$0.50 \pm 0.05 \pm 0.02$
$D^0 \varrho^-$	$0.828\, [a_1 + 0.450\, a_2\, (f_D/200)]^2$	1.02	$1.37 \pm 0.18 \pm 0.05$
$D^0 a_1^-$	$0.898\, [a_1 + 0.317\, a_2\, (f_D/200)]^2$	1.03	
$D^{*0} \pi^-$	$0.315\, [a_1 + 0.886\, a_2\, (f_{D^*}/230)]^2$	0.48	$0.52 \pm 0.08 \pm 0.02$
$D^{*0} \varrho^-$	$0.926\, [a_1^2 + 0.456\, a_2^2\, (f_{D^*}/230)^2 + 1.292\, a_1 a_2\, (f_{D^*}/230)]$	1.26	$1.51 \pm 0.30 \pm 0.02$
$D^{*0} a_1^-$	$1.296\, [a_1^2 + 0.128\, a_2^2\, (f_{D^*}/230)^2 + 0.269\, a_1 a_2\, (f_{D^*}/230)]$	1.36	$1.89 \pm 0.53 \pm 0.08$

ratios are $R_1 = 1.11 \pm 0.23$ and $R_2 = 1.15 \pm 0.34$, in agreement with our predictions, although the errors are still large. We may also consider the corresponding ratios for pairs of class I transitions to final states which differ only in their light meson. For instance, we find

$$R_3 = \left(\frac{f_\varrho}{f_\pi}\right)^2 \frac{\mathrm{B}(\bar{B}^0 \to D^+ \pi^-)}{\mathrm{B}(\bar{B}^0 \to D^+ \varrho^-)} \approx 1.03 \quad [1.06],$$

$$R_4 = \left(\frac{f_\varrho}{f_\pi}\right)^2 \frac{\mathrm{B}(\bar{B}^0 \to D^{*+} \pi^-)}{\mathrm{B}(\bar{B}^0 \to D^{*+} \varrho^-)} \approx 0.88 \quad [0.88], \tag{58}$$

which in the heavy-quark limit become equal unity, too. The experimental values are $R_3 = 0.95 \pm 0.24$ and $R_4 = 0.99 \pm 0.25$, again in agreement with our predictions.

Similar ratios can be taken for class II amplitudes; however, since the relevant matrix elements involve current matrix elements between a heavy and a light meson, the theoretical predictions are considerably more model dependent. Based on the results of our two models, we expect

$$R_5 = \frac{\mathrm{B}(\bar{B} \to \bar{K} \, J/\psi)}{\mathrm{B}(\bar{B} \to \bar{K}^* J/\psi)} \approx 0.62 \quad [0.32],$$

$$R_6 = \frac{\mathrm{B}(\bar{B} \to \bar{K} \, \psi(2\mathrm{S}))}{\mathrm{B}(\bar{B} \to \bar{K}^* \psi(2\mathrm{S}))} \approx 0.54 \quad [0.23]. \tag{59}$$

The strong model dependence does not allow for a test of the factorization hypothesis in this case. The corresponding experimental values are $R_5 = 0.58 \pm 0.11$ and $R_6 = 0.44 \pm 0.31$, where we have averaged the results for $\bar{B}^0$ and B^- decays (accounting for the different lifetimes) when available. Although the experimental errors are sizable, the data seem to prefer the NRSX model.

We now turn to the actual values of the phenomenological parameters a_1^{eff} and a_2^{eff}. From the theoretical point of view, the cleanest determination of a_1^{eff} is from the class of decays $\bar{B}^0 \to D^{(*)+} h^-$, where h^- is a light meson ($h = \pi$, ϱ or a_1). The relevant $\bar{B}^0 \to D^{(*)+}$ transition form factors are known from the analysis of semileptonic B decays using the heavy-quark effective theory, and the decay constants of the light mesons are experimentally known with good accuracy. Also, these transitions have a similar decay kinematics, so that we may expect that they are characterized by the same value of a_1^{eff}. Performing a fit to the experimental data, we obtain[e]

$$a_1^{\mathrm{eff}}|_{Dh} = 1.08 \pm 0.04 \quad [0.98 \pm 0.04]. \tag{60}$$

---

[e]Since we are neglecting final-state interactions, the parameters a_i^{eff} are real numbers, and by convention we take a_1^{eff} to be positive.

The coefficient a_1^{eff} can also be determined from the decays $\bar{B} \to D^{(*)}D_s^{(*)-}$, which are characterized by a quite different decay kinematics. In principle, it would be interesting to investigate whether the resulting value is different in the two cases, i.e. whether there is an observable process dependence of the phenomenological parameter. In practise, this cannot be done because of the large uncertainty in the values of the decay constants of charm mesons. From a fit to the data, we find

$$a_1^{\text{eff}}|_{DD_s} = 1.10 \pm 0.07 \pm 0.17 \quad [1.05 \pm 0.07 \pm 0.16], \tag{61}$$

where the second error accounts for the uncertainty in $f_{D_s^{(*)}}$. In both cases, (60) and (61), the data support the theoretical expectation that a_1^{eff} is close to unity [see (25)].

A value for the parameter $|a_2^{\text{eff}}|$ (but not the relative sign between a_2^{eff} and a_1^{eff}) can be obtained from the class II decays $\bar{B} \to \bar{K}^{(*)}J/\psi$ und $\bar{B} \to \bar{K}^{(*)}\psi(2S)$. From a fit to the six measured branching ratios, we extract

$$|a_2^{\text{eff}}|_{K\psi} = 0.21 \pm 0.01 \quad [0.29 \pm 0.01]. \tag{62}$$

This result is more strongly dependent on the form-factor model used, which is not surprising given that class II decays involve heavy-to-light transition matrix elements. As we have seen above, the NRSX model may be the more trustable one; still, we believe the difference between the two results provides a realistic estimate of the theoretical uncertainty.

A determination of a_2^{eff} from decays with a rather different kinematics is possible by considering the class III transitions $B^- \to D^{(*)0}h^-$ with $h = \pi$ or ϱ. Moreover, because of the interference of a_1 and a_2 amplitudes, these decays are sensitive to the relative sign of the QCD coefficients. From the theoretical point of view, it is of advantage to normalize the branching ratios to those of the corresponding $\bar{B}^0$ decays, which are class I transitions. The theoretical predictions for these ratios are of the form

$$\frac{\text{B}(B^- \to D^{(*)0}h^-)}{\text{B}(\bar{B}^0 \to D^{(*)+}h^-)} = \frac{\tau(B^-)}{\tau(\bar{B}^0)}\left[1 + 2x_1\frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} + x_2^2\left(\frac{a_2^{\text{eff}}}{a_1^{\text{eff}}}\right)^2\right], \tag{63}$$

where x_1 and x_2 are process-dependent parameters depending on the ratio of some hadronic form factors and decay constants ($x_1 = x_2$ except for the decay $B^- \to D^{*0}\varrho^-$). For the ratios of branching fractions on the left-hand side, we use recent CLEO data reported in Ref. 86, which are more accurate than the previous world averages presented in Tables 6–10. Performing a fit to the data, we extract the ratio $a_2^{\text{eff}}/a_1^{\text{eff}}$ for each channel. The results are collected

Table 11: Ratios of non-leptonic decay rates of charged and neutral B mesons [86], and the corresponding values for $a_2^{\mathrm{eff}}/a_1^{\mathrm{eff}}$

Experimental Ratios	Predictions for x_i	$a_2^{\mathrm{eff}}/a_1^{\mathrm{eff}}$
$\dfrac{\mathrm{B}(B^- \to D^0\pi^-)}{\mathrm{B}(\bar{B}^0 \to D^+\pi^-)} = 1.73 \pm 0.25$	1.127 [0.729]	$0.24 \pm 0.08 \pm 0.02$
		$[0.38 \pm 0.13 \pm 0.03]$
$\dfrac{\mathrm{B}(B^- \to D^0\varrho^-)}{\mathrm{B}(\bar{B}^0 \to D^+\varrho^-)} = 1.19 \pm 0.24$	0.587 [0.450]	$0.10 \pm 0.18 \pm 0.03$
		$[0.13 \pm 0.24 \pm 0.04]$
$\dfrac{\mathrm{B}(B^- \to D^{*0}\pi^-)}{\mathrm{B}(\bar{B}^0 \to D^{*+}\pi^-)} = 1.64 \pm 0.28$	1.361 [0.886]	$0.18 \pm 0.08 \pm 0.02$
		$[0.27 \pm 0.12 \pm 0.03]$
$\dfrac{\mathrm{B}(B^- \to D^{*0}\varrho^-)}{\mathrm{B}(\bar{B}^0 \to D^{*+}\varrho^-)} = 1.71 \pm 0.36$	$x_1 = 0.759$ [0.646]	$0.35 \pm 0.17 \pm 0.03$
	$x_2 = 0.813$ [0.675]	$[0.41 \pm 0.20 \pm 0.04]$

in Table 11, where the second error results from the uncertainty in the lifetime ratio [87] $\tau(B^-)/\tau(\bar{B}^0) = 1.06 \pm 0.04$. Taking the average, and using (60), we find

$$\left.\frac{a_2^{\mathrm{eff}}}{a_1^{\mathrm{eff}}}\right|_{Dh} = 0.21 \pm 0.05 \quad [0.31 \pm 0.08]\,,$$

$$a_2^{\mathrm{eff}}|_{Dh} = 0.23 \pm 0.05 \quad [0.30 \pm 0.08]\,. \tag{64}$$

The value of a_2^{eff} is in good agreement with that obtained in (62). Given the different decay kinematics in the two processes, this observation is quite remarkable.

The magnitude and, in particular, the positive sign of a_2^{eff} are of great importance for the theoretical interpretation of our results. We find that in non-leptonic B decays the two parameters a_1^{eff} and a_2^{eff} have the same sign, meaning that the corresponding amplitudes interfere constructively. This finding is in stark contrast to the situation encountered in D-meson decays, where a similar analysis yields [13,57]

$$a_1^{\mathrm{eff}}|_{\mathrm{charm}} = 1.10 \pm 0.05\,, \qquad a_2^{\mathrm{eff}}|_{\mathrm{charm}} = -0.49 \pm 0.04\,, \tag{65}$$

indicating a strong destructive interference. Since most D decays are (quasi)

two-body transitions, this effect is responsible for the observed lifetime difference between D^+ and D^0 mesons [88]: $\tau(D^+)/\tau(D^0) = 2.55 \pm 0.04$. In B decays, on the other hand, the majority of transitions proceeds into multi-body final states, and moreover there are many B^- decays (such involving two charm quarks in the final state) where no interference can occur. The relevant scale for multi-body decay modes may be significantly lower than m_b, leading to destructive interference (see Figure 2). Therefore, the observed constructive interference in the two-body modes is not in conflict with the fact that $\tau(B^-) > \tau(\bar{B}^0)$.

The values for a_2^{eff} extracted from $\bar{B} \to \bar{K}^{(*)}\psi$ and $\bar{B} \to D^{(*)}h$ decays in (62) and (64) indicate that non-factorizable contributions (at the scale $\mu = m_b$) are small in these processes. Using $a_2^{\text{eff}}|_{K\psi} = c_2(m_b) + \zeta_{K\psi}c_1(m_b) = 0.21 \pm 0.05$ and $a_2^{\text{eff}}|_{Dh} = c_2(m_b) + \zeta_{Dh}c_1(m_b) = 0.23 \pm 0.05$ with conservative errors, and combining these with the values of the Wilson coefficients given in Table 1, we find

$$\zeta_{K\psi} = 0.44 \pm 0.05\,, \qquad \varepsilon_8^{(BK,\psi)}(m_b) = 0.11 \pm 0.05\,,$$

$$\zeta_{Dh} = 0.46 \pm 0.05\,, \qquad \varepsilon_8^{(BD,h)}(m_b) = 0.13 \pm 0.05\,. \qquad (66)$$

Hence, within errors there is no experimental evidence for a process dependence of the value of ζ, in accordance with our expectation stated in (27).

9 Tests of Factorization and Extraction of Decay Constants

Based on the factorization hypothesis, we have made in the previous section a number of predictions for the ratios of hadronic decay rates. Within the current experimental uncertainties, these predictions agree well with the available data. In this section, we shall discuss a particularly clean method to test the factorization hypothesis. As suggested by Bjorken [52], we make use of the close relationship between semileptonic and factorized hadronic amplitudes by dividing the non-leptonic decay rates by the corresponding differential semileptonic decay rates evaluated at the same q^2. This method provides a direct test of the factorization hypothesis; moreover, it may be used to determine some interesting decay constants [103,104]. Assuming that factorization holds, we have

$$R_M^{(*)} = \frac{\text{B}(\bar{B}^0 \to D^{(*)+}M^-)}{\text{dB}(\bar{B}^0 \to D^{(*)+}\ell^-\bar{\nu})/\text{d}q^2\Big|_{q^2=m_M^2}} = 6\pi^2 f_M^2\, |a_1^{\text{eff}}|^2\, |V_{ij}|^2\, X_M^{(*)}\,, \quad (67)$$

with $a_1^{\text{eff}} \approx 1$. Here f_M is the decay constant of the meson M, and V_{ij} is the appropriate CKM matrix element (depending on the flavour quantum numbers of the meson M). To determine this ratio experimentally, one needs the

Table 12: Values of the differential semileptonic branching ratio used in the factorization tests, as compiled in Ref. 81

q^2	$\dfrac{\mathrm{dB}(\bar{B} \to D^* \ell \bar{\nu})}{\mathrm{d}q^2}$ $[10^{-2} \text{ GeV}^{-2}]$
m_π^2	0.237 ± 0.026
m_ϱ^2	0.250 ± 0.030
$m_{a_1}^2$	0.335 ± 0.033
$m_{D_s}^2$	0.483 ± 0.033
$m_{D_s^*}^2$	0.507 ± 0.035

values of the differential semileptonic branching ratio at various values of q^2. They have been determined for $\bar{B} \to D^* \ell \bar{\nu}$ decays in Ref. 81, using a fit to experimental data. We collected their results in Table 12.

Neglecting the lepton mass, we obtain for a pseudoscalar meson P:

$$
\begin{aligned}
X_P &= \frac{(m_B^2 - m_D^2)^2}{[m_B^2 - (m_D + m_P)^2]\,[m_B^2 - (m_D - m_P)^2]} \left| \frac{F_0(m_P^2)}{F_1(m_P^2)} \right|^2 , \\
X_P^* &= \left[m_B^2 - (m_{D^*} + m_P)^2 \right] \left[m_B^2 - (m_{D^*} - m_P)^2 \right] \\
&\quad \times \frac{|A_0(m_P^2)|^2}{m_P^2 \sum_{i=0,\pm} |H_i(m_P^2)|^2} .
\end{aligned}
\tag{68}
$$

The helicity amplitudes $H_0(q^2)$ and $H_\pm(q^2)$ are defined in the Appendix. For the special case that the pseudoscalar meson is a pion, it is an excellent approximation to expand the quantities X_π and X_π^* in powers of m_π^2/m_B^2, yielding [57]

$$
\begin{aligned}
X_\pi &\simeq 1 + \frac{4m_\pi^2 m_B m_D}{(m_B^2 - m_D^2)^2} \approx 1.001 , \\
X_\pi^* &\simeq 1 + \frac{4m_\pi^2 m_B m_{D^*}}{(m_B^2 - m_{D^*}^2)^2} - \frac{4m_\pi^2}{(m_B - m_{D^*})^2} \approx 0.994 .
\end{aligned}
\tag{69}
$$

Neglecting the tiny deviation from unity, we obtain

$$
R_\pi = R_\pi^* = 6\pi^2 f_\pi^2 \, |a_1^{\mathrm{eff}}|^2 \, |V_{ud}|^2 \approx |a_1^{\mathrm{eff}}|^2 \times 0.96 \text{ GeV}^2 .
\tag{70}
$$

This prediction may be compared with the experimental value $R_\pi^* = 1.18 \pm 0.22$ GeV^2 obtained by combining the $\bar{B}^0 \to D^{*+}\pi^-$ branching ratio from

Table 6 with the value for the differential semileptonic branching ratio at $q^2 = m_\pi^2$ given in Table 12. This yields $a_1^{\text{eff}} = 1.11 \pm 0.10$, in good agreement with the expectation based on factorization. An even cleaner test of factorization is obtained when (67) is evaluated for a vector or pseudovector meson, in which case one has exactly [105]

$$X_V = X_V^* = 1\,. \tag{71}$$

Since the lepton pair created by the $(V - A)$ current carries spin one, its production is kinematically equivalent to that of a (pseudo-) vector particle with four-momentum q_μ. For a ϱ meson in the final state, for instance, we thus obtain

$$R_\varrho = R_\varrho^* = 6\pi^2 f_\varrho^2 \, |a_1^{\text{eff}}|^2 \, |V_{ud}|^2 \approx |a_1^{\text{eff}}|^2 \times 2.48 \text{ GeV}^2\,, \tag{72}$$

to be compared with the experimental value $R_\varrho^* = 2.92 \pm 0.71$ GeV^2. This gives $a_1^{\text{eff}} = 1.09 \pm 0.13$, again in good agreement with the expectation based on factorization. In principle, eqs. (70) and (72) offer the possibility for four independent determinations of the QCD parameter a_1^{eff}. Good agreement among the extracted values supports the validity of the factorization approximation in B decays. Already at the present level of accuracy, it shows that there is little room for final-state interactions affecting the magnitude of the considered decay amplitudes.

From the kinematic argument about the equivalence of the lepton pair in the semileptonic decay and the spin-1 meson in the hadronic decay, it follows that (67) is valid separately for longitudinal and transverse polarization of the D^* meson in the final state. Thus, the polarization of the D^* meson produced in the non-leptonic decay $\bar{B}^0 \to D^{*+}V^-$ should be equal to the polarization in the corresponding semileptonic decay $\bar{B} \to D^*\ell\bar{\nu}$ at $q^2 = m_V^2$. However, in order to turn this prediction into a test of the factorization hypothesis one would have to determine the polarization of the D^* meson with high precision. This is so because in the semileptonic as well as in the non-leptonic case the D^* polarization at the points $q^2 = 0$ and $q^2 = q_{\text{max}}^2$ is determined by kinematics alone to be 100% longitudinal and 1/3 longitudinal, respectively. This shows that for a stringent test of the factorization hypothesis at small q^2 one must determine the transverse polarization contribution with a small relative error. Especially for the semileptonic decays, a precision measurement of the q^2 dependence of the polarization appears to be a complicated task, however. Still, we can make a rather precise prediction for this quantity using heavy-quark symmetry. In the heavy-quark limit, the ratio of transverse to longitudinal

Table 13: Theoretical predictions for the ratio $\Gamma_{\rm T}/\Gamma_{\rm tot}$ at fixed q^2, where $\Gamma_{\rm tot} = \Gamma_{\rm T} + \Gamma_{\rm L}$

q^2	0	m_ϱ^2	$m_{a_1}^2$	$m_{D_s^*}^2$	$q_{\rm max}^2$
$\Gamma_{\rm T}/\Gamma_{\rm tot}$	0	12 ± 1	26 ± 2	48 ± 1	$2/3$

polarization at some fixed q^2 is simply given by

$$\frac{\Gamma_{\rm T}}{\Gamma_{\rm L}} = \frac{4q^2(m_B^2 + m_{D^*}^2 - q^2)}{(m_B - m_{D^*})^2[(m_B + m_{D^*})^2 - q^2]} \,. \tag{73}$$

Including the leading symmetry-breaking corrections to this result [106−108], one obtains the numbers shown in Table 13. For the polarization of the D^* meson in the decay $\bar{B}^0 \to D^{*+}\varrho^-$, the CLEO Collaboration finds [81] $\Gamma_{\rm T}/\Gamma_{\rm tot} = (7 \pm 5 \pm 5)\%$, in agreement with our prediction of 12% transverse polarization for the semileptonic decay at $q^2 = m_\varrho^2$. However, in order for this test to be sensitive to deviations from factorization, the experimental uncertainty will have to be reduced substantially . The situation may be more favourable in the decay $\bar{B}^0 \to D^{*+}D_s^*$ with predicted 48% of transverse polarization, hopefully allowing for a measurement with smaller relative uncertainties.

We shall now discuss an alternative use of (67). Assuming the validity of the factorization hypothesis with a fixed value for $a_1^{\rm eff}$, one may employ this relation for the determination of unknown decay constants [103,104]. In particular, we can use it to determine the decay constants of the D_s and D_s^* mesons. As explained above, in the latter case we simply have $X_{D^*} = X_{D_s^*}^* = 1$. But also in the case of D_s mesons we obtain essentially model-independent predictions. Defining the mass ratio $x = \frac{1}{2}(m_{D^{(*)}} + m_{D_s})/m_B$, we find in the heavy-quark limit [57] ($x \approx 0.36$ and $x \approx 0.38$, respectively)

$$X_{D_s} = \frac{1 - 3x^2 + 2x^3}{1 - 3x^2 - 2x^3} \approx 1.36 \,,$$

$$X_{D_s}^* = \frac{1 - x - 2x^2}{1 - x + 2x^2} \approx 0.37 \,. \tag{74}$$

These values are close to the predictions of the NRSX model, which are

$$X_{D_s} \approx 1.33 \,, \qquad X_{D_s}^* \approx 0.39 \,. \tag{75}$$

Using these values (allowing for a theoretical error of ± 0.03) together with $|a_1^{\rm eff}| = 1.08 \pm 0.04$, we obtain the theoretical predictions:

$$R_{D_s} = (87.2 \pm 6.7)\, f_{D_s}^2 \,,$$

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$$R^*_{D_s} = (25.6 \pm 2.7)\, f^2_{D_s}\,,$$

$$R_{D^*_s} = R^*_{D^*_s} = (65.6 \pm 4.8)\, f^2_{D^*_s}\,. \tag{76}$$

By averaging the experimental data on $\bar{B}^0$ and B^- decays into two charm mesons (taking into account the differences in the B-meson lifetimes), we obtain

$$\mathrm{B}(\bar{B} \to DD^-_s) = (0.95 \pm 0.24)\%\,,$$

$$\mathrm{B}(\bar{B} \to DD^{*-}_s) = (1.00 \pm 0.30)\%\,,$$

$$\mathrm{B}(\bar{B} \to D^*D^-_s) = (1.03 \pm 0.27)\%\,,$$

$$\mathrm{B}(\bar{B} \to D^*D^{*-}_s) = (2.26 \pm 0.60)\%\,. \tag{77}$$

Using the last two averaged decay rates gives the experimental ratios

$$R^*_{D_s} = (2.13 \pm 0.58)\ \mathrm{GeV}^2\,, \qquad R^*_{D^*_s} = (4.46 \pm 1.22)\ \mathrm{GeV}^2\,. \tag{78}$$

Comparing this with the theoretical predictions in (76), we find

$$f_{D_s} = (288 \pm 42)\ \mathrm{MeV}\,, \qquad f_{D^*_s} = (261 \pm 36)\ \mathrm{MeV}\,. \tag{79}$$

More precise determinations of the decay constants are possible if, instead of using (67), we consider ratios of non-leptonic decay rates, comparing processes involving D_s and D^*_s mesons with those involving the light mesons π and ϱ. These processes involve a similar kinematics, so that the ratios of the corresponding decay rates are sensitive to the same form factors, however evaluated at different q^2 values. This method has the advantage that in the ratios the phenomenological parameter a_1^{eff} cancels; similarly, we may hope that some of the experimental systematic errors cancel. Using the NRSX model, we find

$$\frac{\mathrm{B}(\bar{B}^0 \to D^+ D^-_s)}{\mathrm{B}(\bar{B}^0 \to D^+ \pi^-)} = 1.01 \left(\frac{f_{D_s}}{f_\pi}\right)^2\,,$$

$$\frac{\mathrm{B}(\bar{B}^0 \to D^{*+} D^-_s)}{\mathrm{B}(\bar{B}^0 \to D^{*+} \pi^-)} = 0.72 \left(\frac{f_{D_s}}{f_\pi}\right)^2\,,$$

$$\frac{\mathrm{B}(\bar{B}^0 \to D^+ D^{*-}_s)}{\mathrm{B}(\bar{B}^0 \to D^+ \varrho^-)} = 0.74 \left(\frac{f_{D^*_s}}{f_\varrho}\right)^2\,,$$

$$\frac{\mathrm{B}(\bar{B}^0 \to D^{*+} D^{*-}_s)}{\mathrm{B}(\bar{B}^0 \to D^{*+} \varrho^-)} = 1.68 \left(\frac{f_{D^*_s}}{f_\varrho}\right)^2\,. \tag{80}$$

Combining these predictions with the average experimental branching ratios in (77), we find the rather accurate values

$$f_{D_s} = (234 \pm 25) \text{ MeV}, \qquad f_{D_s^*} = (271 \pm 33) \text{ MeV}. \qquad (81)$$

The result for f_{D_s} is in excellent agreement with the value $f_{D_s} = 241 \pm 37$ MeV in (42), extracted from leptonic decays of D_s mesons. The ratio $f_{D_s^*}/f_{D_s} = 1.16 \pm 0.19$, which cannot be determined from leptonic decays, is in good agreement with theoretical expectations [109,110].

Along these lines, there are numerous other possibilities for extracting information on decay constants and current matrix elements. In particular, the decay constants of P-wave particles like a_0, a_1, K_0^* and K_1 are of interest due to their sensitivity to relativistic quark motion, which allows for a test of hadron models. For instance, one may use the ratio

$$\frac{\text{B}(\bar{B}^0 \rightarrow D^{*+} a_1^-)}{\text{B}(\bar{B}^0 \rightarrow D^{*+} \varrho^-)} \approx 1.18 \left(\frac{f_{a_1}}{f_\varrho} \right)^2 \qquad (82)$$

to determine the pseudovector meson decay constant f_{a_1}, which is defined by $\langle 0 \,|\, \bar{q}\gamma_\mu\gamma_5 q \,|a_1\rangle = \epsilon_\mu m_{a_1} f_{a_1}$. From the experimental results, we obtain

$$f_{a_1} = (1.22 \pm 0.19) f_\varrho = (256 \pm 40) \text{ MeV}, \qquad (83)$$

which agrees with the large value derived from τ decays shown in Table 4. The fact that the decay constant of a P-wave meson is of the same order of magnitude as those of the corresponding S-wave mesons is quite remarkable. It shows that mesons containing light quarks are highly relativistic systems; for non-relativistic constituent quarks one would expect the wave function of a state with non-zero orbital momentum to vanish at the origin, leading to a vanishing decay constant.

10 B Decays to Baryons

We end our discussion of non-leptonic processes with a remark on the particularly interesting case of B decays into baryon-antibaryon pairs. For these transitions it is probably not sufficient to apply factorization in the way described so far. The relevant flavour flow diagram (for a $b \rightarrow c$ transition) is shown in Figure 3. In contrast with the situation realized in B decays into mesons, the c and d quarks produced in the weak interaction now end up in the same hadron. It is, therefore, appropriate to rewrite the effective weak
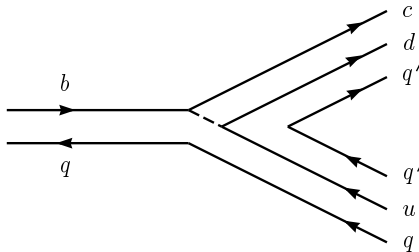
Figure 3: Flavour flow diagram for a B decay into a baryon-antibaryon pair.

Hamiltonian in a different form using charge-conjugate fields and a Fierz re-ordering[111]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} \left[c_-(\mu) \, (cd')^\dagger_{3^*} (ub)_{3^*} + c_+(\mu) \, (cd')^\dagger_6 (ub)_6 \right] + \text{h.c.}$$

$$+ \text{penguin operators}, \tag{84}$$

where $(cd')_{3^*} = \varepsilon_{kij} \, \bar{c}^c_i \, (1 - \gamma_5) \, d'_j$ is an $(S - P)$ colour-antitriplet diquark current. The colour-sextet current $(cd')_6$ is given by a similar expression. The appearance of the coefficients $c_-(\mu)$ and $c_+(\mu)$ shows that perturbative QCD interactions affect the colour-antitriplet and sextet channels differently: the QCD force is attractive $(c_- > 1)$ in the antitriplet and repulsive $(c_+ < 1)$ in the sextet channel.

From the very existence of baryons it is evident that, besides the perturbative enhancement, there must also be a strong long-distance binding force between two quarks in a colour-antitriplet state. This can be taken into account by introducing diquark fields and their couplings to the local currents occurring in (84). QCD sum rules predict that these diquark couplings are large[112]. This result has been confirmed by the magnitude of $\Delta S = 1$ transitions, where diquark effects play a dominant role[51]. Thus, we expect B decays to baryon-antibaryon pairs to proceed predominantly via the formation of a diquark-antidiquark state, followed by the creation of a quark-antiquark pair in the colour field of these diquarks. This decay mechanism implies interesting selection rules, which allow for a number of predictions[113,114]. For instance, the charm baryon in baryonic $b \to c$ transitions is built from (cd) spin-zero states. Thus, in $\bar{B}$ decays one should not find a (cuu) nor a (cus) nor a (css) baryon, as long as final-state interactions may be neglected. In $b \to u$ transitions one naturally obtains a $\Delta I = \frac{1}{2}$ selection rule, because a spin-zero (ud) diquark necessarily has isospin zero. No Δ resonances should therefore be produced in

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these decays, but there is no such suppression of $\bar{\Delta}$ production. A comparison of these predictions with experimental data, once these will become available, should provide information about the creation process of quark-antiquark pairs inside hadrons. For an estimate of the expected branching ratios, the reader is referred to Ref. 114.

11 Summary

We have presented an overview of the theory and phenomenology of exclusive hadronic decays of B mesons, concentrating on two-body modes. Such decays are strongly influenced by the long-range QCD colour forces. Theoretically, their description involves hadronic matrix elements of local four-quark operators, which are notoriously difficult to calculate. The factorization approximation is used to relate these matrix elements to products of current matrix elements. Conventionally, the factorized decay amplitudes depend on two phenomenological parameters a_1 and a_2, which are connected with the Wilson coefficients $c_i(\mu)$ appearing in the effective weak Hamiltonian. We have shown that (except for decays into two vector mesons) this approach can be generalized in a natural way to include the dominant non-factorizable contributions to the decay amplitudes. In the generalized factorization scheme, the effective parameters $a_1^{\rm eff}$ and $a_2^{\rm eff}$ become process-dependent. However, using the large-N_c counting rules of QCD we have argued that in energetic two-body decays of B mesons $a_1^{\rm eff} \approx 1$ and $a_2^{\rm eff} \approx c_2(m_b) + \zeta\, c_1(m_b)$, where $\zeta = O(1/N_c)$ is a dynamical parameter. Moreover, we have shown that the process dependence of ζ is likely to be very mild, so that it can be taken to be a constant for a wide class of two-body decays. These theoretical expectations are supported by the data. From a fit to the world average branching ratios of two-body decay modes, we obtain $a_1^{\rm eff} \approx 1.08$ and $a_2^{\rm eff} \approx 0.21$, corresponding to $\zeta \approx 0.45$. There is no evidence for a process dependence of these parameters; in particular, the values obtained for $a_2^{\rm eff}$ from the decays $\bar{B} \to \bar{K}^{(*)}\psi$ and $B^- \to D^{(*)0}h^-$, where $h = \pi$ or ϱ, are in good agreement with each other.

The obvious interpretation of the fact that the ratio $a_2^{\rm eff}/a_1^{\rm eff}$ is positive and the value of the parameter ζ close to the "naive" factorization prediction $\zeta = 1/3$ is that, in energetic two-body decays of B mesons, a fast moving colour-singlet quark pair interacts little with soft gluons. Hence, factorization works at a high scale of order m_b. The situation is different from that encountered in the much less energetic decays of D mesons, where one finds a negative value of $a_2^{\rm eff}/a_1^{\rm eff}$, corresponding to $\zeta \approx 0$. Since the energy release in charm decays is much less, even soft gluons can rearrange the quarks, and the effective factorization scale is lower. According to the relation between the factorization

scale and the ratio of the phenomenological parameters exhibited in Figure 2, this leads to a negative value of $a_2^{\mathrm{eff}}/a_1^{\mathrm{eff}}$ and thus a smaller value of ζ.

The most important ingredient of factorized decay amplitudes are the hadronic form factors parametrizing the hadronic matrix elements of quark currents. In the case of the heavy-to-heavy transitions $\bar{B} \to D$ and $\bar{B} \to D^*$, heavy-quark symmetry implies simple relations between the various form factors. Incorporating the leading symmetry-breaking corrections using the heavy-quark effective theory, it has become possible to extract all $\bar{B} \to D^{(*)}$ form factors from semileptonic decay data with good precision. The use of heavy-quark symmetry constitutes a significant improvement over earlier estimates of non-leptonic amplitudes, which were based on model calculations of the relevant form factors. As a consequence, for all class I decays considered in this article the factorized decay amplitudes can be predicted without any model assumptions.

There has not been similar progress in the calculation of current matrix elements between heavy and light mesons. For these transitions we must still rely on phenomenological models. Consequently, the theoretical predictions for class II decay amplitudes involve larger theoretical uncertainties. In order to get an idea about the amount of model dependence, we have considered two different quark models and compared their results. We find that it is possible to determine the parameter a_2^{eff} with a theoretical accuracy of about 25%. In the context of each model, a large set of class II branching ratios can be reproduced within the experimental errors using a fixed value of a_2^{eff}.

We have discussed various tests of the (generalized) factorization hypothesis by considering ratios of decay rates, and by comparing non-leptonic decay rates with semileptonic rates evaluated at the same value of q^2. Within the present experimental uncertainties, there are no indications for any deviations from the factorization scheme in which a_1^{eff} and a_2^{eff} are treated as process-independent hadronic parameters. Accepting that this scheme provides a useful phenomenological concept, exclusive two-body decays of B mesons offer a unique opportunity to measure the decay constants of some light or charm mesons, such as the a_1, D_s and D_s^*. In particular, on the basis of the experimental data available today, we find $f_{D_s} = (234 \pm 25)$ MeV and $f_{D_s^*} = (271 \pm 33)$ MeV. The result for f_{D_s} is in excellent agreement with the value extracted from leptonic decays of D_s mesons. The ratio $f_{D_s^*}/f_{D_s} = 1.16 \pm 0.19$ is in good agreement with theoretical expectations.

Acknowledgments

Appendix

We collect our definitions for the weak decay form factors, which parametrize the hadronic matrix elements matrix elements of flavour-changing vector and axial currents between meson states. For the transition between two pseudoscalar mesons, $P_1(p) \to P_2(p')$, we define

$$\langle P_2(p')| V_\mu |P_1(p)\rangle = \left((p+p')_\mu - \frac{m_1^2 - m_2^2}{q^2}\, q_\mu\right) F_1(q^2) + \frac{m_1^2 - m_2^2}{q^2}\, q_\mu\, F_0(q^2)\,,$$

(A.1)

where $q_\mu = (p - p')_\mu$ is the momentum transfer. For the transition of a pseudoscalar into a vector meson, $P_1(p) \to V_2(\epsilon, p')$, we define

$$\langle V_2(\epsilon, p')| V_\mu |M_1(p)\rangle = \frac{2i}{m_1 + m_2}\, \epsilon_{\mu\nu\alpha\beta}\, \epsilon^{*\nu} p'^\alpha p^\beta\, V(q^2)\,,$$

$$\langle V_2(\epsilon, p')| A_\mu |M_1(p)\rangle = \left[(m_1 + m_2)\, \epsilon_\mu^*\, A_1(q^2) - \frac{\epsilon^* \cdot q}{m_1 + m_2}\, (p+p')_\mu\, A_2(q^2)\right.$$

$$\left. - 2m_2\, \frac{\epsilon^* \cdot q}{q^2}\, q_\mu\, A_3(q^2)\right] + 2m_2\, \frac{\epsilon^* \cdot q}{q^2}\, q_\mu\, A_0(q^2)\,,$$

(A.2)

where ϵ_μ is the polarization vector, satisfying $\epsilon \cdot p' = 0$. Here, the form factor $A_3(q^2)$ is given by the linear combination

$$A_3(q^2) = \frac{m_1 + m_2}{2m_2}\, A_1(q^2) - \frac{m_1 - m_2}{2m_2}\, A_2(q^2)\,. \qquad (A.3)$$

Moreover, in order for the poles at $q^2 = 0$ to cancel, we must impose the conditions

$$F_1(0) = F_0(0)\,, \qquad A_3(0) = A_0(0)\,. \qquad (A.4)$$

The helicity amplitudes $H_0(q^2)$ and $H_\pm(q^2)$ are given by the following combinations of form factors [115] (K denotes the momentum of the daughter meson in the parent rest frame):

$$H_\pm(q^2) = (m_1 + m_2)\, A_1(q^2) \mp \frac{m_1 K}{m_1 + m_2}\, V(q^2)\,,$$

$$H_0(q^2) = \frac{1}{2m_1\sqrt{q^2}}\left[(m_1^2 - m_2^2 - q^2)(m_1 + m_2)\, A_1(q^2) \right.$$
$$\left. - \frac{4m_1^2 K^2}{m_1 + m_2}\, A_2(q^2) \right]. \tag{A.5}$$

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