

Black Holes: A Window into A New Theory of Space Time

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Abstract

S. Chandrasekhar wrote in the prologue to his book on black holes, "The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time." In this contribution I briefly discuss recent developments in fundamental theory and black holes that vindicate this statement in a modern perspective. I also include some of my reminiscences of Chandra.

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1 Reminiscences of S. Chandrasekhar

My first reaction, when I was asked to give this talk to commemorate Subrahmaniyan Chandrasekhar, was one of hesitation. Chandra, as he was known to us at the University of Chicago, was a legendary figure during his lifetime and one of the most distinguished astrophysicists of the 20th century. His career spanned over 60 productive years and given the volume, range and extraordinary scholarship of his work he was an institution in himself. It is difficult to imagine the extent of his achievement, especially within the exacting standards he set for himself. A lot has been written about Chandra and Kameshwar Wali has written a very readable biography of Chandra. Here I would like to add a few more strokes to the portrait of a man who was mentally a very youthful and intense person even though he often compared himself to Coleridge's 'Ancient Mariner'. I will take this opportunity to narrate my first and my last meetings with him.

When I reached the University of Chicago in the fall of 1978, I wanted to learn about Einstein's General Theory of Relativity because that was a missing component of my education in physics. Fortunately Chandra was offering a course on the subject during the semester and that was an excellent opportunity. So I mentioned my interest to him. Only later I found out that the classes were at 8.15 am in the morning and almost impossible for me to attend. When I did not show up for the first class he was very upset. Later that day he saw me as I was riding in a crowded elevator. The elevator was so crowded that he could not come in as the door opened. However in those brief moments when the door remained open he saw me and pointing his umbrella at me said, "You did not come because you are interested in fashionable physics". The elevator door had closed by then and I walked to Eckart hall for the weekly colloquium. During the colloquium tea (which is held before the colloquium) I went up to him and tried to explain why I could not come to his lecture and that annoyed him even more. He walked away then reappeared and this is what he said, "You must have heard of Johann Sebastian Bach. He used to wake up early every morning and travel 16 miles to play the organ. But then neither you nor I are Bach", and then he again walked away. That was my first encounter with Chandra and my first glimpse of his sense of discipline and humility. As time went by we developed a friendly relationship, but he always looked upon me as a fashionable high energy physicist.

My last meeting with Chandra was in 1991 when I visited the University of Chicago. There was the usual appointment with his secretary, and at the appointed hour he was waiting in his office. His mannerism had a certain invariance about it. So did the arrangement in his office: the portrait of Sir Isaac on the right hand wall, the bust of Beethoven on the shelf behind. Books, volumes of the astrophysical journal, manuscripts in calligraphic handwriting, were all in perfect arrangement. There was a perfection and order in that office that I have never seen anywhere else. He began by asking about how I was doing and about the Tata institute and the Institute for Advanced Study where I was spending the year. I wanted to be over with all this as I was keen to show him a copy of a recent work I had done on a subject that he was very interested in: black holes. He took the paper, first turned to the references and asked, "Have you quoted my book?" I said no. "Then why do you expect me to read your paper". I was surprised at his response. I still wonder why it mattered so

much to him that we did not refer to his book. He then quickly flipped through the pages and said, "There cannot be a black hole in 2-dims.". When I told him about string theory and the dilaton that makes the solution possible he responded, "You are throwing new words at me".

There was a silence afterwards and then he started speaking again, softly and kindly reminiscing in a different vein, "You know Sir Isaac never felt threatened by any of his contemporaries". The rest of what he said I do not remember accurately, but it was to the effect that Newton was such a towering genius that none of his contemporaries bothered him. He then mentioned how he has been working out the proofs of the propositions in the Principia and how Newton's proof was always better than the one he would construct. I realised that Chandra was focused on Newton during that time.

Since my visits to the University of Chicago were decreasing in frequency, I had carried a camera with me to take some pictures of Chandra. I wanted to photograph him with the portrait of Newton in the background. To my surprise Chandra took down the portrait and placed it over the volumes of Newton's collected works that were on his side table. Then he started hesitating to stand besides the portrait of Newton! He felt so humble doing that. So I first took a picture of Newton and then finally a picture of Chandra besides the portrait of Newton. An hour had gone by and I had to leave for another engagement. That was the last time I saw Chandra.

When I went back to Princeton, I came across V.I. Arnold's book, 'Newton and Hooke, Barrow and Huygens'. It was shocking to read about Newton's feelings towards Hooke, to the extent that when Newton became the President of the Royal Society, he destroyed every known portrait of Hooke. I was bothered by this and wrote to Chandra. I never had another opportunity to find out how he felt about this.

This last encounter with Chandra was a revelation for me and it confirmed my feeling that Chandra had felt for many years that the scientific community was not responding to him in the way he had wished. That the changing fashions of science had left him lonesome. There was little room for a man of his perfection. Besides this, it also revealed to me that he had a fighting spirit till the very end and that he too had his share of the common feelings that in part shape the lives of most working scientists. It is this touch of commonality that I find missing in the portraits of Chandrasekhar in the literature.

2 Black Holes and the Information Paradox

In the following I would like to give a glimpse of some of the developments in fundamental theory and the subject of black holes. I had described the information paradox and related issues during my talk but cannot resist from including more recent developments in this contribution.

What are black holes?

Black holes are objects which divide space time into 2 regions which are distinguished by a surface called the horizon. If there is a light source inside the black hole then the light rays will be bent inwards at the horizon and will never be seen from the outside. If one shines

light at the black hole from the outside it will seem to disappear at the horizon and that is why the term black hole. According to the General Theory of Relativity black holes are likely to be the end point of the gravitational collapse of massive stars and there is mounting evidence that such objects do exist in the universe. There are observations of black holes in binary systems and also as Active Galactic Nuclei (AGN).

Besides being exotic objects which may hold a key to many unanswered questions about the observed universe, black holes are of great interest to theoretical physicists. When one quantizes matter that interacts with a black hole according to the usual laws of quantum mechanics one arrives at the conclusion, due to Hawking, that a black hole is actually a black body whose temperature is of pure quantum mechanical origin, $T = \hbar\kappa/2\pi$, where κ is the surface gravity of the black hole. The black hole is in thermal contact with the in-falling matter and there is a flux of Hawking radiation characterised by the above temperature. Hence, in contrast to the classical theory where in-falling matter disappears into a black hole, the quantum mechanical wave function of the in-falling matter extends 'across' the horizon. Further, the black hole is endowed with an entropy defined by the well known formula of Hawking and Bekenstein, $S = A/4G\hbar$, where A is the area of the horizon of the black hole and G is Newton's constant. The black hole behaves like a macroscopic object obeying the laws of thermodynamics.

For a black hole of a few solar masses, the temperature $T \sim 10^{-8}$ degrees Kelvin and is obviously too small to be of any practical significance. However black hole thermodynamics raises very basic questions about the applications of quantum mechanics to black holes. Let me explain the issue. Since black holes can emit Hawking radiation, they can evaporate by losing their mass, and in case they evaporate completely, the final state of the system would be simply thermal radiation. Now this is problematic, because presumably the black hole was formed by collapsing matter which was initially described by a quantum mechanical wave function. Such a wave function would describe many degrees of freedom, but the phase correlations between the degrees of freedom would evolve in a unitary way, according to the standard laws of quantum mechanics. In case the black hole evaporates completely, and the final state is purely thermal radiation then one is saying that the phase correlations are even in principle completely lost. This is clearly a violation of unitarity as we understand it in the standard formulation of quantum mechanics. It is called the 'information paradox' of black hole physics.

Closely related to the above paradox is the question of the statistical basis of the black hole entropy in accordance with Boltzmann's formula $S = k \ln(\Omega)$, where Ω is the number of microscopic states that constitute the black hole.

There is one more aspect of black holes that is quite disturbing. The solutions that describe black holes have a curvature singularity, where the standard laws of physics break down. The discussion of black hole thermodynamics is not affected by the singularity because in most cases the singularity is within the horizon (cosmic censorship!). However the question of singularities has to be addressed in a fundamental theory.

From the above it is quite clear that the application of quantum mechanics to black holes reveals a possible inadequacy of either our current notions of space time or of the standard laws of quantum mechanics. Many turning points in the history of science abound in similar

circumstances when two or more accepted theories lead to contradictory answers for certain phenomena. Both special relativity and quantum mechanics grew out of the paradoxes that arose in the applications of classical mechanics and classical electrodynamics to phenomena like the propagation of light, black body radiation and the stability of atoms. As we all know it was classical mechanics that was replaced by quantum mechanics as the basic theory and classical mechanics came to be understood as a limiting case of the former. The measure of the deviation was provided by Planck's constant.

We see a similar situation in the application of quantum mechanics and general relativity to black hole physics. Hawking's view is that quantum mechanics needs modification in the presence of black holes. A different view has been advocated by 't Hooft, Susskind and many other high energy physicists, who would like to retain the unitary formulation of quantum mechanics and provide a standard statistical explanation of Hawking radiation in an S-matrix framework. Recent developments seem to indicate that the unitary formulation of quantum mechanics is retained and it is general relativity that is likely to be replaced as a fundamental theory of space time. The theory that replaces general relativity is called M-theory. It a theory that is presently being constructed and whose perturbative corners are described by the 5 consistent superstring theories in 10-dim. Of course there is one essential difference between today's efforts and the revolutions of special relativity and quantum mechanics in that the latter were relatively quickly confirmed by experiment. Perhaps in this respect M-theory may eventually have a historical parallel with general relativity itself. Even though M-theory is not in principle inconsistent with any known experimental fact, there is presently no prediction of M-theory that we can directly test experimentally. This is due to the fact that the theory is not yet completely formulated and also to the fact that perturbation theory does not seem adequate to make contact with the phenomenology of elementary particles. This difficulty of our situation makes the information paradox especially important to tackle because it is more an issue of the logical consistency of a framework rather than the agreement of predictions with experiment. If M-theory can resolve the information paradox then it would be very encouraging for our theoretical efforts. In what follows we shall see that there is good evidence that M-theory is indeed poised to resolve the information paradox.

3 The Setting: M-theory

M-theory resides in 11 space time dimensions, and its low energy limit is described by 11 dim. supergravity theory. Its relation to the type 2A superstring theory in 10 dim. involves a compactification of the 11th dim. to a circle of radius $r(\lambda) = \lambda^{2/3}$, where λ is the coupling constant of 2A theory. The type 2A theory is one of the 5 consistent string theories in 10 dim. These theories represent different corners of a large phase diagram which is supposed to describe M-theory. One can device precise connections between these string theories using duality transformations. An essential role in this is played by certain elementary solitons of the theory called D-branes. D-branes are domain walls of dimensions ranging from 0, ..., 9. A 0-brane is a point like object, a 1-brane is a string like object, a 2-brane is a membrane and so on. These domain walls are precisely defined by the fact that open strings end on

them and the end points of the open string can move freely inside the D-brane. Hence the name D-brane, "D" stands for the Dirichlet boundary conditions satisfied by the open strings in directions transverse to the brane. In this sense they are very much like the quarks in gauge theories. They also carry integer units of charge (called the Ramond charge) which can be measured by a tensor gauge field associated with the brane. These solitons have a further remarkable property which says that their mass and charge satisfy a precise relation, $M = Q/\lambda$, and that this important relation is exactly true in the quantum theory due to supersymmetry. One non-perturbative formulation of M-theory, called M(atrrix) theory in fact formulates the fundamental theory entirely in terms of the open strings that connect the simplest branes, namely the "0-branes". This is an expression of reductionism taken to its extreme. I will not go into all these exciting developments here but suffice it to say that in the setting that we have sketched above it is indeed possible to give a 'constituent' model of a black hole which can give a standard statistical basis to black hole thermodynamics in a unitary theory.

4 D-brane Constituent Model of a Black Hole

Let me now explain a specific constituent model, that describes the 5-dim. black hole of type 2B string theory. This string theory is a close relative of the 2A theory and is related to it by a duality transformation. If one compactifies this 10-dim. theory on a 5-dim. torus, the long wavelength limit is described by a certain supergravity theory, which has black hole solutions. These solutions are characterized by various integer charges. Among these the stable black holes (those which satisfy the Bogomol'nyi-Prasad-Sommerfield (BPS) bound) are described by 3 positive integer charges. Let us call them Q_1, Q_5 and n . Q_1 and Q_5 are the 'electric' and 'magnetic' charges corresponding to a tensor (2-form) gauge field. The mass of this black hole is given by a linear combination of the charges, and its entropy is given by $S = 2\pi\sqrt{Q_1 Q_5 n}$. This stable black hole has zero Hawking temperature, but the non-zero entropy signals a highly degenerate configuration. Small deviations from this stable state are obtained by giving small values to other charges that characterize the solution space. These excited black holes have a non-zero Hawking temperature and will certainly decay to the stable black hole by the emission of Hawking radiation. If one scatters low energy particles from this black hole one can compute, using the standard wave equation, the absorption cross-section and the corresponding decay rate. The absorption cross-section at long wave lengths turns out to be equal to the area of the horizon of the black hole.

Now let me discuss the constituent model. It turns out that the constituents are an assembly of 1-branes and 5-branes. There are Q_1 1-branes and Q_5 5-branes. The 5-branes are wrapped on the 5-torus and the 1-branes are wrapped on one of the circles of the 5-torus. To an observer in the 5 physical non-compact dimensions this collection looks like a point. The collection of 1 and 5 branes interacts via open strings between them, very much like the interaction of quarks by gluon strings. Hence the low energy dynamics is described by a $U(Q_1) \times U(Q_5)$ gauge theory. This gauge theory resides in the space time that is common to all the branes, and in our example since the branes have only one dimension in

common, the gauge theory is 2-dimensional. It also turns out that this gauge theory has $N=4$, supersymmetry. One of the implications of string duality is that the same system can be described in terms of 0-branes and 4-branes, which is described by a large N matrix model.

We have a non-abelian gauge theory at the service of quantum gravity!

Before I proceed let me pause and explain the last statement. To illustrate the point consider the interaction between 2 D-branes, separated by a certain distance r . This distance is measured in the units of the "string length", which is a new length scale in the problem. It is related to Newton's constant, which we would like to take as a derived constant in the present framework. Since these objects carry energy, their interaction will be mediated by gravitons and other modes of the closed string. That is standard. As the distance between the branes shrinks, the closed string description is very complicated and involves the exchange of all the infinite number of massive modes of the string. However the propagation of a closed string between the branes can also be visualized as a virtual open string. While the long range interactions of the branes are best described by gravitons, the short distance interactions between the branes, in the limit of $r \rightarrow 0$, are best described by massless open strings, which are the gauge fields. It is for this reason that the bound state properties of the assembly of branes which constitute the black hole is described by the infra-red properties of the above mentioned non-abelian gauge theory. Another intriguing implication of this picture is that since the small separations of the branes are measured in terms of masses of the open strings that stretch between them, the coordinates of the open string (gauge fields in the transverse direction to the brane) are themselves space-time coordinates. But these are matrices and in general do not commute! Except when the branes are weakly interacting. This has led to the far reaching suggestion of Witten that our usual notions of spacetime have to be replaced, at distances smaller than the 'string' length, by a non-commutative geometry. The gauge theory that describes the assembly of D-branes actually embodies the space time degrees of freedom. This idea finds its simplest expression when applied to an assembly of 0-branes as in M(atrrix) theory.

Let me continue the discussion of the black hole bound state. To describe a macroscopic black hole we clearly need a large number of these branes and in fact we have to choose Q_1 and Q_5 to be comparable and large. Let us call that large number N . A sensible limit is one in which gN is held fixed as $N \rightarrow \infty$ and $g \rightarrow 0$. Here g is the coupling constant of the string theory. The gauge theory then has a systematic expansion in powers of $1/N$. The analogy with $SU(N)$ QCD in the large N limit is instructive. As is well known the low energy effective lagrangian of QCD is the chiral model of mesons whose expansion parameter is given by the inverse of the pion coupling constant $f_\pi \sim 1/N$. The baryon is an N quark bound state interacting via gluons and it is a soliton solution of the chiral lagrangian with $M \sim N$. The baryon is analogous to the black hole in that it is composed of N^2 open string degrees of freedom and it is also a classical solution of a low energy supergravity lagrangian. Newton's coupling scales as $G \sim 1/N^2$. The mass of the black hole scales as $M \sim N^2$. Just like in QCD, where meson-baryon couplings are of order $(1/N)^0$ and of the same order as the pion kinetic energy term, the closed string- black hole couplings are also of order $(1/N^2)^0$, of the same order as the graviton kinetic energy term. In both cases the

interaction is of order one and that is why there is a non-trivial scattering. Also, the size of the baryon is independent of N and so is the area of the horizon of the black hole. However the analogy is partial because the lowest lying collective modes of the baryon are described by the collective coordinates of the flavour group, and hence the degeneracy of the ground state does not increase exponentially, which is characteristic of the black hole. The reason behind this is the fact that the collective modes of the black hole are described by an effective string theory. The effective string theory also incorporates small deviations from extremal and stable black holes and can be used to study black hole thermodynamics for very small values of the Hawking temperature.

The effective string theory picture of the collective modes of the 5-dim. black hole is most easily understood in the limit of weak coupling when the D-brane assembly does not actually form a black hole, because the Schwarzschild radius sits within the characteristic length scale of the basic theory, namely the string length. However ignoring the difficulties of the strongly coupled regime of the gauge theory (which actually describes the black hole regime) one can do various calculations. The most important set of calculations, besides the ones that calculate the entropy of the excited black hole, set out to compare the Hawking decay rates of various particles that interact with the black hole. In all cases the calculations done with the effective string matched with those done using standard methods in relativity. An important conceptual point here is that the answers of black hole thermodynamics are obtained by using the standard rules of quantum mechanics. One calculates the relevant S-matrix for the interaction of a closed string with the effective string that describes the collective modes of the black hole. Then the absolute value square of the S-matrix is averaged over the initial micro-states and the final micro-states states are summed over. These micro-states are simply built out of the oscillators of the effective string.

Presently there is much interest in putting many of these calculations on a firm footing, especially because their validity in the black hole regime where $gN \sim 1$, is far from obvious. There are indications that the agreements at weak coupling may continue to hold at strong coupling. This indicates that the D-brane bound state somehow knows about the geometry of the black hole. To explicitly see the emergence of the geometry of the black hole by constructing the effective theory of a strongly interacting probe is a challenge for the future.

In our view, which is shared by many working in the subject, the information paradox is likely to be resolved in the framework which we have tried to give a glimpse of. In the process we have begun to understand how the conventional ideas of space time and general relativity have to be replaced at a fundamental level. They emerge at scales larger than the string length, from a more fundamental non-commutative geometry in which space time is described by non-commuting matrices.

The unfolding story about the fundametal theory and its applications to black hole physics, that I have tried to sketch above, in this very concise account, is the work of a whole community of people who were and are still called 'string theorists'. Below I give a

few references that may help navigate the interested reader through the literature.

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