View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by CERN Document Server

Higher Fock states and power counting in exclusive P-wave quarkonium decays

Jan Bolz[a], Peter Kroll

Fachbereich Physik, Universität Wuppertal
D-42097 Wuppertal, Germany
E-mail: kroll@theorie.physik.uni-wuppertal.de

and

Gerhard A. Schuler[b]

Theory Division, CERN,
CH-1211 Geneva 23, Switzerland
E-mail: Gerhard.Schuler@cern.ch

Abstract

Exclusive processes at large momentum transfer Q factor into perturbatively calculable short-distance parts and long-distance hadronic wave functions. Usually, only contributions from the leading Fock states have to be included to leading order in $1/Q$. We show that for exclusive decays of P-wave quarkonia the contribution from the next-higher Fock state $|\mathrm{Q}\overline{\mathrm{Q}}\mathrm{g}\rangle$ contributes at the same order in $1/Q$. We investigate how the constituent gluon attaches to the hard process in order to form colour-singlet final-state hadrons and argue that a single additional long-distance factor is sufficient to parametrize the size of its contribution. Incorporating transverse degrees of freedom and Sudakov factors, our results are perturbatively stable in the sense that soft phase-space contributions are largely suppressed. Explicit calculations yield good agreement with data on χ_{cJ} decays into pairs of pions, kaons, and etas. We also comment on J/ψ decays into two pions.

[a] Supported by Deutsche Forschungsgemeinschaft.
[b] Heisenberg Fellow.

CERN–TH/97–64
April 1997

1 Introduction

Exclusive reactions at large momentum transfer Q can be calculated in perturbative QCD (pQCD) owing to a factorization theorem [1], which separates the short-distance physics of the partonic subreactions at the scale Q from the longer-distance physics associated with the binding of the partons inside the hadrons. The full amplitude is given as a sum of terms, where each term factors into two parts, a hard-scattering amplitude T_H, calculable in perturbative QCD, and wave functions $\psi(x_i, \mathbf{k}_{\perp\mathbf{i}})$ for each hadron H. The importance of the various terms depends on their scaling with $1/Q$. For the contribution with the weakest fall-off with Q (leading-twist contribution), the amplitude T_H describes the scattering of clusters of *collinear* partons from the hadron and is given by *valence-parton* scatterings only. Hence the only non-perturbative input required are the distribution amplitudes $\phi_H(x_i, Q)$ for finding valence quarks in the hadron, each carrying some fraction x_i of the hadron's momentum. The distribution amplitudes represent wave functions integrated over transverse momentum $\mathbf{k}_\perp$ up to a factorization scale μ_F of order Q.

Corrections to this standard hard-scattering approach (sHSA) (lowest-order in $\alpha_s(Q)$ calculations in the collinear approximation using valence Fock states only) can be of perturbative origin ($\propto \alpha_s(Q)$) or power-like ($\propto 1/Q$). The latter may be classified as follows:

1. Corrections arising from the overlap of the soft wave functions.
2. Corrections associated with the transverse momentum of the partons inside the hadrons.
3. Corrections from higher Fock states.

In order to show that predictions of the sHSA are reliable, two conditions have to be met. First, one has to prove that both the $O(\alpha_s(Q))$ corrections and the $1/Q$ corrections are small. Second, one has to ensure that the perturbative calculation of T_H does not receive major contributions from phase space regions, where the virtualities of the internal partons become soft and perturbation theory is not applicable [2].

In recent years there has been progress in our understanding which of the above-mentioned corrections are important. On the one hand, the HSA has been modified through the incorporation of tranverse degrees of freedom and gluonic radiative corrections in form of a Sudakov factor [3]. Within this modified hard-scattering approach (mHSA), the perturbative contribution to exclusive observables can be calculated self-consistently, because the Sudakov factor suppresses contributions from soft phase-space regions. In particular, the mHSA provides a description of the pion–photon transition form factor, which is both reliable and in agreement with data, already at momentum transfers as low as 1 GeV [4, 5, 6].

On the other hand, the overlap of the initial- and final-state pion wave functions, representing a soft contribution of higher-twist type, is still sizeable in the few GeV region in the calculation of the pion elastic form factor and just suffices to fill the gap between the perturbative prediction and the experimental data [7, 4]. Since the overlap contribution, while commonly neglected, is an essential ingredient of the HSA [1], the smallness of the perturbative contribution to the pion form factor should not be regarded as a failure but rather as consistent within the entire approach.

In a recent paper [8] we have shown that the hierarchy of the $1/Q$ expansion for exclusive decays of heavy quarkonia is different from the canonical one: for P-wave states, of $1/Q$ compared to the contribution from the leading Fock state. (Q is of the order of the heavy-quark mass m_c.) This is analogous to the case of *inclusive* decays of heavy quarkonia, although there the long-distance matrix elements are organized into a hierarchy according to their scaling with v, the typical velocity of the heavy quark in the quarkonium [9].

For the P-wave charmonium state χ_{cJ}, the Fock-state expansion starts as

$$|\chi_{cJ}(J^{++})\rangle = O(1)\,|c\bar{c}_1({}^3P_J)\rangle + O(v)\,|c\bar{c}_8({}^3S_1)\,g\rangle + O(v^2)\ , \tag{1.1}$$

where the subscript c specifies whether the $c\bar{c}_c$ is in a colour-singlet ($c = 1$) or colour-octet ($c = 8$) state. The angular-momentum state of the $c\bar{c}$ pair is denoted by the spectroscopic notation ${}^{2S+1}L_J$. Contrary to the case of S-wave decays, *two* 4-fermion operators contribute to the decay rate of P-wave states into light hadrons at leading order in v

$$\begin{aligned}\Gamma[\chi_{cJ} \to \mathrm{LH}] = {} & \frac{c_1}{m_c^4}\,\langle\chi_{cJ}|\mathcal{O}_1({}^3P_J)|\chi_{cJ}\rangle \\ & + \frac{c_8}{m_c^2}\,\langle\chi_{cJ}|\mathcal{O}_8({}^3S_1)|\chi_{cJ}\rangle + O(v^2\Gamma)\ .\end{aligned} \tag{1.2}$$

The term involving the colour-singlet matrix element is the one familiar from the quark-potential model. Indeed, up to corrections of order v^2, one has $\langle\chi_{cJ}|\mathcal{O}_1({}^3P_J)|\chi_{cJ}\rangle = 9|R'_P(0)|^2/(2\pi)$. The decays of P-wave (and higher orbital-angular-momentum states) probe components of the quarkonium wave function that involve dynamical gluons. However, the contribution to the inclusive annihilation rate from the higher Fock state $|c\bar{c}g\rangle$ is parametrized through a single number, namely the expectation value of the octet operator between the χ_{cJ} state, i.e. the colour-octet matrix element in (1.2).

Consider now the case of exclusive decays of χ_{cJ}. For definiteness take $\chi_{cJ} \to \pi\pi$ and define the amplitude M_J through

$$\Gamma[\chi_{cJ} \to \pi\pi] = \frac{a_J}{m_c}\,|M_J|^2\ , \tag{1.3}$$

where the a_J are real numbers. The amplitude is the sum of a colour-singlet and a colour-octet part, $M_J = M_J^{(1)} + M_J^{(8)}$, corresponding to the leading and the subleading Fock state in (1.1), respectively. The singlet amplitude factors into the hard amplitude $T_{HJ}^{(1)}$ describing the subprocess

$$c\bar{c}_1({}^3P_J) \to q\bar{q}'_1({}^1S_0) + q'\bar{q}_1({}^1S_0) \tag{1.4}$$

and the wave functions for the leading Fock states of the two pions and the χ_{cJ}. The χ_{cJ} wave function can be taken in the non-relativistic limit, in which the charm (anticharm) momentum is given by $\frac{1}{2}p \pm K$ and terms linear in the relative momentum K have to be kept. The only long-distance information required is hence the singlet decay constant $f_J^{(1)}$ or, equivalently, the derivative of the non-relativistic $c\bar{c}$ wave function at the origin in coordinate space, $R'_P(0) \propto f_J^{(1)}\sqrt{m_c}$. The colour-singlet amplitude thus takes on the form

$$M_J^{(1)} \sim m_c\,\alpha_s^2(m_c)\left(\frac{f_\pi}{m_c}\right)^2\frac{f_J^{(1)}}{m_c^2}\,I_J^{(1)}\ , \tag{1.5}$$

where f_J is a convolution of the pion distribution amplitudes with a hard kernel, $f_J = \int \mathrm{d}x\mathrm{d}y \phi_\pi(x)\phi_\pi(y) f_J^{(1)}(x,y)$.

For the calculation of the colour-octet amplitude two new ingredients enter, the octet wave function and the problem of colour conservation. Consider the wave function of the $|\mathrm{c}\overline{\mathrm{c}}_8\mathrm{g}\rangle$ state first. It is important to realize that in the $|\mathrm{c}\overline{\mathrm{c}}_8\mathrm{g}\rangle$ Fock state not only the $\mathrm{c}\overline{\mathrm{c}}$ pair is in a colour-octet state, but also the three particles, c, $\overline{\mathrm{c}}$ and g, are in an S-state. Hence orbital angular momenta are not involved and the transverse degrees of freedom can be integrated over. Therefore, we only have to operate with a distribution amplitude $\Phi_J^{(8)}(z_1, z_2, z_3)$ ($z_1 + z_2 + z_3 = 1$, where z_1, z_2, z_3 are the plus light-cone fractions of c, $\overline{\mathrm{c}}$, and g, respectively), that is, as usual, subject to the condition $\int \mathrm{d}z_1\mathrm{d}z_2\Phi_J^{(8)} = 1$, and an octet decay constant $f_J^{(8)}$ for each J. The colour-octet component of the $\chi_{\mathrm{c}J}$ is then given by

$$|\chi_{\mathrm{c}J}^{(8)}\rangle = \frac{t^a_{\mathrm{c}\overline{\mathrm{c}}}}{2} f_J^{(8)} \int \mathrm{d}z_1\mathrm{d}z_2\Phi_J^{(8)}(z_1, z_2, z_3)\, S_{J\nu}^{(8)} , \tag{1.6}$$

where $t = \lambda/2$ is the Gell-Mann colour matrix and a the colour of the gluon. In the following, we will make the plausible assumption that the colour-octet $\chi_{\mathrm{c}J}$ states differ only by their spin wave functions, i.e. the distribution amplitudes as well as the decay constants are assumed to be the same for all $\chi_{\mathrm{c}J}$ states, $f_J^{(8)} = f^{(8)}$. The covariant spin wave functions in (1.6) are readily constructed

$$S_{0\nu}^{(8)} = \frac{1}{\sqrt{6}} (\not{p} + M_0)\,(p_\nu/M_0 - \gamma_\nu), \quad S_{2\nu}^{(8)} = \frac{1}{\sqrt{2}} (\not{p} + M_2)\,\varepsilon_{\mu\nu}\gamma^\mu . \tag{1.7}$$

Owing to the non-relativistic expansion we can employ several simplifications. First, the three partons of the $|\mathrm{c}\overline{\mathrm{c}}\mathrm{g}\rangle$ Fock state can be taken to be collinear up to corrections of order v. Then, the c and $\overline{\mathrm{c}}$ three-momenta scale as $m_\mathrm{c}v$ and differ by at most $m_\mathrm{c}v^2$, i.e. $z_1 = z_2$ up to $O(v^2)$. And finally, the gluon momentum $|\vec{k}|$ is peaked at a value of the order of the binding energy $\epsilon = M_\mathrm{c} - 2m_\mathrm{c}$, where M_c is the average charmonium mass. Hence we can assume a δ-function-like $\mathrm{c}\overline{\mathrm{c}}\mathrm{g}$ distribution amplitude, $z_1 = z_2 = (1-z)/2$, $z_3 = z$, where $z \simeq \epsilon/M_\mathrm{c} \simeq 0.15$. Therefore, analogously to the singlet case, the only long-distance information on the octet wave function that enters the final result is the octet decay constant $f^{(8)}$. Identifying the binding energy with $m_\mathrm{c}v^2$ ($v^2 \sim 0.3$), we find $z_3 \sim v^2/2 \sim 0.15$, in accordance with the above estimate.[1]

Next consider the problem of colour conservation. The obvious solution to colour conservation seems to be to demand one of the final-state pions to be also in a higher Fock state. The Fock-state expansion of the pion is governed by the ratio of the QCD scale and the hard scale $Q \sim m_\mathrm{c}$ of the process, $\lambda = \Lambda_{\mathrm{QCD}}/Q$. The (multipole-based) Fock-state expansion of the pion starts as follows

$$\begin{aligned} |\pi(0^{-+})\rangle = O(1)\,|\mathrm{q}\overline{\mathrm{q}}_1({}^1S_0)\rangle + O(\lambda)\,|\mathrm{q}\overline{\mathrm{q}}_8({}^1P_1)\mathrm{g}\rangle \\ +O(\lambda^2)\,\left\{|\mathrm{q}\overline{\mathrm{q}}_8({}^3S_1)\mathrm{g}\rangle + \ldots\right\} + O(\lambda^3) . \end{aligned} \tag{1.8}$$

Both octet terms give corrections of the same order λ^2 with respect to the leading one: the extra suppression of the magnetic dipole transition for ${}^3S_1$ (λ^2 rather than λ as for an

[1] Since we are working to lowest order in the v-expansion we may simply take $p_{\mathrm{c},\overline{\mathrm{c}}} = z_{1,2}p$, $k = z_3 p$ in the actual calculation; the amount of off-shellness is $p_\mathrm{c}^2 - m_\mathrm{c}^2 \sim m_\mathrm{c}^2 v^2$ and $k^2 \sim m_\mathrm{c}^2 v^4$. Correspondingly, the gluon has three polarization states, cf. (1.7).

The octet amplitude will then look as follows:

$$M_J^{(8)}[^1P_1] \sim m_c\,\alpha_s^2(m_c)\,\frac{f_\pi}{m_c}\,\frac{f_\pi^{(P,8)}}{m_c^2}\,\frac{f^{(8)}}{m_c^2}\,\frac{I_J^{(P,8)}}{m_c\lambda^2}$$
$$M_J^{(8)}[^3S_1] \sim m_c\,\alpha_s^2(m_c)\,\frac{f_\pi}{m_c}\,\frac{f_\pi^{(S,8)}}{m_c^3}\,\frac{f^{(8)}}{m_c^2}\,\frac{I_J^{(S,8)}}{\lambda^2}\ , \tag{1.9}$$

where $I_J^{(8)}$ is a convolution of the hard process

$$c\overline{c}_8(^3S_1) \to q\overline{q}'_8(^1P_1, {}^3S_1) + q'\overline{q}_1(^1S_0)\ , \tag{1.10}$$

the ordinary pion distribution amplitude and the distribution amplitude for the higher Fock states of the pion. We have indicated the extra suppression of these higher Fock states by assuming the octet decay constant to scale as $f_\pi^{(P,8)} \sim \lambda f_\pi$, $f_\pi^{(S,8)} \sim \lambda^2 f_\pi$, and indicated explicitly the extra suppression of P-wave production. The factor λ^2 ($\propto 1/m_c^2$) appears as a consequence of this particular solution of colour conservation: the constituent gluon merely acts as a spectator which runs from the χ_{cJ} to one of the pions without changing its momentum.

Yet, we disregard this possibility of colour conservation for the following two reasons. First, it requires the specification of two new distribution amplitudes for the two higher Fock states of the pion. Second, the Fock-state expansion (1.8) for the pion might be badly convergent. Even if a Fock-state expansion existed, it need not obey the usual multipole expansion assumed in (1.8). If we do not want to work with explicit higher Fock components of the pion, we have to answer two questions: what the constituent gluon of the $|c\overline{c}g\rangle$ state couples to, and what determines the probability, i.e. what the analogue of the charmonium $f^{(8)}$ is on the pion side. To this end we invoke two assumptions.

First, we take QCD perturbation theory to be valid down to virtualities of the order of $zm_c^2 \sim (m_c v)^2$ (see Figs. 4 and 5 and Appendix). Then we can attach the gluon of the $|c\overline{c}_8 g\rangle$ state (in all possible ways and with a coupling $\alpha_s^{\rm soft}$) to the hard process leading to the Feynman diagrams shown in Figs. 4 and 5. Thus the hard process is

$$c\overline{c}_8(^3S_1)g \to q\overline{q}'_1(^1S_0) + q'\overline{q}_1(^1S_0)\ . \tag{1.11}$$

Second, the transverse motion of the valence quark and antiquark determines the importance of the momentum distribution of the constituent gluon. It is clear that the diagrams corresponding to (1.11) must contain contributions, which, to leading order in α_s and z, constitute the two higher Fock states $|q\overline{q}g\rangle$ of the pion. In the approximation of collinear light quarks, one thus encounters singular integrals. These infrared singularities precisely correspond to the long-distance wave functions describing the higher Fock states of the pion. In a collinear calculation one hence needs distribution amplitudes for the higher Fock states into which the singularities can be absorbed. In our ansatz we keep the (non-perturbative) transverse motion of the pion's valence constituents (quark and antiquark). Then all integrals are finite. More precisely, gluon momentum configurations are selected such that the q and $\overline{q}$ relative momentum "matches" the one prescribed by the pion wave function. In this sense one can say that our approach leads to the dynamical generation of higher Fock components. We emphasize that also in this scheme of colour conservation the colour-octet contribution is not suppressed by powers of $1/m_c$

Sect. 4.

In our previous calculation [8], we employed the collinear approximation and simply regularized the singular integrals through a cut-off, which corresponds to an average transverse momentum of the quarks inside the pions. The final results for the colour-octet contribution to the decay amplitude then basically depended on a single parameter

$$\kappa = \sqrt{\alpha_{\rm s}^{\rm soft} f^{(8)}}/\varrho^2 \ . \tag{1.12}$$

Here $\alpha_s^{\rm soft}$ denotes the coupling of the gluon of the $|{\rm c\bar{c}g}\rangle$ Fock state to the hard process, and $f^{(8)}$ is the octet decay constant. There is an additional contribution from terms not singular for $\varrho \to 0$, which is proportional to $\sqrt{\alpha_{\rm s}^{\rm soft} f^{(8)}}$. It is, however, rather small, less than about 10%.

In this work we are going to extend our previous one by working within the mHSA. This leads to three major improvements. First, the scale of the coupling constant is not fixed in the collinear approximation, implying a big uncertainty. In contrast, the scales of α_s are fixed in the mHSA. Second, in the collinear approximation there is usually no suppression of the end-point regions of the distribution amplitude, while these infrared regions are explicitly suppressed in the mHSA through a Sudakov form factor. And finally, no ad-hoc regularization of singular integrals appearing in the calculation of the octet contribution is necessary. Rather, the momentum distribution of the octet gluon is linked to the $k_\perp$ dependence of the pion wave function.

The paper is organized as follows: In Sect. 2 we present our ansatz for the pion wave function and in Sect. 3 we introduce the mHSA in the case of the colour-singlet decay contribution to $\chi_{{\rm c}J} \to \pi^+\pi^-$. In Sect. 4 and Appendix, a detailed description of the colour-octet contribution and a discussion of the results for the decay widths are given. In Sect. 5 we present results for the $\chi_{{\rm c}J}$ decays into pairs of kaons and etas. Finally, in Sect. 6 we comment on J/ψ decays, before we end our paper with a summary and some conclusions (Sect. 7).

2 The pion wave function

We write the pion's $|{\rm q\bar{q}}\,\rangle$ Fock state, which is the leading one, in a covariant fashion

$$|\,\pi\,\rangle = \frac{\delta_{ij}}{\sqrt{3}} \int \frac{{\rm d}x\,{\rm d}^2\mathbf{k}_\perp}{16\pi^3}\,\Psi_\pi(x,k_\perp)\,S_\pi \tag{2.1}$$

where the spin wave function is defined as (the pion mass is neglected throughout)

$$S_\pi = \frac{1}{\sqrt{2}} \not{p}\gamma_5 \ . \tag{2.2}$$

Integrating the light-cone wave function Ψ_π over $k_\perp$, the transverse momentum of the quark with respect to the pion momentum, up to a factorization scale μ_F, one arrives, up to a constant dimensionful factor $f_\pi/(2\sqrt{6})$, at the distribution amplitude $\phi_\pi(x,\mu_F)$. The constant factor plays the role of the configuration space wave function at the origin; $f_\pi = 130.7\,$MeV is the usual pion decay constant.

scale μ_F can be characterized by non-perturbative coefficients B_n (see [1] and references therein):

$$\phi_\pi(x,\mu_F) = \phi_{\rm AS}(x)\left[1 + \sum_{n=2,4,\ldots}^{\infty} B_n(\mu_0)\left(\frac{\alpha_{\rm s}(\mu_F)}{\alpha_{\rm s}(\mu_0)}\right)^{\gamma_n} C_n^{(3/2)}(2x-1)\right]. \tag{2.3}$$

The odd expansion terms do not appear since, in the isotopic limit, the pion distribution amplitude is symmetric $\phi_\pi(x) = \phi_\pi(1-x)$. In (2.3) $\alpha_{\rm s}$ is the strong coupling constant, and μ_0 a typical hadronic scale, $0.5 \lesssim \mu_0 \lesssim 1\,{\rm GeV}$. Since the anomalous dimensions γ_n in (2.3) are positive fractional numbers increasing with n, higher-order terms are gradually suppressed and any distribution amplitude evolves into $\phi_{\rm AS}(x) = 6x(1-x)$ asymptotically, i.e. for $\ln(Q^2/\Lambda^2_{\rm QCD}) \to \infty$. The asymptotic (AS) distribution amplitude itself shows no evolution.

From the investigation of the pion–photon transition form factor $F_{\gamma\pi}(Q^2)$ [4, 5, 6, 10, 11] it follows that the form of the pion distribution amplitude is very close to the asymptotic form. Also recent QCD sum-rule analyses [12] lead to that result. Hence, all terms in the expansion (2.3) of the pion distribution amplitude with $n \geq 2$ will provide only small corrections to exclusive observables and it is legitimate to truncate that expansion at the second term (note that the frequently used Chernyak–Zhitnitsky distribution amplitude [13] is given by $B_2 = 2/3$ and $B_n = 0$ for $n \geq 4$) and consider now B_2 as the only soft parameter.

In the mHSA we consider the Fourier transform of Ψ_π to transverse coordinate space which, following [4, 7], is written as

$$\hat\Psi_\pi(x,\mathbf{b},\mu_F) = \frac{f_\pi}{2\sqrt{6}}\,\phi_\pi(x,\mu_F)\,\hat\Sigma_\pi(x,b,\mu_F)\,. \tag{2.4}$$

The dependence on the transverse separation $\mathbf{b}$, canonically conjugated to $\mathbf{k}_\perp$, is thereby chosen to be of a simple Gaussian form

$$\hat\Sigma_\pi(x,b,\mu_F) = 4\pi\,\exp\left[-\frac{x(1-x)\,b^2}{4\,a_\pi^2(\mu_F)}\right]. \tag{2.5}$$

Here, the momentum fraction x and $\mathbf{b}$ (or $\mathbf{k}_\perp$) refer to the quark; the antiquark momentum is characterized by $1-x$ and $\mathbf{b}$ (or $\mathbf{k}_\perp$) throughout. The pion's transverse size parameter $a_\pi = a_\pi(\mu_F)$ is fixed from the process $\pi^0 \to \gamma\gamma$ [14]. That constraint leads to the closed formula $1/a_\pi^2 = 8\,(1 + B_2(\mu_F))\,\pi^2 f_\pi^2$ under the assumption $B_n = 0$ for $n \geq 4$ (for the asymptotic distribution amplitude ($B_2 = 0$) $a_\pi = 0.861\ {\rm GeV}^{-1}$ [4]). The scale dependence of a_π is an approximation sufficient for our purpose. Using (2.1)–(2.5) we obtain the following expressions for the probability of finding the pion in its valence Fock state, for the mean transverse momentum and the mean radius of the $\rm q\bar q$ Fock state

$$\begin{aligned}
P_{\rm q\bar q} &= \frac{1}{4}\,\frac{1+18/7B_2^2}{1+B_2}\\
\langle k_\perp^2\rangle &= \frac{4\pi^2}{5}\,f_\pi^2\,(1+B_2)\frac{1-6/7\,B_2+12/7\,B_2^2}{1+18/7\,B_2^2}\\
R^2_{\rm q\bar q} &= \frac{3}{8\pi^2}\,f_\pi^{-2}\,\frac{1+3B_2+54/7B_2^2}{(1+B_2)^2}
\end{aligned} \tag{2.6}$$

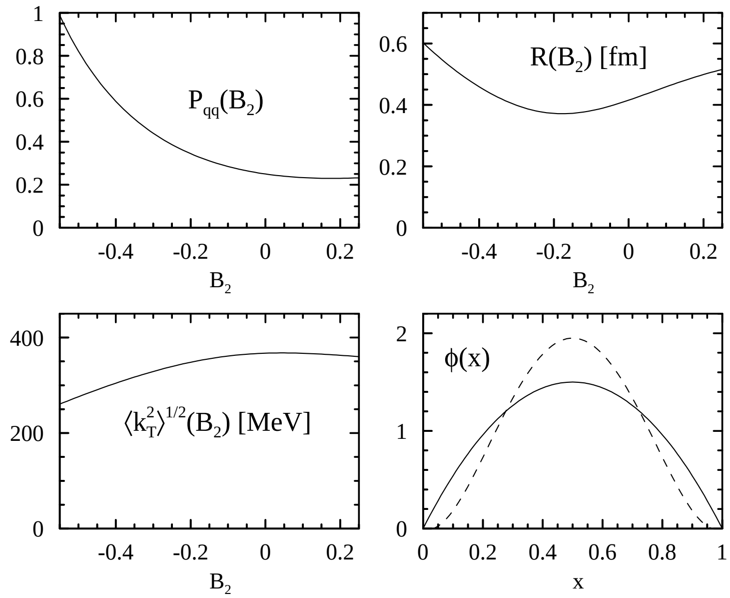


Figure 1: B_2 dependence of the probability $P_{\mathrm{q}\overline{\mathrm{q}}}$ (upper left), the radius R (upper right), r.m.s. transverse momentum $\langle k_\perp^2 \rangle^{1/2}$ (lower left) of the pion's valence Fock state as well as the shape of the distribution amplitude (lower right). In the lower right figure, the solid (dashed) line corresponds to $B_2 = 0$ (-0.2).

As shown in Fig. 1, our wave function has appealing features as a function of B_2. For the asymptotic form of the distribution amplitude we obtain the values $P_{\mathrm{q}\overline{\mathrm{q}}} = 0.25$, $\langle k_\perp^2 \rangle^{1/2} = 367$ MeV and $R_{\mathrm{q}\overline{\mathrm{q}}} = 0.42$ fm. We remark that the latter is considerably smaller than the pion's charge radius of 0.66 fm [15] to which all the Fock states contribute. Since $P_{\mathrm{q}\overline{\mathrm{q}}}$ is much smaller than unity, higher Fock states are important components of the pion and, of course, have been seen for instance in Drell–Yan dilepton $(\mu^+\mu^-)$ production in pion–proton collisions [16]. At large Bjorken x, where the valence quarks dominate, our wave function, for small B_2, is consistent with the valence quark distribution function as derived in [16] (see [4]). For large negative values of B_2, i.e. at a very low scale, the wave function bears resemblance to a constituent wave function with a probability close to unity and a valence Fock state radius that is approximately equal to the charge radius of the pion.

In exclusive reactions, contributions from higher Fock states of the pion are usually neglected since they are suppressed by inverse powers of the relevant hard scale, see (1.8). As explained in the introduction, we do not make use of explicit Fock-state wave functions of the pion. Rather we generate the contribution corresponding to a constituent gluon of the pion dynamically. This is achieved in two steps: (i) the constituent gluon of the χ_{cJ} is coupled perturbatively ($\propto \alpha_s^{\mathrm{soft}}$) to the hard process; (ii) the dependence of the momenta of the pion's valence constituents (i.e. the q, $\overline{\mathrm{q}}$ momenta) on $k_\perp$ is kept. Since it is the non-zero value of $k_\perp$ that regularizes the propagators associated with the "would-be" constituent gluon of the pion, the $k_\perp$ dependence of the wave function of the $\mathrm{q}\overline{\mathrm{q}}$ valence Fock state determines the size of the $\mathrm{q}\overline{\mathrm{q}}\mathrm{g}$ contribution in the pion.

3 The colour singlet contribution and the mHSA

In [8] we have analysed the colour singlet contribution to $\chi_{cJ} \to \pi^+\pi^-$ already within the mHSA. Here, we will give a more detailed description of the calculation.

The colour singlet components of the χ_{cJ} are expressed in terms of non-relativistic wave functions

$$
\begin{aligned}
|\,\chi_0^{(1)}\,\rangle &= \frac{\delta_{ij}}{\sqrt{3}} \int \frac{\mathrm{d}^3k}{16\pi^3 M_0} \tilde{\Psi}_0^{(1)}(k)\, S_0^{(1)}, \\
|\,\chi_2^{(1)}\,\rangle &= \frac{\delta_{ij}}{\sqrt{3}} \int \frac{\mathrm{d}^3k}{16\pi^3 M_2} \tilde{\Psi}_2^{(1)}(k)\, S_2^{(1)}\,,
\end{aligned}
\tag{3.1}
$$

where the $\tilde{\Psi}_J^{(1)}$ represent reduced wave functions, i.e. full wave functions with a factor of the relative momentum K_μ removed from them ($Kp = 0$ and, for instance, $K_\mu = (0, \mathbf{k})$ in the meson rest frame). The $S_J^{(1)}$ denote the covariant spin wave functions

$$
\begin{aligned}
S_0^{(1)} &= \frac{1}{\sqrt{2}} \left[\not{p} + M_0 + 2\not{K}\right] \not{K}, \\
S_2^{(1)} &= \frac{1}{\sqrt{2}} \left[(\not{p} + M_2)\,\gamma_\rho + \frac{2}{M_2}\left[(\not{p} + M_2)\,K_\rho - \not{p}\not{K}\gamma_\rho\right]\right] \varepsilon^{\rho\sigma} K_\sigma.
\end{aligned}
\tag{3.2}
$$

The spin wave functions represent an expansion upon powers of K up to terms of $O(K^2)$ [17] [2]. The derivative of the non-relativistic $c\bar{c}$ wave function at the origin is introduced by

$$
R_P'(0) = \imath\,\frac{16}{3}\,\pi^{3/2}\sqrt{m_c} \int \frac{\mathrm{d}k k^4}{16\pi^3 M_0} \tilde{\Psi}_0^{(1)} = \imath\sqrt{16\pi m_c} f_0^{(1)}\,. \tag{3.3}
$$

In the non-relativistic approximation one has $M_0 \simeq M_2 \simeq 2m_c$ and the same normalization of the two wave functions (usually unity) $P_0^{c\bar{c}} = P_2^{c\bar{c}}$. In this case one finds $\tilde{\Psi}_2 = \sqrt{3}\tilde{\Psi}_0$ and the same relation between the two singlet decay constants $f_J^{(1)}$.

The starting point of the calculation of the colour singlet decay amplitude within the modified HSA is the convolution with respect to the momentum fractions x, y and transverse separation scales $\mathbf{b}_1, \mathbf{b}_2$ of the two pions

$$
\begin{aligned}
M^{(1)}(\chi_{cJ} \to \pi^+\pi^-) = -\imath\,&\frac{32\sqrt{2}\,\pi^{3/2}\,|R_P'(0)|}{3\sqrt{3}\,m_c^{7/2}}\sigma_J \int_0^1 \mathrm{d}x\mathrm{d}y \int \frac{\mathrm{d}^2\mathbf{b}_1}{(4\pi)^2}\frac{\mathrm{d}^2\mathbf{b}_2}{(4\pi)^2} \\
&\hat{\Psi}_\pi^*(y, \mathbf{b}_2)\,\hat{T}_{HJ}(x, y, \mathbf{b}_1, \mathbf{b}_2)\,\hat{\Psi}_\pi(x, \mathbf{b}_1)\,\exp[-S(x, y, \mathbf{b}_1, \mathbf{b}_2, t_1, t_2)]\,,
\end{aligned}
\tag{3.4}
$$

which adapts the method proposed by [3] to our case of exclusive charmonium decays ($\sigma_0 = 1, \sigma_2 = \sqrt{3}/2$). $\hat{T}_{HJ}$ is the Fourier transform of the hard-scattering amplitude T_{HJ} with the $k_\perp$-dependence retained

$$
\hat{T}_{HJ}(x, y, \mathbf{b}_1, \mathbf{b}_2) = \int \frac{\mathrm{d}^2\mathbf{k}_{\perp 1}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{k}_{\perp 2}}{(2\pi)^2}\, T_{HJ}(\mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2})\,\exp\left[-\imath\,\mathbf{k}_{\perp 1}\!\cdot\!\mathbf{b}_1 - \imath\,\mathbf{k}_{\perp 2}\!\cdot\!\mathbf{b}_2\right]. \tag{3.5}
$$

T_{HJ} is to be calculated from the graphs shown in Fig. 2 and reads

[2] The spin wave functions can be written in a more compact form: $S_J^{(1)} = \tilde{S}_J^{(1)} + \{\tilde{S}_J^{(1)}, \not{K}\}/M_J$ where $\tilde{S}_J^{(1)}$ represents the $O(K)$ spin wave function.

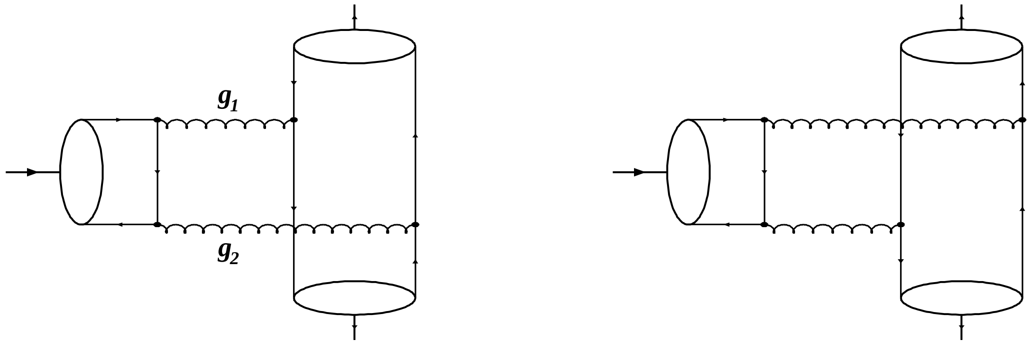


Figure 2: Feynman graphs for the colour-singlet decay $\chi_{cJ} \to \pi\,\pi$ $(J=0,2)$.

$$T_{HJ}(x,y,\mathbf{k}_{\perp 1},\mathbf{k}_{\perp 2}) = \frac{\alpha_s(t_1)\,\alpha_s(t_2)}{(\tilde{g}_1^2+\imath\epsilon)\,(\tilde{g}_2^2+\imath\epsilon)\,N}\left(1+\frac{(-2)^{J/2}}{2}\frac{(x-y)^2}{N}\right) . \tag{3.6}$$

With

$$\mathbf{K}_\perp = \mathbf{k}_{\perp 1} - \mathbf{k}_{\perp 2} \tag{3.7}$$

the virtualities of the internal quark and the two gluons are

$$\begin{aligned} N &= x(1-y)+(1-x)y+\,\mathbf{K}_\perp^2/(2m_c^2)\,, \\ \tilde{g}_1^2 &= xy-\mathbf{K}_\perp^2/(4m_c^2)\,, \\ \tilde{g}_2^2 &= (1-x)(1-y)-\mathbf{K}_\perp^2/(4m_c^2)\ , \end{aligned} \tag{3.8}$$

where a common factor $4m_c^2$ is pulled out. The convolution formula (3.4) involves two independent transverse separation scales b_1 and b_2, each associated with one of the two pions. The b_i also provide the factorization scales $\mu_{Fi} = 1/b_i$ $(i=1,2)$ of the pion wave functions. Distinct from these scales are the renormalization scales t_j $(j=1,2)$ corresponding to the virtualities of the hard gluons.

The fact that T_{HJ} depends on $\mathbf{k}_{\perp 1}$ and $\mathbf{k}_{\perp 2}$ only in the combination $\mathbf{K}_\perp$ implies the following result for the Fourier transform of the hard-scattering amplitude ($\tilde{b} \equiv 2m_c b_1$)

$$\begin{aligned} \hat{T}_{HJ}(x,y,\mathbf{b}_1,\mathbf{b}_2) = 4\,m_c^2\,\alpha_s(t_1^2)\alpha_s(t_2^2)\delta^{(2)}(\mathbf{b}_1-\mathbf{b}_2)\Bigg\{&\frac{-\imath/4}{(1-x)(1-y)-xy} \\ &\left[\frac{\mathrm{H}_0^{(1)}(\sqrt{xy}\,\tilde{b})}{x+y}\left(1+\frac{(-2)^{j/2}(x-y)^2}{2(x+y)}\right)\right. \\ &\left.-\frac{\mathrm{H}_0^{(1)}(\sqrt{(1-x)(1-y)}\,\tilde{b})}{2-x-y}\left(1+\frac{(-2)^{j/2}(x-y)^2}{2(2-x-y)}\right)\right] \\ &+\frac{1}{\pi}\frac{\mathrm{K}_0(\sqrt{(x(1-y)+(1-x)y)/2}\,\tilde{b})}{(x+y)(2-x-y)}\left(1+\frac{(-2)^{j/2}(x-y)^2}{(x+y)(2-x-y)}\right) \\ &+\frac{1}{8\pi}\frac{(-2)^{j/2}(x-y)^2}{(x+y)(2-x-y)}\,\frac{\tilde{b}\,\mathrm{K}_1(\sqrt{(x(1-y)+(1-x)y)/2}\,\tilde{b})}{\sqrt{(x(1-y)+(1-x)y)/2}}\Bigg\} \end{aligned} \tag{3.9}$$

the appearance of the δ-function, which simplifies the numerical work enormously, means that the two pions emerge from the decay with identical transverse separations.

The novel ingredient of the mHSA is the Sudakov factor $\exp[-S]$, which takes into account those gluonic radiative corrections not accounted for in the QCD evolution of the wave function. In next-to-leading-log approximation the Sudakov exponent reads

$$
\begin{aligned}
S(x, y, \mathbf{b}_1, \mathbf{b}_2, t_1, t_2) &= s(x, b_1, 2m_c) + s(1-x, b_1, 2m_c) \\
&+ s(y, b_2, 2m_c) + s(1-y, b_2, 2m_c) - \frac{4}{\beta} \log \frac{\log(t_1/\Lambda_{\rm QCD}) \log(t_2/\Lambda_{\rm QCD})}{\log(1/(b_1\Lambda_{\rm QCD})) \log(1/(b_2\Lambda_{\rm QCD}))}, \qquad (3.10)
\end{aligned}
$$

where the function $s(x, b, Q)$, originally derived by Botts and Sterman [3] and later on slightly improved, can be found, for instance, in [18]. The last term in (3.10) arises from a renormalization group transformation from the factorization scales μ_{Fi} to the renormalization scales t_j at which the hard amplitude $\hat{T}_H$ is evaluated. The renormalization scales t_j, entering $\hat{T}_H$ as the arguments of the two strong coupling constants α_s, are chosen as

$$
t_1 = \max\{4xy\, m_c^2, 1/b_1^2, 1/b_2^2\}, \qquad t_2 = \max\{4(1-x)(1-y)\, m_c^2, 1/b_1^2, 1/b_2^2\}, \qquad (3.11)
$$

thus avoiding large logs from higher-order pQCD. In the limit $b_1 \to 1/\Lambda_{\rm QCD}$ the t_j may in principle approach $\Lambda_{\rm QCD}$ at which the one-loop expression for α_s is diverging. Still the integral appearing in (3.4) is regular since the Sudakov factor compensates the α_s singularities. Actually the suppression of the end-point regions is so strong that the bulk of the perturbative contribution is accumulated in regions of small values of α_s. The physical picture behind the Sudakov suppression is that $q\bar{q}$ pairs with large mutual separation tend to radiate so many gluons that it becomes impossible for the hadronic state to remain intact and that the exclusive process can take place.

Replacing $\exp[-S]$ by 1 and ignoring the transverse momenta in T_H, one finds from (3.4) the colour singlet decay amplitude within the standard HSA as derived by Duncan and Mueller [19]. In that approach the renormalization scale is taken as the charm quark mass and customarily identified with the factorization scale.

In terms of the amplitude (3.4) the χ_{cJ} decay widths into pions are given by

$$
\begin{aligned}
\Gamma[\chi_{c0} \to \pi^+\pi^-] &= \frac{1}{32\,\pi\, m_c} \left|M(\chi_{c0} \to \pi^+\pi^-)\right|^2 \\
\Gamma[\chi_{c2} \to \pi^+\pi^-] &= \frac{1}{240\,\pi\, m_c} \left|M(\chi_{c2} \to \pi^+\pi^-)\right|^2 . \qquad (3.12)
\end{aligned}
$$

The results obtained within the mHSA can be cast into the form

$$
\Gamma[\chi_{cJ} \to \pi^+\pi^-] = \frac{f_\pi^4}{m_c^8} |R'_P(0)|^2 \alpha_s(m_c)^4 \left|a_J^{(1)} + b_J^{(1)} B_2(\mu_0) + c_J^{(1)} B_2(\mu_0)^2\right|^2, \qquad (3.13)
$$

where the coefficients a_J, b_J, and c_J are complex-valued. Written in the form (3.13) we may then immediately find the prediction for a distribution amplitude with a given value of the expansion coefficient B_2 at the scale μ_0. It is still convenient to divide out the fourth power of α_s at the fixed scale m_c in (3.13), since the main effect of the strong coupling is thus made explicit. The actual effective renormalization scale μ_R (i.e. the mean value of hence $\alpha_s = 0.43$ for $B_2(\mu_0) = 0$).

In the following we will choose as central value $R'_P(0) = 0.22$ GeV$^{5/2}$ and $m_c = 1.5$ GeV, which is consistent with a global fit of charmonium parameters [20] as well as results for charmonium radii from potential models [21]. Moreover, we use the one-loop expression for α_s with $\Lambda_{\rm QCD} = 200$ MeV and four light flavours. As we have demonstrated in [8] by varying the input parameters m_c and $R'_P(0)$, B_2, and $\Lambda_{\rm QCD}$, the colour-singlet contribution alone, i.e. a calculation based on the assumption that the χ_{cJ} is a pure $c\bar{c}$ state, is insufficient to explain the observed decay rates.

4 The colour octet contribution

Recently, the importance of higher Fock states in understanding the production and the *inclusive* decays of charmonia has been pointed out [9]. The heavy-quark mass allows for a systematic expansion of both the quarkonium state and the hard, short-distance process and, hence, of the inclusive decay rate or the production cross section. The expansion parameter is provided by the velocity v of the heavy quark inside the meson. The crucial observation is that, for inclusive decays (and also production rates) of the χ_{cJ}, both states in (1.1) contribute at the same order in v and, hence, the inclusion of the "octet mechanism", i.e. the contribution from the $|c\bar{c}_8 g\rangle$ state, is necessary for a consistent description. (Without its inclusion the factorization of the decay width into long- and short-distance factors is spoiled by the presence of infrared-sensitive logarithms.) The inclusion of the octet mechanism is also necessary in exclusive charmonium decays in order to get a consistent description. This is so because the usual suppression of higher Fock states in exclusive reactions does not appear for the decay of P-wave charmonium: it is compensated for by the P-wave nature of the $c\bar{c}_1$ Fock state [8].

As a consequence of employing the collinear approximation we encountered in [8] a number of singular integrals, which we regularized by a parameter ϱ related to the (neglected) transverse momentum of the internal quarks and gluons. However, within the modified HSA where the dependence on the partonic transverse momenta is taken into account, all integrals are finite and there is no longer any need for the parameter ϱ. Apart from that, the modified HSA also provides an explicit prescription for the renormalization scales entering the strong coupling constant such that α_s is no longer a free parameter. Altogether, the modified HSA not only guarantees a self-consistent and thus reliable calculation of the perturbative contribution, it also helps in the present case to diminish uncertainties of the collinear approximation.

As explained in the introduction, the Fock-state gluon has a typical momentum fraction $z \sim m_c v^2/2m_c$ of the χ_{cJ} momentum. We treat z as a small number. We have checked that the results are nearly z-independent for $0.1 < z < 0.3$. Hence the $|c\bar{c}g\rangle$ wave function $\Phi_J^{(8)}$ of (1.6) can be taken as exhibiting a δ-function-like peak at the value $z_3 = z$. Moreover, in our results we will neglect all terms of order z^3 and higher. This is in line with the non-relativistic expansion of the charmonium decay rate since z scales as v^2. Apart from z, the only free parameter on the charmonium side is $f^{(8)}$, the colour octet wave function at the origin.

The details of the calculation of the colour octet contribution within the modified HSA can be found in the Appendix. We have divided the diagrams contributing to the colour octet decay into eleven groups, numbered by the index i. The contribution of each group are determined by the virtualities of the internal quark and gluon lines (see Appendix) and depend non-trivially on the integration variables. As already discussed in Sect. 3 we may simply evaluate the running coupling constant by its one-loop expression at the scales t_{ij} because its singularity (to be reached for b_1 or $b_2 \to \Lambda_{\rm QCD}$) is compensated by the Sudakov factor.

We now want to direct the reader's attention to the question of gauge invariance. Treating the problems as presented above, the results we obtain are gauge invariant to order z^2 (with z being the fraction of the χ_{cJ} momentum carried by the valence gluon), which is sufficient, because we neglect terms of $O(z^3)$ in our results. Eventual violations of gauge-invariance at order z^3 may be traced back to the general problem to describe a constituent within the parton model. This conjecture is supported by the following observation we made: if one changes the c-quark propagator mass in groups 8–10 from the usual value m_c to $(1-z)m_c$, we end up with completely gauge-invariant results. Admittedly, this choice is by no means mandatory and we do not apply it actually, but it reveals that the violations of gauge invariance to $O(z^3)$ are related to the neglect of binding effects for which that particular choice of the mass in the c-quark propagators seems to compensate for.

In order to combine colour singlet and octet contribution we expand the colour-octet decay amplitude (A.1) in a fashion analogous to the colour singlet case:

$$\frac{1}{\sqrt{\lambda_J \pi m_c}}\frac{1}{|\,R_p'(0)\,|}\left(\frac{m_c^4}{\alpha_s(m_c)\,f_\pi}\right)^2 M^{(8)}(\chi_{cJ}\to\pi^+\pi^-) = \left(a_J^{(8)} + b_J^{(8)}B_2 + c_J^{(8)}B_2^2\right)\,, \tag{4.1}$$

where $\lambda_0 = 32$ and $\lambda_2 = 240$. Thus, to obtain the sum of singlet and octet contributions, we simply have to add the coefficients, e.g. $a_J^{(1+8)} = a_J^{(1)} + a_J^{(8)}$.

First, we determine a value for the unknown decay constant $f^{(8)}$, which is contained in the coefficients $a_J^{(8)}$, $b_J^{(8)}$ and $c_J^{(8)}$, from a fit of our predictions to the experimental data on the decay widths, using the asymptotic pion wave function ($B_2 = 0$). We obtain

$$f^{(8)} = 1.46\times 10^{-3}\,\mathrm{GeV}^2\,, \tag{4.2}$$

which is larger than the result estimated in Ref. [8]. This is mainly a consequence of the treatment of the strong coupling constant α_s. In Ref. [8] we have chosen 0.45 for the coupling of the two hard gluons and assumed $\alpha_s^{\rm soft}\approx\pi$ for the coupling of the soft χ_{cJ} Fock-state gluon to the hard process: $\alpha_s(t_1)\alpha_s(t_2)\sqrt{\alpha_s(t_3)} = 0.45^2\sqrt{\pi}\approx 0.36$. In the present analysis based on the mHSA we may extract effective values for the strong coupling constant. We distinguish the hard α_s, evaluated at the scales t_{i1} and t_{i2} from the soft one at t_{i3}. Using the asymptotic form of the pion distribution amplitude we obtain

$$(\alpha_s^{\rm hard})^2(\alpha_s^{\rm soft})^{1/2} = \begin{cases} 0.492^2\times\sqrt{0.557} = 0.181 \ \text{ in } \chi_0\to\pi^+\pi^- \\ 0.509^2\times\sqrt{0.566} = 0.195 \ \text{ in } \chi_2\to\pi^+\pi^- \end{cases}. \tag{4.3}$$

The smaller values of the products of strong couplings have to be compensated for by a larger value of $f^{(8)}$. Observe that the coupling $\alpha_s^{\rm soft}$ of the constituent gluon of the χ_{cJ} is only slightly larger than the coupling of the "hard" gluons. This is caused by two effects: (i) the dynamical setting of the scales makes $\alpha_s^{\rm hard}$ larger than the naive, collinear estimate $\alpha_s^{\rm hard} = \alpha_s^{\rm hard}(m_c)$; (ii) the Sudakov factor is effective enough to suppress the

	$J=0$	$J=2$
$a_J^{(1)}$	$23.36 + 14.67\,\imath$	$5.14 + 3.42\,\imath$
$a_J^{(8)}$	$33.90 + 15.89\,\imath$	$11.60 + 4.12\,\imath$
$a_J^{(1+8)}$	$57.26 + 30.56\,\imath$	$16.74 + 7.54\,\imath$

Table 1: Contributions to a_J (singlet, octet, and their sum for $f^{(8)} = 1.46 \times 10^{-3}\,\mathrm{GeV}^2$).

soft phase-space regions, in which α_s^{soft} would become large. The fact that this coupling is not probed at large values gives us confidence in our perturbative treatment of the χ_{cJ} cosntituent gluon.

The question remains whether the result (4.2) is physically sensible or not. Therefore, we are going to estimate the probability $P_{c\bar{c}g}$ to find the χ_{cJ} in the colour-octet state from the following examplary parametrization for the $c\bar{c}g$ wave function

$$\Psi_0^{(8)}(z_i, \mathbf{k}_{\perp i}) = N z_1 z_2 z_3^2 \;\; \exp\left\{-2a_\chi^2 m_c^2 \left[(z_3 - z)^2 + (z_1 - z_2)^2\right]\right\} \\ \times \exp\left\{-a_\chi^2 \sum k_{\perp i}^2\right\} \;, \tag{4.4}$$

This ansatz combines the known asymptotic behaviour of a distribution amplitude for a $q\bar{q}g$ Fock state with a mass-dependent exponential and a Gaussian $k_\perp$ dependence in analogy to the Bauer–Stech–Wirbel parametrization of charmed-meson wave functions [22]. The mass-dependent exponential guarantees a pronounced peak of the distribution amplitude at $z_1 \simeq z_2 \simeq (1-z)/2$. The δ-function-like distribution amplitude used in the estimate of the colour-octet contribution appears as the peaking approximation to this function. Since the $c\bar{c}g$ Fock state is an S-wave state we assume its radius to be equal to that of the S-state charmonia, namely 0.42 fm [21]. In this case the transverse size parameter a_χ takes the value 1.23 GeV^{-1}. The probability of the colour-octet Fock state is then found to be

$$P_{c\bar{c}g} = \left(f^{(8)}/2.1 \times 10^{-3}\,\mathrm{GeV}^2\right)^2 \;. \tag{4.5}$$

This relation is obtained with $z = 0.15$; it is however practically independent of the exact value of z. We stress again that the ansatz (4.4) is only an example for a possible wave function. Thus, our result of about 1/2 for $P_{c\bar{c}g}$ (from (4.2) in (4.5)) is not in apparent disagreement with expectation and we may conclude that the result (4.2) corresponds to a large but not unphysical value for $P_{c\bar{c}g}$.

Using the value (4.2) the coefficients a_J add up as shown in Table 1. Our results for the decay widths are presented in Table 2. Note that in the colour octet decay amplitude there are terms contributing to decays into neutral pions only (see group 11 in Fig. 5). We may thus find deviations from 1/2 for the ratio of the decay width into neutral pions over that into charged pions. In experiment as well as in our calculation these deviations are found to be small. As discussed before we perform a peaking approximation in the momentum fraction z carried by the χ_{cJ} Fock-state gluon. As in Ref. [8] we set $z = 0.15$. We have checked that our results are only weakly dependent on the actual value of z within the range between 0.1 and 0.2 and all our conclusions remain valid.

In Fig. 3 we sketch the dependence of the $\chi_{c0} \to \pi^+\pi^-$ decay width on the expansion parameter B_2 in the allowed B_2 range [8]. We dispense with a corresponding plot for χ_{c2}

		PDG [23]	BES [24]
$\Gamma[\chi_{c0} \to \pi^+\pi^-]$ [keV]	45.4	105 ± 47	64 ± 21
$\Gamma[\chi_{c2} \to \pi^+\pi^-]$ [keV]	3.64	3.8 ± 2.0	3.04 ± 0.73
$\Gamma[\chi_{c0} \to \pi^0\pi^0]$ [keV]	23.5	43 ± 18	
$\Gamma[\chi_{c2} \to \pi^0\pi^0]$ [keV]	1.93	2.2 ± 0.6	

Table 2: Results for the χ_{cJ} decay widths into pions ($f^{(8)} = 1.46 \times 10^{-3}\,\mathrm{GeV}^2\,;\, B_2 = 0$) in comparison with experimental data. The BES result for $\Gamma[\chi_{c0} \to \pi^+\pi^-]$ is evaluated with the BES result for the total width. In the other cases the PDG average values for the total widths are used.

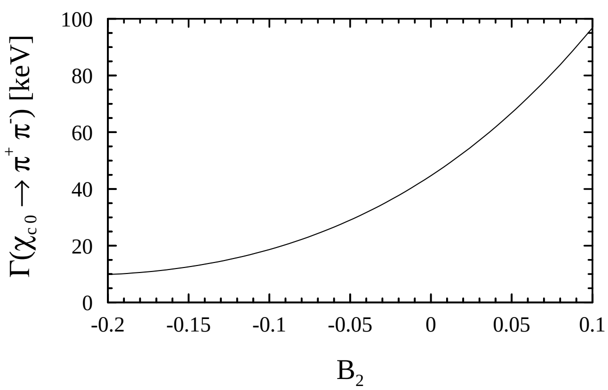


Figure 3: Dependence of the prediction for the $\chi_{c0} \to \pi^+\pi^-$ decay width on the expansion parameter B_2 of the pion distribution amplitude.

As can be seen from Fig. 3 the dependence on B_2 is quite strong, i.e. pushing B_2 from the central value zero to its upper limit $B_2 = 0.1$ doubles the width. If the octet decay constant is determined by some other method, the $\chi_{cJ} \to \pi\pi$ decays constitute a severe test of the pion distribution amplitude.

We conclude from these considerations that it is possible to explain the experimental data on $\chi_{cJ} \to \pi^+\pi^-$ within a perturbative approach, using the asymptotic form of the pion distribution amplitude, provided colour-octet contributions are included. The numerical results of our new calculation within the mHSA are similar to our old ones [8], obtained within the collinear approximation. The new calculation within the mHSA improves, however, the old one in various respects: (i) the scales of the coupling constant are determined, (ii) soft phase-space regions are suppressed, and (iii) all propagators associated with the constituent gluon are regularized through the non-zero transverse momenta of the pion's constituents.

5 χ_{cJ} decays into other light pseudoscalars

The considerations made so far for the case of the pion can straightforwardly be generalized to the decays of χ_{cJ} into the other light pseudoscalar mesons, e.g. etas and kaons. The η meson is a linear combination of an SU(3) octet and singlet component

$$|\,\eta\,\rangle = \cos\theta_P\,|\,\eta_8\,\rangle \;-\; \sin\theta_P\,|\,\eta_1\,\rangle\,. \tag{5.1}$$

For the wave functions of η_1 and η_8 we follow [4] and make the same ansatz (2.1)–(2.5) for it as for the pion (with $a_\eta = a_\pi$). The decay constants f_{η_1} and f_{η_8} as well as the mixing angle θ_P have been determined from a fit to the η–γ and η'–γ transition form-factor data [4]:

$$f_{\eta_8} = 145 \pm 3\,\mathrm{MeV}; \qquad f_{\eta_1} = 136 \pm 10\,\mathrm{MeV}; \qquad \theta_P = -18^\circ \pm 2^\circ\,. \tag{5.2}$$

Since the strong interaction is flavour-blind, the hard scattering amplitudes for the decays of χ_{cJ} into η_1 and η_8 pairs equal both that of the $\pi^0\pi^0$ channel (with group 11 included), i.e. the only modification with respect to the pion case is to replace f_π^2 by $\cos^2\theta_P f_{\eta_8}^2 + \sin^2\theta_P^2 f_{\eta_1}^2 = (144\,\mathrm{MeV})^2$ in Eq. (3.13). In addition we take into account the η mass in the phase space factor, $[1 - m_\eta^2/m_c^2]^{1/2}$, of the decay width (3.12), (3.13). Using, as in [4], $B_2^\eta = 0$ for the η distribution amplitude, we obtain the χ_{cJ} decay widths into pairs of etas as quoted in Table 3. With that asymptotic form of the η distribution amplitude, fair agreement with the data is achieved. On the other hand, a fit of B^η to the $\Gamma[\chi_{cJ} \to \eta\eta]$ data yields a value of -0.036 for it and leads to a slightly better agreement with the data.

The kaon state is also written in the form (2.1) - (2.5), with f_π exchanged for $f_K = 157.8$ MeV. Again in order to simplify matters, we assume $a_K = a_\pi$. Since the kaon consists of up and strange quarks with different masses one may no longer expect the kaon distribution amplitude ϕ_K to be symmetric under exchange $x \leftrightarrow 1-x$. Therefore, one has to include in the expansion (2.3) also the antisymmetric terms

$$\phi_K(x,\mu_F) = \phi_{\mathrm{AS}}(x)\,\left[1 + \sum_{n=1,2,\ldots}^{\infty} B_n^K(\mu_0)\left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)}\right)^{\gamma_n} C_n^{(3/2)}(2x-1)\right]\,, \tag{5.3}$$

	$\Gamma[\chi_{cJ} \to K^+K^-]$ [keV]		$\Gamma[\chi_{cJ} \to \eta\eta]$ [keV]	
	$J=0$	$J=2$	$J=0$	$J=2$
$B_2^K = -0.176$	22.4	1.68		
$B_2^K = -0.100$	38.6	2.89		
$B_2^\eta = -0.036$			24.0	1.91
$B_2^\eta = 0.000$			32.7	2.66
PDG [23]	99 ± 49	3.0 ± 2.2	35 ± 20	1.6 ± 1.0
BES [24]	52 ± 17	1.04 ± 0.43		

Table 3: Results for the χ_{cJ} decay widths into kaons and etas in comparison with experimental data ($f^{(8)} = 1.46 \times 10^{-3}\,\mathrm{GeV}^2$, $B_1^K = 0$).

where e.g. the anomalous dimension γ_1 is 32/81 [1].

Information on the antisymmetric part of the kaon distribution amplitude may be extracted from the valence quark distribution functions of the kaon at large x, where the contributions from Fock components higher than the valence Fock state are negligible. In particular the ratio of strange antiquarks over up quarks is sensitive to the amount of asymmetry in the kaon distribution amplitude. Unlike the cases of the nucleon and the pion, no systematic model-independent analysis of the kaon quark distribution functions exists so far. From calculations of the kaon's valence quark distributions within the Nambu–Jona–Lasinio model [25], there is evidence that at $x \approx 0.8$ there are about twice as many strange antiquarks as u quarks inside the K^+. Truncating (5.3) at $n = 2$ one sees that such a ratio can be accommodated by a coefficient B_1^K of order -0.1. About the same value of B_1^K is found in an instanton model [26].

Due to the symmetry of the hard scattering amplitude for $\chi_{cJ} \to K^+K^-$ under the simultaneous exchanges $x \leftrightarrow 1-x$ and $y \leftrightarrow 1-y$, B_1^K will enter the decay amplitude only quadratically, i.e. in (3.13) and (4.1) the only additional terms are $d_J^{(c)}(B_1^K)^2$. For $|B_1^K| \leq 0.1$ the contributions from the antisymmetric part of the distribution amplitude therefore provide only tiny corrections to the decay of χ_{cJ} into kaons and can be neglected[3]. Hence, the hard scattering amplitude for χ_{cJ} decays into K^+K^- has the same form as the one for the decays into charged pions. A inspection of Table 3 reveals that already a small negative B_2^K value of the order of -0.1 to -0.2 is sufficient to obtain fair agreement between the phase-space-corrected mHSA result and the available data. Admittedly, a precise value of B_2^K cannot be determined at present because the BES data and the PDG average values only agree within large errors; we therefore present results for two different B_2^K values in Table 3. Despite this uncertainty it seems that the kaon distribution amplitude is somewhat narrower than the asymptotic one (which is favoured for the pion). This finding appears to be plausible considering the comparatively large strange quark mass[4]. For $B_1^K = 0$, $B_2^K = -0.1$ we obtain from (2.6) the following properties of the

[3]This was already observed by the authors of [13].

[4]Remember that the non-relativistic case of infinitely heavy quarks is described by a δ-function-like distribution amplitude.

radius 0.44 fm.

6 Comments on $J/\psi \to \pi^+\pi^-$ decays

It has long been known that there exists a problem with the pion form factor. The value $sF_\pi = 0.95 \pm 0.08\,\mathrm{GeV}^2$ at $s = M^2_{J/\psi}$ deduced from the decay $J/\psi \to \pi^+\pi^-$ is much larger than the space-like one at the equivalent Q^2 scale ($Q^2 = -s$): $Q^2 F_\pi = 0.35 \pm 0.10\,\mathrm{GeV}^2$. Admittedly, the space-like data [27] suffer from large systematical errors. The discrepancy is even more severe for $\psi(2S) \to \pi\pi$ ($sF_\pi = 2.67 \pm 0.87\,\mathrm{GeV}^2$). Unfortunately, the data on the time-like form factor measured in $\mathrm{e}^+\mathrm{e}^- \to \pi^+\pi^-$ [28], suffer from low statistics and are inconclusive. While, within the large experimental errors, these data are in agreemnt with the value for the time-like pion form factor as extracted from $J/\psi \to \pi\pi$, a much smaller value (close to that of the space-like form factor) cannot be excluded. These two differences between the time-like and space-like pion form factors pose a true challenge for the HSA. Within the *standard* HSA there is no difference between the form factor in the two regions (except for interchanging Q^2 with $-s$). While possible explanations for a larger value of the form factor in the time-like region than in the space-like one within the mHSA have come up [29], the large values of the pion form factor extracted from $\psi(nS) \to \pi\pi$ still lack a decent explanation. Here we want to propose a solution to this puzzle.

The one-to-one correspondence between sF_π and $J/\psi \to \pi\pi$ is based on the conventional assumption that in a leading-twist analysis the only contribution to $J/\psi \to \pi\pi$ comes from the electromagnetic process with one intermediate photon. The hard process is $O(\alpha_s^2\alpha_{\mathrm{em}}^2)$. Purely hadronic processes, seemingly larger by factors $\alpha_s^p/\alpha_{\mathrm{em}}^2$, are usually supposed to be suppressed by powers of $\Lambda_{\mathrm{QCD}}/m_{\mathrm{c}}$. Moreover, the rate for the $O(\alpha_s^6)$ contribution via three intermediate gluons is zero if the light-quark masses are assumed to be zero [30].

Recent developments [9] in the theory of heavy quarkonia have shown that the appropriate expansion parameter is not $\Lambda_{\mathrm{QCD}}/m_{\mathrm{c}}$, as is the case for light systems, but rather the velocity v of the heavy quark (or antiquark) in the bound state. The dominant Fock state of the J/ψ is $|(\mathrm{c\bar{c}})_1(^3S_1)\rangle$, a colour-singlet $\mathrm{c\bar{c}}$ pair in a spin-triplet S-wave state. This state decays to a pair of pions via a virtual photon and its rate is of order $\alpha_{\mathrm{em}}^2\alpha_s^2 v^3$. (The factor v^3 arises from the squared wave function). Contributions from higher Fock states are suppressed by powers of v and α_s^{soft}. Factors of v give the probability to find these soft gluons in the quarkonium, while factors of α_s^{soft} are associated with the coupling of the soft gluons to the decay process.

Corrections to the electromagnetic decay of the valence Fock state first arise at relative order v^4 from the $|(\mathrm{c\bar{c}})_8(^3S_1)\mathrm{gg}\rangle$ Fock state and the $|(\mathrm{c\bar{c}})_8(^3P_J)\mathrm{g}\rangle$ Fock state. In both cases the hard process is of order α_s^4, while the soft part scales as $v^3(v^2\alpha_s^{\mathrm{soft}})^2$ ($v^3v^2(v^2\alpha_s^{\mathrm{soft}})$) for the former (latter) Fock state. (The former contains two soft gluons, the latter only one, but it is a P-wave annihilation. For comparison, in the conventional HSA (all gluons are purely perturbative and the valence Fock state $\mathrm{c\bar{c}}_1(^3S_1)$ is the only relevant one), the diagrams corresponding to $|(\mathrm{c\bar{c}})_8(^3S_1)\mathrm{gg}\rangle$ are of order α_s^8. The rate for the $O(\alpha_s^6)$ diagrams corresponding to $|(\mathrm{c\bar{c}})_8(^3P_J)\mathrm{g}\rangle$ vanishes for $m_{\mathrm{q}} \to 0$.) Since $v^2 \sim 0.3$ for charmonium, these contributions from the higher Fock states can easily be as large as (or even larger than) the electromagnetic decay of the valence Fock state. From these considerations it dubious.

7 Summary

In this paper we have presented a detailed analysis of χ_{cJ} decays into light pseudoscalar mesons within the framework of the modified HSA. For sufficiently heavy-quark masses, quarkonia are almost non-relativistic, and corrections to the quark-potential model description can be organized into an expansion in v, the typical velocity of the charm quark in the meson. Recently it has been shown [9] that, for inclusive decays, the contribution from the higher Fock state $|c\overline{c}_8(^3S_1)g\rangle$ is not suppressed by v with respect to the contribution from the valence Fock state $|c\overline{c}_1(^3P_J)\rangle$ (i.e. the quark-potential-model Fock state). Here we found a similar situation for exclusive χ_{cJ} decays: the colour-octet contribution (i.e. the one arising from the higher Fock state $|c\overline{c}g\rangle$ with $c\overline{c}$ in a colour-octet state) is not suppressed by powers of either v or $1/m_c$. Hence the usual suppression of higher Fock states for exclusive reactions does not hold in this case.

Owing to the v-expansion, the wave function of the $|c\overline{c}g\rangle$ Fock state of the χ_{cJ} is, to leading order in v, completely fixed apart from a single long-distance parameter, namely the colour-octet decay constant $f^{(8)}$. However, in our approach $f^{(8)}$ remains the only parameter of the colour-octet contribution to the decay rate $\chi_{cJ} \to M\overline{M}$ ($M = \pi$, K, η). We have given arguments against the use of higher Fock states for the pion. Therefore we conserve colour (an issue irrelevant for inclusive decays) by coupling the constituent gluon of the $|c\overline{c}g\rangle$ Fock state of the χ_{cJ} to the hard process. This becomes a sensible proceeding in the mHSA. The two reasons for this are as follows.

First, we keep the transverse momentum of the pion's valence-quark momenta. In this way, no singular integrals occur. The regularization depends on the pion wave function, but the latter is quite well constrained from an analysis of the photon–pion transition form factor in the mHSA. Secondly, incorporation of a Sudakov factor ensures that the coupling α_s of the constituent gluon to the hard process never becomes large. The coupling is evaluated dynamically and the Sudakov factor suppresses soft phase-space regions where the scale of α_s would become so small that a perturbative treatment were no longer justified.

In our estimates of the decay rates $\chi_{cJ} \to \pi\pi$, KK, and $\eta\eta$, the colour-octet contribution turned out to be of great importance in order to establish contact with the experimental data. The value of the single long-distance parameter $f^{(8)}$ came out very reasonable, consistent with the expectation from v scaling. Since the $|Q\overline{Q}g\rangle$ Fock state of χ_{QJ} is neither v- nor power-suppressed, we expect also a large fraction of the $\chi_{bJ} \to \pi\pi$ widths to originate from the $|b\overline{b}g\rangle$ state. This is indeed the case in our approach. Predictions for bottomonium decays are quoted in Table 4. At present there is no data to compare with.

In the case of exclusive J/ψ decays, contributions from higher Fock states (and, hence, from colour-octet contributions) first start at $O(v^4)$ and are often also power-suppressed. They can therefore be neglected in most reactions, e.g. in baryon–antibaryon decay channels, which are dominated by the contributions from $c\overline{c}$ annihilations through three gluons. Indeed, a recent calculation [31] along the lines proposed here provides good results for many $B\overline{B}$ channels. We have pointed out that the situation is different for the J/ψ decay into two pions. This decay is customarily assumed to be dominated by the electromag-

$f^{(8)}\,[\mathrm{GeV}^2]$	$\Gamma[\chi_{b0} \to \pi^+\pi^-]$	$\Gamma[\chi_{b2} \to \pi^+\pi^-]$	$\Gamma[\chi_{b0} \to \pi^0\pi^0]$	$\Gamma[\chi_{b2} \to \pi^0\pi^0]$
1.46×10^{-3}	20.6	1.69	10.5	0.88
5×10^{-3}	152	13.8	78.2	7.27

Table 4: Decay widths in eV for P-wave bottomonia for two values of $f^{(8)}$ ($m_b = 4.5\,\mathrm{GeV}$; $R'_P(0) = 0.7\,\mathrm{GeV}^{5/2}$).

netic decay of $c\bar{c}$ annihilation into a photon, since the three-gluon contributions cancel to zero. We have described the dominant hadronic decay channels arising from higher Fock states. These are likely to be responsible — at least partially — for the large difference between the value of the pion form factor in the space-like region and its value deduced from J/ψ decays into pions. Our considerations may also have consequences for the decays $\psi(nS) \to \rho\pi$ and for the process $\gamma\gamma \to \pi\pi$ where, within the HSA, a substantial part of the cross-section is related to the pion form factor in the time-like region.

A Appendix: Calculation of the colour-octet contribution within the mHSA

The colour-octet decay amplitude can be written in a form similar to (3.4) :

$$M^{(8)}(\chi_{cJ} \to \pi^+\pi^-) = \int_0^1 \mathrm{d}x\mathrm{d}y \int \frac{\mathrm{d}^2\mathbf{b}_1}{4\pi}\frac{\mathrm{d}^2\mathbf{b}_2}{4\pi}\,\hat{\Psi}^*_\pi(y,\mathbf{b}_2)\,\hat{\Psi}_\pi(x,\mathbf{b}_1) \times \sum_{i=1}^{10} \hat{T}_i^{(J)}(x,y,\mathbf{b}_1,\mathbf{b}_2)\,\exp\left[-S(x,y,\mathbf{b}_1,\mathbf{b}_2,t_{i1},t_{i2})\right] \; . \tag{A.1}$$

The sum runs over ten groups of graphs (see Figs. 4 and 5) in the case of charged pions. For the $\pi^0\pi^0$ final state the graphs of group 11 contribute as well. Each group contains a certain number of Feynman graphs, which differ only by permutations of either the two-pion states, the light quark and antiquark lines, or the attachment of the gluons from the charmonium side. Note that for those Feynman graphs where the $c\bar{c}$ pair annihilates into one gluon, the momentum fractions of c and $\bar{c}$ do not appear explicitly and hence only the momentum fraction z carried by the constituent gluon remains. For those graphs where $c\bar{c}$ annihilate into two gluons (groups 8–10) we assume that c and $\bar{c}$ carry each a momentum fraction $(1-z)/2$ of the charmonium momentum.

In Eq. (A.1) the $\hat{T}_i^{(J)}$ denote the Fourier-transformed hard scattering amplitude of group i for the colour-octet decay of the χ_{cJ} in transverse coordinate space. The hard scattering amplitudes $T_i^{(J)}$ are given in Figs. 4 and 5.

In the expressions for the $T_i^{(J)}$ given in Figs. 4 and 5, we absorb the combinatorial and colour factors as well as the colour-octet χ_{cJ} decay constant and α_s into the factors

$$\kappa_0 = \frac{64\,\pi^{5/2}\,f^{(8)}}{9\sqrt{6}\,m_c^3}\alpha_s(t_{i1})\alpha_s(t_{i2})\sqrt{\alpha_s(t_{i3})}\,,$$
$$\kappa_2 = \frac{64\,\pi^{5/2}\,f^{(8)}}{9\sqrt{2}\,m_c^3}\alpha_s(t_{i1})\alpha_s(t_{i2})\sqrt{\alpha_s(t_{i3})}\,. \tag{A.2}$$

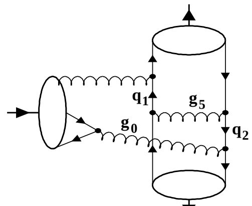
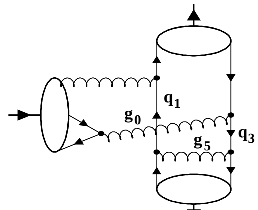
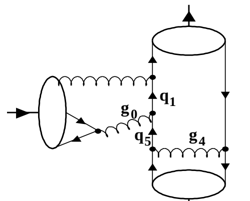
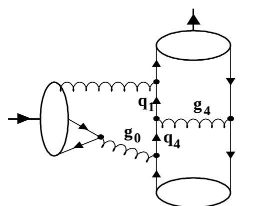
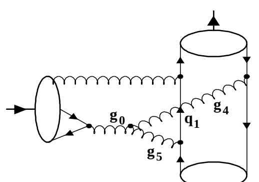
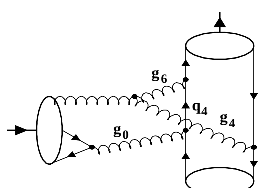
 **Group 1**	$g_0^2 = (1-z)^2 M^2$ $g_5^2 = (z-x_1)(z-y_1)M^2 - \mathbf{K}_\perp^2$ $q_1^2 = z(z-x_1)M^2 - \mathbf{k}_{\perp 1}^2$ $q_2^2 = (1-z)(y_1-z)M^2 - \mathbf{k}_{\perp 2}^2$	$T_1^{(0)} = \frac{\kappa_0\, M^3}{(1-z)^2}\, \frac{2z(1-z)-(x_1-z)(y_1-z)}{(q_1^2+i\epsilon)(q_2^2+i\epsilon)(g_5^2+i\epsilon)}$ $T_1^{(2)} = \frac{\kappa_2\, M^3}{(1-z)^2}\, \frac{z(1-z)+(x_1-z)(y_1-z)}{(q_1^2+i\epsilon)(q_2^2+i\epsilon)(g_5^2+i\epsilon)}$
 **Group 2**	$g_0^2 = (1-z)^2 M^2$ $g_5^2 = (z-x_1)(z-y_1)M^2 - \mathbf{K}_\perp^2$ $q_1^2 = z(z-x_1)M^2 - \mathbf{k}_{\perp 1}^2$ $q_3^2 = (1-z)(x_1-z)M^2 - \mathbf{k}_{\perp 1}^2$	$T_2^{(0)} = -\frac{8\,\kappa_0}{z\,(1-z)^3\, M}\, \frac{1}{g_5^2+i\epsilon}$ $T_2^{(2)} = \frac{8\,\kappa_2}{z\,(1-z)^3\, M}\, \frac{1}{g_5^2+i\epsilon}$
 **Group 3**	$g_0^2 = (1-z)^2 M^2$ $g_4^2 = x_2 y_2 M^2 - \mathbf{K}_\perp^2$ $q_1^2 = z(z-x_1)M^2 - \mathbf{k}_{\perp 1}^2$ $q_5^2 = x_2 M^2 - \mathbf{k}_{\perp 1}^2$	$T_3^{(0)} = \frac{8\,\kappa_0\, M^3}{(1-z)^2}\, \Big[\frac{2(1-z)-(x_1-z)}{(q_1^2+i\epsilon)(q_5^2+i\epsilon)(g_4^2+i\epsilon)} + (z\leftrightarrow 1-z)\Big]$ $T_3^{(2)} = \frac{8\,\kappa_2\, M^3}{(1-z)^2}\, \Big[\frac{(1-z)+(x_1-z)}{(q_1^2+i\epsilon)(q_5^2+i\epsilon)(g_4^2+i\epsilon)} + (z\leftrightarrow 1-z)\Big]$
 **Group 4**	$g_0^2 = (1-z)^2 M^2$ $g_4^2 = x_2 y_2 M^2 - \mathbf{K}_\perp^2$ $q_1^2 = z(z-x_1)M^2 - \mathbf{k}_{\perp 1}^2$ $q_4^2 = (1-z)(y_2-z)M^2 - \mathbf{k}_{\perp 2}^2$	$T_4^{(0)} = -\frac{\kappa_0}{z\,(1-z)^3\, M}\, \frac{1}{g_4^2+i\epsilon}$ $T_4^{(2)} = \frac{\kappa_2}{z\,(1-z)^3\, M}\, \frac{1}{g_4^2+i\epsilon}$
 **Group 5a**	$g_0^2 = (1-z)^2 M^2$ $g_4^2 = x_2 y_2 M^2 - \mathbf{K}_\perp^2$ $g_5^2 = (z-x_1)(z-y_1)M^2 - \mathbf{K}_\perp^2$ $q_1^2 = z(z-x_1)M^2 - \mathbf{k}_{\perp 1}^2$	$T_{5a}^{(0)} = \frac{9\,\kappa_0\, z M^3}{(1-z)^2}\, \frac{2(1-z)+(z-x_1)}{(q_1^2+i\epsilon)(g_4^2+i\epsilon)(g_5^2+i\epsilon)}$ $T_{5a}^{(2)} = \frac{9\,\kappa_2\, z M^3}{2\,(1-z)^2}\, \frac{2(1-z)+(z-x_1)}{(q_1^2+i\epsilon)(g_4^2+i\epsilon)(g_5^2+i\epsilon)}$
 **Group 5b**	$g_0^2 = (1-z)^2 M^2$ $g_4^2 = x_2 y_2 M^2 - \mathbf{K}_\perp^2$ $g_6^2 = (z-x_2)(z-y_2)M^2 - \mathbf{K}_\perp^2$ $q_4^2 = (1-z)(y_2-z)M^2 - \mathbf{k}_{\perp 1}^2$	$T_{5b}^{(0)} = \frac{9\,\kappa_0\, M^3}{(1-z)}\, \frac{2\,z+(y_2-z)}{(q_4^2+i\epsilon)(g_4^2+i\epsilon)(g_6^2+i\epsilon)}$ $T_{5b}^{(2)} = \frac{9\,\kappa_2\, M^3}{2\,(1-z)}\, \frac{2\,z+(y_2-z)}{(q_4^2+i\epsilon)(g_4^2+i\epsilon)(g_6^2+i\epsilon)}$

Figure 4: Overview of groups 1–5 of Feynman graphs contributing to the colour-octet decay amplitude ($M \equiv 2\,m_c$, $x_1 \equiv x$, $x_2 \equiv 1-x$, $\mathbf{K}_\perp$ is defined in (3.7)).

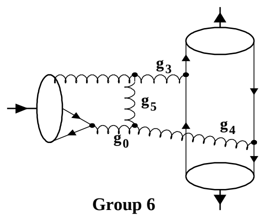
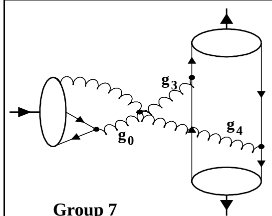
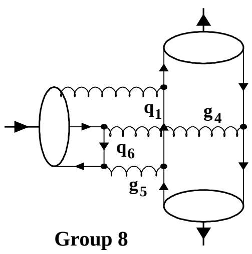
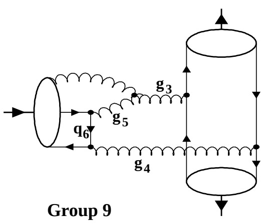
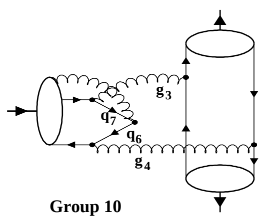
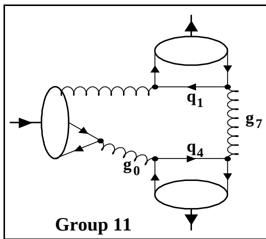
 Group 6	$g_0^2=(1-z)^2M^2$ $g_3^2=x_1y_1M^2-\mathbf{K}_\perp^2$ $g_4^2=x_2y_2M^2-\mathbf{K}_\perp^2$ $g_5^2=(z-x_1)(z-y_1)M^2-\mathbf{K}_\perp^2$	$T_6^{(0)}=\frac{9\,\kappa_0\,M^3}{(1-z)^2}\,\frac{1-(z-x_2)(z-y_2)+(x_2-y_2)^2}{(g_3^2+i\epsilon)(g_4^2+i\epsilon)(g_5^2+i\epsilon)}$ $T_6^{(2)}=\frac{9\,\kappa_2\,M^3}{2\,(1-z)^2}$ $\frac{(z-x_1)(z-y_1)+2\,z(1-z)+4\,x_1x_2-2\,x_1y_2}{(g_3^2+i\epsilon)(g_4^2+i\epsilon)(g_5^2+i\epsilon)}$
 Group 7	$g_0^2=(1-z)^2M^2$ $g_3^2=x_1y_1M^2-\mathbf{K}_\perp^2$ $g_4^2=x_2y_2M^2-\mathbf{K}_\perp^2$	$T_7^{(0)}=\frac{18\,\kappa_0\,M}{(1-z)^2}\,\frac{1}{(g_3^2+i\epsilon)(g_4^2+i\epsilon)}$ $T_7^{(2)}=-\frac{9\,\kappa_2\,M}{2\,(1-z)^2}\,\frac{1}{(g_3^2+i\epsilon)(g_4^2+i\epsilon)}$
 Group 8	$g_4^2=x_2y_2M^2-\mathbf{K}_\perp^2$ $g_5^2=(z-x_1)(z-y_1)M^2-\mathbf{K}_\perp^2$ $q_1^2=z(z-x_1)M^2-\mathbf{k}_{\perp 1}^2$ $q_6^2=\big([x_2(z-y_1)+y_2(z-x_1)]$ $+(1-z)^2/2\big)M^2/2-\mathbf{K}_\perp^2$	$T_8^{(0)}=\frac{9\,\kappa_0\,zM^5}{(g_4^2+i\epsilon)(g_5^2+i\epsilon)(q_1^2+i\epsilon)}$ $\frac{1+z/2-x_1}{q_6^2-M^2/4+i\epsilon}$ $T_8^{(2)}=\frac{9/2\,\kappa_2\,zM^5)}{(g_4^2+i\epsilon)(g_5^2+i\epsilon)(q_1^2+i\epsilon)}$ $\frac{1+z/2-x_1}{q_6^2-M^2/4+i\epsilon}$
 Group 9	$g_3^2=x_1y_1M^2-\mathbf{K}_\perp^2$ $g_4^2=x_2y_2M^2-\mathbf{K}_\perp^2$ $g_5^2=(z-x_1)(z-y_1)M^2-\mathbf{K}_\perp^2$ $q_6^2=\big([x_2(z-y_1)+y_2(z-x_1)]$ $+(1-z)^2/2\big)M^2/2-\mathbf{K}_\perp^2$	$T_9^{(0)}=9\,\kappa_0M^5$ $\frac{z+z^2/2-zx_1/2+x_1y_2-y_1(x_1-y_1)}{(q_6^2-M^2/4+i\epsilon)(g_3^2+i\epsilon)(g_4^2+i\epsilon)(g_5^2+i\epsilon)}$ $T_9^{(2)}=9/2\,\kappa_2M^5$ $\frac{z+z^2/2-zx_1/2+x_1y_2-y_1(x_1-y_1)}{(q_6^2-M^2/4+i\epsilon)(g_3^2+i\epsilon)(g_4^2+i\epsilon)(g_5^2+i\epsilon)}$
 Group 10	$g_3^2=x_1y_1M^2-\mathbf{K}_\perp^2$ $g_4^2=x_2y_2M^2-\mathbf{K}_\perp^2$ $q_6^2=\big([x_2(z-y_1)+y_2(z-x_1)]$ $+(1-z)^2/2\big)M^2/2-\mathbf{K}_\perp^2$ $q_7^2=\big([x_1(z-y_2)+y_1(z-x_2)]$ $+(1-z)^2/2\big)M^2/2-\mathbf{K}_\perp^2$	$T_{10}^{(0)}=-\frac{\kappa_0\,M^5}{4\,(g_3^2+i\epsilon)(g_4^2+i\epsilon)}$ $\frac{2\,x_1x_2+z^2+2\,z}{(q_6^2-M^2/4+i\epsilon)(q_7^2-M^2/4+i\epsilon)}$ $T_{10}^{(2)}=\frac{\kappa_0\,M^5}{4\,(g_3^2+i\epsilon)(g_4^2+i\epsilon)}$ $\frac{2\,x_1x_2+z^2+2\,z}{(q_6^2-M^2/4+i\epsilon)(q_7^2-M^2/4+i\epsilon)}$
 Group 11	$g_0^2=(1-z)^2M^2$ $g_7^2=z(1-z)M^2$ $q_1^2=z(z-x_1)M^2-\mathbf{k}_{\perp 1}^2$ $q_4^2=(1-z)(y_2-z)M^2-\mathbf{k}_{\perp 2}^2$	$T_{11}^{(0)}=-\frac{12\,\kappa_0\,M}{(1-z)^2}\,\frac{1}{(q_1^2+i\epsilon)(q_4^2+i\epsilon)}$ $T_{11}^{(2)}=-\frac{9\,\kappa_2\,M}{(1-z)^2}\,\frac{1}{(q_1^2+i\epsilon)(q_4^2+i\epsilon)}$

Figure 5: Overview of groups 6–11 of Feynman graphs contributing to the colour-octet decay amplitude ($M\equiv 2\,m_c$, $x_1\equiv x$, $x_2\equiv 1-x$, $\mathbf{K}_\perp$ is defined in (3.7)).

The renormalization scales t_{ij} appearing in the various amplitudes, depend on the kinematics of the specific group. The virtualities of the two hard internal gluons connecting the external lines determine the renormalization scales t_{i1} and t_{i2}, respectively. In analogy to (3.11) these scales are chosen as the maximum of $1/b_1^2$, $1/b_2^2$ and the corresponding gluon virtuality (at $\mathbf{k}_{\perp 1} = \mathbf{k}_{\perp 2} = \mathbf{0}$). In addition there is the constituent gluon from the χ_{cJ} colour-octet Fock state coupling to the hard part. The relevant scale t_{i3} is chosen as the maximum of $1/b_1^2$, $1/b_2^2$ and the virtuality of the adjacent internal quark or gluon line (at $\mathbf{k}_{\perp 1} = \mathbf{k}_{\perp 2} = \mathbf{0}$).

We take into account the $k_\perp$ dependence only in the denominators of the $T_i^{(J)}$, i.e. we neglect corrections of $O(k_\perp^2/m_c^2)$. In the collinear approximation we employed in [8], propagator singularities are touched within the range of the momentum fraction integrals which cannot be regularized in the usual way with the $\imath\epsilon$ prescription. The difficulty arises from the gluon propagator g_5 (see Figs. 4 and 5). In [8] we regularized the corresponding integrals by replacing $\mathbf{K}_\perp$ in g_5^2 through the r.m.s. transverse momentum $\langle k_\perp^2\rangle^{1/2}$ of the quarks inside the pions. Here, in the modified mHSA all integrals appearing in (A.1) are either finite or can be regularized with the $\imath\epsilon$ prescription. In some of the $T_i^{(J)}$ there are x-dependent terms in the numerators, which will cancel some of the propagator denominators if the transverse momentum is neglected. In these cases we actually let these terms cancel, thus avoiding some singularities. This procedure is equivalent to neglecting $O(k_\perp^2/m_c^2)$ terms. It is applied, for instance, in group 2 through which only the dependence on the transverse momentum difference $\mathbf{K}_\perp$ (see (3.7)) is left in $T_2^{(J)}$. As already encountered in the case of the colour singlet case the corresponding Fourier-transformed hard amplitude $\hat{T}_2^{(J)}$ then includes a δ-function $\delta^{(2)}(\mathbf{b_1} - \mathbf{b_2})$. Similar situations occur in group 4 and in parts of groups 1, 3, 5 and 8.

The Fourier transform of propagators regularizable with the $\imath\epsilon$ prescription read

$$\int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\, \frac{\exp[-\imath\,\mathbf{k}_\perp\!\cdot\mathbf{b}]}{s - \mathbf{k}_\perp^2 + \imath\epsilon} = \begin{cases} -\frac{\imath}{4}\,\mathrm{H}_0^{(1)}(\sqrt{s}b) & \text{for } s > 0 \\ -\frac{1}{2\pi}\,\mathrm{K}_0(\sqrt{-s}b) & \text{for } s < 0 \end{cases} \,. \tag{A.3}$$

Note that the inclusion of $\mathbf{k}_\perp$ weakens the $1/s$ singularity to a logarithmic one. In the general case of two independent $\mathbf{b}$-variables the integration over the relative angle θ between $\mathbf{b}_1$ and $\mathbf{b}_2$ can be analytically performed by means Graf's theorem. For instance,

$$\begin{aligned}\int_0^{2\pi} d\theta\,\mathrm{H}_0^{(1)}(\sqrt{s}\,|\mathbf{b}_1 - \mathbf{b}_2|) &= \sum_{l=-\infty}^{\infty}\int_0^{2\pi} d\theta\,\cos(l\theta)\,\mathrm{H}_l^{(1)}(\sqrt{s}\,\max(b_1,b_2))\,\mathrm{J}_l(\sqrt{s}\,\min(b_1,b_2)) \\ &= 2\pi\mathrm{H}_0^{(1)}(\sqrt{s}\,\max(b_1,b_2))\,\mathrm{J}_0(\sqrt{s}\,\min(b_1,b_2))\,. \end{aligned} \tag{A.4}$$

In most of the cases one thus finds analytical expressions for the $\hat{T}_i^{(J)}$, except for group 1, where $T_1^{(J)}$ depends on $\mathbf{k}_{\perp 1}$, $\mathbf{k}_{\perp 2}$ and $\mathbf{K}_\perp$. Here, $\hat{T}_1^{(J)}$ includes the integral

$$\begin{aligned}\int_0^\infty & b_0 db_0\,\mathrm{H}_0^{(1)}(\sqrt{(z-x)(z-y)+\imath\epsilon}\,M b_0) \\ &\times\,\mathrm{H}_0^{(1)}(\sqrt{z(z-x)+\imath\epsilon}\,M\max(b_1,b_0))\,\mathrm{H}_0^{(1)}(\sqrt{(1-z)(z-y)+\imath\epsilon}\,M\max(b_2,b_0)) \\ &\quad\times\,\mathrm{J}_0(\sqrt{z(z-x)+\imath\epsilon}\,M\min(b_1,b_0))\,\,\mathrm{J}_0(\sqrt{(1-z)(z-y)+\imath\epsilon}\,M\min(b_2,b_0))\,. \end{aligned} \tag{A.5}$$

To proceed we split the range of integration at some large value $\bar{b}_0$ of b_0. Actually $\bar{b}_0$ is chosen in such a way that all arguments of the Bessel functions $\mathrm{H}_0^{(1)}$ and J_0 are greater expressions and (A.5) is solved. For the remainder of the integration region ($\bar{b}_0 \leq b_0$) the integral is evaluated numerically.

References

[1] S.J. Brodsky and G.P. Lepage, Phys. Rev. **D22** (1980) 2157.

[2] N. Isgur and C. Llewellyn Smith, Nucl. Phys. **B317** (1989) 526.

[3] J. Botts and G. Sterman, Nucl. Phys. **B325** (1989) 62;
H.-N. Li and G. Sterman, Nucl. Phys. **B381** (1992) 129.

[4] R. Jakob, P. Kroll and M. Raulfs, J. Phys. **G22** (1996) 45; P. Kroll, Proceedings of the PHOTON95 Workshop, Sheffield (1995), eds. D.J. Miller et al. (World Scientific, Singapore, 1997), p. 244,

[5] S. Ong, Phys. Rev. **D52** (1995) 3111.

[6] I.V. Musatov and A.V. Radyushkin, JLAB-THY-97-07, February 1997, hep-ph/9702443.

[7] R. Jakob and P. Kroll, Phys. Lett. **B315** (1993) 463 and **B319** (1993) 545(E).

[8] J. Bolz, P. Kroll and G.A. Schuler, Phys. Lett. **B392** (1997) 198.

[9] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D51** (1995) 1125.

[10] A.V. Radyushkin and R.T. Ruskov, Phys. Lett. **B374** (1996) 173.

[11] P. Kroll and M. Raulfs, Phys. Lett. **B387** (1996) 848.

[12] V. Braun and I. Halperin, Phys. Lett. **B328** (1994) 457; A.P. Bakulev and S.V. Mikhailov, Mod. Phys. Lett. **A11** (1996) 1611.

[13] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. **112** (1984) 173.

[14] S.J. Brodsky, T. Huang and G.P. Lepage, Particles and Fields 2, eds. Z. Capri and A.N. Kamal (Banff Summer Institute, 1983), p. 143.

[15] S.R. Amendolia et al., Phys. Lett. **B146** (1984) 116 and **B178** (1986) 435.

[16] P.J. Sutton, A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Rev. **D45** (1992) 2349; M. Glück, E. Reya and A. Vogt, Z. Phys. **C53** (1992) 651.

[17] F. Hussain, J.G. Körner and G. Thompson, Ann. Phys. (N.Y.) **206** (1991) 334;
J. Bolz, P. Kroll and J.G. Körner, Z. Phys. **A350** (1994) 145.

[18] M. Dahm, R. Jakob and P. Kroll, Z. Phys. **C68** (1995) 595.

[19] A. Duncan and A.H. Mueller, Phys. Lett. **B93** (1980) 119.

[20] M.L. Mangano and A. Petrelli, Phys. Lett. **B352** (1995) 445.

[22] M. Wirbel, B. Stech and M. Bauer, Z. Phys. **C29** (1985) 637.

[23] Particle Data Group: Review of Particle Properties, Phys. Rev. **D54** (1996) 1.

[24] Y. Zhu for the BES coll., talk given at the XXVIII Int. Conf. on High Energy Physics, 25–31 July 1996, Warsaw, Poland.

[25] T. Shigetani, K. Suzuki and H. Toki, Phys. Lett. **B308** (1993) 383; Nucl. Phys. **A579** (1994) 413.

[26] A.E. Dorokhov private discussion and Nuovo Cimento **109** (1996) 391.

[27] C.J. Bebek et al., Phys. Rev. **D13** (1976) 25 and Phys. Rev. **D17** (1978) 1693.

[28] D. Bollini et al., Lett. Nuovo Cim. **14** (1975) 418.

[29] T. Gousset and B. Pire, Phys. Rev. **D51** (1995) 15.

[30] S.J. Brodsky and G.P. Lepage, Phys. Rev. **D24** (1981) 2848.

[31] J. Bolz and P. Kroll, WU B 97-2, February 1997, hep-ph/9703252.