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EMITTANCE MEASUREMENTS IN THE CERN PS COMPLEX

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The LHC project, with its demand for small emittance beams and their monitoring, constitutes a challenge for the instruments of the CERN PS complex used hitherto to measure emittances of generally larger beams with less precision. Among these instruments, one (Beamscope) measures the betatron amplitude distribution, while the other two (SEM-grids, fast wire scanner) are recording the projected beam density. Emittances are quoted as derived from one standard width of the projected beam density, and the goal is to provide figures which refer to this definition regardless of the measurement instrument and method. The paper recalls the principles of these measurement systems and describes the mathematical methods applied to eliminate noise and measurement errors, to transform between the phase plane and its projection and the formalism used to determine the matching between machines.

Keywords: Beam instrumentation; SEM (Secondary Emission Monitor); PSB machine; PS machine

1 INTRODUCTION

The measurement of emittance consists, in most cases, of a determination of transverse beam dimensions and requires the knowledge of the optical parameters at the location of the instrument. The methods are based on different functional principles: (i) devices of destructive type recording betatron amplitude distributions (Beamscope, flip targets, scrapers), (ii) devices of nearly non-destructive type measuring beam profiles in the real space (SEM-grids, fast wire scanner), and (iii) devices sweeping the beam over an array of point-like detectors

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(e.g. Faraday cups), which are also fully destructive. SEM-grids allowing only single passes of beams operate only in transport lines, while instruments of group (i) work in circular machines only. The Faraday cups require tens of μs beam pulse duration and are used to measure the Linac emittances. We exclude them from further description as the initial emittances of the hadron beams are determined by the physics of the multi-turn injection process in the PS Booster.

The remaining classes (i) and (ii) have their natural definition of emittance resulting from the measured entities: class (i) easily yields an emittance confining a given fraction (e.g. 95%) of all particles having betatron amplitudes less or equal to that limit; class (ii) suggests straightforward evaluation of the variance of the profile and an emittance expressed by this quantity. Nearly all instruments belong to class (ii), apart from the Beamscope[1] device of the PS Booster. To enable the PS to monitor emittance conservation and identify possible blow-ups during transfers and acceleration, an on-line conversion between the two emittance definitions has to be provided.

Theoretically a rotationally symmetric betatron phase space distribution and its projected density are related by the Abel transform.[2] However, in the horizontal dimension its application is unreliable for two reasons: (i) the Abel transform is, strictly speaking, not meaningful in the presence of momentum spread and finite lattice dispersion; (ii) it requires precise knowledge of the center of the beam which is blurred by the momentum spread.

SEM-grids are the most widely used instruments for transverse emittance measurements in the PS. A new standard method using SEM-grids to measure emittances at the entrance of the PS and in the transport lines of the PS complex has been put into operation in the framework of the rejuvenation of the PS control system.[3] The method involves a wide range of statistical tools to analyze the SEM-grid measurements and reliably derive the transverse beam dimensions. The numerical treatment first eliminates faulty wires. Then, it smoothes the beam profile using a choice of curve fitting functions to reduce the noise effect on wire signals, and carries out a treatment of the beam tails to establish the base line. In order to permit a diagnostic of the quality of the measurement, none of the profile measurements is automatically discarded. It is left to the user to keep or discard the proposed results. In the presence of lattice dispersion the momentum width of the beam is folded with the betatron width, and the emittance obtained from the measured profile may no longer be meaningful. It may then be necessary to unfold the momentum component from that of the beam profile in order to obtain the betatron beam width.

2 BEAM DISTRIBUTION AND EMITTANCE

The distribution in the phase plane of a Gaussian beam can be expressed in terms of the Courant–Snyder invariant

$$\varepsilon_x = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2$$

as

$$P_\varepsilon(\varepsilon_x) = \frac{\beta_x}{2\sigma_x^2} \exp\left(-\frac{\beta_x \varepsilon_x}{2\sigma_x^2}\right),$$

where ε_x is the transverse beam emittance.

Similarly, the distribution can be rewritten in terms of the betatron amplitude a,

$$a = \sqrt{\beta_x \varepsilon_x},$$

yielding the betatron amplitude distribution

$$P_a(a) = \frac{a}{\sigma_x^2} \exp\left(-\frac{a^2}{2\sigma_x^2}\right).$$

The r.m.s. value of this distribution expressed with respect to the r.m.s. value of the beam profile density is

$$\sigma_a = \sqrt{\frac{4-\pi}{2}}\sigma_x = 0.66\sigma_x.$$

One usually prefers to define the beam emittance in terms of σ_x in real space (beam profile) rather than in terms of σ_a for reasons of simplicity. In particular:

(i) $\pm 2\sigma_x$ contain 95% of the particle beam,
(ii) $\pm 2\sigma_a$ contain 57.6% of the particle beam,
(iii) $\pm 3.74\sigma_a$ contain 95% of the particle beam.

This leads to the different definitions for the transverse beam emittances, e.g.:

$$\varepsilon_x^{(\sigma_x)} \equiv \frac{\sigma_x^2}{\beta_x} = \left(\frac{2}{4-\pi}\right)\frac{\sigma_a^2}{\beta_x} = 2.33\frac{\sigma_a^2}{\beta_x}$$

and

$$\varepsilon_x^{(95\%)} \equiv \frac{6\sigma_x^2}{\beta_x} = \left(\frac{12}{4-\pi}\right)\frac{\sigma_a^2}{\beta_x} = 13.98\frac{\sigma_a^2}{\beta_x},$$

where $\varepsilon_x^{(95\%)} = \varepsilon_x^{(3.74\sigma_a)}$. The link between the 95%-emittance and the σ_x-emittance definitions (for a Gaussian distribution) is

$$\varepsilon_x^{(95\%)} = 6\varepsilon_x^{(\sigma_x)} = \frac{3}{2}\varepsilon_x^{(2\sigma_x)}.$$

3 THE MEASUREMENT SYSTEMS

3.1 Beamscope

Beamscope (BEtatron AMplitude Scraping by Closed Orbit PErturbation) is a device developed more than a decade ago at the CERN PS Booster. It consists basically of a three simultaneously pulsed dipoles in the horizontal and vertical plane in 4 rings exciting a fast rising local orbit bump which drives the beam into a precision scraper where it is lost within a few ms; the dipole excitation current, the beam current and its derivative are recorded by transient digitizers (Figure 1).

After a fair amount of processing using resident databases (e.g. for the magnetization curves of the 24 dipoles involved) and a lattice code, applying corrections for eddy currents in the vacuum pipe, the bump amplitude at the scraper location is computed. The beam radius is taken as the difference between the bump amplitudes at 95% circulating and at zero current. The latter is best found from the betatron amplitude distribution, which is computed from the recorded beam current derivative. This works very well in the vertical plane but turned out to be unreliable in the radial one because of (i) the dependence of the beam center (or closed orbit) on particle momentum and (ii) the reaction of the radio-frequency beam control system to the change in

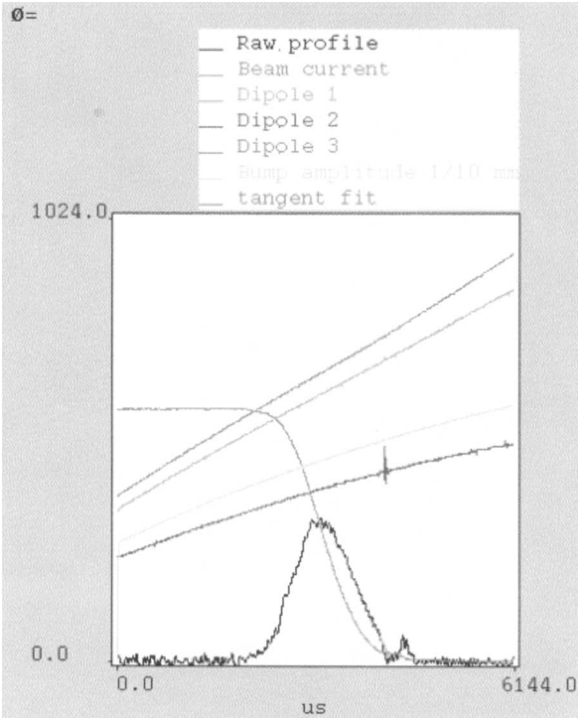

FIGURE 1 Beamscope monitor window of the five acquired signals plus the computed bump amplitude.

circumference due to the orbit bump. While for simple evaluation of the 95% emittance this problem can be circumvented by taking the average of the two measurements (in which case the exact beam center is not needed), the Abel transform of these data sometimes generates erroneous results. For these reasons a conversion scheme has been chosen based on fitting the widely used binomial distribution to the measured data with subsequent analytical computation of all information wanted from the fitted function.[4–6]

The display of results (Figure 2) shows the calibrated amplitude distributions for the selected one or all rings and the physical and normalized emittances computed from the beam radii comprising 95% of the betatron amplitude distribution. Computation of the emittances

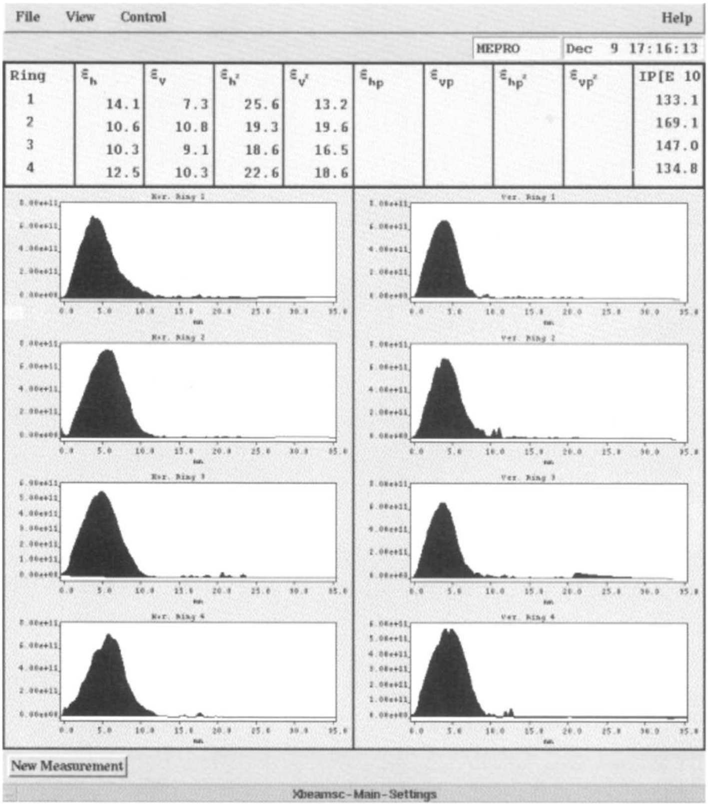

FIGURE 2 Operations new Beamscope display of betatron amplitude distributions of the four CERN PS Booster rings and their 95% amplitude emittances. Projected density calculations are in the process of implementation.

defined by the projected distributions is optional because it is time-consuming. The user interface has been changed recently in order to integrate better into the standard control system architecture.

3.2 SEM-grids

A SEM-grid (Secondary Emission Monitor) consists of an array of ribbons or wires placed in the beam path. Profile shapes are obtained by measuring the secondary electron current from each wire due to the impact of the particles. Beam profiles are then sampled by the array of evenly separated ribbons or wires. SEM-grids can only be used to measure single passage beams, in which case it is practically non-destructive. The beam profiles are measured using either low resolution SEM-grids (2.5 mm ribbon step size) or high resolution SEM-'fil's (0.35 or 0.5 mm wire step sizes), adequate for small beam size of, say, 3 mm, as required for the LHC.

3.3 Fast Wire Scanner

The fast wire scanners of the PS consist of a single wire moving at a speed of up to 20 m/s across the beam. A scintillator and a photomultiplier detect the secondary particles produced in the interaction of the beam with the wire. The signal is sampled against the wire position and yields the beam profile. Fast wire scanners are used to measure circulating beams only. The measurement is nearly non-destructive, the effect of Coulomb scattering on the beam being small.[7]

4 SIGNAL PROCESSING AND METHOD APPLIED

4.1 Beamscope

Beam emittances measured with the Beamscope device may be expressed as

$$\varepsilon_x^{(95\%)} = \frac{a_L^2}{\beta_x},$$

where $\pm a_L$ is the beam size. Assuming a Gaussian beam the relationship between the beam size and the r.m.s. value of the betatron amplitude distribution is $a_L = 3.74\sigma_a$.

4.1.1 Abel Transform

The projected density $p_x(x)$ of a rotationally symmetric phase space density distribution $P_a(a)$ onto the coordinate axis x can be found simply to be

$$p_x(x) = \frac{1}{\pi}\int_x^{\infty} \frac{P_a(a)}{\sqrt{a^2 - x^2}}\,\mathrm{d}a,$$

in which a is the betatron amplitude.

Numerical evaluation of the Abel transform has been included into the Beamscope processing software to obtain the projected density $p_x(x)$.

4.1.2 Ivanov Fit

This method is based on the fitting of a distribution[5]

$$P_a(a) = \frac{m}{\pi} a(1 - a^2)^{m-1}$$

of phase space amplitudes $a = \sqrt{u^2 + v^2}$, where u and v are the coordinates x and x' normalized to the limiting amplitude x_L ($u = x/x_L$, $v = \beta_x x'/x_L$). Its integral I_a describes the normalized beam current as a function of the normalized position u of a scraping target:

$$I_a(u) = 2\pi \int_0^u P_a(a)\,\mathrm{d}a = 1 - (1 - u^2)^m.$$

To fit this expression to the measured (sampled) beam current I_d, it is evaluated at a given number of equidistant arguments $x(i)$. Measured data $I_d(i)$ are also evaluated at these points by three-point Lagrange interpolation between samples. Differences $I_d(i) - I_a(i)$ are summed separately to obtain $S_+(k, j)$ (for i's where $I_d(i) > I_a(i)$) and $S_-(k, j)$ (for i's where $I_d(i) < I_a(i)$); $m = j/2$, j and k are iteration parameters, where k represents a variation of the (initially somewhat arbitrary) limiting amplitude $x_L(k) = x_L^{\mathrm{in}} - (k-1)\,\mathrm{d}x$, $\mathrm{d}x$ denoting the step between the discrete arguments $x(i)$. The procedure starts with $j = k = 1$ and consists in finding the value of j for which the sums over positive and negative differences are equal. Then these sums are computed for increasing m's, until a minimum is found. The final fitted function is then given by the above equation and the r.m.s. width of the projected density is found from the expression $\sigma = x_L/\sqrt{2(m+1)}$.

The principal attraction of this method is, beside the obvious desirability of an analytical description by a handy class of functions, that it does not require precise knowledge of the beam center. In this respect it matches well the problematic of Beamscope measurements in the radial plane. In practical implementation the measured data set is already the average over two measurements, one with inward and one with outward bump amplitude.

4.2 SEM-grids

When measured with SEM-grids or fast wire scanners the transverse emittance may be quoted in terms of the r.m.s. value σ_x of the profile distribution obtained from the projected transverse phase space density distribution onto the plane of the profile. This definition appears as the most appropriate to characterize the density of the bulk of the beam. In the presence of dispersion the betatron r.m.s. beam width $\sigma_{\beta x}$ is to be used instead of the r.m.s. value σ_x to derive the σ-emittance

$$\varepsilon_x^{(\sigma_x)} = \frac{\sigma_{\beta x}^2}{\beta_x},$$

where β_x is the beta-function at the monitor location. In transfer lines, the emittance of the beam may be obtained from beam profiles measured at three different SEM-grids with known transfer matrices between the detectors.

The betatron r.m.s. beam width may be obtained by subtracting the contribution δ of the momentum dispersion from the beam profile using the quadratic formula

$$\sigma_{\beta x}^2 = \sigma_x^2 - D_x^2 \sigma_\delta^2,$$

in which D_x is the dispersion function in the plane of the profile and σ_δ is the r.m.s. value of the momentum dispersion.

4.2.1 Beam Profile Treatment

The accuracy of the emittance figures depends on how well the transverse beam profile, from which the beam sizes have to be derived, is known. SEM-grid wire signals f_i are approximations of the unknown beam profile distribution. Erroneous wire output signals due to faulty wires or bad gain of the amplifier would cause gross errors in the evaluation of the beam size. Hence, any data which are too far away from the rest of the sample are discarded. Moreover, random errors of the wire signals could strongly influence the value of the beam size derived by direct analysis of the measured beam profile. In addition to the random errors, possible systematic errors could shift the base line, which will render the beam size calculations meaningless. A method for the search of the base line is described in Ref. [3].

(i) When no hypothesis is made on the shape of the data fit, flexible approximation of beam profile data by means of spline functions may be used. Assume that approximate values f_i of the beam profile distribution $f(x)$ are known at the wire locations x_i. A cubic spline approximation of $f(x)$ consists of polynomials of degree three. They are pieced together so that their values and those of their first two derivatives match at the points x_i:

$$f(x) = \sum_{j=0}^{3} a_i^{(j)} (x_i - x)^j.$$

The expressions for the coefficients $a_i^{(j)}$ of the spline function may be found in Ref. [3]. Spline approximation is in some sense the best fitting of beam profiles because it provides smooth approximating curves yielding minimal oscillations.

(ii) When physical considerations suggest that the beam profile should be of Gaussian shape, a good alternate model may be obtained by fitting a Gaussian to the measured beam profile

$$f(x) = a_0 \exp\left(-\frac{(x - a_2)^2}{a_1^2}\right),$$

where the a_i's are to be determined. The general approach for deriving least squares estimates for Gaussian curves is to minimize the sum of squared residuals by successive improvements to initial guesses a_{i_0}. The r.m.s. value of the beam profile is then $\sigma_x = a_1/\sqrt{2}$.

4.2.2 Emittance and Matching

The transverse beam emittance ε and matching can be obtained from beam profiles measured at three different beam profile monitors in a transport channel (Figure 3):

$$\varepsilon = \frac{w_{i\beta}^2}{\beta_i}, \quad w_{i\beta}^2 = w_i^2 - D_i^2 \left(\frac{\Delta p}{p}\right)^2,$$

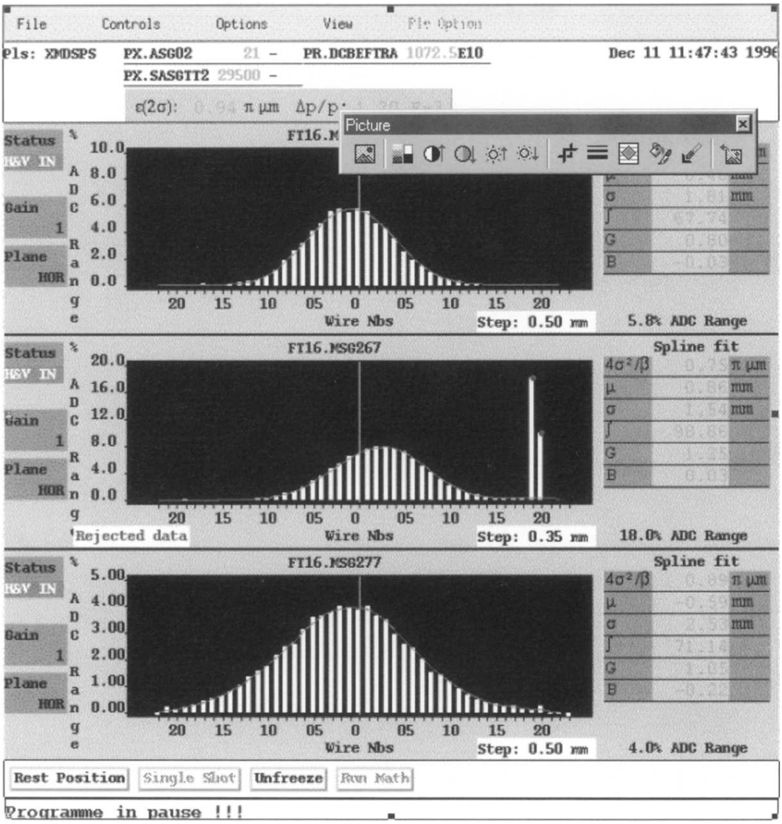

FIGURE 3 Beam profile and emittance measurements at the three SEM-grids in the CERN PS to SPS transport channel.

in which, α_i, β_i, D_i are the optical parameters of the beam, $\Delta p/p$ is the half-momentum spread and w_i is the beam half-width (defined as twice the r.m.s. value) at the ith monitor position.

The emittance increase due to mismatch after filamentation in the downstream circular accelerator may be written

$$\frac{\Delta\varepsilon}{\varepsilon} = |\vec{k}|\left(\frac{|\vec{k}|}{2} + \sqrt{1 + \frac{|\vec{k}|^2}{4}}\right),$$

where $\vec{k}$ is the complex mismatch vector defined as

$$\vec{k} = \frac{1}{\sqrt{\beta_{i0}\beta_i}}(\beta_{i0} - \beta_i + j(\alpha_i\beta_{i0} - \alpha_{i0}\beta_i)).$$

Note that the modulus $|\vec{k}|$ of the mismatch vector is equal to $1/\sqrt{2}$ for a 100% emittance blow-up. Likewise, the mismatch and emittance may be conveniently expressed in terms of Hereward's normalized betatron phase space parameters B_i and G_i:

$$\vec{k} = \frac{1}{\sqrt{G_i}}(G_i - 1 + jB_i),$$

in which

$$B_i = \alpha_i \frac{\beta_{i0}}{\beta_i} - \alpha_{i0}, \qquad G_i = \frac{\beta_{i0}}{\beta_i},$$

where α_{i0} and β_{i0} are the Twiss parameters of the lattice. The beam emittance may be rewritten using the normalized betatron phase space parameters as

$$\varepsilon = G_i \frac{w_{i\beta}^2}{\beta_{i0}}.$$

The matching parameters at the first monitor location depend on the half beam size measured at the three monitors and can be cast in the form

$$B_1 = \frac{1}{2\sin\Delta\mu_{23}}\left(a\frac{\sin\Delta\mu_{13}}{\sin\Delta\mu_{12}} - b\frac{\sin\Delta\mu_{12}}{\sin\Delta\mu_{13}}\right),$$

$$G_1^2 = 1 - B_1^2 + \frac{1}{\sin\Delta\mu_{23}}\left(a\frac{\cos\Delta\mu_{13}}{\sin\Delta\mu_{12}} - b\frac{\cos\Delta\mu_{12}}{\sin\Delta\mu_{13}}\right),$$

where $\Delta\mu_{ij}$ is the phase advance between the monitors i and j and

$$a = 1 - \frac{\beta_{10}w_2^2}{\beta_{20}w_1^2}, \qquad b = 1 - \frac{\beta_{10}}{\beta_{30}}\frac{w_3^2}{w_1^2}.$$

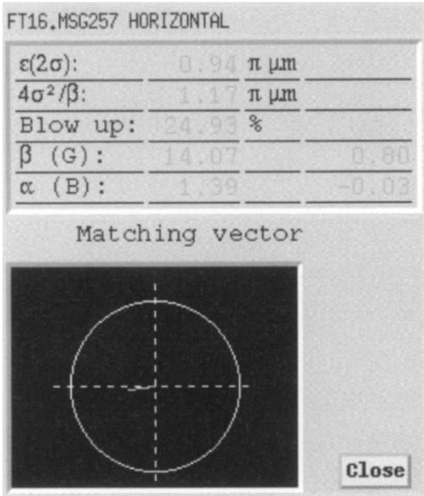

FIGURE 4 Mismatch and emittance blow-up estimation at the first monitor in the CERN PS to SPS transport channel (the area inside the circle corresponds to emittance blow-ups below 100%).

The betatron phase space parameters reduce to $B_i = 0$ and $G_i = 1$ when the beam is perfectly matched. Figure 4 shows a polar plot of the mismatch vector $\vec{k}$.

4.3 Fast Wire Scanners

The data processing consists first of calculating the real wire position data (from laboratory calibrations), then of evaluating the r.m.s. beam size. Assuming that the transverse particle density is a near-Gaussian distribution with possible bias in the tails due to instrumental errors, the excessive influence of profile tails may be discarded by subtracting systematically a 7.4% offset from the profile data.[8] Indeed, given a Gaussian distribution with r.m.s. value σ_x and a bias expressed as a fraction ν of the peak distribution value, a new r.m.s. value $\sigma_{\nu x}$ may be calculated from the distribution obtained by ignoring the part of the Gaussian below the bias-line. Computations yield

$$\frac{\sigma_{\nu x}^2}{\sigma_x^2} = 1 - \frac{4\nu(-\ln\nu)^{3/2}}{3\left(\sqrt{\pi}\,\mathrm{erf}(\sqrt{-\ln\nu}) - 2\nu\sqrt{-\ln\nu}\right)}.$$

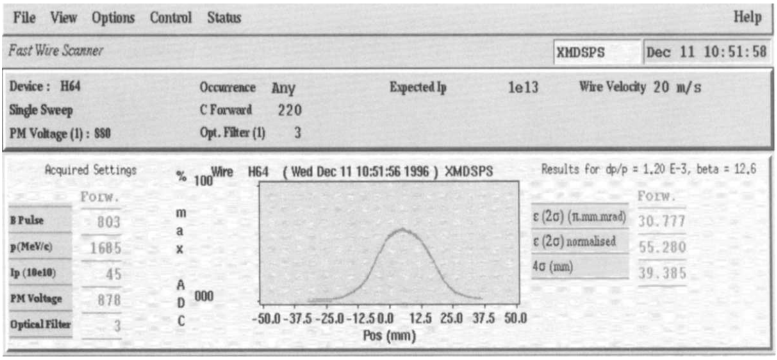

FIGURE 5 Beam emittance measurement using a fast wire scanner in the CERN PS.

Taking $\nu = 0.074$ (i.e. a 7.4% offset) yields $\sigma_{\nu x}/\sigma_x = 0.85$, which gives an r.m.s. value 15% smaller than the true r.m.s. value for negligible bias. Thus, the beam dimensions used for emittance calculations must be corrected by the corresponding multiplying factor.

Like the SEM-grid devices, the transverse beam emittance figures obtained from fast wire scanner instruments are quoted in terms of the r.m.s. value $\sigma_{\nu x}$ of the measured profile (Figure 5).

5 CONCLUSION AND OUTLOOK

Three groups of instruments – SEM-grids, fast wire scanners and Beamscope in the PSB – have been upgraded to cope with the demands of the LHC era. During the test in December 1993 of the PS complex as injector for LHC, a number of emittance measurements have been carried out, comparing the PSB Beamscope, the SEM-grids at the PS entrance, and the PS fast wire scanner. The results are shown in *these Proceedings*, Fig. 4 of Ref. [9]. Horizontal emittance figures given by the Beamscope are systematically smaller than those delivered by the SEM-grids and the fast wire scanner, while vertical emittances given by the Beamscope agree reasonably with those measured by the SEM-grids and the wire scanner. The smaller horizontal Beamscope emittance results are believed to be due to an instrument fault related to the reaction of the RF system to the orbit change in the presence of beam loss. For this reason a test wire scanner has recently been installed in the Booster ring and a 'Blade scanner' is under development at TRIUMF in the frame of the collaboration with CERN. While development of some instruments has been almost halted during the conversion of the PS control system, the increased computing power available now will certainly contribute to further improvements.

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