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Quantum N = 2 Supersymmetric Black Holes in the S-T Model

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ABSTRACT

We consider axion-free quantum corrected black hole solutions in the context of the heterotic S-T model with half the $N = 2, D = 4$ supersymmetries unbroken. We express the perturbatively corrected entropy in terms of the electric and magnetic charges in such a way, that target–space duality invariance is manifest. We also discuss the microscopic origin of particular quantum black hole configurations. We propose a microscopic interpretation in terms of a gas of closed membranes for the instanton corrections to the entropy.

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1 Introduction

Black holes play an important role in string theory, and in recent times there has been considerable progress in the understanding of microscopic and macroscopic properties of supersymmetric black holes in string theory (for a review see [1]).

It is well known from classical general relativity that non-rotating black hole solutions can be parametrized in terms of electric and magnetic charges and the ADM mass, only. In the context of string theory, it has been shown in [2] that four-dimensional non-rotating black hole solutions in the BPS limit depend classically only on the bare quantized charges on the horizon. Thus, the black hole solutions in the BPS limit are, on the horizon, independent of the values of the moduli at spatial infinity. In [3] it has been shown how one can understand this result from a supersymmetric point of view: On the horizon the central charge of the extended supersymmetry algebra acquires a minimal value and thus the extremization of the central charge provides the specific moduli values on the horizon [3, 4]. Moreover, the entropy of certain supersymmetric black hole configurations can depend on additional topological data. In the context of a Calabi–Yau compactification these can be, for instance, the intersection numbers, the second Chern class and the Euler number [5].

Although the BPS limit of black hole solutions in four dimensions with $N \geq 4$ is by now well understood [6], new features of black hole physics arise in four-dimensional $N = 2$ string theory. In particular there exists a large number of different $N = 2$ string vacua so that the extreme black hole solutions depend on the specific details of the particular $N = 2$ string model. Consequently the same features are present for the $N = 2$ entropy formula.

The $N = 2$ central charge and the $N = 2$ BPS mass spectrum can be directly calculated form the $N = 2$ holomorphic prepotential. Therefore the parameters of the prepotential of a given $N = 2$ string model determine the black hole entropy as well as the values of the scalar fields on the horizon.

If one considers four-dimensional $N = 2$ heterotic string compactifications on $K3 \times T_2$ with $N_V + 1$ vector multiplets (including the graviphoton), the classical prepotential is completely universal and corresponds to a scalar non-linear σ-model based on the coset space $\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2,N_V-1)}{SO(2)\times SO(N_V-1)}$. The corresponding classical $N = 2$ black hole entropy and the moduli on the horizon have been computed explicitly in [7, 8].

Since in heterotic $N = 2$ string compactifications the dilaton can be described by a vector multiplet, the heterotic prepotential receives perturbative quantum corrections only at the one-loop level [9, 10]; in addition there are non-perturbative contributions. The perturbative (and non-perturbative) corrections generically split into a cubic polynomial, a constant term and an infinite series of polylogarithmic terms. Thus, quantum black hole solutions are generically determined by an infinite set of integer numbers. Hence, the extremization problem of the quantum corrected $N = 2$ central charge is, in general, difficult to solve. Nevertheless, we will be able to give explicit examples, where all the perturbative quantum corrections are taken into account, and where the extremization problem can still be solved completely.

In [5, 11] a simple implicit formula for the black hole entropy in terms of the heterotic string coupling and the target–space duality invariant inner product of charges has been given, which holds to all orders in perturbation theory. This result is the starting point of the present paper and we will discuss it in the context of the heterotic S-T model.

The paper is organized a follows: In the first section we will briefly introduce the $N = 2$ vector couplings, the $N = 2$ central charge and the related Bekenstein-Hawking entropy in terms of the $N = 2$ prepotential. In section 3 we introduce the heterotic S-T model, its perturbative and non-perturbative quantum corrections and the corresponding transformation laws under perturbative target–space duality. In section 4 we discuss axion-free black holes in the S-T model. We treat most of the cases explicitly in terms of target–space duality invariant combinations of quantized charges. In one case, we also discuss the implicit axion-free black hole entropy in the S-T model including all perturbative and non-perturbative quantum corrections. Then we solve this case for a special weak coupling limit near the line of gauge symmetry enhancement $S = T$ in moduli space. Section 5 is devoted to the 10 and 11 dimensional configurations that yield the black hole solutions upon compactification. Finally in section 6 we propose a microscopic interpretation for the Bekenstein-Hawking entropy in terms of an intersection of M-branes living in a gas of closed membranes. In the last section we summarize our results.

2 N = 2 supergravity and special geometry

The vector couplings of $N = 2$ supersymmetric Yang-Mills theory are encoded in a holomorphic function $F(X)$, where X denotes the complex scalar fields of the vector supermultiplets. With local supersymmetry this function depends on one extra field, in order to incorporate the graviphoton. The theory can then be encoded in terms of a holomorphic function $F(X)$ which is homogeneous of second degree and depends on complex fields X^I with $I = 0, 1, \dots N_V$. Here N_V counts the number of physical vector multiplets.

The resulting *special geometry* [13, 15] can be defined more abstractly in terms of a symplectic section V, also referred to as period vector: a $(2N_V+2)$-dimensional complex symplectic vector, expressed in terms of the holomorphic prepotential F according to

$$V = \begin{pmatrix} X^I \\ F_J \end{pmatrix} , \tag{2.1}$$

where $F_I = \partial F/\partial X^I$. The N_V physical scalar fields of this system parametrize an N_V-dimensional complex hypersurface, defined by the condition that the section satisfies a constraint

$$\langle \bar V, V\rangle \equiv \bar V^{\mathrm{T}}\Omega V = -i, \tag{2.2}$$

with Ω the antisymmetric matrix

$$\Omega = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} . \tag{2.3}$$

The embedding of this hypersurface can be described in terms of N_V complex coordinates z^A $(A=1,\ldots,N_V)$ by letting the X^I be proportional to some holomorphic sections $X^I(z)$ of the complex projective space. In terms of these sections the X^I read

$$X^I = e^{\frac{1}{2}K(z,\bar z)}\, X^I(z) \,, \tag{2.4}$$

where $K(z,\bar z)$ is the Kähler potential, to be introduced below. In order to distinguish the sections $X^I(z)$ from the original quantities X^I, we will always explicitly indicate their z-dependence. The overall factor $\exp[\frac{1}{2}K]$ is chosen such that the constraint (2.2) is satisfied. Furthermore, by virtue of the homogeneity property of $F(X)$, we can extract an overall factor $\exp[\frac{1}{2}K]$ from the symplectic sections (2.1), so that we are left with a holomorphic symplectic section. Clearly this holomorphic section is only defined projectively, i.e. modulo multiplication by an arbitrary holomorphic function. On the Kähler potential these projective transformations act as Kähler transformations, while on the sections V they act as phase transformations.

The resulting geometry for the space of physical scalar fields belonging to vector multiplets of an $N=2$ supergravity theory is a special Kähler geometry, with a Kähler metric $g_{A\bar B} = \partial_A\partial_{\bar B}K(z,\bar z)$ following from a Kähler potential of the special form

$$K(z,\bar z) = -\log\Big(i\bar X^I(\bar z)F_I(X^I(z)) - iX^I(z)\bar F_I(\bar X^I(\bar z))\Big). \tag{2.5}$$

A convenient choice of inhomogeneous coordinates z^A are the *special* coordinates, defined by

$$X^0(z) = 1\,, \quad X^A(z) = z^A\,, \qquad A = 1,\ldots,N_V. \tag{2.6}$$

In this parameterization the Kähler potential can be written as [14]

$$K(z,\bar{z}) = -\log\Big(2(\mathcal{F}+\bar{\mathcal{F}}) - (z^A-\bar{z}^A)(\mathcal{F}_A-\bar{\mathcal{F}}_A)\Big)\,, \tag{2.7}$$

where $\mathcal{F}(z) = i(X^0)^{-2}F(X)$.

We should point out that it is possible to rotate the basis specified by (2.1) by an $Sp(2N_V+2,\mathbf{Z})$ transformation in such a way that it is no longer possible to associate them to a holomorphic function [16]. The supergravity Lagrangian is then expressed entirely in terms of the symplectic section $V=(P^I, iQ_J)^T$, without restricting its parametrization so as to correspond to a prepotential $F(X)$ [16].

The target–space duality group Γ is a certain subgroup of $Sp(2N_V+2,\mathbf{Z})$. Under target–space duality transformations, the period vector V transforms as a symplectic vector:

$$\tilde{X}^I \;=\; U^I{}_J\,X^J + Z^{IJ}\,F_J\,, \qquad\qquad \tilde{F}_I \;=\; V_I{}^J\,F_J + W_{IJ}\,X^J\,, \tag{2.8}$$

where U, V, W and Z are constant, real, $(N_V+1)\times(N_V+1)$ matrices, which have to satisfy the symplectic constraint

$$\mathcal{O}^{-1} = \Omega\,\mathcal{O}^{\mathrm{T}}\,\Omega^{-1} \qquad \text{where} \quad \mathcal{O} = \begin{pmatrix} U & Z \\ W & V \end{pmatrix}\,. \tag{2.9}$$

Finally consider $N=2$ BPS states, whose masses are equal to the central charge Z of the $N=2$ supersymmetry algebra. In terms of the magnetic/electric charges (p^I, q_J) and the period vector $V=(X^I,F_J)^T$ the BPS masses take the following form [16]:

$$M^2_{BPS} = |Z|^2 = e^{K(z,\bar{z})}|q_I X^I(z) - p^I F_I(z)|^2 = e^{K(z,\bar{z})}\,|\mathcal{M}(z)|^2. \tag{2.10}$$

It follows that M^2_{BPS} is invariant under symplectic transformations (2.8).

In the symplectic basis where the symplectic section V is given by $V=(P^I, iQ_J)^T$, the BPS mass takes the following form [16]:

$$M^2_{BPS} = |Z|^2 = e^{K(z,\bar{z})}|M_I P^I(z) + iN^I Q_I(z)|^2 = e^{K(z,\bar{z})}\,|\mathcal{M}(z)|^2. \tag{2.11}$$

We will choose $V=(P^I, iQ_J)^T$ in such a way that the symplectic quantum numbers (N^I, M_J) and the charges (p^I, q_J) are related as follows,

$$N^I \;=\; (p^0, q_1, p^2, \ldots, p^{N_V}), \qquad M_J \;=\; (q_0, -p^1, q_2, \ldots, q_{N_V}) \quad . \tag{2.12}$$

The BPS mass formula (2.10), when evalutated on the horizon of a BPS black hole, also yields its entropy. On the horizon, the moduli fields take their fixed values, and these fixed values can be determined by solving a set of $2N_V+2$ extremisation conditions [3].

In a suitable basis Y, given by $Y^I = \bar{Z} X^I$ [5], these $2N_V + 2$ extremisation equations are then given by

$$Y^I - \bar{Y}^I \;=\; ip^I, \qquad\qquad F_I - \bar{F}_I \;=\; iq_I \quad , \tag{2.13}$$

and the Bekenstein-Hawking entropy reads

$$S_{BH} \;=\; \pi\,|Z|^2_{|\text{fix}} = \pi\, \left(|Y^0|^2 e^{-K(z,\bar{z})}\right)_{|\text{fix}} = i\pi\, \left(\bar{Y}^I F_I(Y^I) - Y^I \bar{F}_I(\bar{Y}^I)\right)_{|\text{fix}} . \tag{2.14}$$

These expressions are valid on the horizon or as double extreme black holes [3]. For a discussion of more general black holes, where one replaces the charges by harmonic functions, see [12].

3 The S-T model

3.1 General formulae

In the following, we will focus on the two parameter model [17, 19, 21, 24] based on a type IIA compactification on a Calabi–Yau space given by a degree 12 hypersurface in the weighted projective space $\mathbf{P}^4_{(1,1,2,2,6)}$ with Hodge numbers $(h^{1,1}, h^{2,1}) = (2, 128)$ and Euler number $\chi = 2(h^{1,1} - h^{2,1}) = -252$. On the type II side, the vector multiplet prepotential is given by [22, 21, 24]

$$\begin{aligned} \mathcal{F}_{II} \;=\;& -t_1(t_2)^2 - \frac{2}{3}(t_2)^3 - c + \frac{1}{8\pi^3} \sum_{j\geq 0, k\geq 1} n_{k,j} Li_3(e^{-2\pi(jt_1 + kt_2)}) \\ & + \frac{1}{8\pi^3} n_{0,1} Li_3(e^{-2\pi t_1}) \quad , \end{aligned} \tag{3.1}$$

where $c = \frac{\chi\zeta(3)}{16\pi^3}$. Here, $t^1 = iz^1$ and $t^2 = iz^2$ denote the two coordinates of the Kähler cone. The instanton numbers $n_{k,j}$ can be found in [22, 24]. Note that $n_{0,1} = 2$ as well as $n_{k,j} \geq 0$.

This model has a dual description [17] in terms of a certain compactification of the heterotic $E_8 \times E_8$ string on first a torus T_2 and then on $K3$. This is the so-called heterotic S-T model with

$$S \;=\; -iz^1, \qquad\qquad T \;=\; -iz^2. \tag{3.2}$$

The dilaton S is related to the tree-level coupling constant and to the theta angle by $S = 4\pi/g^2 - i\theta/2\pi$.

In order to relate the type II description to its dual heterotic description, the type II coordinates t_1 and t_2 must be mapped to the heterotic coordinates S and T. Based on the physical requirement that the non-perturbative duality transformations should preserve the positivity of Re S, it has been argued in [19, 21] that the correct identification is given by

$$t_1 = S - T \quad , \quad t_2 = T \quad . \tag{3.3}$$

In the following, we will take this to be the correct identification. Thus, in the chamber Re $S >$ Re T, the heterotic prepotential[2] reads for $T > 1$

$$\mathcal{F}_{\text{het}} = -ST^2 - \alpha T^3 - c + \frac{1}{8\pi^3} \sum_{j\geq 0, k\geq 1} n_{k,j} Li_3(e^{-2\pi(jS+(k-j)T)}) - \frac{\beta}{4\pi^3} Li_3(e^{-2\pi(S-T)}), \tag{3.4}$$

where $\alpha = -\frac{1}{3}$ and $\beta = -\frac{1}{2} n_{0,1} = -1$. The S-T model possesses an $S \leftrightarrow T$ exchange symmetry [19], which is reflected in the instanton coefficients which satisfy $n_{k,j} = n_{k,k-j}$ [22].

At $S = T$, there is a genuine gauge symmetry enhancement [20]. A $U(1)$ group gets enhanced to an $SU(2)$ and four additional hypermultiplets become massless at this point in moduli space. Three of them belong to the adjoint representation of $SU(2)$. The $SU(2)$ can then be completely higgsed away. On the type II side this amounts to an extremal transition to a Calabi–Yau threefold with Hodge numbers $(h^{1,1}, h^{2,1}) = (1, 129)$ and Euler number $\tilde{\chi} = -256$ [20].

In the standard perturbative regime $S \to \infty$ with T finite, the heterotic prepotential is given by

$$\mathcal{F}_{\text{het}} = -ST^2 - \alpha T^3 - c - \frac{1}{4\pi^3} \sum_{k\geq 1} c(k) Li_3(e^{-2\pi kT}) \quad , \tag{3.5}$$

whith $n_{k,0} = -2c(k) > 0$. Note that at $T \approx 1$, $\partial_T^2 \mathcal{F}_{\text{het}}$ develops a singularity proportional to $\log(T-1)$ [18]. In the vicinity of the wall $S = T \to \infty$, on the other hand, it follows from

$$\begin{aligned} Li_3(e^{-x}) &= p(x) + q(x) \log x \quad , \quad x \to 0 \quad , \\ p(x) &= \zeta(3) - \frac{\pi^2}{6} x + \frac{3}{4} x^2 + \mathcal{O}(x^3) \quad , \\ q(x) &= -\frac{1}{2} x^2 + \mathcal{O}(x^3) \end{aligned} \tag{3.6}$$

that

$$\mathcal{F}_{\text{het}} = -ST^2 - \alpha T^3 - \tilde{c} + \frac{\beta}{2\pi} (S-T)^2 \log(S-T) \quad , \tag{3.7}$$

[2]In the following we will only specify the prepotential in this particular chamber.

where $\tilde{c} = c + \frac{\beta\zeta(3)}{4\pi^3} = \frac{\tilde{\chi}\zeta(3)}{16\pi^3}$. Here, we have also omitted terms which are linear and quadratic in $(S-T)$.

Finally, consider writing (3.4) as

$$\mathcal{F}_{\rm het} = -ST^2 + f(S,T). \tag{3.8}$$

Here $f(S,T)$ encodes all perturbative and non-perturbative quantum corrections and may be expanded in powers of $e^{-2\pi S}$, as follows [23, 24, 25]

$$f(S,T) = \sum_{k=0}^{\infty} f_k(T)\, e^{-2\pi k S}, \tag{3.9}$$

where $f_0(T) \equiv h(T)$ encodes all the perturbative quantum corrections in the standard weak coupling limit $S \to \infty$. It follows that the prepotential $F(Y) = -i(Y^0)^2\mathcal{F}_{\rm het}$ and its periods $F_I(Y)$ are given by

$$\begin{aligned}
F(Y) &= -i(Y^0)^2\left[-ST^2 + f(S,T)\right] \quad , \\
F_0 &= iY^0\left[-ST^2 - 2f + Tf_T + Sf_S\right] \quad , \\
F_1 &= Y^0\left[T^2 - f_S\right] \quad , \\
F_2 &= Y^0\left[2ST - f_T\right] \quad .
\end{aligned} \tag{3.10}$$

In special coordinates, the associated Kähler potential reads

$$K(S,\bar{S},T,\bar{T}) = -\log(S+\bar{S}+\Delta) - \log(T+\bar{T})^2. \tag{3.11}$$

Here, Δ contains perturbative and non-perturbative corrections and is defined as follows:

$$\Delta(S,\bar{S},T,\bar{T}) = \frac{2(f+\bar{f}) - (T+\bar{T})(f_T+\bar{f}_{\bar{T}}) - (S+\bar{S})(f_S+\bar{f}_{\bar{S}})}{(T+\bar{T})^2} \quad . \tag{3.12}$$

In the standard weak coupling limit these corrections reduce to the Green–Schwarz term [9]

$$\lim_{S\to\infty}\Delta(S,\bar{S},T,\bar{T}) = V_{GS}(T,\bar{T}) = \frac{2(h+\bar{h}) - (T+\bar{T})(h_T+\bar{h}_{\bar{T}})}{(T+\bar{T})^2}, \tag{3.13}$$

and in the classical limit these quantum corrections vanish (by definition). The true target–space duality invariant perturbative string coupling constant is given by [9]

$$\frac{8\pi}{g^2_{pert}} = S + \bar{S} + V_{GS}(T,\bar{T})\ . \tag{3.14}$$

3.2 Perturbative target–space duality transformations

The target–space duality group Γ is a certain subgroup of $Sp(6,\mathbf{Z})$. At the perturbative level, these duality transformations amount to $PSL(2,\mathbf{Z})_T$ transformations of the modulus T, which are generated by $T \rightarrow T+i$ and $T \rightarrow 1/T$. The latter transformation will be of special interest in the following.

Consider the perturbative BPS mass

$$M^2_{BPS} = |Z|^2 = e^K |q_I X^I - p^I F_I|^2 = e^K |M_I P^I + N^I i Q_I|^2 \quad , \tag{3.15}$$

where the section $V = (P^I, iQ_J)^T$ is given by

$$\begin{aligned} V &= (P^I, iQ_J)^T = (1, T^2, iT, i(ST^2 + 2h - Th_T), iS, -2ST + h_T)^T \quad , \\ h &= -\alpha T^3 - c - \frac{1}{4\pi^3} \sum_{k \geq 1} c(k) Li_3(e^{-2\pi k T}) \quad , \end{aligned} \tag{3.16}$$

and where

$$\begin{aligned} M_I &= (q_0, -p^1, q_2) \quad , \\ N^I &= (p^0, q_1, p^2) \quad . \end{aligned} \tag{3.17}$$

The duality transformation $T \rightarrow 1/T$ acts as follows [21, 24] on the section V given in (3.16)

$$V \rightarrow \mathbf{S}_1 V \quad , \quad \mathbf{S}_1 = \begin{pmatrix} U & Z \\ W & V \end{pmatrix} \quad , \quad U = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

$$Z = 0 \ , \ W = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad V = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad . \tag{3.18}$$

It follows from (3.18) that [21, 24]

$$\begin{aligned} S &\rightarrow S - i + \frac{1}{T^2}(2h - Th_T + i) \quad , \\ h &\rightarrow \frac{h}{T^4} + \frac{i}{2T^4} - \frac{i}{T^2} + \frac{i}{2} \quad , \\ h_T &\rightarrow -\frac{h_T}{T^2} + \frac{4h}{T^3} + \frac{2i}{T^3} - \frac{2i}{T} \quad . \end{aligned} \tag{3.19}$$

Next, consider taking T to be real. In the region Re $T > 1$, both h and h_T are real. Then, it follows from (3.19) that

$$\begin{aligned}
\text{Re } S &\rightarrow \text{Re } S + \frac{1}{T^2}(2h - T h_T) \quad , \\
\text{Re } h &\rightarrow \frac{\text{Re } h}{T^4} \quad , \\
\text{Re } h_T &\rightarrow -\frac{\text{Re } h_T}{T^2} + \frac{4\text{Re } h}{T^3} \quad , \\
\frac{\text{Re }(h - T h_T)}{T^2} &\rightarrow \frac{\text{Re }(h - T h_T)}{T^2} - \frac{2\text{Re }(2h - T h_T)}{T^2} \quad .
\end{aligned} \tag{3.20}$$

In the region Re $T < 1$, on the other hand, both h and h_T acquire imaginary parts, as can be seen from (3.19).

The charges (M_I, N^J) transform as follows under (3.18)

$$M \rightarrow U^{T,-1} M - W N \quad , \quad N \rightarrow U N \quad . \tag{3.21}$$

This should be contrasted with the classical transformation law, which follows from (3.21) by setting $W = 0$.

It will turn out to be convenient to introduce the $O(2,1)$ scalar product [8]

$$\langle N, N \rangle = (N^2)^2 + N^0 N^1 = (p^2)^2 + p^0 q_1 \tag{3.22}$$

Note that $\langle N, N \rangle$ is invariant under both classical and perturbative target space duality transformations [8].

The perturbative entropy of $N = 2$ supersymmetric quantum black holes in the BPS limit is, in target–space duality invariant form, given as follows [5]

$$S_{BH} = \frac{8\pi^2}{g^2_{pert}{}_{|\text{fix}}} \langle N, N \rangle, \tag{3.23}$$

with g_{pert} defined in (3.14) and with the fields taking their fixed values on the horizon.

4 Axion-free black holes in the S-T model

In this section, we will compute the entropy for certain classes of BPS black hole solutions. We will take T to be real in the following. Moreover, we will first consider the region Re $S >$ Re T with Re $T > 1$. Then, it is possible to have perturbative axion-free solutions in this region of moduli space. Axion-free solutions are solutions with Re $z^A = 0$, that is, Im $S =$ Im $T = 0$. In the region Re $T < 1$, on the other hand, it is not any longer possible to set Im $S = 0$, as can be seen from (3.19).

For the axion-free solutions in the region Re $T > 1$, the extremisation conditions (2.13) yield (with $z^A = Y^A/Y^0$)

$$Y^0 = \frac{1}{2}(\lambda + ip^0) \quad , \quad z^A \lambda = ip^A \quad , \quad F_I - \bar{F}_I = iq_I \quad , \tag{4.1}$$

where $\lambda = Y^0 + \bar{Y}^0$. Thus one can consider three different cases: (i) $\lambda \neq 0, p^0 \neq 0$, (ii) $\lambda = 0, p^0 \neq 0$ and (iii) $\lambda \neq 0, p^0 = 0$. We will discuss each of these cases in the following.

4.1 The axion-free S-T black hole with $\lambda \neq 0, p^0 \neq 0$

In this subsection, we will be interested in perturbative axion-free black hole solutions in the region $S \gg T > 1$ with $\lambda \neq 0, p^0 \neq 0$. The extremisation conditions (4.1) then yield

$$\frac{1}{\lambda^2} \;=\; \frac{q_1}{p^0(p^2)^2}, \qquad S \;=\; \frac{p^1}{\lambda} = \frac{p^1}{p^2}\sqrt{\frac{q_1}{p^0}}, \qquad T \;=\; \frac{p^2}{\lambda} = \sqrt{\frac{q_1}{p^0}} \tag{4.2}$$

as well as

$$q_0 \;=\; -\frac{p^1 q_1}{p^0} - 2\lambda h + \lambda T h_T, \qquad\qquad q_2 \;=\; 2\,\frac{p^1 q_1}{p^2} - p^0 h_T \quad , \tag{4.3}$$

with h given in (3.16). For real T, h and h_T are also real. Solving (4.3) for h and h_T yields

$$\begin{aligned}
h_T \;&=\; 2\frac{p^1 q_1}{p^0 p^2} - \frac{q_2}{p^0} \quad , \\
h \;&=\; \frac{1}{2\lambda p^0}\left(p^1 q_1 - p^2 q_2 - p^0 q_0\right) \quad , \\
2h - T h_T \;&=\; -\frac{1}{\lambda p^0}\left(p^1 q_1 + p^0 q_0\right) \quad , \\
h - T h_T \;&=\; \frac{1}{2\lambda p^0}\left(-3p^1 q_1 + p^2 q_2 - p^0 q_0\right) \quad .
\end{aligned} \tag{4.4}$$

Note that (4.4) relates infinite sums over polylogarithmic functions (appearing on the left hand side) to simple expressions on the right hand side. Moreover, it is possible to determine the parameter λ completely in terms of the charges, because the perturbative quantum corrections are independent of the dilaton ($\frac{\partial}{\partial S} h(T) = 0$). Including non-perturbative corrections encoded in $f(S,T)$ destroys this property of the black hole solution. In this more general case λ remains an undetermined parameter.

Let us now check the target–space duality transformation properties of (4.4) under $p^0 \leftrightarrow q_1$, that is, under $T \to 1/T$. It follows from (3.20) that the left hand side of (4.4) transforms as

$$\begin{aligned} \mathrm{Re}\, h &\rightarrow \frac{p^0}{2q_1^2\lambda}(p^1 q_1 - p^2 q_2 - p^0 q_0) \quad , \\ \mathrm{Re}\, h_T &\rightarrow -\frac{q_2}{q_1} - 2\frac{p^0 q_0}{q_1 p^2} \quad . \end{aligned} \tag{4.5}$$

The right hand side of (4.4) transforms in the same way, provided the electric and magnetic charges transform as follows:

$$p^0 \leftrightarrow q_1, \quad p^2 \rightarrow p^2, \quad q_0 \leftrightarrow -p^1, \quad q_2 \rightarrow q_2. \tag{4.6}$$

Note that these are the classical transformation laws associated to $T \rightarrow 1/T$ (cf. eq. (3.21)). Similarly, it follows from (3.20) that S transforms as

$$\mathrm{Re}\, S \rightarrow -\frac{q_0}{p^2}\sqrt{\frac{p^0}{q_1}} \quad , \tag{4.7}$$

which is also consistent with the transformation behaviour of $S = p^1/\lambda$ under (4.6). Note that in the classical limit the dilaton is only invariant under target–space duality transformations if the additional charge constraints, given by (4.3), are taken into account.

The perturbative entropy is then given by (3.23) with

$$\frac{8\pi}{g^2_{pert}}_{|\mathrm{fix}} = \frac{1}{2}\sqrt{\frac{q_1}{p^0}}\left(\frac{p^1}{p^2} + \frac{q_2}{q_1} - \frac{p^0 q_0}{q_1 p^2}\right) \quad . \tag{4.8}$$

In the classical limit, on the other hand, we find, for the dilaton on the horizon, that

$$\frac{4\pi}{g^2}_{|\mathrm{fix}} = \frac{p^1}{p^2}\sqrt{\frac{q_1}{p^0}} \tag{4.9}$$

as well as the classical duality invariant charge constraints $p^1 q_1 = -p^0 q_0 = \frac{1}{2}p^2 q_2$, which follow from (4.3).

Note that (4.8) was computed in the region Re $S >$ Re $T > 1$. It is easy to check that the perturbative string coupling constant (4.8), given in terms of the bare charges on the horizon, is invariant under the classical target–space duality transformations (4.6) of the charges. Thus, the entropy formula

$$S_{BH} = \frac{\pi}{2}\sqrt{\frac{q_1}{p^0}}\left(\frac{p^1}{p^2} + \frac{q_2}{q_1} - \frac{p^0 q_0}{q_1 p^2}\right)\left((p^2)^2 + p^0 q_1\right) \tag{4.10}$$

actually holds in the entire chamber Re $S \gg$ Re T. Note that the entropy varies smoothly across the point $T = 1$, where $p^0 = q_1$.

4.2 The axion-free S-T black hole with $\lambda = 0, p^0 \neq 0$

Here, we will be interested in perturbative axion-free black hole solutions in the region $S \gg T > 1$ with $\lambda = 0, p^0 \neq 0$. The extremisation conditions (4.1) then yield $p^A = 0, q_0 = 0$ and

$$\langle N, N\rangle = p^0 q_1 \ , \qquad T = \sqrt{\frac{q_1}{p^0}} \ , \qquad S \ = \ \frac{1}{2}\frac{q_2}{\sqrt{p^0 q_1}} + \frac{1}{2}\sqrt{\frac{p^0}{q_1}}\, h_T \ , \tag{4.11}$$

with h given in (3.16). Under $T \to 1/T$ we have again $p^0 \leftrightarrow q^1$ and $q_2 \to q_2$ as in (4.6). Moreover (3.20) also holds for this solution.

The perturbative string coupling constant on the horizon is now given by

$$\frac{8\pi}{g^2_{pert\,|\text{fix}}} \ = \ \frac{p^0}{q_1}\left(\sqrt{\frac{q_1}{p^0}}\frac{q_2}{p^0} + \text{Re}\ h(\sqrt{q_1/p^0})\right) \ . \tag{4.12}$$

It is easy to check that the perturbative string coupling constant (4.12) is indeed invariant under the classical target–space duality transformations (4.6) of the charges. It follows that the perturbative entropy formula

$$S_{BH} = \pi\ \left(\sqrt{p^0 q_1 q_2^2} + (p^0)^2 \text{Re}\ h(\sqrt{q_1/p^0})\right) \tag{4.13}$$

holds in the entire chamber Re $S \gg$ Re T. Note again that the entropy (4.13) varies smoothly across the point $T = 1$, where $p^0 = q_1$.

In the classical limit the string coupling constant on the horizon and the classical entropy have the following form:

$$\frac{8\pi}{g^2}_{|\text{fix}} = \sqrt{\frac{p^0}{q_1}\frac{q_2}{p^0}} \ , \quad S_{BH}^{class} \ = \ \pi\ \sqrt{p^0 q_1 q_2^2} \ . \tag{4.14}$$

4.2.1 The entropy in the limit $S \approx T \to 0$

In the strong coupling limit $S \approx T \to 0$ the heterotic prepotential is given by $f(S,T)$ only. In particular we find

$$\mathcal{F}_{\text{het}} \ = \ f(0,0) \ = \ \frac{1}{8\pi^3}\zeta(3)\sum_{j,k\geq 0} n_{k,j} \tag{4.15}$$

with $n_{0,0} = -8\pi^3 c - 2\beta$. Since the sum is divergent, this expression is only to be understood in an asymptotic sense. For vanishing S and T the prepotential would diverge, because of this infinite sum. In this limit, the entropy is then given by

$$S_{BH} = \pi\ \left(|Y^0|^2 e^{-K}\right)_{|\text{fix}} = (p^0)^2 \frac{1}{8\pi^2}\zeta(3)\sum_{j,k\geq 0} n_{k,j} \ . \tag{4.16}$$

In section 6, we will discuss a microscopic interpretation for the entropy (4.16).

4.3 The axion-free S-T black hole with $\lambda \neq 0, p^0 = 0$

Next, we will be interested in perturbative axion-free black hole solutions in the region $S > T$ with $\lambda \neq 0, p^0 = 0$. More precisely, we will discuss the general standard weak coupling limit, the general black hole solution including non-perturbative corrections and a special weak coupling limit near $S = T$. This case is analogous to one studied in the context of the S-T-U model, where the fixed points of T and U had to be taken to lay near the wall $T = U$ of perturbative gauge symmetry enhancement [28]. In the case of the S-T model, there is a genuine gauge symmetry enhancement on the wall $S = T$ [20]. We will use the $S \leftrightarrow T$ exchange symmetry of the model in order to determine the entropy in the two chambers $\mathrm{Re}\, S > \mathrm{Re}\, T$ and $\mathrm{Re}\, S < \mathrm{Re}\, T$ near the wall $S = T$.

Recall that, in the chamber $\mathrm{Re}\, S > \mathrm{Re}\, T$, the heterotic prepotential is given by $\mathcal{F}_{\rm het} = -ST^2 + f(S,T)$ with

$$f(S,T) \;=\; -\alpha T^3 - c + \frac{1}{8\pi^3} \sum_{j\geq 0, k\geq 1} n_{k,j} Li_3(e^{-2\pi(jS+(k-j)T)}) - \frac{\beta}{4\pi^3} Li_3(e^{-2\pi(S-T)}). \tag{4.17}$$

For the case $\lambda \neq 0, p^0 = 0$, the extremisation conditions (4.1) then yield

$$q_A \;=\; 0 \quad , \quad Y^A = i\frac{p^A}{2} \quad , \quad Y^0 = \frac{\lambda}{2} \quad , \quad z^A = i\frac{p^A}{\lambda} \quad . \tag{4.18}$$

The parameter λ is determined (in general implicitly) by the constraint

$$i q_0 \;=\; 4\frac{\partial}{\partial\lambda} F(\lambda, p^A) \quad . \tag{4.19}$$

4.3.1 Standard weak coupling limit

In the standard weak coupling limit $S = p^1/\lambda \rightarrow \infty$ with arbitrary but finite $T = p^2/\lambda$, and consequently $f(S,T) \rightarrow h(T)$, the entropy is given by (3.23) with

$$\frac{8\pi}{g^2_{pert}}_{|\rm fix} \;=\; 2\frac{p^1}{\lambda} + \frac{\lambda^2}{(p^2)^2} h(p^2/\lambda) - \frac{\lambda}{p^2} h_T(p^2/\lambda) \quad . \tag{4.20}$$

Using (3.20) it is easy to show that the perturbative string coupling constant on the horizon (4.20) is invariant under target–space duality transformations $T \rightarrow 1/T$ with

$$\lambda \rightarrow (p^2)^2/\lambda, \qquad p^2 \rightarrow p^2, \qquad \frac{p^1}{\lambda} \rightarrow \frac{p^1}{\lambda} + \frac{\lambda^2}{(p^2)^2}(2h - \frac{p^2}{\lambda} h_T). \tag{4.21}$$

In the classical limit these transformations reduce to $p^1 \leftrightarrow q_0$ and $p^2 \rightarrow p^2$. The string coupling constant has the following fixed value in terms of the charges on the horizon:

$$\frac{4\pi}{g^2}_{|\rm fix} \;=\; \frac{p^1}{\lambda}, \qquad \lambda \;=\; \sqrt{-\frac{p^1 (p^2)^2}{q_0}}. \tag{4.22}$$

Thus, the classical entropy of the black hole in terms of the charges is ($q_0 < 0$)

$$S_{BH}^{class} = \pi \sqrt{|q_0| p^1 (p^2)^2}. \tag{4.23}$$

4.3.2 More general axion-free quantum black holes

Let us now consider more general quantum corrected black hole solutions given in terms of $f(S,T)$ and of

$$\begin{aligned} f_S(S,T) &= -\frac{1}{4\pi^2} \sum_{j\geq 0, k\geq 1} n_{k,j} j Li_2(e^{-2\pi(jS+(k-j)T)}) + \frac{\beta}{2\pi^2} Li_2(e^{-2\pi(S-T)}) \quad , \\ f_T(S,T) &= -3\alpha T^2 - \frac{1}{4\pi^2} \sum_{j\geq 0, k\geq 1} n_{k,j}(k-j) Li_2(e^{-2\pi(jS+(k-j)T)}) - \frac{\beta}{2\pi^2} Li_2(e^{-2\pi(S-T)}). \end{aligned} \tag{4.24}$$

The corresponding general axion-free black hole entropy in the S-T model is then given by

$$S_{BH} = \pi \left(|Y^0|^2 e^{-K}\right)_{|\text{fix}} = 4\pi |Y^0|^2 \left(2ST^2 + f - T f_T - S f_S\right)_{|\text{fix}} \quad . \tag{4.25}$$

This entropy contains all perturbative and non-perturbative quantum corrections encoded in $f(S,T)$ and represents the general axion-free entropy in the S-T model. For the case considered here we have $Y^0 = \lambda/2$, $S = p^1/\lambda$ and $T = p^2/\lambda$, and the parameter λ is subject to the constraint

$$q_0 = -\frac{p^1 (p^2)^2}{\lambda^2} - 2\lambda f - \lambda^2 \frac{\partial}{\partial\lambda} f \tag{4.26}$$

with

$$\begin{aligned} \frac{\partial}{\partial\lambda} f &= 3\alpha \frac{(p^2)^3}{\lambda^4} - \frac{\beta}{2\pi^2} \frac{p^1 - p^2}{\lambda^2} Li_2(e^{-2\pi(p^1-p^2)/\lambda}) \\ &\quad + \frac{1}{4\pi^2\lambda^2} \sum_{j\geq 0, k\geq 1} n_{k,j} \left(j p^1 + (k-j) p^2\right) Li_2(e^{-2\pi(jp^1+(k-j)p^2)/\lambda}). \end{aligned} \tag{4.27}$$

The constraint (4.26) can be solved for a special weak coupling limit as we will show next.

4.3.3 Special weak coupling limit

For the present case $Y^0 - \bar{Y}^0 = 0$, using (2.14), the general axion-free entropy can be brought into the following form:

$$S_{BH} = \frac{\pi}{2}\Big(-\lambda q_0 + 3\left(\frac{p^1(p^2)^2}{\lambda} + \alpha \frac{(p^2)^3}{\lambda}\right) - \frac{\beta\lambda}{2\pi^2}(p^1 - p^2) Li_2(e^{-2\pi(p^1-p^2)/\lambda})$$

$$+\frac{\lambda}{4\pi^2}\sum_{j\geq 0,k\geq 1} n_{k,j}\left((k-j)p^2+jp^1\right) Li_2(e^{-2\pi((k-j)p^2+jp^1)/\lambda})\Big) \quad . \tag{4.28}$$

In the perturbative regime $S > T \to \infty, S-T \approx 0$, that is, in the vicinity of the wall $S=T$ the constraint (4.26) can be solved approximately:

$$\begin{aligned} q_0\lambda^2 &= -p^1(p^2)^2-\alpha(p^2)^3+\lambda^3\left(2c+\frac{\beta\zeta(3)}{2\pi^3}\right)-\frac{\beta}{12}(p^1-p^2)\lambda^2-\frac{\beta}{2\pi}(p^1-p^2)^2\lambda \\ &+ \quad \cdots \quad . \end{aligned} \tag{4.29}$$

Here we expanded in $x=2\pi(p^1-p^2)/\lambda$ around $x=0$ using

$$\begin{aligned} Li_3(e^{-x}) &= \zeta(3)-\frac{\pi^2}{6}x+\left(\frac{3}{4}-\frac{\log x}{2}\right)x^2+\mathcal{O}(x^3), \\ Li_2(e^{-x}) &= \frac{\pi^2}{6}+(\log x-1)\,x+\frac{1}{2}x^2+\mathcal{O}(x^3). \end{aligned} \tag{4.30}$$

Note that the logarithmic contributions from the polylogarithms cancel against each other in (4.29). Using that $|p^1-p^2| \ll |p^A| \ll |q_0|$, one can solve (4.29) in terms of the following power series expansion

$$\lambda=\sum_{i=1}^{\infty}\frac{\gamma_i}{(\sqrt{q_0})^i}=\frac{\gamma_1}{\sqrt{q_0}}+\frac{\gamma_2}{q_0}+\ldots \quad . \tag{4.31}$$

Inserting (4.31) into (4.29) and comparing terms yields

$$\gamma_1^2 = -p^1(p^2)^2-\alpha(p^2)^3, \qquad \gamma_2 = \frac{\beta}{4\pi}(p^1-p^2)^2 \quad . \tag{4.32}$$

Choosing again $q_0<0, p^A>0$ it follows that

$$\lambda=\sqrt{\frac{-p^1(p^2)^2-\alpha(p^2)^3}{q_0}}+\frac{\beta}{4\pi}\frac{(p^1-p^2)^2}{q_0}+\cdots \quad . \tag{4.33}$$

In the limit $S > T \to \infty, S-T \approx 0$, the only polylog term contributing to the corresponding quantum corrected entropy is the term $Li_2(e^{-2\pi(S-T)}) \approx 2\pi(S-T)\log 2\pi(S-T)$. It follows that in this limit the entropy can be written as

$$S_{BH}=2\pi\sqrt{|q_0|\,(p^1(p^2)^2+\alpha(p^2)^3)}-\frac{\beta}{4}(p^1-p^2)^2\log\left(\frac{|q_0|(p^1-p^2)^2}{(p^1(p^2)^2+\alpha(p^2)^3)}\right)+\cdots \tag{4.34}$$

Equation (4.34) gives the entropy in the chamber Re $S >$ ReT near the wall ReS = ReT. By utilising the $S \leftrightarrow T$ exchange symmetry of the model, it follows that in the chamber Re $T >$ ReS the entropy near the wall ReT = ReS is given by (4.34) with $p^1 \leftrightarrow p^2$. Note, in particular, that the entropy is finite on the wall $S=T$ and that it varies continuously across the wall $S=T$. A similar effect, which we will briefly describe next, also occurs in 5 dimensions when considering the entropy density of the associated black string.

4.4 The entropy density for the associated 5 dimensional black string

Consider the S-T model with the following prepotential (in the chamber $S > T$)

$$\mathcal{F} = -\left(ST^2 + \alpha T^3\right) \quad , \quad \alpha = -\frac{1}{3} \quad . \tag{4.35}$$

Let R_5 denote the radius of the circle of the compactified 5-th dimension. Then [26]

$$S = R_5 s \quad , \quad T = R_5 t \quad , \tag{4.36}$$

where s and t denote the two moduli fields in 5 dimensions. It follows that

$$\begin{aligned} \mathcal{F} &= -R_5^3\, \mathcal{V} \quad , \\ \mathcal{V} &= d_{\Lambda\Delta\Sigma} t^\Lambda t^\Delta t^\Sigma = st^2 + \alpha t^3 \quad , \quad t^\Lambda = s, t \quad , \end{aligned} \tag{4.37}$$

where $\mathcal{V}$ denotes the prepotential of real special geometry in 5 dimensions. It has to satisfy the additional constraint [26]

$$\mathcal{V} = 1 \quad . \tag{4.38}$$

Consider a black string carrying charges p^1 and p^2. In the M-theory picture these charges are magnetic and carried by two 5-branes, whereas in the heterotic picture p^1 is the charge of the fundamental string, i.e. electric, and p^2 comes from a 5-brane that has been identified with a KK-monopole (see next section). The associated magnetic central charge is given by [26]

$$Z_m = -t_\Lambda p^\Lambda = -\left(p^1 t^2 + p^2(2st + 3\alpha t^2)\right) \quad , \tag{4.39}$$

with $t_\Lambda = d_{\Lambda\Delta\Sigma} t^\Delta t^\Sigma$. It can be derived from the central charge in 4 dimensions, as follows. The 4 dimensional central charge reads

$$Z^{4D} = e^{K/2}\mathcal{M} \quad , \quad \mathcal{M} = q_I X^I - p^I F_I \quad . \tag{4.40}$$

For the case at hand, $p^0 = 0, q_A = 0$, so that

$$\mathcal{M} = R_5^2 \left(\frac{q_0}{R_5^2} - p^1 t^2 - p^2(2st + 3\alpha t^2)\right) \quad . \tag{4.41}$$

The magnetic central charge Z_m in 5 dimensions is related to the 4 dimensional central charge in the following way

$$\begin{aligned} Z_m &= \lim_{R_5 \to \infty} (R_5)^{-1/2}\, Z^{4D} \\ &= \lim_{R_5 \to \infty} (R_5)^{3/2}\, e^{K/2} \left(\frac{q_0}{R_5^2} - p^1 t^2 - p^2(2st + 3\alpha t^2)\right) . \end{aligned} \tag{4.42}$$

Using that $K = -\log(2R_5)^3 - \log \mathcal{V} = -\log(2R_5)^3$, it follows that

$$Z_m = \lim_{R_5\to\infty} (R_5)^{-1/2} Z^{4D} = -p^1 t^2 - p^2(2st + 3\alpha t^2) \quad , \tag{4.43}$$

up to an overall constant factor. This is in accordance with (4.39).

Inserting the constraint (4.38) into (4.39) yields

$$Z_m = -\left(t^2(p^1 + \alpha p^2) + \frac{2p^2}{t} \right) \quad . \tag{4.44}$$

According to [3], the entropy density can be obtained by solving the extremization condition

$$\frac{\partial}{\partial t} Z_m = 0 \quad . \tag{4.45}$$

The extremisation condition (4.45) yields

$$t^3 = \frac{p^2}{p^1 + \alpha p^2} \quad . \tag{4.46}$$

Inserting (4.46) into (4.39) yields the magnetic central charge at the fixed point

$$Z_m|_{\text{fix}} = -3\left((p^2)^2(p^1 + \alpha p^2)\right)^{1/3} = -3\left(d_{\Lambda\Delta\Sigma} p^\Lambda p^\Delta p^\Sigma\right)^{1/3} \quad . \tag{4.47}$$

The 5 dimensional entropy density[3] is then given by

$$S^{5D}_{BH} \propto Z^2_m|_{\text{fix}} \propto \left(d_{\Lambda\Delta\Sigma} p^\Lambda p^\Delta p^\Sigma\right)^{2/3} \quad . \tag{4.48}$$

This is the entropy density in the chamber $s > t$. In the chamber $t > s$, on the other hand, (4.48) holds with $p^1 \leftrightarrow p^2$. Hence, it follows that

$$S^{5D}_{BH} \propto \left(p^1(p^2)^2 + \alpha(p^2)^3\right)^{2/3} \theta(s-t) + \left(p^2(p^1)^2 + \alpha(p^1)^3\right)^{2/3} \theta(t-s) \quad . \tag{4.49}$$

5 The heterotic and type II solutions

In the previous sections we computed the entropy, by solving a set of extremisation conditions, for certain classes of black hole solutions. In this section, we will describe the corresponding black hole and black string solutions. On the heterotic side, we have pure Neveu-Schwarz (NS) solutions, whereas on the type II side they represent intersections of D- or M-branes living in a gas of closed type II strings or closed M-2-branes.

[3]Note that, for extended objects, one usually considers densities instead of total quantities. In analogy to the BPS mass density it is reasonable to discuss the entropy density here.

The general S-T model allows 6 non-vanishing charges. The restriction to the axion-free case gives two constraints, given in equation (4.3). Thus, in the axion-free case we have only 4 independent charges. In the case (i) we kept all 4 charges. The other cases (ii) and (iii) are the least charge configuration where we turned off q_0 or p^0. These cases are especially interesting. In 11 dimensions the case (ii) describes an intersection of membranes and (iii) an intersection of 5-branes. In 4 dimensions both solutions are S-dual to one another. The general configuration, where we keep the charges q_0 and p^0 non-vanishing, describes an interpolation between these least charged solution. We will in the following focus on the case (iii), although on the type II side we will present some speculations about the general solution (case (i)).

We begin with a discussion of the heterotic solutions. On the heterotic side we can only give a microscopic interpretation to the classical solutions. For simplicity we will restrict ourselves to the special case where $p^0 = 0$. In this case it follows from (4.34) that the classical entropy is given by

$$S_{BH}^{class} = 2\pi\sqrt{|q_0\, p^1\, p_2^2|} \quad . \tag{5.1}$$

Note that, on the heterotic side, p^1 is an electric charge. The corresponding solution in 10 dimensions is given by (see the second ref. of [6])

$$\begin{array}{l} ds_{10}^2 = \frac{1}{H_1} du\,(dv + H_0 du) + dy_m dy_m + H_2\left(\frac{1}{H_2}(dx_8 + \vec{V} d\vec{x})^2 + H_2 d\vec{x}\right) \\ H = d(1/H_1) \wedge du \wedge dv +^* dH_2 \wedge du \wedge dv \wedge (dy)_m \qquad , \qquad e^{-2\hat{\phi}} = \frac{H_1}{H_2} \quad , \end{array} \tag{5.2}$$

($\epsilon_{ijk}\partial_j V_k = \partial_i H_2$, $u, v = x_9 \pm t$, $m = 1..4$). This configuration describes a fundamental string lying in a NS 5-brane. In addition, there are momentum modes travelling along the string (boost), and in the transversal space is a KK-monopole. In comparison to the S-T-U model we have identified $T = U$, which means that the harmonic functions related to the 5-brane and to the KK-monopole part have been identified. As a consequence, this classical solution is T-selfdual with respect to the x_8 direction, but concerning the u direction this duality transformation exchanges H_0 with H_1 ($q_0 \leftrightarrow p^1$). When compactifying this solution, one reduces first over the torus (x_8, x_9). This yields a black hole lying in a 4-brane. In a second step one wraps the 4-brane completely over a $K3$ manifold.. The associated scalar fields are then given by

$$S = e^{-2\phi} = e^{-2\hat{\phi}}\sqrt{|G_{rs}|} = \sqrt{\frac{H_0 H_1}{H_2^2}} \quad , \quad T = \sqrt{|G_{rs}|} = \sqrt{\frac{H_0}{H_1}} \quad , \tag{5.3}$$

where G_{rs} denotes the (x_8, u) part of the metric (5.2). For the 4d metric in the Einstein frame one obtains

$$ds^2 = -\frac{1}{\sqrt{H_0 H_1 H_2^2}}dt^2 + \sqrt{H_0 H_1 H_2^2}\, d\vec{x}d\vec{x}\ . \tag{5.4}$$

Thus, by inserting the harmonic functions $H_0 = 1 + \frac{\sqrt{2}q_0}{r}$, $H_1 = 1 + \frac{\sqrt{2}p^1}{r}$, $H_2 = 1 + \frac{\sqrt{2}p^2}{r}$ into (5.4) and by calculating the area of the horizon, one obtains the entropy (5.1). In addition, the scalar fields behave smoothly and take fixed values on the horizon ($r = 0$), which are given in terms of the charges only.

In addition to the quantum corrections (higher genus corrections), described in the previous sections, one has to consider α' corrections as well. These terms do not appear in the prepotential, instead they are related, e.g., to higher curvature corrections. In order to have control over these terms as well, we have to make sure that the curvature in the string frame does not blow up on the horizon. Since the radius of the horizon in the string frame is proportional to the magnetic charge, we can suppress these terms by choosing a sufficiently large charge p^2.

Next, we would like to discuss the solutions on the type II side. The heterotic solution discussed above can be mapped onto the type II side, where the corresponding black hole solution can be interpreted as a compactification of intersecting branes. Both solutions are equivalent, but on the type II side the corrections to the prepotential have a clear geometrical interpretation in terms of the Calabi–Yau threefold. Thus, on the type II side, one can identify the additional states and the statistical interpretation of the entropy is especially clear.

The black hole becomes non-singular if 4 branes intersect each other. If one has less branes intersecting each other, the horizon shrinks to zero size and the black hole becomes singular. Actually, it is not necessary to have additional branes at the intersection, also internal waves (boosts) or KK-monopoles can stabilize the horizon.

We will now discuss the type II analogue of (5.4), and we will mainly do this in the M-theory picture. Since the solution has only two Kähler class moduli, we can only wrap two inequivalent branes around the two non-homologous 4-cycles of the Calabi–Yau threefold. Since on the type II side p^1 and p^2 are magnetic charges, the 11-d brane configuration must contain the intersection of two 5-branes. These two 5-branes intersect over a 3-brane and in order to obtain the electric charge q_0, we make again a boost along the worldvolume of the intersection, i.e. along one of the 3-brane directions. The corresponding metric is given by (in the following we will mainly consider the metric) [29]

$$ds_{11}^2 = \frac{1}{(H^1 H^2)^{1/3}}\left[dudv + H_0 du^2 + dx_5 dx_5 + dx_6 dx_6 + H^1 H^2 d\vec{x}d\vec{x} + H^\Lambda \omega_\Lambda\right] \quad , \tag{5.5}$$

where ω_Λ ($\Lambda = 1,2$) are two 2-dimensional line elements and where $\vec{x} = (x_1, x_2, x_3)$. The boost direction is x_4 ($u, v = x_4 \pm t$), which is parametrized by H_0. The location of the branes can be chosen as follows:

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
boost	$\circ$				$\circ$						
$H_1 - 5 - brane$	$\times$				$\times$	$\times$	$\times$	$\times$	$\times$		
$H_2 - 5 - brane$	$\times$				$\times$	$\times$	$\times$			$\times$	$\times$

(5.6)

where the worldvolume coordinates are indicated by "$\times$", and where "$\circ$" denotes the boost directions.

Next, one has to compactify this configuration on a Calabi–Yau threefold, which yields a string solution in 5 dimensions $(t, x_1 \ldots x_4)$. Ignoring the instanton corrections for a moment, this solution is given by [30]

$$ds^2_{5D} = \frac{1}{(d_{\Lambda\Delta\Sigma}H^\Lambda H^\Delta H^\Sigma)^{1/3}} \left[dudv + H_0 du^2 + d_{\Lambda\Delta\Sigma}H^\Lambda H^\Delta H^\Sigma \, d\vec{x}d\vec{x}\right] \quad , \tag{5.7}$$

where $\Lambda, \Delta, \Sigma = 1, 2$, and where $d_{\Lambda\Delta\Sigma}$ denote the intersection numbers of the Calabi–Yau given in (3.1). The index Λ counts the number of non-trivial 4-cycles of the Calabi–Yau, and in the solution it indicates around which 4-cycle we have wrapped the 5-brane. For this string we can define an entropy density (entropy per string length) and we obtain, after inserting the harmonic functions given after equation (5.4),

$$S^{5D}_{BH} = 2\pi \, (d_{\Lambda\Delta\Sigma}p^\Lambda p^\Delta p^\Sigma)^{2/3} \tag{5.8}$$

which coincides with (4.48). In a second step one has to compactify this string, that is one has to wrap it around the 4-th direction. As a result we obtain the 4-d black hole

$$ds^2_{4D} = -\frac{1}{\sqrt{H_0 \, d_{\Lambda\Delta\Sigma}H^\Lambda H^\Delta H^\Sigma}} dt^2 + \sqrt{H_0 \, d_{\Lambda\Delta\Sigma}H^\Lambda H^\Delta H^\Sigma} \, d\vec{x}d\vec{x} \quad , \tag{5.9}$$

whose 4-d entropy is given by

$$S^{4D}_{BH} = 2\pi \sqrt{|q_0| \, d_{\Lambda\Delta\Sigma}p^\Lambda p^\Delta p^\Sigma} \quad . \tag{5.10}$$

which coincides with the first term in (4.34).

This model shows that compactifying on a Calabi–Yau threefold can stabilize a solution. Since the model under consideration has only two Kähler class moduli, we can only wrap two topological inequivalent (e.g. orthogonal) 5-branes around 4-cycles of the Calabi–Yau threefold. As a consequence all triple intersections are self-intersections, which stabilize the black hole solution. Although the 11-d configuration is singular, the Calabi–Yau compactification makes it non-singular. Because the dependence of the black hole solution on the intersection numbers is given via the expression $d_{\Lambda\Delta\Sigma}H^{\Lambda}H^{\Delta}H^{\Sigma}$, a self-intersection of branes has qualitative the same consequence as the triple intersection of different branes. Thus, in the same way as additional branes, also the self-intersections improve the singularity structure of a black hole.

Finally, we would like to comment on the interpolating case (i). We will again only discuss the intersection part of the solution. In this case, in addition to the magnetic charges we have the electric charges q_1, q_2 as well as the constraints (4.3). Therefore, we need in 11 dimensions a brane solution that interpolates between the 2-brane and the 5-brane. This solution is known and is given by [31]

$$ds^2 = \frac{1}{(H\tilde{H})^{2/3}} \left[\tilde{H}(\mathbf{M_3}) + H(\mathbf{E_3}) + \tilde{H}H(\mathbf{E_5})\right] \quad , \tag{5.11}$$

where $\mathbf{M_3}$ and $\mathbf{E_n}$ denote a 3-dimensional Minkowskian and an n-dimensional Euclidian space, respectively. The harmonic functions H and $\tilde{H}$ are function of the transversal space $\mathbf{E_5}$: $H = 1 + \frac{q}{r^3}$, $\tilde{H} = 1 + \frac{q\cos^2\xi}{r^3}$. For $\xi = 0$ we have a 5-brane, and for $\xi = \pi/2$ we have a 2-brane. We do not wish to discuss this solution in detail, but we would like to point out that an intersecting configuration in terms of these objects along the line of [32] could provide a microscopic picture for the case (i).

In order to understand the instanton corrections to this solution, it is not sufficient to consider the 11-d intersection of 5-branes and their compactification only. Instead, one has to add free membranes in 11 dimensions, which are mapped onto rational curves in the Calabi–Yau threefold. We will discuss this point in the next section.

6 The microscopic picture

It has, for a long time, been an open question how to give the Bekenstein-Hawking entropy a statistical interpretation in terms of a degeneracy of states. Although there has been substantial progress in terms of the D-brane picture [37], it is still a question that deserves further study.

Consider, for example, the special configuration (iii) which, upon compactification on a Calabi–Yau threefold, yields a string in five dimensions. Inspired by the degeneracy of fundamental strings and the "correspondence principle" [34], one could argue that the degeneracy of states of the corresponding (unknown) underlying quantum theory should, for large level N, be of the form

$$d(N) \quad \sim \quad N^{-\gamma/4}\ e^{2\pi\sqrt{\frac{c}{6}N}}. \tag{6.1}$$

Here c and γ are a priori unknown parameters. For the case $\gamma = c+3$, eq. (6.1) describes the degeneracy of a fundamental string with central charge c for large level N [35].

The exponential term in (6.1) is known as the leading term and the polynomial term as the subleading term. The leading term is well understood in the context of classical solutions of supersymmetric vacua, especially for the BPS saturated case [2, 3, 6, 5]. The subleading term has been recently identified in the context of $N = 2$ supersymmetric heterotic and type II vacua [28]. These subleading corrections occur naturally as quantum or instanton corrections for $N < 3$. However, the configuration (iii) is special, and other configurations (and compactifications) such as, for instance, the case (ii) do *not* share this microscopic picture. Moreover, non-extreme black hole entropies in effective string theories depend on the values of the moduli at infinity [36]. Thus, an interpretation of their entropy in terms of the degeneracy of the spectrum of an underlying quantum theory, such as in eq. (6.1), appears in general to be somewhat problematic.

In the following we will give a microscopic interpretation for certain black hole entropies, that were derived above in the context of $N = 2$ supergravity coupled to two vector multiplets, which arises as a low-energy effective string theory. Such a microscopic interpretation is up to now only possible near particular points in moduli space. In particular we will propose a microscopic picture that gives a statistical/thermodynamical interpretation of the 4-d entropy for the cases (ii) and (iii).

In order to understand the microscopic picture one has to understand the brane picture. We will first consider case (iii), for which there is a clear brane picture. As mentioned in the previous section, the 11-d configuration consists of an intersection of 5-branes and a gas of closed membranes. The intersection part has been discussed in the last section and it give rise to the Bekenstein-Hawking entropy (5.10). A microscopic picture for this part has been given in [27]. Following the ideas given there, the microscopic states can be seen as open membrane states that connect the 5-branes. Since they are massive as long as they are stretched, they will move to the common intersection in order to become massless there. Next, one wraps the 5-branes around 4-cycles of the Calabi–Yau threefold and obtains the black string solution given in (5.7). This string is also the common intersection of all 5-branes, and the open membranes sitting on the common intersection appear now as momentum modes for this string. If one further takes into account that the magnetic charge p^{Λ} can be interpreted as arising when wrapping the 5-branes p^{Λ} times around the Λ-th 4-cycle, one can identify the Bekenstein-Hawking entropy (5.10) with the statistical entropy for the string states of this 5-d black string.

This part of the entropy, associated with the intersection of the 5-branes, gets now corrected by an instanton part. The corresponding microscopic interpretation was given in [28]. Here we will extend this interpretation further. The 11-d origin of the instanton part in the entropy is of a different nature. Turning on the instanton corrections means that we consider the 11-d intersection to live in a gas of closed membranes. When compactifying this configuration, the worldvolume of the closed membranes are completely mapped into the internal space. Two of the three worldvolume coordinates are mapped onto rational curves in the Calabi–Yau threefold and the third one is again identical with the direction of the 5-d black string. The type IIA analogon would be, that we first compactify over this string (the 11-th direction) and obtain in 10-d 4-branes living in a gas of closed strings. In the second step of the compactification, the worldvolumes of these closed strings are mapped onto rational curves in the Calabi–Yau threefold.

Keeping this in mind, there emerges a corresponding thermodynamical picture. The 11-d intersection lives in thermal equilibrium with a gas of free closed membranes. When they touch a 5-brane, they break up into open membranes which move to the common intersection. Eventually, they recombine to escaping closed membranes. The average number of open membranes on the 5-branes is counted by q_0 [27]. In a thermodynamical picture a natural definition of the temperature[4] is given by the radius of the 5-d black string, i.e. $T^2 \sim 1/R_5^2 = (d_{\Lambda\Delta\Sigma} p^\Lambda p^\Delta p^\Sigma)^{\frac{1}{3}}/q_0$ (see eq. (5.7)). By keeping the magnetic charges at some generic value, the temperature is directly related to the average number of states on the intersection, i.e. to q_0. There are now two special cases:

a) the zero temperature limit ($R_5 \to \infty$): In this limit all Kähler class moduli are large and hence all instanton corrections are suppressed. The black hole states are given by the open membranes living on the common intersection. Or in the thermodynamical language, all membranes are condensed - there are no free membranes.

b) the infinite temperature limit ($R_5 \ll 1$): There are no open membranes on the 5-branes (the Kähler class moduli are small). In this case the instanton corrections yield the dominant part and the black hole states consist of a "hot gas" of closed membranes, which are mapped into the internal space. The total number of these states is related to the sum over all rational curves. Note that this sum is in general infinite. On the other hand the charges are bounded from below by the brane tension or the zero point oscillations. Equivalently the temperature is bounded from above. And any non-vanishing value of the charges will regularize the instanton sum.

[4]One should keep in mind that in this picture the temperature has nothing to do with the Hawking temperature of non-extremal black holes. We only want to give a statistical/thermodynamical picture of the 11-d configuration.

In this picture, the transition between the two cases is smooth. The reason for this is that we have, so far, implicitly assumed that the magnetic charges take some finite value. As a consequence we were able to change the values of all Kähler class moduli in the same way, i.e. we went up and down in the Kähler cone. Going down in the Kähler cone means that we heat up the system, which takes us into the instantonic region. By going up in the Kähler cone, on the other hand, we cool down the system - all open membranes condense and we are in the intersection region.

This situation changes, however, drastically if we allow that also one of the magnetic charges becomes very small. In this case we are approaching a wall of the Kähler cone ($t_1 \to 0$), where one of the 4-cycles vanish. At this point, the system undergoes a phase transition, a vanishing 4-cycle "is replaced" by an emerging 3-cycle beyond the wall. We do not wish to discuss this phase transition in detail here. But if we approach this point, which on the heterotic side corresponds to $S \simeq T$, the entropy gets logarithmic corrections as given in eq. (4.34).

Finally, let us discuss a problem related to the $\zeta(3)$ terms in the prepotential (3.1). As a consequence also the entropy contains terms wich are proportional to $\zeta(3)$. Since this irrational number cannot be expressed in terms of rational numbers or factors of π, it seems to be difficult to give the entropy a statistical interpretation. In order to address this question we can go to a region in moduli space where only these terms contribute. This is shown in eq. (4.16), where we took $p^A \simeq 0$ and $p^0 \gg 1$. The 11-d starting point for this limit is an intersection of two 2-branes embedded in a gas of closed membranes. This is a configuration dual to the 5-brane case discussed above. As before, in this picture we have open membranes sitting on the intersection. The contributions proportional to $\zeta(3)$ can now be extracted if we go to the hot temperature limit, i.e. $R_5 \ll 1$, which in this case correspond to $p^0 \gg 1$. Again the dominant part is given by the gas of closed membranes. This pure instantonic part yields the entropy contribution (4.16), and we have to face the problem of interpreting $\zeta(3)$. Interestingly, this term also appears in the statistical entropy that counts the number of free bosons and fermions (ideal gas) living in the membrane worldvolume, which is given by [33]

$$S_{stat} = \frac{7}{8\pi}\zeta(3) N L^2 T^2 \quad , \tag{6.2}$$

where N denotes the number of states of free bosons that should be equal to the number of fermions, L^2 is the spacial volume (which should be normalized properly) and T^2 is the membrane tension. The membrane tension is related to the string tension by [33]: $T^2 = T^1/L = 1/(2\pi\alpha' L)$. So, by comparing this statistical entropy with (4.16) and by setting $\alpha' = L$, we see that both expressions coincide up to integers. This coincidence suggests, that the $\zeta(3)$ terms in the entropy (4.16) count the number of worldvolume states of the compactified M-2-branes.

7 Conclusions

In this paper we investigated axion-free quantum black hole solutions in the $N = 2$ supersymmetric heterotic S-T model. For these solutions we discussed the entropy in target–space duality invariant form as well as the scalar fields on the horizon. The entropy in this model is determined by 4 independent charges. If we keep all 4 charges the entropy is given by eq. (4.10). This result takes into account *all* the perturbative corrections appearing in the prepotential. Next, we considered two special classes of solutions, whose entropy is given in terms of 3 charges only. In 11 dimensions, these two cases correspond to intersections of only membranes or only 5-branes. For the first case the entropy is given in (4.13) and for the second one in (4.25). In the latter case we also included the non-perturbative corrections. However, this latter solution depends on a constrained parameter. We expanded this solution around a vanishing 4-cycle and found logarithmic corrections for the entropy (subleading terms).

In the second part we considered the corresponding 10-d (heterotic) or 11-d (M-theory) configurations. In the context of M-theory we proposed a microscopic interpretation for the entropy formulae. In this picture we have in 11 dimensions an intersection of two branes living in a gas of free closed membranes. When compactifying this configuration the intersecting branes are wrapped around inequivalent cycles and the free closed membranes are mapped onto rational curves of the CY-threefold. An interesting feature of this model is that, although the 11-d solution is singular, the compactification on a CY-threefold stabilizes this solution. A torus compactification, on the other hand, yields a singular configuration in 4 dimensions.

Finally, we discussed a thermodynamical picture for the intersection of branes living in a gas of membranes. The number of open membranes attached to the intersection of 2- or 5-branes depends on the point in the Kähler cone. If we move up in the Kähler cone, the number increases (the closed membranes "condense") and going down has the consequence that all open membranes "evaporate" from the intersection. Deep inside the cone we reach a pure instantonic region. Here, for the case of intersecting membranes, we proposed a microscopic interpretation for the $\zeta(3)$ terms in the entropy in terms of worldvolume states of membranes.

To conclude, the microscopic picture of the quantum black hole solutions we have investigated is not yet complete. But it is encouraging that, at least at special points in moduli space, a reasonable statistical interpretation of the entropy, including quantum corrections, is possible.

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