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N=2 WORLDSHEET INSTANTONS YIELD CUBIC SELF-DUAL YANG-MILLS *

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Abstract

When the gauge instantons on the N=2 string worldsheet are properly included in the sum over topologies, the breaking of $SO(2,2)$ Lorentz symmetry in $\mathbb{R}^{2,2}$ is parametrized by a *spacetime twistor* containing the string coupling and theta angle. The resulting (tree-level) effective action for the open string is not Yang's but Leznov's *cubic* action for self-dual Yang-Mills in a light-cone gauge. In the closed case, Plebański's action for self-dual gravity gets modified analogously. In contrast to the N=1 NSR string, picture-changing is not locally invertible, but produces a semi-infinite tower of massless physical states with ever-increasing spin, perhaps related to harmonic superspace. A truncation yields the two-field action of Chalmers and Siegel.

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Generally, the field or wave function of any bosonic string has only a single component. Usually, this implies that the ground state is a scalar. However, the N=2 string describes self-dual Yang-Mills theory (open) or self-dual gravity (closed) [1, 2]. The description of either theory by a single component requires a unitary light-cone gauge, where the single polarization transforms nonlinearly under Lorentz transformations. There are two well-known types of light-cone gauges for self-dual theories, which are "dual" to each other in the sense of trading a constraint with the field equation. For self-dual Yang-Mills theory, the Yang gauge [3] results in a nonlinear field equation resembling that of a two-dimensional Wess-Zumino model, while the Leznov gauge [4, 5] gives a quadratic field equation. The description of self-dual Yang-Mills theory originally found from the N=2 string was in the Yang gauge [3]. Furthermore, in the closed string case the analog of the Yang gauge for gravity, the Plebański gauge [6], was found. However, there was no reason in principle why the string should prefer these gauges. More recently, a gauge other than the Plebański gauge was found in the closed string case by including the previously ignored worldsheet $U(1)$ instantons [7]. In this paper we identify that gauge as the gravity analog [8] of the Leznov gauge, and find also the Leznov gauge for the open string case. Furthermore, the two string couplings, associated with loop and instanton number, are identified as the two-component commuting spinor (twistor) that chooses the arbitrary self-dual lightlike plane with respect to which the gauge is defined.

Strings with two world-sheet supersymmetries in the NSR formulation are built from an N=2 world-sheet supergravity multiplet containing the metric h_{mn}, an abelian gauge field A_m and two charged Majorana gravitini $\chi_m^{\pm}$. In a self-dual $(2,2)$ metric background $\mathcal{M}$ [1] the matter sector consists of the string coordinates X and their charged NSR partners ψ. Specializing to flat space, $\mathcal{M} = \mathbb{R}^{2,2}$, we write

$$X^{a\dot{a}} \;=\; \sigma_\mu^{a\dot{a}} X^\mu \;=\; \begin{pmatrix} X^0{+}X^3 & X^1{+}X^2 \\ X^1{-}X^2 & X^0{-}X^3 \end{pmatrix} \quad , \qquad a \in \{+,-\} \quad \dot{a} \in \{\dot{+},\dot{-}\} \;, \qquad (1)$$

with a set of chiral gamma matrices σ_μ, $\mu = 0, \ldots, 3$, appropriate for a spacetime metric $\eta_{\mu\nu} = \mathrm{diag}(-+-+)$. Note that we are employing the van der Waerden index notation, splitting $SO(2,2)$ vector indices μ into two $SL(2,\mathbb{R})$ spinor indices, a and $\dot{a}$. Spinor indices are raised and lowered by the epsilon tensor, and vectors have an $SL(2,\mathbb{R}) \times\ SL(2,\mathbb{R})'$ invariant length-squared of

$$\eta_{\mu\nu}\, X^\mu X^\nu \;=\; -\tfrac{1}{2}\, \epsilon_{ab}\, \epsilon_{\dot{a}\dot{b}}\, X^{a\dot{a}}\, X^{b\dot{b}} \;=\; -\det X^{a\dot{a}} \quad . \qquad (2)$$

[1]Metric with (r,r) signature have been termed 'Kleinian' [9]. For r=2, Kleinian self-duality implies Ricci flatness and $SL(2,\mathbb{R})$ holonomy. Such spaces are called half-flat or hypersymplectic and possess one complex and two real structures, all covariantly constant [9].

For later use, we define the following bilinears constructed from two vectors k and p,

$$
\begin{aligned}
p^a \wedge k^b &= \epsilon_{\dot{a}\dot{b}}\, p^{a\dot{a}}\, k^{b\dot{b}} \\
p^{(+} \wedge k^{-)} &= p^+ \wedge k^- + p^- \wedge k^+ \\
p^{[+} \wedge k^{-]} &= p^+ \wedge k^- - p^- \wedge k^+ \;=\; \eta_{\mu\nu}\, p^\mu k^\nu \quad ,
\end{aligned} \tag{3}
$$

with contracted $SL(2,\mathbb{R})'$ indices suppressed. We have diagonalized L_{+-} as one of the $SL(2,\mathbb{R})$ *boost* generators, while the rotation is off-diagonal and recovered from the nilpotent light-cone combinations L_{++} and L_{--}. The three antisymmetric bilinears in (3), $p^+ \wedge k^+$, $p^- \wedge k^-$, and $p^{(+} \wedge k^{-)}$, are still invariant under a smaller "Lorentz group" $\mathcal{L} = GL(1,\mathbb{R}) \times SL(2,\mathbb{R})'$, where the $GL(1,\mathbb{R})$ factor is created by L_{++}, L_{--}, or L_{+-}, respectively.

We are considering n-point amplitudes of oriented open strings. In writing down the Brink-Schwarz action [10], we must pick some real or complex structure in $\mathbb{R}^{2,2}$, with respect to which the action is neutral. Consequently, only $\mathcal{L}$ invariance is manifest. After superconformal gauge fixing, string theory demands to average over matter fields (X,ψ) and the ghost sector, to integrate over (metric, susy and gauge) moduli, and to sum over worldsheet topologies parametrized by the genus $g \in \frac{1}{2}\mathbb{Z}_+$ and the gauge instanton number $c \in \mathbb{Z}$. As is customary, contributions from different topologies are weighted by powers of the string coupling e and the theta phase $e^{i\theta}$. However, since we are working in a *real* basis, the latter combine to monomials in $\sin\theta$ and $\cos\theta$. The full string amplitude reads [11, 12, 7]

$$
\begin{aligned}
A^{(n)}(k_1,\ldots,k_n) &= \sum_{g,c} \binom{2j}{j+c}\, e^j\, \sin^{j-c}\tfrac{\theta}{2}\, \cos^{j+c}\tfrac{\theta}{2} \int dm_h\, dm_A \\
&\quad \times \Big\langle\, V(k_1)\ldots V(k_n)\, \mathcal{A}\, \mathcal{P}_+^{j-c}\, \mathcal{P}_-^{j+c} \,\Big\rangle_{g,c}(m)
\end{aligned} \tag{4}
$$

where $j \equiv 2g-2+n$, and a few remarks are in order.

- Chan-Paton factors are implied but suppressed.
- For $|c| > j$, unbalanced susy ghost zero modes make the correlator vanish, thus cutting the instanton sum to $c = -j,\ldots,+j$.
- Integration over bosonic moduli cover the metric moduli (of dimension $3g-3+n$) as well as the gauge moduli (of dimension $g-1+n$). The latter are given by the gauge field holonomies $\oint A$ and contain the spectral flow of the vertex operators at the punctures [13].

- $\mathcal{A}$ denotes antighost zero mode insertions, which arise from proper gauge-fixing in the presence of moduli; their precise form is not relevant here.

- $\mathcal{P}_\pm$ are two *picture-changing operators*, originating from susy antighost insertions plus susy moduli integration. It is important to note that $\mathcal{P}_\pm$ are *not invertible* as local operators [14], in contrast to the case of the N=1 NSR string.

- $\langle \ldots \rangle_{g,c}(m)$ signifies a correlator of the (matter plus ghost) conformal field theory on a worldsheet with fixed moduli m and topology (g, c).

It remains to characterize the physical string states, whose vertex operators $V(k)$ appear in (4). As for the N=1 NSR string, the (open-string) BRST cohomology displays the phenomenon of picture degeneracy, here parametrized by a pair (π_+, π_-) of picture charges. In the picture $(-1, -1)$, the BRST analysis is easy [15] and leads (modulo the usual ghost zero mode doubling) to a single massless physical state [1]

$$|{-1}, -1; k\rangle \;=\; V(k)\, |0, 0; 0\rangle \qquad \text{with} \quad \eta_{\mu\nu} k^\mu k^\nu = 0 \tag{5}$$

corresponding to a massless spacetime field $\phi(x)$. We take the vertex operators in (4) from this "canonical" picture. The study of other pictures is simplified by the properties of the picture-changing operators $\mathcal{P}_\pm$ and a spectral-flow operator $\mathcal{S}(\alpha)$, $\alpha \in \mathbb{R}$, which commute with one another (on the cohomology) and with the BRST operator. They shift the picture numbers as follows,

$$\begin{aligned} \mathcal{P}_+ \,|\pi_+, \pi_-\rangle \;&=\; |\pi_+{+}1, \pi_-\rangle \quad , \qquad \mathcal{P}_- \,|\pi_+, \pi_-\rangle \;=\; |\pi_+, \pi_-{+}1\rangle \quad , \\ \mathcal{S}(\alpha)\, |\pi_+, \pi_-\rangle \;&=\; |\pi_+{+}\alpha, \pi_-{-}\alpha\rangle \quad , \end{aligned} \tag{6}$$

but do not lead to a commutative triangle in the (π_+, π_-) plane since [7, 16]

$$\tilde{\mathcal{P}}^+ \;\equiv\; \mathcal{P}_-\, \mathcal{S}(+\tfrac{1}{2}) \;\neq\; \mathcal{P}_+\, \mathcal{S}(-\tfrac{1}{2}) \;\equiv\; \tilde{\mathcal{P}}^- \quad . \tag{7}$$

The reason is that $\mathcal{S}(\alpha)$ carries global boost charge $q = \alpha$ with respect to L_{+-} whereas $\mathcal{P}_\pm$ are neutral. Therefore, the *compound* picture-changing operators $\tilde{\mathcal{P}}^\pm$ change (π_+, π_-, q) by $(+\frac{1}{2}, +\frac{1}{2}, \pm\frac{1}{2})$ and may be employed to reach from the canonical $(-1, -1, 0)$ all diagonal higher pictures, i.e.

$$\pi \;\equiv\; \pi_+ + \pi_- \;=\; -2, -1, 0, +1, \ldots \qquad \text{and} \qquad \Delta \;\equiv\; \pi_+ - \pi_- \;=\; 0 \quad . \tag{8}$$

Subsequent application of $\mathcal{S}(\alpha)$ keeps π but shifts $\Delta \to 2\alpha$. Although it has not been proved, we assume that all physical states are obtained in this way.

Unexpectedly, we have arrived at a proliferation of physical states at higher pictures. Distributing the indices $\pm$ in the (symmetric) product of $2j$ compound picture-changing operators, one gets a $(2j{+}1)$-plet of states with $q{-}\frac{1}{2}\Delta = -j,\ldots,+j$ on each line of pictures with $\pi = 2(j{-}1)$. Clearly, these are spin j multiplets of the broken $SL(2,\mathbb{R})$, and $\tilde{\mathcal{P}}^{\pm}$ itself transforms as an $SL(2,\mathbb{R})$ doublet, mapping $\pi \to \pi{+}1$ and spin j to spin $j{+}\frac{1}{2}$ states [7, 16]. In detail (suppressing Chan-Paton labels),

$$\begin{array}{lrcl} \pi = -2: & |{-1},{-1};k\rangle & \longleftrightarrow & \phi(x) \\ \pi = -1: & \tilde{\mathcal{P}}^{a}\,|{-1},{-1};k\rangle & \longleftrightarrow & \phi^{a}(x) \\ \pi = \ \ 0: & \tilde{\mathcal{P}}^{(a}\tilde{\mathcal{P}}^{b)}\,|{-1},{-1};k\rangle & \longleftrightarrow & \phi^{(ab)}(x) \\ \pi = +1: & \tilde{\mathcal{P}}^{(a}\tilde{\mathcal{P}}^{b}\,\tilde{\mathcal{P}}^{c)}\,|{-1},{-1};k\rangle & \longleftrightarrow & \phi^{(abc)}(x) \\ \vdots & \vdots & & \vdots \end{array} \quad . \tag{9}$$

Moreover, since spectral flow by an *integral* α is just a singular gauge transformation [13], the vertex operator for a state with integer global boost charge q creates a gauge instanton of charge $c{=}q$ at the puncture,

$$\mathcal{S}(\alpha{=}q)\,|c{=}0\rangle \ \sim\ |c{=}q\rangle \quad . \tag{10}$$

Hence, world-sheet gauge instantons fill up the $SL(2,\mathbb{R})$ multiplets. Our assignment of $SL(2,\mathbb{R})$ spin to individual states leads to total picture numbers of

$$\pi_{tot} \ = \ 2j_{tot} - 2n \ = \ 4g-4 \qquad \text{and} \qquad \Delta_{tot} \ = \ -2c \tag{11}$$

in a correlator for topology (g,c). This selection rule is also evident from (4).

Apparently, we find no physical states for $\pi < -2$ but a growing number for $\pi > -2$. Are they physically distinct? At this stage, two answers seem possible. Interpretation one [16] sees higher-picture states $\phi^{(a_1 a_2 \ldots a_{2j})}$ as spacetime derivatives of the single scalar field ϕ present in the canonical $\pi{=}-2$ picture. It is supported by the fact that the states $(\tilde{\mathcal{P}}^{\pm})^{2j}\,|{-1},{-1};k\rangle$ are polynomials of degree $2j$ in $k^{a\dot{a}}$, and suggests a worldsheet–spacetime correspondence

$$\tilde{\mathcal{P}}_a \qquad \Longleftrightarrow \qquad \zeta^{\dot{a}}(x)\,\frac{\partial}{\partial x^{a\dot{a}}} \tag{12}$$

with some commuting $SL(2,\mathbb{R})'$ spinor $\zeta(x)$. In this way, all pictures yield the same physics, provided $\zeta(x)$ does not represent a new degree of freedom. It is tempting to relate $\zeta^{\dot{a}}$ with extra harmonic degrees of freedom, as present in harmonic superspace [18], and interpret picture-changing as some kind of supersymmetry. Interpretation two confides that there is indeed an *infinity* of physical states, since non-invertible picture-changing cannot identify $\phi, \phi^a, \phi^{(ab)}, \ldots$ [17]. However, this tower can be consistently truncated as we shall see below.

Using the compound picture-changing operators (7), we condense the instanton sum in (4),

$$A^{(n)}(k_1,\ldots,k_n) \;=\; \sum_g \int dm_h\, dm_A \Big\langle\, V(k_1)\ldots V(k_n)\; \mathcal{A}\;\; [v^a \tilde{\mathcal{P}}_a]^{2j} \,\Big\rangle_{g,c=0} \;, \tag{13}$$

by introducing [7]

$$\begin{pmatrix} v^+ \\ v^- \end{pmatrix} \;=\; \sqrt{e}\, \begin{pmatrix} +\cos\frac{\theta}{2} \\ -\sin\frac{\theta}{2} \end{pmatrix} \quad . \tag{14}$$

As the notation suggests, $A^{(n)}$ is $SL(2,\mathbb{R})$ (and thus $SO(2,2)$) invariant, once we insist that v^a transforms as a fundamental spinor so that the combination

$$\hat{\mathcal{P}} \;\equiv\; \epsilon_{ab}\, v^a\, \tilde{\mathcal{P}}^b \;=\; \sqrt{e}\, \Big[\cos\tfrac{\theta}{2}\, \mathcal{P}_+\, \mathcal{S}(-\tfrac12) \;+\; \sin\tfrac{\theta}{2}\, \mathcal{P}_-\, \mathcal{S}(+\tfrac12)\Big] \tag{15}$$

is a Lorentz singlet. This implies that we can not only rotate away the theta angle, but also boost the string coupling to an arbitrary positive value [5]! Obviously, the gauge instanton contributions ($c{\neq}0$) in (4) are vital for reclaiming $SO(2,2)$ Lorentz symmetry. In fact, the instanton creation and destruction operators $\mathcal{S}(\pm1)$ may be interpreted [7] as the $SL(2,\mathbb{R})$ generators $L_{\pm\pm}$ needed to restore $\mathcal{L} \to SO(2,2)$.

Any choice of couplings $(e,\theta) \leftrightarrow v^a$ picks a particular real structure in spacetime and breaks $SO(2,2) \to \mathcal{L}$. Since $SL(2,\mathbb{R}) = SO(2,1)$, the unbroken generator $v^a v^b L_{ab}$ is associated with the *null* vector $\vec{n} = v{\times}v$ in $\mathbb{R}^{2,1}$. Consequently, the moduli space of real structures is the light cone in $D = 2{+}1$, parameterized by (e,θ) where e is nothing but the "time" component of the lightlike $\vec{n}$. Since the zero instanton sector of (4) is neutral under the L_{+-} boost, it corresponds to a *spacelike* (2+1)-vector and can therefore not be isolated for any value of (e,θ). We may, however, eliminate all but the minimal (or maximal) instanton sector from the amplitude (4) by taking $\theta{=}0 \leftrightarrow v^-{=}0$ (or $\theta{=}\pi \leftrightarrow v^+{=}0$) in (13). This amounts to picking a light-cone frame in spacetime, as only $\tilde{\mathcal{P}}_+ \sim \zeta^{\dot a}\partial_{+\dot a}$ (resp. $\tilde{\mathcal{P}}_- \sim \zeta^{\dot a}\partial_{-\dot a}$) will appear.

It is straightforward to compute tree-level n-point amplitudes $A_0^{(n)}$, with the instanton sum ranging from $2{-}n$ to $n{-}2$. From consistency of duality with the absence of massive states, and also from the topological description [11, 19], we know that $A_0^{(n)} = 0$ for $n{>}3$ in all instanton sectors. The tree-level three-point function comes out as [20]

$$A_0^{(3)} \;=\; \Big\langle\; V^A(k_1)\; V^B(k_2)\; V^C(k_3) \;\;\; \hat{\mathcal{P}}^2 \;\Big\rangle_{0,0}$$

$$
\begin{aligned}
&= \Big\langle V^A(k_1)\, V^B(k_2)\, V^C(k_3) \Big[+ v^+ v^+ \,\mathcal{P}_+\, \mathcal{P}_+\, \mathcal{S}(-1) \\
&\qquad\qquad\qquad\qquad\qquad -2 v^+ v^- \,\mathcal{P}_+\, \mathcal{P}_- \\
&\qquad\qquad\qquad\qquad\qquad + v^- v^- \,\mathcal{P}_-\, \mathcal{P}_-\, \mathcal{S}(+1) \Big] \Big\rangle_{0,0} \\
&= ief^{ABC} \Big[\cos^2 \tfrac{\theta}{2}\; k_1^- \wedge k_2^- \;+\; \sin\tfrac{\theta}{2} \cos\tfrac{\theta}{2}\; k_1^{(+} \wedge k_2^{-)} \;+\; \sin^2 \tfrac{\theta}{2}\; k_1^+ \wedge k_2^+ \Big]
\end{aligned}
\tag{16}
$$

using the notation of (3). Employing the broken Lorentz generators, this may be written as

$$
A_0^{(3)} \;=\; e^{\theta (L_{++} - L_{--})/2}\; e^{(\ln \mu e) L_{+-}}\; A_0^{(3)}\Big|_{\theta=0, e=1/\mu}
\tag{17}
$$

where

$$
A_0^{(3)}\Big|_{\theta=0, e=1/\mu} \;=\; i\mu f^{ABC} k_1^- \wedge k_2^-
\tag{18}
$$

is the result from the $c{=}-1$ sector only. We have scaled e to a reference value $1/\mu$, fixing a mass scale μ.

This three-point function (and the vanishing of the higher tree-level amplitudes) may be obtained from Leznov's spacetime action [4, 5]

$$
S_L \;=\; \mathrm{tr} \int d^4x \,\Big[\tfrac{1}{2}\, \phi \Box \phi \;+\; \tfrac{i}{3\mu}\, \phi\, (\partial_+{}^{\dot a} \phi)\, (\partial_{+\dot a} \phi) \Big]
\tag{19}
$$

for a Lie-algebra-valued anti-hermitian field ϕ. Even better does a related two-field action

$$
S_{CS} \;=\; \mathrm{tr} \int d^4x \,\Big[\tilde\phi \Box \phi \;+\; \tfrac{i}{\mu}\, \tilde\phi\, (\partial_+{}^{\dot a} \phi)\, (\partial_{+\dot a} \phi) \Big]
\tag{20}
$$

proposed by Chalmers and one of the authors [21], because it permits us to absorb μ into the fields by

$$
\phi \;\to\; \mu\, \phi \quad , \qquad \tilde\phi \;\to\; \tfrac{1}{\mu}\, \tilde\phi \quad .
\tag{21}
$$

Both actions describe self-dual Yang-Mills at tree-level and in a light-cone gauge. Their one-loop amplitudes differ by a factor of 2. Beyond one loop, however, S_{CS} yields vanishing amplitudes whereas S_L does not. For further comparison and the relation to maximally helicity violating pure Yang-Mills amplitudes see [21].

Another light-cone gauge, proposed by Yang [3], leads to a *non-polynomial* effective action [22]

$$
S_Y \;=\; \mathrm{tr} \int d^4x \,\Big[\tfrac{1}{2}\, \phi \Box \phi \;+\; \tfrac{i}{3\mu}\, \phi\, (\partial_{(+}{}^{\dot a} \phi)\, (\partial_{-)\dot a} \phi) \;+\; O(\phi^4) \Big]
\tag{22}
$$

which was shown to reproduce the zero-instanton ($c{=}0$) sector of the $N{=}2$ open string tree-level amplitudes [1, 2]. Yang's action (22) also has a two-field relative [21]. A point we make in this paper is that the full string amplitude necessarily receives gauge instanton contributions, which shift the spacetime effective action from the Yang to the Leznov type. Not only is the latter *polynomial* (cubic) but it also allows us to parametrize the explicit breaking of $SO(2,2)$ Lorentz invariance with a *spacetime twistor* made from the coupling constants (e,θ).

At tree level, the closed string is basically the square of the open string, so our considerations apply just as well to the closed $N{=}2$ string. Here, the instanton contributions are labelled by a pair (c_L, c_R), and are summed over independently for left- and right-movers [12]. The zero-instanton sector is known [1] to be described by the (cubic) Plebański action [6]

$$S_P \;=\; \int d^4x \, \Big[\tfrac12 \, \varphi \Box \varphi \;+\; \tfrac{2}{3\mu^3}\, \varphi \, (\partial_+{}^{\dot a} \partial_-{}^{\dot b} \varphi) \, (\partial_{+\dot a} \partial_{-\dot b} \varphi) \Big] \quad , \tag{23}$$

but the instanton sum again corrects this to a Leznov-type action,

$$S'_L \;=\; \int d^4x \, \Big[\tfrac12 \, \varphi \Box \varphi \;+\; \tfrac{2}{3\mu^3}\, \varphi \, (\partial_+{}^{\dot a} \partial_+{}^{\dot b} \varphi) \, (\partial_{+\dot a} \partial_{+\dot b} \varphi) \Big] \tag{24}$$

or its two-field cousin using a Lagrange multiplier field [21]. Here, φ denotes the Kähler deformation of self-dual gravity [1].

It is possible to obtain the two-field action (20) (after the rescaling (21)) by an appropriate assignment of spacetime fields to string vertex operators. Let us indicate π charges by braced subscripts, i.e. $V_{(\pi)}$. Note that only the highest component of each $SL(2,\mathbb{R})$ multiplet appears, as picked out by the twistor v^a. Since ϕ and $\tilde\phi$ carry $GL(1,\mathbb{R})$ charges $q{=}-2$ and $q{=}+2$, respectively, and the number of Lagrange multiplier legs $\tilde\phi$ in non-vanishing amplitudes is $1{-}g$ while the total picture increases by 4 units per loop, we are forced to associate

$$\phi \;\longleftrightarrow\; V_{(0)} \qquad \text{and} \qquad \tilde\phi \;\longleftrightarrow\; V_{(-4)} \quad . \tag{25}$$

This seems problematic because there are no physical states in the picture $\pi=-4$. Yet, it is possible to define such states by creating a pairing

$$\Big\langle \, V_{(-4-\pi)}(-k) \; V_{(\pi)}(k) \, \Big\rangle_0 \;=\; 1 \tag{26}$$

via the reflection symmetry (nonlocal on the worldsheet)

$$\pi \;\leftrightarrow\; -4-\pi \quad , \qquad \Delta \;\leftrightarrow\; -\Delta \quad , \qquad q \;\leftrightarrow\; -q \quad , \qquad k \;\leftrightarrow\; -k \quad . \tag{27}$$

The correspondence (25) allow us to identify (spacetime) helicity s with $GL(1,\mathbb{R})$ charge q and picture number π through $s=\frac12 q = 1{+}\frac12\pi$, and the maximal helicity condition [21] is nothing but (11).

In order to restrict the helicities to $|s| \leq 1$, a truncation to $-4 \leq \pi \leq 0$ is necessary. Employing the vanishing of all tree-level (n>3)-point functions [19] and the BRST-exactness of $\partial\mathcal{P}_\pm$ and $\partial\mathcal{S}$,

$$\begin{aligned} 0 &= \Big\langle\, V^1_{(-2)} \ldots V^{n-1}_{(-2)}\, V^n_{(-2)}\, \mathcal{A}\, [\mathcal{P}_{(+1)}]^{2n-4} \,\Big\rangle_{g=0} \\ &= \Big\langle\, V^1_{(-2)} \ldots V^{n-1}_{(-2)}\, V^n_{(2n-6)}\, \mathcal{A} \,\Big\rangle_{g=0} \quad , \end{aligned} \tag{28}$$

we may set $V_{(\pi>0)}$ to zero, keep only the pictures from -4 to 0. At the loop level, moving all $\mathcal{P}$ onto a single vertex produces a $V_{(4g+2n-6)}$, which would kill the correlator for $n > 3-2g$, according to our truncation $\pi \leq 0$. As the explicit non-vanishing of the one-loop three-point function [23] shows, however, this argument is too naive, and we expect it to be modified by the appearance of contact terms and moduli boundary terms.[2] A further restriction to the extremal helicities $s = \pm 1$ (i.e. $\pi = -4, 0$ only) yields the two-field action (20).

It is instructive to work out the 4D mass dimensions of worldsheet objects. After rescaling (21), we learn from (20) that $[\phi] = 0$ and $[\tilde{\phi}] = 2$. Since the vertex operators multiply the spacetime fields in the worldsheet action, it follows that $[V_{(0)}] = 0$ and $[V_{(-4)}] = -2$ which means that $[\mathcal{P}] = +\frac{1}{2}$, as is appropriate for spacetime supersymmetry.

It may well be that the N=2 string contains more than just self-dual Yang-Mills or gravity. If the semi-infinite picture tower of massless physical states is for real, its proper spacetime identification presumably requires to consider more general than self-dual backgrounds.

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