Solitons in the supersymmetric extensions of the Standard Model

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Abstract

All supersymmetric generalizations of the Standard Model allow for stable non-topological solitons of the Q-ball type which may have non-zero baryon and lepton numbers, as well as the electric charge. These solitons can be produced in the early Universe, can affect the nucleosynthesis, and can lead to a variety of other cosmological consequences.

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Supersymmetric generalizations of the Standard Model (SSM) involve a complicated scalar potential that depends on a large number of variables. Although the details of such a potential depend on a model, a generic feature of all SSM is the presence of the tri-linear couplings of the type $H\Phi\phi$, where Φ is a left-handed squark ($\tilde{Q}_L$) or slepton ($\tilde{L}_L$) doublet, and ϕ is the corresponding right-handed singlet ($\tilde{q}_R$ or $\tilde{l}_R$) of the SU(2). These terms arise from the Yukawa couplings in the superpotential, as well as from the supersymmetry breaking terms. We will show that such cubic interactions lead to the appearance of non-topological solitons in the spectrum of the SSM. Solitons of this type, dubbed *Q-balls* [2], can have a non-zero baryon or lepton number, or electric charge. They can lead to interesting cosmological consequences and may provide new constraints on the parameters of the SSM.

We argue that B and L balls created in the early Universe can also play an important role in the synthesis of nuclei by producing lumps of nuclear matter prior to the onset of the standard nucleosynthesis. This opens a new possibility for the production of heavy elements through fission of the quark matter lumps that are left over after the decay of the squark and slepton Q-balls.

It was shown [3] that very small Q-balls (Q-beads) with charges $Q \sim 1$ can exist, despite the fact that the usual thin-wall approximation breaks down for small Q. A new formalism [3] that has been developed to analyse these solitons gives an adequate description of Q-beads as long as the charge and the tri-linear couplings in the potential are sufficiently small. Such small-charge solitons are of particular interest for the phenomenology of the MSSM, because the leptonic and baryonic beads can be absolutely stable due to a combination of several conservation laws. They could be produced in large quantities in the early Universe and can contribute to dark matter.

Finally, a $B \neq 0$, $L \neq 0$ soliton interacts as a leptoquark, which has intriguing implications.

1   Q-balls with many flavors

We begin with a straightforward generalization of Coleman's discussion of Q-balls [2] to the case that involves several scalar fields with different charges.

Let us consider a field theory with a scalar potential $U(\varphi) \equiv U(\varphi_1, ..., \varphi_n)$ which has a

global minimum at $\varphi = 0$; $U(0) = 0$. Let $U(\varphi)$ have an unbroken global U(1) symmetry at the origin, where $\varphi = 0$. The scalar fields φ_i have charges q_i with respect to this $U(1)$, and at least one of q_i $(i = 1, ..., n)$ is not equal to zero.

The charge (taken to be positive for definiteness) of some field configuration $\varphi(x, t)$ is

$$Q = \sum_k q_k \frac{1}{2i} \int \varphi_k^* \overleftrightarrow{\partial_t} \varphi_k \, d^3x \tag{1}$$

Clearly, a configuration $\varphi(x, t) \equiv 0$ has zero charge, so the solution that minimizes the energy

$$E = \int d^3x \left[\frac{1}{2} \sum_k |\dot{\varphi}_k|^2 + \frac{1}{2} \sum_k |\nabla \varphi_k|^2 + U(\varphi) \right], \tag{2}$$

and has a given charge $Q > 0$, must differ from zero in some (finite) domain. We will use the method of Lagrange multipliers to look for the minimum of E at fixed Q. One must find an extremum of

$$\begin{aligned} \mathcal{E}_\omega &= E + \omega \left[Q - \sum_k q_k \frac{1}{2i} \int \varphi_k^* \overleftrightarrow{\partial_t} \varphi_k \, d^3x \right] \tag{3} \\ &= \int d^3x \, \frac{1}{2} \sum_k |\partial_t \varphi_k - i\omega q_k \varphi_k|^2 + \int d^3x \left[\frac{1}{2} \sum_k |\nabla \varphi_k|^2 + \hat{U}_\omega(\varphi) \right] + \omega Q, \tag{4} \end{aligned}$$

where ω is a Lagrange multiplier, and

$$\hat{U}_\omega(\varphi) = U(\varphi) - \frac{1}{2}\omega^2 \sum_k q_k^2 |\varphi_k|^2. \tag{5}$$

Variations of $\varphi(x, t)$ and those of ω can now be treated independently, the usual advantage of the Lagrange method.

We are looking for a solution that extremizes $\mathcal{E}_\omega$, while all the physical quantities, including the energy, E, are time-independent. To minimize the first term in equation (4), the only one that appears to depend on time explicitly, one must choose

$$\varphi_k(x, t) = e^{iq_k \omega t} \varphi_k(x), \tag{6}$$

where $\varphi_k(x)$ is real and independent of time. We conclude that Q-balls with many flavors are solitons built of fields that rotate in the internal space with velocities proportional to their charges. For the solution (6), equation (1) yields

$$Q = \omega \sum_k q_k \int \varphi_k^2(x) \, d^3x \tag{7}$$

It remains to find an extremum of the functional

$$\mathcal{E}_\omega = \int d^3x \left[\frac{1}{2} \sum_k |\nabla \varphi_k(x)|^2 + \hat{U}_\omega(\varphi(x)) \right] + \omega Q, \tag{8}$$

with respect to ω and the variations of $\varphi(x)$ independently. We can first minimize $\mathcal{E}_\omega$ for a fixed ω, while varying the shape of $\varphi(x)$. The solution to this part of the problem [3] is just a bounce $\bar{\varphi}_\omega(x)$ associated with tunneling in $d = 3$ Euclidean dimensions [4, 5, 6, 7] in the potential $\hat{U}_\omega(\varphi)$. The problem is, therefore, reduced to that which is more familiar and better developed. This analogy was used in Ref. [3] to prove the existence and the classical stability of the solitons in the limit of small charge. For large Q, the existence proof was given in Ref. [2]. From Ref. [6] we know that the solution is spherically symmetric: $\bar{\varphi}(x) = \bar{\varphi}(r)$, $r = \sqrt{\vec{x}^2}$. This implies, in particular, that the ground state soliton has zero angular momentum.

For a Q-ball to exist, the following condition (*cf.* Ref. [2]) must be satisfied:

$$\mu^2 = 2U(\varphi) \left/ \left(\sum_k q_k \varphi_{k,0}^2 \right) \right. = \min, \quad \text{for } |\vec{\varphi}_0|^2 > 0. \tag{9}$$

As discussed below, if $U(\varphi) \left/ \left(\sum_k q_k \varphi_{k,0}^2 \right) \right.$ has a global minimum at $\varphi_k = \varphi_{k,0} \neq 0$, then Q-balls are stable with respect to decay into the φ quanta. However, if condition (9) is satisfied in the sense of a local minimum, then the corresponding soliton is metastable and can either dissociate into φ particles through tunneling, or evolve into a different soliton with lower value of μ.

2 Thin-wall approximation for large Q-balls

For clarity, in this section we assume that $\varphi_1 \times \varphi_2 \times ... \times \varphi_n = 0$ implies $U(\varphi) = 0$. Relaxing this constraint is straightforward and amounts to allowing Q-balls of different radii made of

different subsets of fields to overlap. In some sense, the condition stated above defines an "irreducible" Q-ball and will simplify the algebra.

For large Q, the solution that minimizes the energy can be approximated [2] by a thin-wall ball of charged matter with a radius R: $\varphi_i(r) \approx \varphi_0 \theta(R-r)$. (Note that we use a single radius R for all flavors, which is the simplification due to restricting our discussion to irreducible Q-balls only.) One can eliminate ω from the expression for the energy using constraint (7) and minimize E with respect to the volume $V = 4\pi R^3/3$ of the soliton.

$$E \approx \frac{Q^2}{2(\sum_k q_k \varphi_{k,0}^2)V} + U(\varphi_0)V + \text{surface energy (neglected)} = min \qquad (10)$$

for $V \equiv 4\pi R_0^3/3 = Q/\sqrt{2U(\varphi_{k,0})(\sum q_k \varphi_{k,0}^2)}$ and

$$M_Q = E_{min} = Q\sqrt{2U(\varphi_0) \left/ \left(\sum_k q_k \varphi_{k,0}^2\right)\right.} \qquad (11)$$

The energy per unit charge, $M_Q/Q \approx \sqrt{2U(\varphi_0)/(\sum_k q_k \varphi_{k,0}^2)}$, is less than the mass of the lightest of the φ_k particles, if condition (9) is satisfied in the *strong* sense: that is if the minimum is global. In this case, the Q-ball is stable with respect to its decay to φ particles.

For large Q, the surface energy is small and can be neglected. For smaller Q, the surface energy becomes more important. A naive application of the thin-wall formalism seems to imply that only the Q balls with a large enough charge, $Q > Q_{min}$, can exist. This constraint, however, is merely an artifact of the thin-wall approximation. The latter fails to account correctly for the energies of the wall and the interior when they become inseparable, that is in the "thick-wall" case. Q-balls of small charges have been proven to exist [3]. There is no classical lower limit of the charge Q of a (classically) stable Q-ball. However, quantum consistency requires charge quantization in units of the charge of the φ field. Therefore, $Q \geq 1$. Also, in the limit $Q \to 1$, quantum corrections can significantly modify semiclassical results (at least, we do not have a proof to the contrary [3]).

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3   Beyond the thin-wall approximation: Q-beads

If Q is small, ω becomes large[1] [3]. For large ω, the bounce in the potential $\hat{U}_\omega(\varphi)$ cannot be analysed using the thin-wall approximation. A "thick-wall" approximation [7, 8] can be used instead. We will briefly summarize the results of Ref. [3] relevant to our discussion.

For a single scalar field with a potential[2] $U(\varphi) = \frac{1}{2}M^2\varphi^2 - A\varphi^3 + \lambda_4\varphi^4$, one has to calculate the bounce in the effective potential

$$\hat{U}_\omega(\varphi) = \frac{1}{2}(M^2 - \omega^2)\varphi^2 - A\varphi^3 + \lambda_4\varphi^4 \tag{12}$$

and then minimize $\mathcal{E}_\omega$ in equation (8) with respect to ω. The thick-wall approximation [3] is applicable and the minimum exists if

$$Q \ll \frac{3S_\psi M}{A} \times \min\left(\frac{1}{\sqrt{\lambda_4}}, \frac{M}{2A}\right) \tag{13}$$

where $S_\psi \approx 4.85$. The small Q soliton has a mass M_Q and a size R_Q:

$$M_Q \approx QM \left[1 - \frac{1}{6}\epsilon^2 - \frac{1}{8}\epsilon^4 - O(\epsilon^6)\right] \tag{14}$$

$$R^{-1} \sim (M^2 - \omega^2)^{1/2} \approx \epsilon M \left(1 + \frac{1}{2}\epsilon^2 + \frac{7}{8}\epsilon^4 + O(\epsilon^6)\right) \tag{15}$$

where $\epsilon = (QA^2/3S_\psi M^2) < \frac{1}{2}$ by virtue of the constraint (13).

Generalization of this discussion to the case of many scalar fields is straightforward and involves finding the bounce in the potential (5). For a complicated scalar potential, as that of the MSSM, this can be done numerically, for example, using the Improved Action method [9].

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[1]This is not in contradiction with equation (7). As ω increases, $(\int \bar{\varphi}_\omega^2)$ for the bounce $\bar{\varphi}_\omega^2(x)$ in the potential $\hat{U}_\omega(\varphi)$ decreases faster than $(1/\omega)$; see discussion in Ref. [3].

[2]The φ^3 term should be thought of as a $U(1)$-symmetric cubic interaction, *e. g.*, $(\varphi^\dagger\varphi)^{(3/2)}$. In the MSSM, the tri-linear couplings of the Higgs field to squarks and sleptons yield the requisite cubic terms, whose "flavor structure" is discussed in the next section.

4 B and L balls in the MSSM

Every supersymmetric generalization of the MSSM must have Yukawa couplings of the Higgs fields H_1 and H_2 to quarks and leptons which arise from the superpotential of the form

$$W = yH_2\Phi\phi + \tilde{\mu}H_1H_2 + ... \tag{16}$$

Here Φ stands for either a left-handed quark ($\tilde{Q}_L$), or a lepton ($\tilde{L}_L$) superfield, and ϕ denotes $\tilde{q}_R$ or $\tilde{l}_R$, respectively. The corresponding scalar potential must, therefore, have cubic terms of the form $y\tilde{\mu}H_2\Phi\phi$. In addition, there are soft supersymmetry breaking terms of the form $yAH_1\Phi\phi$. This is a generic feature of all SSM.

For squarks and sleptons, there are several abelian symmetries[3] that are suitable for building Q-balls. These are $U(1)_B$, $U(1)_{L_i}$ and $U(1)_E$, associated with the conservation of baryon number, three types of lepton numbers, and the electric charge. Although we discussed only the case of a global $U(1)$ symmetry, Q-balls can be constructed for a local $U(1)$ as well [10]. In the case of a local symmetry, Q-balls are stable as long as their charge is less than some maximal value [10].

In the MSSM, Q-balls are allowed, therefore, to have a baryon number, a lepton number, and an electric charge. As a toy model, one can consider a potential for the Higgs field, H, and the sleptons, $\tilde{L}_L$ and $\tilde{l}_R$, with a scalar potential

$$U = m_H^2|H|^2 + m_L^2|\tilde{L}_L|^2 + m_l^2|\tilde{l}_R|^2 - yA(H\tilde{L}_L^*\tilde{l}_R + c.c.) + y^2(|H^2\tilde{L}_L^2| + |H^2\tilde{l}_R^2| + |\tilde{L}_L^2\tilde{l}_R^2|) + V_D, \tag{17}$$

where $V_D = (g_1^2/8)[|H|^2 - |\tilde{L}_L|^2]^2 + (g_2^2/8)[|H|^2 + |\tilde{L}_L|^2 - 2|\tilde{l}_R|^2]^2$ is the contribution of the gauge the D-terms. For simplicity, we neglected the Higgs VEV. Nevertheless, this toy model is instructive because it allows for some non-topological solitons with the same quantum numbers as those in the MSSM. The potential is invariant under the global $U(1)_L$ symmetry ($\tilde{L}_L \to \exp\{i\theta\}\tilde{L}_L$ and $\tilde{l}_R \to \exp\{i\theta\}\tilde{l}_R$) associated with the lepton number conservation. Both $\tilde{L}_L$ and $\tilde{l}_R$ have a unit charge with respect to this $U(1)$, while the Higgs field is $U(1)_L$ invariant.

It is convenient to write

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[3]The case of non-abelian Q-balls associated with squarks and sleptons will be discussed elsewhere.

$$\begin{cases} H & = & F \ sin\xi \\ \tilde{L}_L & = & F \ \cos\xi \ \sin\theta \\ \tilde{l}_R & = & F \ \cos\xi \ \cos\theta \end{cases} \qquad (18)$$

The condition (9) is satisfied, and a Q-ball with mass $M_Q = \mu Q$ exists, if μ^2 in equation (11) is minimized at some value of $F \neq 0$.

$$\mu^2 = \frac{2U}{|\tilde{L}_L|^2 + |\tilde{l}_R|^2} = \frac{1}{\cos^2\xi}[\gamma_2(m_i^2, \xi.\theta) - yA\gamma_3(\xi, \theta) \, F \ + \ \gamma_4(\xi, \theta) \, F^2], \qquad (19)$$

where γ_2 and γ_4 are non-negative functions of masses and mixing angles, $\gamma_3 = \cos^2\xi \sin\xi \sin(2\theta)$. The minimum of μ^2 in (19) is achieved at $F \neq 0$ if $yA \neq 0$. The origin is not a local minimum. Therefore, in our toy model, L balls exist no matter how small the tri-linear couplings might be, as long as they are non-zero. The same is true of the baryonic balls built of squarks.

Of course, in the full MSSM there can be other fields that carry the same charge. Therefore, the local minimum of energy corresponding to a particular set of fields may not be the global minimum. For example, an electrically neutral selectron L ball, $\{H, \tilde{e}_L, \tilde{e}_L\}$, will be in competition with a sneutrino ball, $\{H, \tilde{\nu}_L, \tilde{\nu}_L\}$. However, since the origin is not a local minimum of (19) for $yA \neq 0$, there is always a stable Q-ball with a given lepton (baryon) number[4].

Having convinced ourselves that non-topological solitons exist in the MSSM, we will now discuss some of the phenomenological consequences. Large Q-balls are extended objects and cannot be produced in a collider. As follows from equation (15), Q-beads, with charge of order a few, are also extended objects, whose size is large in comparison to their De Broglie wavelength. The probability of producing them in a collider experiment is, probably, exponentially suppressed by their size and is likely to be negligible. This question, however, is by no means obvious and deserves a more careful analysis because, if the Q-beads can be created in a collider, their signatures could be spectacular. For example, a soliton with both $B \neq 0$ and $L \neq 0$ would interact as a leptoquark.

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[4]This would not necessarily be the case if one of the sleptons or squarks had its tri-linear coupling equal to zero (and was sufficiently light). However, as far as we know, this cannot happen in a realistic model, where the cubic couplings are allowed by the gauge symmetry, and are also required in order to break the continious R symmetry explicitly.

In the early Universe, the non-topological solitons can be created in the course of a phase transition [11, 13] via the Kibble mechanism ("solitogenesis"), or they can be produced in a fusion process reminiscent of nucleosynthesis [12, 13] ("solitosynthesis"). Their subsequent evolution can lead to interesting cosmological phenomena [14].

Since the baryon and lepton asymmetries are small (if not zero), it is the statistical fluctuations of charge that play a major role in the formation of the baryonic and leptonic balls. The rate of such fluctuations was estimated in Ref. [13] for a particular model. A typical soliton number to entropy ratio was found to be $Y_Q \equiv n_Q/s \sim c\, Q^{-3/2} \exp(-Q)$, where c is a dimensionless number ($c \sim 10^{-3}$ for the model discussed in Ref. [13]). Although this estimate is expected to break down for small Q, it is clear on general grounds that the small-charge solitons can be produced in greater numbers than the large Q-balls. In a separate work, we will discuss the details of the B and L-ball production at high temperature [15]. In any case, small and moderately large solitons can be produced in great numbers at high temperatures in the early Universe.

Stability of very small solitons, for example those with a unit charge[5], can be guaranteed merely by some combination of the conservation laws, regardless of the soliton mass. For example, an electrically neutral, $SU(2)$ singlet, $L = 1$ bead with zero spin cannot decay because of the lepton number and the angular momentum conservation. There is simply no state in the MSSM spectrum, except for the soliton sector, that would have these quantum numbers. Although caution is urged in applying the semiclassical treatment to Q-beads of a unit charge, there is no obvious reason to exclude these objects as candidates for dark matter.

Large minimal-energy B and L-balls built of squarks and sleptons can be stable against decay into their constituent scalar fields, but they can still evaporate into the fermions that carry B and L, quarks and leptons [16]. According to Ref. [16], the evaporation proceeds from the surface of the Q-ball and the rate is proportional to the surface area, rather than the volume of the Q-ball. This is due to the exclusion principle for fermions. Inside the Q-ball, the

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[5]Semiclassical results can be modified noticeably by quantum corrections if $Q = 1$ [3]. For instance, the soliton mass can receive order 1 corrections in this limit. On the other hand, since the size of a $Q = 1$ soliton is still large in comparison to its De Broglie wavelength (equation (15)), the semiclassical treatment of $Q = 1$ beads might still be appropriate. Since we know of no alternative to the semiclassical description of solitons, we will proceed keeping in mind this caveat.

Dirac sea of quarks and leptons fills up until the Fermi pressure prevents further production of these particles via the decay of the squarks and sleptons. The fermionic decay products can still leak through the surface of the Q-ball, and the evaporation proceeds slowly, at the rate proportional to the surface area. The evaporation rate would be proportional to the volume of the Q-ball if it were to decay into scalar particles. However, we saw that this is forbidden by the energy conservation for the Q-balls of minimal energy. Gauge fields carry no $(B - L)$ charge and cannot facilitate the evaporation.

In the MSSM, the processes that can lead to B and L balls evaporation into quarks and leptons are mediated by gauginos (and gluinos) and, if the gaugino mass is larger than μ, they can be further suppressed. The lifetimes of baryonic and leptonic balls built of sparticles are model-dependent and will be analysed elsewhere [15] for a variety of the MSSM parameters.

Those B and L solitons that decay at temperatures T above 1 GeV, probably, have no observable consequences. However, a remarkable transformation can take place for a Q-ball that survived to a temperature of order Λ_{QCD}. We recall that the interior of a large evaporating Q-ball is populated with a high density of quarks that fill the Dirac sea up to the energies of order μ. If the Q-ball survives to temperatures below Λ_{QCD}, then the population of quarks fostered inside the sparticle ball can remain bound, now by the QCD forces, even after the sparticle structure, which kept them together originally, disappears. At $T \gg 1$ GeV, such a conglomerate of nuclear matter would thermalize without a trace. However, at lower temperatures, heavy nuclei can form as vestiges of sparticle Q-balls.

Since the statistical fluctuation mechanism [13] is probably the most likely source of solitons at the electroweak scale temperatures, a comparable numbers of baryon (lepton) and anti-baryon (anti-lepton) balls will be produced. Those Q-balls that have a lifetime of order 10^{-6} s or more, will give birth to heavy nuclei ($A \sim Q$) of matter and anti-matter, with some excess for $B > 0$. The excess of $B > 0$ nuclei can survive the subsequent annihilation.

This allows for a highly non-standard synthesis of heavy nuclei in the early Universe, such that they are already present at the time $t \sim 1$ s, when the standard nucleosynthesis is supposed to commence. Fission of heavy nuclei can also be the source of additional lighter elements, in particular, ^4He, which are copiously produced in nuclear decays. Details of this

and other cosmological implications of the MSSM solitons will be analyzed elsewhere [15].

Q-balls with lifetime longer than 1 second are probably disallowed, at least if they can be produced in substantial quantities. Their decay products can cause an unacceptable increase in entropy, or disturb the spectrum of the microwave background radiation.

In summary, non-topological solitons with non-zero baryon and lepton number, as well as the electric charge, are generically present in the spectrum of the MSSM and other models with low-energy supersymmetry. Production of these objects in the early Universe and can have a number of important cosmological ramifications.

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