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A Case for 14 Dimensions

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ABSTRACT

Extended superalgebras of types A, B, C, heterotic and type-I are all derived as solutions to a BPS equation in 14 dimensions with signature $(11,3)$. The BPS equation as well as the solutions are covariant under $\mathrm{SO}(11,3)$. This shows how supersymmetries with $N \leq 8$ in four dimensions have their origin in the same superalgebra in 14D. The solutions provide different bases for the same superalgebra in 4D, and the transformations among bases correspond to various dualities.

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1 64 supercharges

It is well known that in four flat dimensions there cannot be more than eight conserved real (Majorana) supercharges. If one imagines that the 4D theory comes from some fundamental theory, then the fundamental theory apparently may not have more than 32 *real* supercharges in its flat limit: If it had more than 32 it would imply more than $N = 8$ in 4D. The physical basis for this assertion is that in flat 4D spacetime there cannot be massless interacting particles with helicity higher than 2 and/or that there is only one graviton. For $N \geq 8$, a supermultiplet that includes the graviton in 4D necessarily contradicts these facts.

A caveat in this argument is that there may be different sets of 32 supercharges that are equivalent to each other under duality symmetries from the point of view of a lower dimensional effective theory. We suggest that the existence of dualities may allow 64 supercharges, and that in dimensions 10 to14 one can embed three sets 32_A,32_B and 32_C as different projections of the 64, which form three distinct superalgebras of types A, B, C. First examples of theories containing the $32_{A,B}$ supercharges are 10D supergravity/superstrings of type-IIA,B. In 11D supergravity there is 32_A. For dimensions 9 and below the distinction between $32_{A,B}$ disappears, but the T-duality which is well known for $d \leq 9$ actually performs the transformation $32_A \longleftrightarrow 32_B$. This duality was interpreted as a transformation of 64 spinors among themselves by a transformation of hidden dimensions [1]. Since the various types of non-perturbative dualities map different forms of theories into each other it is not clear whether the overall theory behind all of it is a theory with 64 supercharges or a theory with only 32 of them.

Are there 64 supercharges rather 32? What mechanism generates effectively 32 supercharges in different sectors? As suggested below $64 \rightarrow 32$ happens naturally through a BPS condition that can be formulated as a covariant equation in 14 dimensions and which has three distinct branches of solutions labelled by A, B, C. In this way we answer a related set of questions: Is there a single set of 528 bosons in the superalgebras A, B, C or are there three sets $528_{A,B,C}$ that are mapped to each other by dualities. If they are different, what is the subset of bosons that is common to various sectors?

In this paper we explore further the possibility that the unknown fundamental theory behind non-perturbative string theory and its dualities may be a theory more usefully formulated in higher dimensions, perhaps in 14D. By recognizing its hidden dimensions one may better understand its overall structure as well as its low energy properties. Since the fundamental theory is unknown we concentrate only on properties of its supersymmetries. We assume that in the fundamental theory 32_A and 32_B are two distinct sets mapped to each other by a transformation that is interpreted as T-duality from the point of view of effective theories in lower dimensions. Since T-duality maps small to large distances, our assumption implies that some of the 32_A supercharges that govern supersymmetry at large distances get mapped to some of the 32_B supercharges that govern supersymmetry at small distances, and vice versa. In this paper we show how to embed type A, B, C, heterotic and type-I superalgebras *covariantly* in the framework of 14 dimensions with signature (11,3) and how to recognize the 14 dimensions when the theory is compactified to lower dimensions.

2 From 10 to 14 dimensions.

The $A(B)$ superalgebra in 10D contains two 16-component Majorana-Weyl spinors of opposite (same) chirality $32_{A(B)} = 16_L + 16_{R(L)}$. The A superalgebra may be rewritten as an 11D superalgebra using a single 32 component spinor. In M-theory [2], the anticommutator has all possible 528_A extensions corresponding to the 11D momentum and the 2 and 5 branes [3] $528_A = 11+55+462$. This hides a 12D structure of the form $528_A = 66+462$ which corresponds to a 12D superalgebra [4]-[10]

$$\{Q_\alpha, Q_\beta\} = \gamma_{\alpha\beta}^{M_1M_2}\ Z_{M_1M_2}\ + \gamma_{\alpha\beta}^{M_1\cdots M_6}\ \ Z^+_{M_1\cdots M_6} \tag{1}$$

where α labels the 32 component Weyl spinor of SO(10,2). The (10,2) signature is necessary to have a real spinor[1]. The 12th gamma matrix is the $32{\times}32$ identity matrix $\gamma^{0'} = 1_{32}$ since it is in the Weyl sector[2].

[1]Bott periodicity indicates that the reality properties of spinors are similar for $\mathrm{SO}(10,1) \sim \mathrm{SO}(2,1) = \mathrm{SL}(2,R)$ and $\mathrm{SO}(10,2) \sim \mathrm{SO}(2,2) = \mathrm{SL}(2,R)\times\mathrm{SL}(2,R)$ and $\mathrm{SO}(11,2) \sim \mathrm{SO}(3,2) = \mathrm{Sp}(4,R)$ and $\mathrm{SO}(11,3) \sim \mathrm{SO}(3,3) = \mathrm{SL}(4,R)$.

[2]In terms of 11D one obtains $Z_{M_1M_2} \to P_\mu \oplus Z_{\mu_1\mu_2}$ and $Z^+_{M_1\cdots M_6} \to Z_{\mu_1\cdots\mu_5}$. If nonzero, the 2 and 5 branes are sources coupled to the 11D antisymmetric gauge potential $A_{\mu_0\mu_1\mu_2}$ and its magnetic dual $A_{\mu_0\mu_1\mu_2\cdots\mu_5}$. The Lorentz generator $L_{\mu_1\mu_2}$ is outside of this algebra since $Z_{\mu_1\mu_2}$ interpreted as above cannot coincide with $L_{\mu_1\mu_2}$. Therefore the 11 or

The 10D type-IIB superalgebra is not included in the 11D or 12D superalgebra above. The B superalgebra with its 528_B bosonic generators can be written in a form that exhibits higher dimensions [1]

$$\left\{Q_{\bar{\alpha}\bar{a}}, Q_{\bar{\beta}\bar{b}}\right\} = (i\tau_2\tau_i)_{\bar{a}\bar{b}}\left[\bar{\gamma}^{\bar{\mu}}_{\bar{\alpha}\bar{\beta}}\ P^i_{\bar{\mu}} + \bar{\gamma}^{\bar{\mu}_1\cdots\bar{\mu}_5}_{\bar{\alpha}\bar{\beta}}\ \ X^i_{\bar{\mu}_1\cdots\bar{\mu}_5}\right] + \bar{\gamma}^{\bar{\mu}_1\bar{\mu}_2\bar{\mu}_3}_{\bar{\alpha}\bar{\beta}}\ \ (i\tau_2)_{\bar{a}\bar{b}}\ Y_{\bar{\mu}_1\bar{\mu}_2\bar{\mu}_3}. \tag{2}$$

where $\bar{\alpha}, \bar{\beta} = 1, 2, \cdots, 16$ and $\bar{a}, \bar{b} = 1, 2$ are spinor indices, while $\bar{\mu} = 0, 1, \cdots, 9$ and $i = 0', 1', 2'$ are vector indices for $SO(9,1) \times SO(2,1)$. The τ_i are given by Pauli matrices $\tau_i = (-i\tau_2, \tau_1, \tau_3)$. The $P^i_{\bar{\mu}}$ is a SO$(2,1)$ triplet containing the momentum $p_{\bar{\mu}}$ and two 1-brane sources $w^{1,2}_{\bar{\mu}}$ that couple to the two antisymmetric gauge potentials $B^{1,2}_{\mu_0\mu_1}$ in type-IIB superstring theory [3]. In [1] the SO$(2,1)$ symmetry was interpreted as Lorentz transformations

12D superalgebra is not related to the superconformal algebra that has operators labelled in a similar way. There is an extended superconformal algebra with the same structure [11] but it has different physical content. One may wonder about the closure and Jacobi identities since such properties determine the representations of the superalgebra. For our present discussion this question may be left open since there are various possibilities [12]. The simplest case is to take Abelian extensions corresponding to a linearized flat limit of the theory. A model for a curved space may correspond to OSp(1/32) which satisfies all Jacobi identities. Intermediate cases are obtained by considering various contractions of OSp(1/32). However, there are more possibilities since the operators may close into an even larger set of operators in the quantum theory of interacting p-branes. The anticommutator considered in our discussion applies to all cases.

[3] It may seem unusual that the 10D momentum operator is a member of the SO$(2,1)$ triplet $P^i_{\bar{\mu}}$, but this is clearly true. This SO$(2,1)$ should not be confused with the U duality group SL(2,Z). U acts on the gauge potentials $B^{1,2}_{\mu_0\mu_1}$ and sources $w^{1,2}_{\bar{\mu}}$ but leaves the metric and momentum $p_{\bar{\mu}}$ invariant. The transformation known as S duality fits in the maximal compact subgroup of both the SO$(2,1)$ and SL(2,Z). What is the relation between these groups? To distentagle them one may expand $P^i_{\bar{\mu}} = p_{\bar{\mu}}v^i + w^1_{\bar{\mu}}v^i_1 + w^1_{\bar{\mu}}v^i_2$ by using a basis of three orthogonal SO$(2,1)$ vectors (v^i, v^i_1, v^i_2) where v^i is timelike and $v^i_{1,2}$ are spacelike. Then $p_{\bar{\mu}}$ is the 10D momentum and v^i is a 3D "momentum" [13] both of which are singlet under SL(2,Z) while the spacelike vectors $(w^1_{\bar{\mu}}, w^2_{\bar{\mu}})$ and (v^i_1, v^i_2) form doublets under SL(2,Z) such that $P^i_{\bar{\mu}}$ is a singlet of SL(2,Z). This is the B basis that exhibits the Lorentz symmetry SO$(2,1)$ while hiding the duality symmetry. In the the rest frame of the internal "momentum" one may take $v^i = (1,0,0)$ so that the superalgebra takes the more familiar form $\left\{Q^{\bar{a}}_{\bar{\alpha}}, Q^{\bar{b}}_{\bar{\beta}}\right\} = \bar{\gamma}^{\bar{\mu}}_{\bar{\alpha}\bar{\beta}}\ (i\tau_2\tau_i)^{\bar{a}\bar{b}} p_{\bar{\mu}}v^i + \cdots = \bar{\gamma}^{\bar{\mu}}_{\bar{\alpha}\bar{\beta}}\ \delta^{\bar{a}\bar{b}}p_{\bar{\mu}} + \cdots$. This latter form exhibits the SL(2,Z) basis rather than the Lorentz basis. Therefore the duality and the Lorentz bases are related to each other by a boost in SO(2,1).

This is a general situation that applies in every dimension as explained in [15]. Namely, the basis in which the momentum is a singlet under dualities is defined to be the duality in 3 additional dimensions beyond the usual 10D. In the present paper it will be more properly interpreted as the $\mathrm{SL}(2,R)_+$ embedded in $\mathrm{SO}(2,2) = \mathrm{SL}(2,R)_+ \times \mathrm{SL}(2,R)_-$. Then the triplet index i will be replaced by the self dual $\mathrm{SO}(2,2)$ tensor $[mn]_+$ which is a triplet of $\mathrm{SL}(2,R)_+$ when the vector index $m = 11, 12, 13, 14$ spans the extra 4 dimensions with signature $(+,-,+,-)$.

Based on T duality we suggest that the A, B superalgebras are different sectors of the same fundamental theory as explained in the previous section. To unify them in the same theory one is led to 64 supercharges classified as the spinor in 14D with signature $(11,3)$ as a generalization of S theory [1]. This is possible since $\mathrm{SO}(11,3)$ also has a Majorana-Weyl spinor of dimension 64 (see footnote 1).

There should exist a mechanism that provides two distinct projectors that cuts down 64 to the branches $32_{A,B}$. We will see soon that there is a third distinct projector that leads to a third branch C with 32_C fermions. The heterotic and type-I superalgebras are secondary branches attached to the main A, B, C branches. As will be shown, all such branches and sub-branches may be embedded *covariantly* in $\mathrm{SO}(11,3)$.

We remind the reader of the non-covariant A and B projectors. By using the $\mathrm{SO}(11,2)$ 64×64 gamma matrices Γ_M as in the appendix of [1], the A projector is given by $1+\Gamma_{13}$. It distinguishes the spacelike 13th dimension. The B projector is $(1+\Gamma_{11}\Gamma_{12}\Gamma_{13}) = (1+\Gamma_0\cdots\Gamma_9)$. It distinguishes the (2,1) from the (9,1) dimensions. Thus, as exhibited in the superalgebras above, they are covariant under the groups $A = \mathrm{SO}(10,2)$ and $B = \mathrm{SO}(9,1)\times\mathrm{SO}(2,1)$ respectively, while they are both embedded in $\mathrm{SO}\,(11,2)$. How do these fit in 14D? It is possible to go up one more dimension because the 64 component real spinor may also be regarded as the Weyl spinor of $\mathrm{SO}(11,3)$. In the Weyl sector the timelike 14th gamma matrix is given by the identity matrix $\Gamma_{14} = 1_{64}$. Then the projector to the A sector is really lightlike $(\Gamma_{14}+\Gamma_{13})$ leaving behind $\mathrm{SO}(10,2)$ covariance as desired. From the point of view of $14\mathrm{D} \rightarrow (9,1)+(2,2)$ note that the A projector is a lightlike vector embedded in $(2,2)$. This projector cuts down 64 to 32_A which consists of two opposite chirality 16 component spinors in 10D.

basis. In the A or B bases the momentum is a member of a multiplet that transforms under an internal Lorentz group. In the rest frame of an internal momentum one recovers the duality basis. Hence A and B bases are connected to the duality basis with boosts and they are related to each other by T-duality.

For the B sector consider also $\mathrm{SO}(11,3) \longrightarrow \mathrm{SO}(9,1) \times \mathrm{SO}(2,2)$ and note that there are two possibilities for embedding the $\mathrm{SO}(2,1)$ that appears in eq.(2). It could be identified either with the vectorial $\mathrm{SL(2,R)}_V$ or with the chiral-like $\mathrm{SL}(2,R)_+$ embedded in $\mathrm{SO}(2,2)$

$$SO\,(2,2) = SL(2,R)_+ \times SL(2,R)_- \supset SL(2,R)_V. \tag{3}$$

However, as we will see in footnote 5 below, there is no difference of content between the two and the proper interpretation is the $\mathrm{SL}(2,R)_+$. This chiral embedding is fully covariant under $\mathrm{SO}(9,1)\times$ $\mathrm{SO}(2,2)$. The B projector which is consistent with this invariance is $(1+\Gamma_{11}\Gamma_{12}\Gamma_{13}\Gamma_{14}) = (1+\Gamma_0\cdots\Gamma_9)$.

Are there any other main branches? A main branch will be defined as a superalgebra that has symmetries that cannot be contained as a subgroup of the symmetries of another branch. The symmetries should be realized on only 32 *real* fermions, not 64. Only symmetries of the form $\mathrm{SO}(n,1)\times$ $\mathrm{SO}(11-n,2)$ or $\mathrm{SO}(n,3)\times$ $\mathrm{SO}(11-n,0)$ that fit within $\mathrm{SO}(11,3)$ need to be considered[4]. Since the symmetries $\mathrm{SO}(10,2)$ and $\mathrm{SO}(9,1)\times\mathrm{SO}(2,2)$ are not contained in each other, the A, B branches are distinct main branches. We have found that there is only one other main branch with symmetry $\mathrm{SO}(3,3)\times$ $\mathrm{SO}(8)$ that satisfies the C superalgebra

$$\{Q_{\alpha a}, Q_{\beta b}\} = \gamma^{\mu}_{\alpha\beta}\gamma^{ij}_{ab}P_{\mu ij} + \gamma^{\mu_1\mu_2\mu_3}_{\alpha\beta}\delta_{ab}T^{+}_{\mu_1\mu_2\mu_3} + \gamma^{\mu_1\mu_2\mu_3}_{\alpha\beta}\gamma^{ijkl}_{ab}Z^{+}_{\mu_1\mu_2\mu_3 ijkl} \tag{4}$$

Here α,β are spinor indices for $\mathrm{SO}(3,3)=\mathrm{SL}(4,R)$, and a,b are 8_+ spinor indices for $\mathrm{SO}(8)$. Both of these spinors are Majorana-Weyl, therefore there are $4\times8=32$ real components. The indices μ, i are vector indices for $\mathrm{SO}(3,3)$, $\mathrm{SO}(8)$ respectively. The gamma matrices $\gamma^{\mu_1\mu_2\mu_3}_{\alpha\beta}, \gamma^{ijkl}_{ab}$ are automatically self dual in the indices $[\mu_1\mu_2\mu_3]$ and $[ijkl]$ since they are in the Weyl sectors. Therefore the tensors $T^{+}_{\mu_1\mu_2\mu_3}$, $Z^{+}_{\mu_1\mu_2\mu_3 ijkl}$ are self dual in the corresponding indices. The number of bosons may be counted as follows: for $P_{\mu ij}$ $6\times28=168$, for $T^{+}_{\mu_1\mu_2\mu_3}$ $\frac{1}{2}\frac{6\times5\times4}{1\times2\times3}=10$, for $Z^{+}_{\mu_1\mu_2\mu_3 ijkl}$ $10\times\frac{1}{2}\frac{8\times7\times6\times5}{1\times2\times3\times4}=350$. The total is 528_C.

The heterotic and type-I superalgebras in 10D may be obtained as sub-branches of the main branches. Also different compactifications are sub-

[4]We have in mind a theory of various interacting p-branes formulated in some way in terms of $X^M(\tau,\sigma_1,\cdots,\sigma_p)$, where the coordinate index M provides the basis for $\mathrm{SO}(n,m)$ transformations. When we refer to "dimensions" we mean X^M, and the symmetries we are discussing are rotations of these coordinates.

branches. We will present them below as sub-branches in a 14D covariant formalism of the main branches.

3 14D covariance

The superalgebra with 64 spinors may be written covariantly for $\mathrm{SO}(11,3)$ in the form $\{Q_\alpha, Q_\beta\} = S^{64}_{\alpha\beta}$ with

$$S^{64}_{\alpha\beta} = \Gamma^{\bar{M}_1\bar{M}_2\bar{M}_3}_{\alpha\beta} E_{\bar{M}_1\bar{M}_2\bar{M}_3} + \Gamma^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}_{\alpha\beta} F^+_{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}, \tag{5}$$

where $\bar{M}$ labels the vector of $\mathrm{SO}(11,3)$. The tensor F^+ is self dual because the seven index antisymmetric gamma matrix in the Weyl sector is automatically self dual in 14D. This superalgebra contains 2080 bosons (=364+1716). If reduced to 13D one obtains antisymmetric tensors with 2,3,6 indices corresponding to the decomposition 2080 = 78+286+1716.

We suggest that, independent of the details of the fundamental theory, the mechanism that cuts 64→ 32 can be formulated as a $\mathrm{SO}(11,3)$ covariant BPS equation in the form

$$\det\left(S^{64}_{\alpha\beta}\right) = 0. \tag{6}$$

The multiplicity of the zero eigenvalues of this equation corresponds to the number of supercharges that vanish on the BPS states.

3.1 A branch

The solution of eq.(6) corresponding to the A branch may be written in a 14D covariant notation as follows

$$\begin{aligned} \left(S^{64}_A\right)_{\alpha\beta} &= \Gamma^{\bar{M}_1\bar{M}_2\bar{M}_3}_{\alpha\beta} Z_{\bar{M}_1\bar{M}_2} V_{\bar{M}_3} + \Gamma^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}_{\alpha\beta} Z^+_{\bar{M}_1\bar{M}_2\cdots\bar{M}_6} V_{\bar{M}_7}\,, \\ V_{\bar{M}} V^{\bar{M}} &= Z_{\bar{M}_1\bar{M}_2} V^{\bar{M}_2} = Z^+_{\bar{M}_1\bar{M}_2\cdots\bar{M}_6} V^{\bar{M}_6} = 0, \end{aligned} \tag{7}$$

where $V_{\bar{M}}$ is a 14D lightlike vector and the antisymmetric tensors Z, Z^+ are orthogonal to it. Thanks to orthogonality one may factor out the lightlike projector $\Gamma_{\bar{M}} V^{\bar{M}}$ as an overall factor in S^{64}_A, showing that 32 supercharges vanish. Although orthogonality allows components in Z, Z^+ that point along $V_{\bar{M}}$, those drop out due to the antisymmetric indices $\bar{M}_k$ on

$\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\bar{M}_3}$, $\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}$. The remaining effective subspace of $(10,2)$ indices orthogonal to $V_{\bar{M}}$ give precisely 528_A bosons. $Z^{+}_{\bar{M}_1\bar{M}_2\cdots\bar{M}_6}$ is self dual in the $(10,2)$ subspace because the six-index gamma matrices in the 14D Weyl basis $\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}V_{\bar{M}_7}$ are automatically self dual in the $(10,2)$ subspace thanks to the lightlike vector.

Using the SO$(11,3)$ covariance one can choose a special frame in which $V_{\bar{M}}$ points along the 13th+14th dimensions with signature $(1,1)$. Then $\Gamma_{\bar{M}}V^{\bar{M}} \sim \Gamma_{13}+\Gamma_{14}$ and S_A^{64} takes the block diagonal form $S_A^{64} = (S_A^{32}, 0)$ where we have used $\Gamma^{14} = 1_{64}$. Then S_A^{32} is precisely the right hand side of eq.(1).

In a general frame the full SO$(11,3)$ covariance is possible provided the lightlike vector is an operator rather than a fixed vector frozen in some direction. The presence of such operators lead to more general superalgebras. Examples, possible interpretations and physical roles of such operators may be found in [14] [13].

3.2 B branch

Introduce four orthogonal SO$(11,3)$ unit vectors $V^m_{\bar{M}}$, with $m = 1,2,3,4$. Let V^1, V^3 be spacelike and V^2, V^4 timelike. Then the m label defines an auxiliary SO$(2,2)'$. Construct SO$(11,3)\times$ SO$(2,2)'$ covariant antisymmetric tensors

$$F_{\bar{M}\bar{N}}^{[mn]_+} = V^m_{[\bar{M}}V^n_{\bar{N}]} + \frac{1}{2}\varepsilon^{mn}{}_{pq}V^p_{[\bar{M}}V^q_{\bar{N}]}. \tag{8}$$

They are *self dual* tensors for the auxiliary SO$(2,2)'$ =SL$(2,R)'_+ \times$ SL$(2,R)'_-$, so they form a *triplet* of the auxiliary SL$(2,R)'_+$. The covariant superalgebra for the B sector can now be written by giving the solution to the BPS equation in the form

$$\begin{aligned}\left(S_B^{64}\right)_{\alpha\beta} \;=\;& \Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}\left(Z^{[mn]_+}_{\bar{M}_1\bar{M}_2\cdots\bar{M}_5}F_{\bar{M}_6\bar{M}_7[mn]_+} + Y_{\bar{M}_1\bar{M}_2\bar{M}_3}\,V^1_{\bar{M}_4}V^2_{\bar{M}_5}V^3_{\bar{M}_6}V^4_{\bar{M}_7}\right)\\ &+\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\bar{M}_3}\left(P^{[mn]_+}_{\bar{M}_1}F_{\bar{M}_2\bar{M}_3[mn]_+} + Y_{\bar{M}_1\bar{M}_2\bar{M}_3}\right),\end{aligned} \tag{9}$$

where the bosons P, Y, Z are orthogonal to all four vectors

$$P^{[pq]_+}_{\bar{M}_1}V^{\bar{M}_1}_m = Y_{\bar{M}_1\bar{M}_2\bar{M}_3}V^{\bar{M}_3}_m = Z^{[pq]_+}_{\bar{M}_1\bar{M}_2\cdots\bar{M}_5}V^{\bar{M}_5}_m = 0. \tag{10}$$

Due to orthogonality, the $Y_{\bar{M}_1\bar{M}_2\bar{M}_3}$ term in S_B^{64} is proportional to the projector

$$\left(1+V^1V^2V^3V^4\right), \tag{11}$$

where the $V^m \equiv V_{\bar{M}}^m\Gamma^{\bar{M}}$ anticommute $V^1V^2 = -V^2V^1$. Also, because of the self duality of $F_{\bar{M}_6\bar{M}_7[mn]_+}$ the other terms may be multiplied by the same projector without changing anything. Therefore, the full S_B^{64} is proportional to the same projector, signaling the vanishing of 32 supercharges.

The projector is invariant under $\mathrm{SO}(11,3)\times\ \mathrm{SO}(2,2)'$. This symmetry can be used to gauge fix the $V_{\bar{M}}^m$ to a special frame $V_{\bar{M}}^m \to \delta_{\bar{M}}^{10+m}$ in which the orthogonal 14D vectors $V_{\bar{M}}^1, V_{\bar{M}}^2, V_{\bar{M}}^3, V_{\bar{M}}^4$ point along the 11th,12th,13th and 14th dimensions respectively. In this frame the auxiliary $\mathrm{SO}(2,2)'$ coincides with the $\mathrm{SO}(2,2)$ embedded in $\mathrm{SO}(11,3)$. The projector becomes $(1+\Gamma_{11}\Gamma_{12}\Gamma_{13}\Gamma_{14}) = (1+\Gamma_0\cdots\Gamma_9)$, which is recognized as the B projector of the previous section. The superalgebra then collapses to the non-covariant form in eq.(2) with remaining explicit symmetry $\mathrm{SO}(9,1)\times\ \mathrm{SO}(2,2)$, after replacing the triplet index i with the triplet index $[mn]_+$.

In the general frame there are 32_B fermions and 528_B bosons P,Y,Z covariantly embedded in $\mathrm{SO}(11,3)$. In addition there are also the 56(=4×14) components of the vectors $V_{\bar{M}}^m$. For the full covariance to be valid these must be operators rather than fixed vectors, as in the examples of [14] [13].

3.3 C branch

Consider 8 orthogonal spacelike unit vectors $V_{\bar{M}}^i$. There is an auxiliary $\mathrm{SO}(8)'$ defined on the indices i. The C branch solution to the BPS equation is covariant under $\mathrm{SO}(11,3)\times\mathrm{SO}(8)'$

$$\begin{aligned}\left(S_C^{64}\right)_{\alpha\beta} &= \Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\bar{M}_3}\left(P_{\bar{M}_1ij}V_{\bar{M}_2}^iV_{\bar{M}_3}^j + T_{\bar{M}_1\bar{M}_2\bar{M}_3}^+\right) && (12)\\ &+\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}\ Z_{\bar{M}_1\bar{M}_2\bar{M}_3ijkl}^+V_{\bar{M}_2}^iV_{\bar{M}_3}^jV_{\bar{M}_2}^kV_{\bar{M}_3}^l \\ &+\frac{1}{7!}\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}P_{\bar{M}_1}^{i_8i_1}\ V_{\bar{M}_2}^{i_2}\cdots V_{\bar{M}_7}^{i_7}\varepsilon_{i_8i_1\cdots i_7} && (13)\end{aligned}$$

where the tensors P,T,Z are orthogonal to the $V_{\bar{M}}^i$. There are self duality conditions: $Z_{\bar{M}_1\bar{M}_2\bar{M}_3ijkl}^+$ is $\mathrm{SO}(8)'$ self dual in the $[ijkl]$ indices, and the tensor

$T^{+}_{\bar{M}_1\bar{M}_2\bar{M}_3}$ satisfies the 14D duality condition

$$T^{+\bar{M}_1\bar{M}_2\bar{M}_3} = \frac{1}{3!}\varepsilon^{\bar{M}_1\bar{M}_2\bar{M}_3\bar{N}_1\bar{N}_2\bar{N}_3K_1\cdots K_8}T^{+}_{\bar{N}_1\bar{N}_2\bar{N}_3}V^1_{\bar{K}_1}\cdots V^8_{\bar{K}_8}. \tag{14}$$

Due to orthogonality the terms involving P can be rewritten in the form

$$\Gamma^{\bar{M}_1\bar{M}_2\bar{M}_3}_{\alpha\beta}P_{\bar{M}_1ij}V^i_{\bar{M}_2}V^j_{\bar{M}_3}\left(1+V^1\cdots V^8\right). \tag{15}$$

where the $V^i \equiv V^i_{\bar{M}}\Gamma^{\bar{M}}$ anticommute among themselves $V^1V^2 = -V^2V^1$. Using the duality properties of T, Z given above, the remaining terms in S^{64}_C may be multiplied by the same projector without changing anything. Therefore the full S^{64}_C is proportional to the same projector, showing that only 32_C supercharges survive. If one recalls that $\Gamma^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}_{\alpha\beta}$ is self dual in 14D then the presence of the projector forces $Z^{+}_{\bar{M}_1\bar{M}_2\bar{M}_3ijkl}$ to satisfy a duality condition on the $\bar{M}$ indices similar to the one satisfied by $T^{+\bar{M}_1\bar{M}_2\bar{M}_3}$. One may then verify that the number of bosons P, T, Z is 528_C. In addition there are 112 (=8×14) bosons describing the spacelike unit vectors $V^i_{\bar{M}}$.

The projector $(1+V^1\cdots V^8)$ is invariant under $SO(11,3)\times SO(8)'$. Using the symmetry one may gauge fix to $V^i_{\bar{M}} = \delta^i_{\bar{M}}$. The projector becomes $(1+\Gamma^1\cdots\Gamma^8) = (1+\Gamma^9\Gamma^0\Gamma^{11}\Gamma^{12}\Gamma^{13}\Gamma^{14})$ showing that it projects to the self dual sectors of $SO(6,6)\times SO(8)$ which is the surviving symmetry in the special frame. Then the superalgebra C collapses to the non-covariant form of eq.(4).

3.4 Heterotic and type-I sub-branches

Just as the $32_{A,B,C}$ fermions are distinct embeddings in 64, the $528_{A,B,C}$ bosons are distinct embeddings in the 2080. However, some of the fermions and bosons are common among the various sets. To identify the common ones one may use the following method. Consider the A, B subsets in the special frames and use the $SO(9,1)\times SL(2,R)_+\times SL(2,R)_-$ basis embedded in $SO(11,3)$ [5]. The 64 Weyl fermions and 2080(=364+1716) bosons are reclassified as follows

$$64 = (16_+, 2, 1) + (16_-, 1, 2),$$

[5]Due to the lack of space we will not discuss the CB or CA pairs. The method is similar and a useful common basis is $SO(8)\times SO(1,1)\times SO(2,2)$.

$$
\begin{aligned}
364 &= (120,1,1)+(45,2,2)+(10,3,1)+(10,1,3)+(1,2,2)\,, \qquad (16)\\
1716 &= (120,1,1)'+(210,2,2)+(126_+,3,1)+(126_-,1,3)\,.
\end{aligned}
$$

The A projection keeps two 10D fermions of opposite chirality $32_A = (16_+,+,1)+(16_-,1,+)$ where we have denoted each $2=\pm$ and kept only the $+$ component. The A projection gives the bosons

$$
\begin{aligned}
528_A &= (45,+,+)+(10,++,1)+(10,1,++)+(1,+,+) \qquad (17)\\
&\quad +(210,+,+)+(126_+,++,1)+(126_-,1,++)
\end{aligned}
$$

The B projector keeps two 10D fermions of the same chirality which also happen to be singlets of $\mathrm{SL}(2,R)_-$, namely $32_B=(16_+,2,1)$. Then the 528_B bosons are[6]

$$
528_B = \frac{1}{2}\left[(120,1,1)+(120,1,1)'\right]+(10,3,1)+(126_+,3,1)\,. \qquad (18)
$$

By comparing the two sets $528_{A,B}$ one finds that the common subset is the heterotic fermions $(16_+,+,1)$ and bosons $(10,++,1)+(126_+,++,0)$ which form the heterotic superalgebra in 10D. This provides the key for the embedding of the heterotic superalgebra covariantly in 14D. It is obtained by starting with the solution S_B^{64} and keeping only the bosons that couple to the combination $F^+_{\bar M\bar N}\equiv\frac{1}{2}\left(F^{42}_{\bar M\bar N}+F^{41}_{\bar M\bar N}\right)$ while setting all others equal to zero. This combination picks up only the components $(10,++,1)+(126_+,++,1)$ among the bosons when written in the special frame. More simply, one can show that $F^+_{\bar M\bar N}=V_{[\bar M}V'_{\bar N]}$ where $V_{\bar M}\sim\frac{1}{\sqrt2}\left(V^4_{\bar M}+V^3_{\bar M}\right)$ and $V_{\bar M}\sim\frac{1}{\sqrt2}\left(V^2_{\bar M}+V^1_{\bar M}\right)$ are the lightcone combinations which are orthogonal but not parallel to each other $V\cdot V=V'\cdot V'=V\cdot V'=0$. In terms of these the heterotic superalgebra is embedded covariantly in 14D as follows

$$
\begin{aligned}
\left(S^{64}_{Het}\right)_{\alpha\beta} &= \Gamma^{\bar M_1\bar M_2\bar M_3}_{\alpha\beta}P_{\bar M_1}V_{\bar M_2}V'_{\bar M_3}+\Gamma^{\bar M_1\bar M_2\cdots\bar M_7}_{\alpha\beta}\;Z^+_{\bar M_1\cdots\bar M_5}V_{\bar M_6}V'_{\bar M_7}, \qquad (19)\\
P_{\bar M_1}V^{\bar M_1} &= P_{\bar M_1}V'^{\bar M_1} = Z^+_{\bar M_1\cdots\bar M_5}V^{\bar M_5} = Z^+_{\bar M_1\cdots\bar M_5}V'^{\bar M_5} = 0.
\end{aligned}
$$

[6]Can one find a B projector that identifies the $\mathrm{SO}(2,1)$ of eq.(2) with $\mathrm{SL}(2,R)_V$ of eq.(3) rather than $\mathrm{SL}(2,R)_+$? To answer the question reduce the representations of $\mathrm{SL}(2,R)_+\times\mathrm{SL}(2,R)_-$ to $\mathrm{SL}(2,R)_V$ and then pick the the two 16_+ fermions. Evidently the fermions are the same ones since there are only two of them. But then the bosons must also be unambigously the same as those in eq.(18) since they appear in the products of the same fermions that were previously classified as $(16_+,2,1)$. Therefore there is no other B projector, and the symmetry of eq.(2) is automatically $\mathrm{SL}(2,R)_+$ embedded in $\mathrm{SO}(2,2)$, not $\mathrm{SL}(2,R)_V$.

Although orthogonality permits components in P, Z^+ along the lightlike directions V, V', these drop out due to the antisymmetry of the gamma matrices. There remains an effective $(9,1)$ subspace. Z^+ is automatically self dual in the $(9,1)$ subspace thanks to the lightlike nature of the vectors in $\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}\ V_{\bar{M}_6}V'_{\bar{M}_7}$ and the self duality of $\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}$ in the 14D Weyl sector. The number of independent components in P, Z^+ is 136.

Using orthogonality one sees that S_{Het}^{64} is proportional to the projector $(V\cdot\Gamma)\,(V'\cdot\Gamma)$ which is invariant under $\mathrm{SO}(11,3)$. The double lightcone projections cut down 64 to 16 non-zero supercharges. In the special frame the projector becomes $(\Gamma^{12}+\Gamma^{11})\,(\Gamma^{14}+\Gamma^{13})$ showing that the remaining symmetry is $\mathrm{SO}(9,1)$ and that the superalgebra reduces to the usual heterotic superalgebra in the non-zero 16×16 block embedded in 64×64.

The type-I superalgebra may be obtained from the B superalgebra by identifying the two supercharges $(16_+,2,1)$. In the non-covariant eq.(2) this requires a right hand side that is proportional to $\delta_{\bar{a}\bar{b}}$, which means only the $i=0'$ term contributes since $(\tau_2\tau_2)_{\bar{a}\bar{b}}=\delta_{\bar{a}\bar{b}}$. In the $\mathrm{SO}(2,2)$ notation this is equivalent to the term $[mn]_+=[42]_+=[13]_+$. Thus, the covariant embedding of the type-I superalgebra in 14D is

$$\left(S_I^{64}\right)_{\alpha\beta}=\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\bar{M}_3}P_{\bar{M}_1}F^{42}_{\bar{M}_2\bar{M}_3}+\Gamma_{\alpha\beta}^{\bar{M}_1\bar{M}_2\cdots\bar{M}_7}\ Z^+_{\bar{M}_1\cdots\bar{M}_5}F^{42}_{\bar{M}_6\bar{M}_7} \tag{20}$$

Due to orthogonality there is an overall $F^{42}_{\bar{M}\bar{N}}\Gamma^{\bar{M}\bar{N}}$ factor that corresponds to the projector. In the special frame the projector becomes $F^{42}_{\bar{M}\bar{N}}\Gamma^{\bar{M}\bar{N}}=(\Gamma^{14}\Gamma^{12}+\Gamma^{13}\Gamma^{11})$. This seems to cut down 64 to 32, not to 16. However, the non-zero 32×32 sector is equivalent to two identical 16×16 blocks, showing that only 16 supercharges of type-I are non-zero. The two identical block structure is produced because the two 16_+'s were identified.

3.5 Compactifications

Every branch has compactifications down to 4D with $\mathrm{SO}(3,1)$ Lorentz symmetry and the internal symmetries inherited from the A,B,C main branches (see e.g. [14] where the compactified A,B branches in 4D are written out explicitly). Therefore one finds that the same 4D extended superalgebra has several reclassifications which might be called A,B,C, each one containing a maximum of 32 fermions and 528 bosons, corresponding to a maximum of 8 supersymmetries in 4D . Each one has also a duality basis which is obtained by a boost transformation in the internal Lorenz dimensions (see [15] and footnote 3). The maps among these classifications are related to various duality transformations. The compactification sub-branches to various dimensions will not be discussed here since they are obtained by naively compactifying the main branches.

We have shown that all known superalgebras derive from a single one in 14D as solutions to the BPS equation. To exhibit the 14D nature of the solutions some vectors V were needed. If the various vectors V are frozen constants the symmetry is broken to some subgroup of $\mathrm{SO}(11,3)$, and the superalgebra collapses to the familiar one. If the vectors V are also operators then the $\mathrm{SO}(11,3)$ symmetry is not broken, on the contrary there are more symmetries that we called "auxiliary". Possible interpretations of such operators have been given elsewhere in several contexts [15] [14] [13].

Our construction leads us to speculate that a fundamental theory in $(11,3)$ dimensions may be behind string and p-brane duality properties. The presence of extra timelike dimensions seems to pose a problem for such a theory. However, in recent papers [13] we have shown that extra timelike dimensions can be interpreted without the obvious problems. The interpretation is done in the context of models that involve several interacting particles or p-branes forming different physical sectors, each with its own timelike dimension. The superalgebra of such systems has the types of signatures and operators V discussed in this paper. The models have sufficient gauge symmetries to eliminate redundant timelike degrees of freedom. A cosmological scenario may also be invoked to arrive at the single time sector which describes our current universe.

Note Added: While this paper was under preparation ref.[16] appeared. This work was apparently stimulated by a brief announcement in [13] of our present work on $(11,3)$ dimensions. The embedding of 10D Yang-Mills theory in $(11,3)$ dimensions lends support to the ideas expressed here.

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