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Inhomogeneous Pre-Big Bang String Cosmology

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Abstract

An inhomogeneous version of Pre–Big Bang cosmology emerges, within string theory, from quite generic initial conditions, provided they lie deeply inside the weak-coupling, low-curvature regime. Large-scale homogeneity, flatness, and isotropy appear naturally as late-time outcomes of such an evolution.

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1 Introduction

Superstring theory, a supposedly consistent and unified quantum theory of all interactions, suggests, through its peculiar duality symmetries [1], an attractive alternative to standard inflationary cosmology, the so-called Pre- Big Bang (PBB) scenario [2] [3] [4].

The basic postulate of PBB cosmology is that the Universe started its evolution from a very perturbative initial state, i.e. from very weak coupling and very small curvatures (in appropriate units, see below). If initial homogeneity is also assumed, this leads automatically to a super-inflationary epoch during which the accelerated expansion of the Universe is driven by the accelerated growth of the effective (gauge and gravity) coupling, i.e. of the dilaton.

This result looks surprising, at first sight, since the equation of state of the dilaton is not of the kind that one usually associates with inflation. Indeed, inflation only occurs if the geometry is looked at in the so-called String(S)-frame, the one appearing in the Nambu-Goto-Polyakov action, while in the Einstein(E)-frame one rather observes an accelerated contraction. The point here is that, precisely at weak coupling and small curvatures, fundamental strings keep their characteristic size constant and sweep geodesic surfaces with respect to the S-frame metric [5] [6]. Thus, in this regime at least, the latter *is* the correct frame for describing how distances among objects vary w.r.t. their intrinsic size, and for determining what is actually meant by small or large curvatures. As long as curvatures remain small (resp. become $O(\frac{1}{\alpha'})$) in the string frame, the low energy effective action of string theory represents an adequate (resp. becomes an inadequate) description of physics.

Going from one "frame" to another, although not a coordinate transformation, is only a matter of field redefinitions. Hence, any physical observable takes the same value in any such "frame". Whenever useful for computations, any other frame can be used, and we shall take advantage of this possibility below. However, the description of physical phenomena is all but intuitive in other frames and, in order to avoid problems of interpretation, it is best to transform back the results to the string frame at the end of the calculations.

The PBB scenario also needs an "exit assumption" from the PBB phase to standard non-inflationary cosmology. Higher derivative (α') corrections are expected to stop the indefinite growth of curvature (see [7] for recent progress on this issue), while loop corrections and/or a non-perturbative potential are usually assumed to stop the indefinite growth of the coupling itself. If such a "graceful exit" turns out to be a property of string theory, one will obtain an interesting scenario whereby the usually assumed Big Bang initial conditions (a hot, dense and highly-curved state) will be the outcome -rather than the starting point- of inflation. Several interesting phenomenological consequences of the scheme have also been worked out [8].

In spite of the above advantages of the PBB scenario, it is clear that the assumption of homogeneous initial conditions is a difficult one to swallow. After all, inflation is supposed to explain [9] homogeneity (as well as isotropy and flatness)! The purpose of this note is to relax the assumption of initial homogeneity by taking the initial state of the Universe to be in the weak-coupling, weak-curvature regime, but otherwise arbitrary. Generically, PBB behaviour will emerge in suitable regions of space which, eventually, will fill all but an infinitesimal fraction of space. Within those regions, the Universe will appear very homogeneous, isotropic and spatially flat.

The plan of the paper is as follows: in Section 2, I give the general form of the equations in the E-frame and describe some general features of their solutions, which will be confirmed by the construction, in Section 3, of the general analytic asymptotic solution near the "singularity". In Section 4, the results of the previous Sections will be reinterpreted in the physical S-frame and semiquantitative local conditions for the onset of PBB behaviour will be given.

Most of our technical results are not really new, only the physical motivation and interpretation are. In particular, our method can be traced back to the work of Belinskii and Kalatnikov [10] which was made recently more systematic through the so-called gradient expansion method either directly in terms of Einstein's equations [11], or within a Hamilton-Jacobi formulation [12].

2 General properties of the solutions in the E-frame

Having made the basic postulate that the Universe started its (pre) history in a weak-coupling, low-curvature state, we will describe its early evolution in terms of the tree-level, low-energy effective action of string theory.

For simplicity we shall consider only the particular case of a critical superstring theory, with vanishing cosmological constant and six frozen internal dimensions. The effective fourdimensional theory is thus described (in Landau-Lifshitz notations [13]) by the action:

$$\Gamma^S_{eff} = \frac{1}{2}\int dx\sqrt{G}e^{-\phi}\left(R(G) + G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) \tag{2.1}$$

where $G_{\mu\nu}$ is the string-frame (S-frame) metric, ϕ is the dilaton and we have also set to zero the antisymmetric tensor field $B_{\mu\nu}$. In the conformally related Einstein frame (E-frame) defined by the new metric $g_{\mu\nu}$ via:

$$g_{\mu\nu} = e^{-\phi}G_{\mu\nu} \tag{2.2}$$

the action becomes:

$$\Gamma^E_{eff} = \frac{1}{2}\int dx\sqrt{g}\left(R(g) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) \tag{2.3}$$

and the field equations take the familiar form:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}g_{\mu\nu}(\partial\phi)^2 \\ \bigtriangledown^2\phi &\equiv g^{\mu\nu}D_\mu\partial_\nu\phi = 0 \end{aligned} \tag{2.4}$$

where we are using units in which $8\pi G_N = 1$.

We also choose to work in the synchronous gauge, i.e. we fix:

$$g_{00} = -1,\ g_{0i} = 0\ (i = 1, 2, 3), \tag{2.5}$$

so that the dynamical variables become the three-metric g_{ij} and the dilaton ϕ.

The Einstein equations for the system can be readily written. They consist of:

- The Hamiltonian constraint

$$2\dot{\phi}^2 + Tr\chi^2 - \chi^2 = 4\mathcal{R} - 2(\bigtriangledown\phi)^2 \tag{2.6}$$

- The three Momentum Constraints

$$\chi_{,i} - \chi^j_{i;j} = -\dot{\phi}\bigtriangledown_i\phi \tag{2.7}$$

- The dynamical equation for ϕ

$$\ddot{\phi} + \frac{1}{2}\chi\dot{\phi} = \bigtriangledown^2\phi \tag{2.8}$$

- The dynamical equation for g_{ij}

$$\dot{\chi}^j_i + \frac{1}{2}\chi\,\chi^j_i = -2\mathcal{R}^j_i + \bigtriangledown_i\phi\bigtriangledown^j\phi \tag{2.9}$$

where a dot denotes derivative w.r.t. E-frame "cosmic" time τ, covariant derivatives are defined w.r.t. the three-metric g_{ij} (whose three-curvature is denoted by $\mathcal{R}$), and we have defined:

$$\chi_{ij} = \dot{g_{ij}},\ \chi_i^j = \chi_{ik} g^{kj},\ Tr\chi^2 = \chi_i^j \chi_j^i,\ \chi = \chi_i^i = \frac{\dot{g}}{g},\ g = det(g_{ij}). \tag{2.10}$$

It is easy to check that the above equations reduce to the usual cosmological equations once all spatial gradients are set to zero. It is also possible to check that the four constraints, once imposed at an initial time, remain valid at all times thanks to the evolution equations.

The following general inequalities follow as long as the three metric is positive (semi) definite:

$$Tr\chi^2 \geq \frac{1}{3}\chi^2 \geq 0\ , \tag{2.11}$$

where the first equality is reached in the isotropic case.

Combining the trace of (2.9) and (2.6) we obtain a useful equation which does not involve spatial gradients:

$$\dot{\chi} = -\left(\frac{1}{2}Tr\chi^2 + \dot{\phi}^2\right) \leq 0, \tag{2.12}$$

with the equality sign holding iff $\dot{\phi} = \chi_{ij} = 0$.

Equation (2.12), together with the basic postulate of PBB cosmology, is enough to conclude that, on the $\tau = 0$ hypersurface, we can only have regions with $\chi < 0$. Indeed, regions with $\chi > 0$, given Eq. (2.12), will necessarily have to originate from a large curvature phase in the past. Even regions with $\chi = 0$ are excluded, since, going backward in time, they generically came from a positive χ. Furthermore, the Hamiltonian constraint (2.6) implies that χ can only vanish if

$$\mathcal{R} \geq \frac{1}{4}Tr\chi^2 + \frac{1}{2}\dot{\phi}^2, \tag{2.13}$$

i.e. in regions of sufficiently large positive spatial curvature. In sufficiently homogeneous regions, in which fields vary little over a Hubble radius, we get from (2.11), (2.6) the bounds:

$$-\sqrt{3} \leq \rho \equiv \frac{\chi}{\sqrt{Tr\chi^2}} \leq -1. \tag{2.14}$$

We conclude that generic initial data satisfying the PBB postulate can be presented in terms of quasi-homogeneous regions where $-\sqrt{3} < \rho < -1$, separated by inhomogeneous regions where ρ *can* take any negative value larger than $-\sqrt{3}$. Let us see how each one of these regions evolves in time:

The quasi-homogeneous regions with $\rho < 0$ undergo an accelerated contraction in the Einstein frame and are consistent with the PBB postulate. We shall see below (and in Section 3) more precisely how they evolve in the E-frame while, in Section 4, we will reinterpret those results in the S-frame.

Highly inhomogeneous regions are more difficult to deal with, but are not expected [10] [11] [12] to undergo a marked accelerated (or decelerated) evolution.

It is relatively easy to see that, for quasi-homogeneous regions with $\rho < 0$, the approximation of neglecting spatial derivatives improves with time. Leaving to Section 3 the study of the asymptotic solution, we just note that, neglecting spatial gradients, the trace of (2.9) gives:

$$\chi \sim \frac{2}{\tau - \tau_0} \Rightarrow g \sim (\tau - \tau_0)^2, \tau < \tau_0, \tag{2.15}$$

where, in general, τ_0 can depend on x. Invoking now (2.6) and the definition (2.14) we can determine $\dot{\phi}$ up to a sign ambiguity. However, given the fact that we chose $\chi < 0$, and that we want to start the evolution from weak coupling, we have to choose the sign giving:

$$\dot{\phi} = -\frac{\chi}{\sqrt{2}}\sqrt{1 - \rho^{-2}} \Rightarrow \phi \sim \ -c(x) log(\tau_0 - \tau), \ \ c(x) > 0. \tag{2.16}$$

In order to check that spatial gradients are subleading w.r.t. time derivatives as we approach $\tau = \tau_0$ consider, for instance, the ratio (with no sum over i, j implied):

$$r_{ij} \equiv \frac{g^{ij} \bigtriangledown_i \phi \bigtriangledown_j \phi}{\dot{\phi}^2}. \tag{2.17}$$

As we approach the singularity, using (2.15), (2.16), we find:

$$r_{ij} \sim g \cdot g^{ij} \bigtriangledown_i \phi \bigtriangledown_j \phi \sim \epsilon^{ikn}\epsilon^{jlm} g_{kl} g_{nm} \, log^2(\tau_0 - \tau) \to 0, \tag{2.18}$$

unless the contraction of the three-volume is highly anisotropic.

We thus conclude that, within sufficiently homogeneous and isotropic regions, the approximation of neglecting spatial derivatives becomes increasingly good as we approach the singularity at $\tau = \tau_0$. This conclusion will be fully confirmed in the analytic asymptotic solution presented in Section 3.

Let us now try to understand how things evolve as one goes backward in time, $\tau \to -\infty$. It is not possible to obtain an asymptotic solution in this limit by the method used in Section 3. Nevertheless, we can make the following observations: Eqs. (2.8), (2.12), together imply that the only early-time fixed points are either at infinity (corresponding to a singularity) or at $\dot{\phi} = \chi_{ij} = \bigtriangledown^2\phi = 0$. This implies however (neglecting boundary terms) $\bigtriangledown_i\phi = 0$ and thus, using Eq. (2.9), $\mathcal{R}_i^j = 0$, which, in three dimensions, is equivalent to flat space. Thus the only non-singular fixed point is the trivial one.

Homogeneous PBB cosmologies are examples of solutions converging towards this trivial fixed point in the far past. Our basic postulate requires that there should be a *finite-measure* basin of attraction (containing the homogeneous cases) towards this fixed point. By using a reliable linear perturbation theory around the homogeneous solutions, it is not hard to show that such a finite basin should exist, although determining its exact extension is highly non-trivial.

Another related issue is whether, within such a basin of attraction, one is really approaching the trivial Minkowski vacuum with vanishing coupling and curvature. The tricky point here is that curvatures and derivatives of ϕ have to go to zero in *string* units, namely they have to become much smaller than e^{ϕ} in the E-frame. Although we have indications that this is indeed the case, we leave a full discussion of this important question to further investigations.

3 General asymptotic solution in the E-frame

In this Section we present the leading term of the general asymptotic solution of Eqs. (2.6)-(2.9) near the singularity hypersurface $\tau = \tau_0(x)$. The construction is quite straightforward (see e.g. ([10])), if one makes a few approximations in solving the equations and then checks, a posteriori, their validity. Finally, a counting of the number of arbitrary functions contained in the solution shows that we are indeed describing the most general solution.

Inserting (2.15) back into Eq.(2.9) we get, for small enough gradients,

$$\dot{\chi}_i^j + \frac{1}{\tau - \tau_0}\chi_i^j = 0, \tau < \tau_0. \tag{3.1}$$

Integrating this equation twice we get:

$$\chi_i^j = \frac{2\lambda_i^j}{\tau - \tau_0},\ g_{ij}(x,\tau) = \left[exp\left(2\lambda(x)log(1 - \frac{\tau}{\tau_0})\right)\right]_i^k\ g_{kj}(x,0) \tag{3.2}$$

where the space-dependent quantities λ_i^k satisfy:

$$\lambda_i^i = 1,\ \frac{1}{3} \leq \lambda_i^k\lambda_k^i \leq 1,\ \lambda_{ij} \equiv \lambda_i^k g_{kj}(x,0) = \lambda_{ji}. \tag{3.3}$$

At this point we use (2.6) and find, asymptotically:

$$\phi(x,\tau) = \phi(x,0) - \sqrt{2}\sqrt{1-\lambda_i^k\lambda_k^i}\, log(1-\frac{\tau}{\tau_0}). \tag{3.4}$$

Rewriting finally Eq.(2.8) in the form:

$$(\sqrt{g}\dot{\phi})^{\cdot} = \sqrt{g}\, \bigtriangledown^2 \phi, \tag{3.5}$$

we see that it is automatically fulfilled asymptotically since, from (3.2), (3.4), we find $\sqrt{g}\dot{\phi} \sim const(x)$.

A convenient way to present the solution and to check that, indeed, it is general, consists of introducing local coordinates with respect to which the matrix λ and the three-metric are diagonal. Then we can write:

$$g_{ij}(x,\tau) = \sum_a e_i^a(x)e_j^a(x)(1-\frac{\tau}{\tau_0})^{2\lambda_a(x)} \tag{3.6}$$

$$\phi(x,\tau) = \phi(x,0) - \sqrt{2}\sqrt{1-\sum\lambda_a^2}\, log(1-\frac{\tau}{\tau_0}),\ \sum\lambda_a = 1 \tag{3.7}$$

where, at the moment, e_i^a are arbitrary "dreibein" matrices and we note that the parameter ρ of Section 2 is now given by $\rho^{-2} = \sum\lambda_a^2$.

The above solution appears to depend upon thirteen arbitrary functions of space, i.e. nine e_i^a, two of the three λ_a, τ_0, and ϕ_0. However, we have not imposed yet the momentum constraints (2.7). These constraints, being conserved, reduce to their form at $\tau = 0$ and can be used to eliminate three of the nine e_i^a. We are left with ten arbitrary functions. Yet, within the synchronous gauge, we can still perform a four-parameter family of gauge transformations (see, e.g., [13]) allowing us to set $\tau_0(x) = \tau_0$ (independent of x) and to further fix three of the remaining six independent quantities e_i^a. We end up with six (physical) arbitrary functions which is, indeed, the correct number [10] for the problem at hand.

4 Physical reinterpretation in the string frame

In order to discuss the physical implications of our results we transform them back to the string frame. Using Eq. (2.2) we obtain

$$G_{ij}(x,\tau) = \sum_a e_i^a(x)e_j^a(x)\cdot e^{\phi(x,0)}(1-\frac{\tau}{\tau_0})^{2\lambda_a-\sqrt{2}\sqrt{1-\sum\lambda_b^2}}\ , \tag{4.1}$$

Since $g_{00} = -1$ has also been changed into a non-trivial G_{00}, we still have to go to the synchronous gauge in the S-frame. This is easily accomplished, to leading order in the gradients, by changing the time variable from τ to t according to:

$$\sqrt{2}(t_0 - t) = \tau_0 \, e^{\phi(x,0)/2}(\sqrt{2} - \sqrt{1 - \sum \lambda_a^2})^{-1}(1 - \tau/\tau_0)^{1-\frac{1}{\sqrt{2}}\sqrt{1-\sum\lambda_a^2}} \tag{4.2}$$

where we note that, for any choice of the λ_a, $t \to t_0$ as $\tau \to \tau_0$. Finally, after defining approriate S-frame "dreibeins" $E_i^a(x)$, we get:

$$G_{ij}(x,t) = \sum_a E_i^a(x) E_j^a(x)(1 - \frac{t}{t_0})^{2\alpha_a(x)} \tag{4.3}$$

$$\phi(x,t) = \phi(x,0) + \gamma \; log(1 - \frac{t}{t_0}), \tag{4.4}$$

where:

$$\alpha_a = \frac{\lambda_a - \frac{1}{\sqrt{2}}\sqrt{1 - \sum \lambda_b^2}}{1 - \frac{1}{\sqrt{2}}\sqrt{1 - \sum \lambda_b^2}}, \; \gamma = -\frac{\sqrt{2}\sqrt{1 - \sum \lambda_b^2}}{1 - \frac{1}{\sqrt{2}}\sqrt{1 - \sum \lambda_b^2}}. \tag{4.5}$$

The above relations imply:

$$\sum \alpha_a^2 = 1, \; \gamma = -1 + \sum \alpha_a \tag{4.6}$$

so that we find for the ($O(d,d)$-invariant) shifted dilaton [1]:

$$\bar{\phi} \equiv \phi - \frac{1}{2} tr log G \sim -log(1 - \frac{t}{t_0}) + \bar{\phi}(x,0), \tag{4.7}$$

which generalizes the well known S-frame homogeneous cosmology results [2], [3].

Eqs. (4.5) can easily be inverted to express the λ_a's in terms of the α_a's:

$$\lambda_a = \frac{1}{3} + \frac{2}{3}\frac{\alpha_a - \frac{1}{3}\sum \alpha_b}{1 - \frac{1}{3}\sum \alpha_b}. \tag{4.8}$$

Using these equations we can construct, in principle, scale-factor-duality(SFD) related solutions ($\alpha_a \to -\alpha_a$ for some a[2]) even in the inhomogeneous case. Of course, one has to check that the constraints can be simultaneously imposed on each component of the dual pairs.

We are finally in a position to study how the three-volume of a quasi-homogeneous region evolves in the string frame. We find

$$\frac{1}{2}\chi^{(s)} \equiv \frac{1}{2}\frac{\dot{G}}{G} = -\frac{\sum \alpha_a}{t_0 - t} \tag{4.9}$$

which shows that quasi-homogeneous regions undergo *superinflation* in the string frame provided:

$$\sum \alpha_a = \frac{1 - \frac{3}{\sqrt{2}}\sqrt{1 - \sum \lambda_a^2}}{1 - \frac{1}{\sqrt{2}}\sqrt{1 - \sum \lambda_a^2}} < 0. \tag{4.10}$$

This condition is satisfied provided $\rho^{-2} = \sum \lambda_a^2 < 7/9$, which includes all but a small region for the allowed values of this quantity (see (2.14)). Note that the maximal expansion rate is reached for $\sum \alpha_a = -\sqrt{3}$ corresponding to the isotropic case $\lambda_a = 1/3$.

We may also ask about the flatness problem. It is easy to see that the typical quantity measuring flatness, $\frac{\mathcal{R}}{\chi^2}$, contains a logarithmically enhanced term which, for $t \to t_0$, behaves as

$$\frac{\mathcal{R}}{\chi^2} \sim \frac{G^{ij}\, \nabla_i \phi\, \nabla_j \phi}{\dot{\phi}^2} \sim (t - t_0)^{2 - 2\mathrm{Max}(\alpha_a)} log^2(t_0 - t), \tag{4.11}$$

and thus goes to zero almost everywhere thanks to (4.6). The non-enhanced terms are more involved, but certainly behave in the same way for sufficiently isotropic situations. Thus, as expected, spatial flatness is generically achieved after a long PBB era.

We conclude that all but the "skin" of the quasi-homogeneous regions with negative ρ inflates when we look at the geometry in the "right" frame and becomes homogeneous and spatially flat. Presumably, after a while, these regions represent by far the largest fraction of the Universe. Furthermore, the isotropic regions dominate all others, providing also a possible "explanation" for the isotropy and flatness of our observable Universe.

5 Conclusions

In this paper we have relaxed the assumption of homogeneity (and spatial flatness) in the initial conditions for string cosmology. The emerging picture can be described as follows: the primordial state of the Universe is arbitrarily close to flat space-time but not particularly homogeneous, in the sense that time and space derivatives are both extremely small but comparable. Stochastically, however, sufficiently homogeneous and isotropic conditions will emerge in some regions of space and undergo, in the S-frame, an accelerated expansion yielding homogeneity, flatness and isotropy as one approaches a singularity in the future within a finite proper time. Before such a singularity is reached, the low-energy effective action will cease to be valid and, hopefully, the higher-derivative corrections present in string theory will lead to a finite-curvature stringy phase [7] rather than to a genuine singularity.

In this connection, it is amusing to speculate that the famous singularity theorems of Hawking and Penrose [14] will find an unexpected (and welcome!) application in string cosmology: since the conditions of the theorems do probably apply to the low-energy effective action, they will imply the necessary growth of curvature up to the scale at which string corrections invalidate the assumptions made in those theorems, i.e. the unavoidable occurrence of a long, dilaton-driven inflationary phase as long as the initial Universe was indeed at very weak coupling and very nearly flat.

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