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Weakly-Coupled Higgs Bosons and Precision Electroweak Physics

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Abstract

We examine the prospects for discovering and elucidating the weakly-coupled Higgs sector at future collider experiments. The Higgs search consists of three phases: (i) discovery of a Higgs candidate, (ii) verification of the Higgs interpretation of the signal, and (iii) precision measurements of Higgs sector properties. The discovery of one Higgs boson with Standard Model properties is not sufficient to expose the underlying structure of the electroweak symmetry breaking dynamics. It is critical to search for evidence for a non-minimal Higgs sector and/or new physics associated with electroweak symmetry breaking dynamics. An improvement in precision electroweak data at future colliders can play a useful role in confirming the theoretical interpretation of the Higgs search results.

Summary Report
The Weakly-Coupled Higgs Boson and Precision Electroweak Physics Working Group
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ABSTRACT

We examine the prospects for discovering and elucidating the weakly-coupled Higgs sector at future collider experiments. The Higgs search consists of three phases: (i) discovery of a Higgs candidate, (ii) verification of the Higgs interpretation of the signal, and (iii) precision measurements of Higgs sector properties. The discovery of one Higgs boson with Standard Model properties is not sufficient to expose the underlying structure of the electroweak symmetry breaking dynamics. It is critical to search for evidence for a non-minimal Higgs sector and/or new physics associated with electroweak symmetry breaking dynamics. An improvement in precision electroweak data at future colliders can play a useful role in confirming the theoretical interpretation of the Higgs search results.

I. INTRODUCTION

Present day colliders test the Standard Model at an energy scale of order 100 GeV. Precision experiments at LEP, SLC and Tevatron (with some additional measurements at lower energies) have measured more than twenty separate experimental observables, and have confirmed the Standard Model predictions with an accuracy of one part in a thousand [1,2]. A few anomalies in the data could suggest hints of new physics beyond the Standard Model [3], although no deviations have been rigorously confirmed.

Nevertheless, the verification of the Standard Model is not yet complete. Absent to date is any experimental signal that sheds light on the dynamics responsible for electroweak symmetry breaking. Any consistent theory of electroweak symmetry breaking must generate Goldstone bosons which are absorbed by the $W^\pm$ and Z gauge bosons, thereby generating the gauge boson masses. The Standard Model posits that electroweak symmetry breaking is due to the dynamics of a weakly-coupled complex doublet (with hypercharge one) of elementary scalar fields. The physical consequence of this model is the existence of a CP-even neutral Higgs boson with mass roughly of order m_Z. Extensions of this model can easily be constructed, in which the scalar sector is enlarged. The resulting model then contains a *non-minimal* Higgs sector consisting of neutral Higgs bosons (of definite or indefinite CP depending on the model) and charged Higgs bosons [4].

The best motivated non-minimal Higgs sector is the two Higgs doublet model. Starting with two complex scalar doublets of hypercharge ± 1 respectively, one finds a Higgs sector (after three Goldstone bosons are absorbed to give mass to the $W^\pm$ and Z) consisting of five states: a light CP-even Higgs scalar, h^0, a heavy CP-even scalar, H^0, a CP-odd scalar A^0, and a charged Higgs pair, $H^\pm$. This is the Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM) [4–6].

* This is the summary report of the Weakly-Coupled Higgs Boson and Precision Electroweak Physics Working Group. The full list of working group members can be found in the subgroup reports that follow this summary report. This work was supported in part by the U.S. Department of Energy and the National Science Foundation.

In the global fits of LEP, SLC, and Tevatron data based on the Standard Model, there is weak (but non-trivial) sensitivity to the Higgs boson mass by virtue of Higgs mediated radiative corrections. The most recent global fits find that $m_{h^0} < 550$ GeV at 95% CL [2], although some care needs to be taken in interpreting this limit [7]. The potential for improving this bound at future colliders is discussed in Section II. In the context of the Standard Model, the Higgs boson in this mass range is necessarily weakly-coupled. Moreover, such fits also apply to non-minimal Higgs sectors in which the lightest Higgs scalar (h^0) is separated in mass from heavier non-minimal Higgs states. Therefore, there is a strong motivation to conduct a vigorous experimental search for weakly-coupled Higgs bosons at LEP and future colliders.

If a Higgs boson with Standard Model properties were discovered, then one might naively conclude that the search for the model of the elementary particles has been completed. However, theorists strongly believe that the Standard Model cannot be the fundamental model of particles. Apart from the many parameters of the Standard Model which must be inserted by hand (with no explanation), there is a theoretical problem in the Standard Model associated with the very large hierarchy of energy scales. We know that the Planck scale, $M_{\rm PL} \simeq 10^{19}$ GeV, exists in nature; it characterizes the energy scale above which gravitational interactions cannot be neglected relative to the strong and electroweak interactions of the elementary particles. Given the existence of such a large energy scale, one must explain how the scale of electroweak symmetry breaking, which is so small when expressed in units of the Planck scale ($m_Z \simeq 10^{-17} M_{\rm PL}$), could be generated by a fundamental theory of particles that includes gravity. Related to this question is the theoretical problem of generating a "naturally" light Higgs boson (with a mass of order $m_Z \ll M_{\rm PL}$), since in the Standard Model, there is no symmetry that can protect the mass of an elementary scalar from being driven up to $M_{\rm PL}$ via radiative corrections. These problems are intimately connected with the dynamics that generates electroweak symmetry breaking.

Attempts to solve the problem of hierarchy and the related problem of the unnaturally light Higgs boson inevitably lead to the existence of new physics at the 1 TeV energy scale or below. Possible mechanisms invoke either supersymmetry [5] (a symmetry that can protect the masses of elementary scalars) or dynamical electroweak symmetry breaking [8] (which typically eliminates elementary scalar fields completely). Which path nature chooses can only be determined through experimentation. Thus the central goals of the future colliders program are: to explore the dynamics of electroweak symmetry breaking, and to determine its implications for the structure of the Standard Model and the nature of physics that lies beyond the Standard Model.

In this report, we assume that nature chooses a weakly-coupled Higgs sector as the source of electroweak symmetry breaking dynamics. We do not address the alternative approach which invokes strong interaction dynamics as the source of electroweak symmetry breaking. The phenomenology of the electroweak symmetry breaking sector in this latter case is explored by the Strongly Interacting Electroweak Symmetry Breaking Working Group [9]. The focus of this working group is the weakly-coupled Higgs sector of the Standard Model, and possible non-minimal Higgs sector extensions (including the Higgs sector of the MSSM). Although there is considerable freedom for the structure of the scalar sector (even after imposing all known theoretical and phenomenological constraints), models of the scalar sector often exhibit the following structure: (i) the lightest scalar (h^0) is a CP-even neutral Higgs boson with couplings closely approximating those of the Standard Model Higgs boson ($h^0_{\rm SM}$), and (ii) additional Higgs scalars (neutral Higgs bosons with definite or indefinite CP quantum numbers and charged Higgs bosons) are expected to be heavier (perhaps significantly heavier) than h^0, although still weakly-coupled. This is the so-called *decoupling limit* which will be discussed in Section IIIA.[1] In this case, the discovery of $h^0 \simeq h^0_{\rm SM}$ is not sufficient to probe the underlying structure of the electroweak symmetry breaking sector. The essence of the decoupling limit is that the existence of a light CP-even Higgs boson with properties closely approximating those of $h^0_{\rm SM}$ is consistent with many possible non-minimal Higgs sectors. Thus, the discovery of the heavy non-minimal Higgs scalars is essential in order to probe the details of the electroweak symmetry breaking dynamics.

The MSSM provides a natural framework for light elementary Higgs scalars. The Higgs sector of the MSSM is a constrained two-Higgs-doublet model, whose tree-level properties are determined by two free parameter (typically chosen to be the mass of the CP-odd state, A^0, and the ratio of vacuum expectation values, $\tan\beta$). The decoupling limit of the model corresponds to $m_{A^0} \gg m_Z$; in this limit, the properties of h^0 become identical to those of $h^0_{\rm SM}$. Extensions of the MSSM Higgs sector are also possible. For example, the simplest non-minimal supersymmetric extension of the Standard Model (NMSSM) consists of a Higgs sector with two doublets and one singlet of complex Higgs fields [10]. Thus, a detailed exploration of the scalar sector has the potential for probing both the electroweak symmetry breaking dynamics and the underlying supersymmetric structure of the theory.

Present experimental data tells us that the Higgs sector must be compatible with

(i) $\rho \equiv m_W^2/m_Z^2 \cos^2\theta_W \simeq 1$;

(ii) the absence of significant Higgs mediated flavor changing neutral currents;

(iii) the absence of significant virtual charged Higgs mediated effects (which can contribute, *e.g.*, to B^0–$\overline{B^0}$ mixing, $b \to s\gamma$ and $Z \to b\bar{b}$).

Even after imposing such model constraints, there is still significant freedom in the structure of the Higgs sector. Exotic Higgs sectors (beyond those mentioned above) are easily constructed that satisfy all present day phenomenological constraints. Such Higgs sectors could arise in models with extended gauge groups, models with exotic scalar multiplets, or models with a lepton number violating sector (*e.g.*, in R-parity violating models of low-energy supersymmetry [11], in which there

[1] It could be that all scalar states are somewhat close in mass, with no state possessing couplings that match those predicted by the Standard Model. This case is actually simpler to address experimentally and interpret theoretically.

is no distinction between scalar lepton superpartners and Higgs bosons). Sorting out the details of the scalar sector will be one of the fundamental challenges for future collider experimentation.

The Weakly-Coupled Higgs Boson and Precision Electroweak Physics Working Group program consisted of the following tasks:

1. Extend present day precision tests of the Standard Model

   This will serve to tighten constraints on the Higgs sector and perhaps uncover deviations from the Standard Model and provide evidence for new physics beyond the Standard Model.

2. Evaluate the Higgs boson discovery reach of future colliders

   High energy colliders are needed to directly produce the massive Higgs bosons. However, the cleanest decay channels of the Higgs boson usually have rather small branching ratios. Thus, high luminosity is critical to insure that Higgs signals can be extracted from the Standard model backgrounds. In Table I, we list the approved and possible future collider facilities considered in our study. Furthermore, special features of the collider detectors (such as the high resolution for the electromagnetic calorimeter for $h^0_{\rm SM} \to \gamma\gamma$, and high b tagging efficiency) are also required in order to maximize the significance of the Higgs signal. Thus, establishing the discovery reach for future colliders is an important and non-trivial first step in the pursuit of the Higgs boson.

3. Consider precision measurements of h^0 properties

   The discovery of the Higgs boson will complete the experimental verification of the Standard Model. Once the Higgs boson is discovered, one must check that it does indeed possess couplings to particles proportional to their masses. One should quickly be able to verify that the properties of the scalar state roughly match those expected for the Standard Model Higgs boson. More precise measurements may be required to detect deviations of the observed Higgs properties from that of the $h^0_{\rm SM}$. The difficulty of this latter task will depend on how close one is to the decoupling limit (see Section IIIA).

4. Evaluate the potential for direct detection of the non-minimal Higgs states and the measurement their properties

   This is essential for probing the nature of the electroweak symmetry breaking dynamics. In addition, the non-minimal Higgs states may be sensitive to physics associated with the hierarchy problem (for example, the properties of the non-minimal Higgs states in supersymmetric models can provide important checks of the supersymmetric dynamics). The non-minimal Higgs sector imposes the most stringent requirements on the collider facility. To accomplish this task may require the highest energies and luminosities now being considered.

Table I: Approved and possible future collider facilities considered in this study. LEP-2 is currently running, but has not yet reached its design energy and luminosity [12]. Experimentation at the Tevatron Main Injector (M.I.) is often referred to in the text as Run II. Center of mass energy $\sqrt{s}$ and design annual integrated luminosity are specified.

Name	Type	$\sqrt{s}$	Annual $\int \mathcal{L}$
Approved:			
LEP-2	e^+e^-	192 GeV	170 pb^{-1}
Tevatron (M.I.)	$p\bar{p}$	2 TeV	2 fb^{-1}
LHC	pp	14 TeV	100 fb^{-1}
Possible:			
TeV-33	$p\bar{p}$	2 TeV	30 fb^{-1}
NLC†	e^+e^-	0.5–1.5 TeV	50–200 fb^{-1}
(† with $e\gamma, \gamma\gamma, e^-e^-$ options)			
FMC	$\mu^+\mu^-$	0.5–4 TeV	50–1000 fb^{-1}

This report consists of four parts. Following this Introduction, Section II briefly summarizes the results of the Precision electroweak physics subgroup. Section III discusses some theoretical issues that are important for the considerations of the Higgs discovery and properties subgroups. Section IV summarizes the essentials of Higgs phenomenology at future colliders. The conclusions and some final thoughts are given in Section V. The details underlying Sections II and IV can be found in the subgroup reports that follow this summary report [13,14].

II. PRECISION ELECTROWEAK PHYSICS AT FUTURE COLLIDERS

In the electroweak Standard Model, there are two coupling parameters, g and g', of $SU(2)_L \times U(1)_Y$ gauge interactions. The vacuum expectation value of the scalar field, v, sets the mass scale. At tree level, the $W^\pm$ and Z boson masses m_W, m_Z, as well as the weak mixing angle $\sin\theta_W$, are determined by these three parameters. Alternatively, one may use the precisely measured quantities—the electromagnetic coupling constant α, the muon decay constant G_μ and m_Z—as inputs to evaluate the other electroweak parameters. When the radiative corrections are taken into account, the relations among these parameters become dependent on m_t, $m_{h^0_{\rm SM}}$ as well as other possible contributions from new physics. Therefore, precision electroweak measurements not only check the consistency of the Standard Model, but also constrain $m_{h^0_{\rm SM}}$ and other new physics [15,16].

The Precision Electroweak Physics subgroup [13] paid special attention to the measurements of m_W, m_t and $\sin\theta_W$ at future collider experiments. The implications for the constraints on $m_{h^0_{\rm SM}}$ are also discussed.

Currently, the world average values for m_W and m_t are

$$m_W = 80.356 \pm 0.125 \text{ GeV}, \quad m_t = 175 \pm 6 \text{ GeV}. \qquad (1)$$

The precision which can be achieved for m_W and m_t measurements at different colliders is summarized in Table II and Table III, respectively. Table entries are taken from Ref. [13] unless otherwise indicated.

Table II: Expected W mass precision at future colliders.

Collider	δm_W (MeV)
NuTeV [17]	100
HERA (1000 pb^{-1})	60
LEP-2 (4×25 pb^{-1})	144
LEP-2 (4×500 pb^{-1})	40
Tevatron (2 fb^{-1})	35
TeV-33 (10 fb^{-1})	20
LHC (10 fb^{-1})	15
NLC (50 fb^{-1}) [18]	15
FMC (10 fb^{-1}) [19]	20

Table III: Expected top quark mass precision at future colliders.

Collider	δm_t (GeV)
Tevatron (2 fb^{-1})	4
TeV-33 (10 fb^{-1})	2
LHC (10 fb^{-1})	2
NLC (50 fb^{-1}) [18]	0.12
FMC (10 fb^{-1}) [19]	0.2

The weak mixing angle is conveniently defined by

$$\sin^2\theta_{eff}^{lept} = \frac{1}{4}\left(1 - \frac{g_{V\ell}}{g_{A\ell}}\right), \qquad (2)$$

where $g_{V\ell}$ and $g_{A\ell}$ are the effective vector and axial vector coupling constants of the leptons to the Z boson. They are measured with very high precision from Z leptonic decays [1,2] at LEP-I (forward-backward asymmetries) and at SLC (left-right asymmetries). The relation between $\sin^2\theta_{eff}^{lept}$ and the weak mixing angle in the $\overline{\rm MS}$ scheme, $\sin^2\hat{\theta}_W(M_Z)$ is given by

$$\sin^2\theta_{eff}^{lept} \simeq \sin^2\hat{\theta}_W(M_Z) + 0.00028. \qquad (3)$$

A fit to the combined current LEP-I and SLD asymmetry data yields [2]

$$\sin^2\theta_{eff}^{lept} = 0.23165 \pm 0.00024. \qquad (4)$$

The anticipation for the measurement on $\sin^2\theta_{eff}^{lept}$ at future experiments is summarized in Table IV.

The high precision measurements of α, G_μ and m_Z, along with the improved measurements of m_W, m_t and $\sin\theta_W$, may indirect shed light on the Standard Model Higgs boson mass

Table IV: Anticipated precision for $\sin^2\theta_{eff}^{lept}$ measurement at future colliders.

Collider	$\delta\sin^2\theta_{eff}^{lept}$ $(\times 10^{-4})$
SLC2000 [20]	1.2
TeV-33 (10 fb^{-1})	2
LHC (10 fb^{-1})	3
NLC (10 fb^{-1})	0.6

$m_{h^0_{\rm SM}}$. As an illustration, Fig. 1 shows the mass correlation for m_W versus $m_{h^0_{\rm SM}}$ with $m_t = 176 \pm 2$ GeV. A measurement of the m_W with a precision of $\delta m_W = 10$ MeV and of m_t with an accuracy of 2 GeV thus translates into an indirect determination of the Higgs boson mass with a relative error of about

$$\delta m_{h^0_{\rm SM}}/m_{h^0_{\rm SM}} \approx 20\%. \qquad (5)$$

However, it should be noted that to reach such a high precision, other sources of uncertainty, such as $\alpha(m_Z^2)$, α_s and theoretical uncertainties that arise when extracting m_W [21] and m_t [22], must be kept under control.

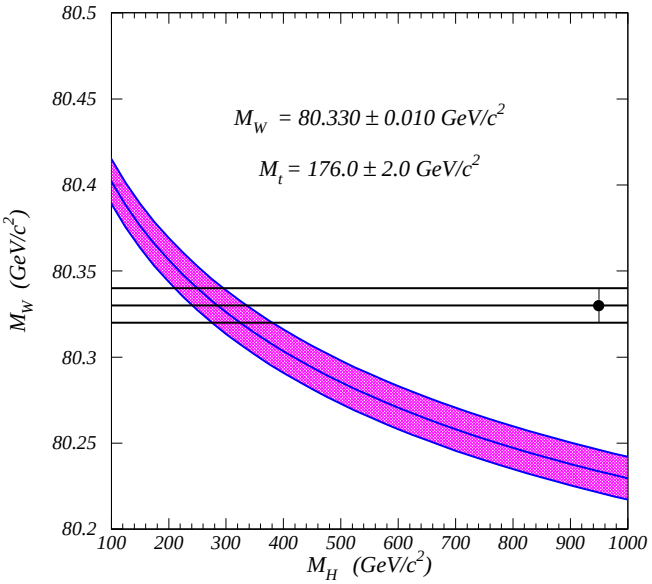

Figure 1: m_W versus $m_{h^0_{\rm SM}}$ for $m_t = 176 \pm 2$ GeV/c^2. The theoretical predictions incorporate the effects of higher order electroweak and QCD corrections.

III. THEORETICAL CONSIDERATIONS FOR THE WEAKLY-COUPLED HIGGS SECTOR

A. The Decoupling Limit

In this section, we discuss the theoretical implications of the discovery of the first neutral Higgs boson, denoted by h^0. Once

this state is discovered, one must check its theoretical interpretation. A Higgs state is predicted to couple to particles with coupling strengths proportional to the particle masses. After its initial discovery, it should be straightforward to verify whether its properties roughly match those expected of the Standard Model Higgs boson, $h^0_{\rm SM}$.

In order to interpret the significance of the first Higgs discovery, it is important to appreciate the concept of the *decoupling limit* [23,24]. First, consider the Standard Model Higgs boson. At tree-level, the Higgs self-coupling is related to its mass. If λ is the quartic Higgs self-interaction strength, then $\lambda = 3m^2_{h^0_{\rm SM}}/v^2$ (where $v \simeq 246$ GeV is the Higgs vacuum expectation value which is fixed by the $W^\pm$ mass: $v = 2m_W/g$). This means that one cannot take $m_{h^0_{\rm SM}}$ arbitrarily large without the attendant growth in λ. That is, the heavy Higgs limit in the Standard Model is non-decoupling. In models of a non-minimal Higgs sector, the situation is more complex. In some models (with the Standard Model as one example), it is not possible to take any Higgs mass much larger than $\mathcal{O}(v)$ without finding at least one strong Higgs self-coupling [23]. In other models, one finds that the non-minimal Higgs boson masses can be taken large at fixed Higgs self-couplings. Such behavior can arise if the model possesses one (or more) additional independent mass parameters beyond the diagonal scalar squared-masses. In the limit where the additional mass parameters are taken large [keeping the dimensionless Higgs self-couplings fixed and $\lesssim \mathcal{O}(1)$], the heavy Higgs states decouple, while both light and heavy Higgs bosons remain weakly-coupled. In this *decoupling limit*, exactly one neutral CP-even Higgs scalar remains light, and its properties are precisely those of the (weakly-coupled) Standard Model Higgs boson.

In this report, we shall always assume that all Higgs scalars are weakly-coupled (hence the name of this working group). Then, the decoupling limit is one where $h^0 \simeq h^0_{\rm SM}$, $m_{h^0} \simeq \mathcal{O}(m_Z)$, and all other non-minimal Higgs states are significantly heavier than m_{h^0}. Squared-mass splittings of the heavy Higgs states are of $\mathcal{O}(m_Z^2)$, which means that all heavy Higgs states are approximately degenerate, with mass differences of order m_Z^2/m_{A^0} (here m_{A^0} is approximately equal to the common heavy Higgs mass scale). In contrast, if the non-minimal Higgs sector is weakly coupled but far from the decoupling limit, then h^0 is not separated in mass from the other Higgs states. In this case, the properties[2] of h^0 differ significantly from those of $h^0_{\rm SM}$.

The decoupling limit arises naturally in many approaches. For example, in models of Higgs doublets (and singlets), with no artificial discrete symmetries imposed, the decoupling limit is reached when off-diagonal Higgs mass parameters are taken large. Naturalness properties suggest that all such parameters should reflect the highest possible energy scale consistent with the model. In models that introduce new TeV scale physics (to explain the dynamics of electroweak symmetry breaking), these mass parameters are expected to be associated with this new physics. The paradigm for this discussion is the MSSM. In the MSSM, the decoupling regime is reached once $m_{A^0} \gtrsim 2m_Z$. The parameter m_{A^0} arises in the MSSM from the supersymmetry breaking sector. The success of the Standard Model in accounting for precision electroweak data suggests that if the MSSM is correct, then the supersymmetry breaking scale is somewhat higher than m_Z (though it must not be much higher than 1 TeV if it is to explain the origin of the electroweak scale). Likewise, one might expect m_{A^0} to also be somewhat higher than m_Z. Thus, in the MSSM, there is some expectation that the Higgs sector approximately satisfies the decoupling limit. Although this argument is clearly not definitive, it will become more persuasive if supersymmetry and/or the non-minimal Higgs sector is not discovered at LEP-2 or the Tevatron.

The phenomenological consequences of the decoupling regime are both disappointing and challenging. In this case, h^0 (once discovered) will exhibit all the expected properties of $h^0_{\rm SM}$. The existence of the non-minimal Higgs sector will still be unconfirmed. It will require precision measurements or the highest energies and luminosities at future colliders to either detect a deviation from Standard Model Higgs physics or to directly detect the non-minimal Higgs states and explore their properties. In contrast, in the non-decoupling regime, more than one Higgs state is expected to populate the mass region where the first Higgs boson is found. The properties of the first Higgs state will show a marked deviation from $h^0_{\rm SM}$ properties. Experiments that can discover the Higgs boson will have access to many scalar sector observables.

B. Implications of a Higgs Discovery for New High Energy Scales

Phenomenologists and experimentalists who plan the Higgs searches at future colliders spend much effort in designing a search for the Standard Model Higgs boson. However, the term "Standard Model Higgs boson" is meaningless unless additional information is provided. This is because the Standard Model itself cannot be a fundamental theory of particle interactions. It must break down once the energy is raised beyond some critical scale Λ. What is the value of Λ? Of course, this is unknown at present. Λ can lie anywhere between a few hundred GeV and the Planck scale ($M_{\rm PL} \simeq 10^{19}$ GeV).

Theorists who study the phenomenology of the Standard Model usually do not need to know the value of Λ. At energy scales below Λ, the new physics beyond the Standard Model decouples, leaving a low-energy effective theory which looks almost exactly like the Standard Model. However, the discovery of the Higgs boson provides an opportunity to probe the value of Λ. Consider the behavior of the quartic Higgs self coupling, λ, as a function of the energy scale. At low-energies, $\lambda = 3m^2_{h^0_{\rm SM}}/v^2$. If one solves the one-loop renormalization group equation for $\lambda(\mu)$, one finds that λ increases with energy scale, μ. Eventually $\lambda(\mu)$ becomes infinite at the so-called Landau pole. Although this behavior could have been an artifact of the one-loop approximation, lattice results confirm that the theory breaks down at scales near the Landau pole [25]. That is, we may associate Λ with the Landau pole. Conversely, fixing the

[2]The basic property of the Higgs coupling strength proportional to mass is maintained. But, the precise coupling strength patterns of h^0 will differ from those of $h^0_{\rm SM}$ in the non-decoupling limit.

value of Λ leads to an upper bound on the low-energy value of λ, or equivalently to an upper bound on $m_{h^0_{\rm SM}}$. For example, if $\Lambda = M_{\rm PL}$, then $m_{h^0_{\rm SM}} \lesssim 200$ GeV [26,27]. Lower values of Λ imply a higher Higgs mass upper bound. Since Λ had better be larger than $m_{h^0_{\rm SM}}$ (since we are assuming the Standard Model is a valid low-energy effective theory over some range of energies), one can deduce an absolute Higgs mass upper bound of about 700–800 GeV. Similar conclusions are reached by lattice computations [25].

The stability of the Higgs potential also places non-trivial constraints on the Higgs mass, due to the large value of the top quark mass. (More refined limits require only a metastable potential with a lifetime that is long compared to the age of the universe.) For example, recent computations of Refs. [28] and [29] show that if $\Lambda = M_{\rm PL}$, then for $m_t = 175$ GeV the Higgs mass must be larger than about 120 GeV. If a Higgs boson were discovered whose mass lies below this limit, then one would conclude that new physics beyond the Standard Model must exist at some scale below $M_{\rm PL}$. As an example, if a Higgs boson of mass 100 GeV were discovered, then new physics beyond the Standard Model must enter at or below an energy scale of order $\Lambda = 1000$ TeV (based on the graphs presented in Ref. [29]). Of course, in this case, if all the new physics were confined to lie in the vicinity of 1000 TeV, then LHC phenomenology would find no deviations from the Standard Model. Thus, physicists who plan searches for the Standard Model Higgs boson are not wasting their time. In particular, even if Λ is rather close to the TeV scale, one would expect the lightest Higgs boson to retain all the properties of the Standard Model Higgs boson.

The MSSM provides a nice illustration of these considerations. A Higgs boson of mass 100 GeV (and with properties approximating those of $h^0_{\rm SM}$) is perfectly consistent in the context of the MSSM. In this case, the Standard Model breaks down at an energy scale far below 1000 TeV, due to the existence of supersymmetric partners whose masses are no heavier than (roughly) 1 TeV. In particular, $h^0 \simeq h^0_{\rm SM}$ in the MSSM as long as $m_{A^0} \gtrsim 2m_Z$, as noted in Section IIIA.

C. Higgs Mass Bounds in Low-energy Supersymmetric Models

If the minimal supersymmetric extension of the Standard Model (MSSM) is correct, then we should identify the scale Λ at which the Standard Model breaks down as the scale of low-energy supersymmetry breaking. In models of low-energy supersymmetry, Λ is presumed to lie between m_Z and about 1 TeV. The mass of the light CP-even neutral Higgs boson, h^0, in the MSSM can be calculated to arbitrary accuracy in terms of two parameters of the Higgs sector, m_{A^0} and $\tan\beta$ [30], and other MSSM soft-supersymmetry-breaking parameters that affect the Higgs mass through virtual loops [31]. If the scale of supersymmetry breaking is much larger than m_Z, then large logarithmic terms arise in the perturbation expansion. These large logarithms can be resummed using renormalization group (RG) methods.

At tree level, the mass of the lightest CP-even Higgs boson of the MSSM is bounded: $m_{h^0} \leq m_Z |\cos 2\beta| \leq m_Z$. If this prediction were exact, it would imply that the Higgs boson must be discovered at the LEP-2 collider (running at its projected maximum center-of-mass energy of 192 GeV, with an integrated luminosity of 150 $\rm pb^{-1}$). Absence of a Higgs boson lighter than m_Z would apparently rule out the MSSM. When radiative corrections are included, the light Higgs mass upper bound is increased significantly. In the one-loop leading logarithmic approximation [31],

$$m^2_{h^0} \lesssim m^2_Z \cos^2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln\left(\frac{M_{\tilde t}^2}{m_t^2}\right) , \qquad (6)$$

where $M_{\tilde t}$ is the (approximate) common mass of the top-squarks. Observe that the Higgs mass upper bound is very sensitive to the top mass and depends logarithmically on the top-squark masses. Although eq. (6) provides a rough guide to the Higgs mass upper bound, it is not sufficiently precise for LEP-2 phenomenology, whose Higgs mass reach depends delicately on the MSSM parameters. In addition, in order to perform precision Higgs measurements and make comparisons with theory, a more accurate result for the Higgs sector masses (and couplings) are required. The formula for the full one-loop radiative corrected Higgs mass has been obtained in the literature, although it is very complicated since it depends in detail on the virtual contributions of the MSSM spectrum [32]. Moreover, if the supersymmetry breaking scale is larger than a few hundred GeV, then RG methods are essential for summing up the effects of large logarithms and obtaining an accurate prediction.

The computation of the RG-improved one-loop corrections requires numerical integration of a coupled set of RG equations [33]. (The dominant two-loop next-to-leading logarithmic results are also known [34].) Although this program has been carried out in the literature, the procedure is unwieldy and not easily amenable to large-scale Monte-Carlo analyses. Recently, two groups have presented a simple analytic procedure for accurately approximating m_{h^0}. These methods can be easily implemented, and incorporate both the leading one-loop and two-loop effects and the RG-improvement. Also included are the leading effects at one loop of supersymmetric thresholds (the most important effects of this type are squark mixing effects in the third generation). Details of the techniques can be found in Ref. [35] and [36], along with other references to the original literature. Here, we simply quote two specific bounds, assuming $m_t = 175$ GeV and $M_{\tilde t} \lesssim 1$ TeV: $m_{h^0} \lesssim 112$ GeV if top-squark mixing is negligible, while $m_{h^0} \lesssim 125$ GeV if top-squark mixing is "maximal". Maximal mixing corresponds to an off-diagonal squark squared-mass that produces the largest value of m_{h^0}. This mixing leads to an extremely large splitting of top-squark mass eigenstates. Current state of the art calculations can obtain a mass bound for the light CP-even Higgs boson of the MSSM that is reliable to within a few GeV. Of course, the bound one finally obtains is very sensitive to the top quark mass, and depends crucially on the upper bound one chooses to place on supersymmetric particle masses. In this report, a conservative bound of $m_{h^0} \lesssim 130$ GeV was used as input to the phenomenological analysis.

The charged Higgs mass is also constrained in the MSSM. At tree level, $m^2_{H^\pm} = m^2_W + m^2_{A^0}$, which implies that charged Higgs bosons cannot be pair produced at LEP-2. Radiative corrections modify the tree-level prediction, but the corrections are typically smaller than the neutral Higgs mass corrections discussed above. Although $m_{H^\pm} \geq m_W$ is not a strict bound when one-loop corrections are included, the bound holds approximately over most of MSSM parameter space (and can be significantly violated only when $\tan\beta$ is well below 1, a region of parameter space that is theoretically disfavored).

The MSSM Higgs mass bounds do not in general apply to non-minimal supersymmetric extensions of the Standard Model. If additional Higgs singlet and/or triplet fields are introduced, then new Higgs self-couplings parameters appear, which are not significantly constrained by present data. These parameters can contribute to the light Higgs masses; [3] the upper bound on these contributions depends on an extra assumption beyond the physics of the TeV scale effective theory. For example, in the simplest non-minimal supersymmetric extension of the Standard Model (NMSSM), the addition of a Higgs singlet superfield adds a new Higgs self-coupling parameter, λ [10]. The mass of the lightest neutral Higgs boson can be raised arbitrarily by increasing the value of λ (analogous to the behavior of the Higgs mass in the Standard Model!). In this case, we must generalize the analysis of Section IIIB and introduce a new scale $\widetilde{\Lambda}$ beyond which the NMSSM breaks down. The upper bound on the Higgs mass then depends on the choice of $\widetilde{\Lambda}$. The standard assumption of theorists who construct low-energy supersymmetric models is that all couplings stay perturbative up to the Planck scale. Choosing $\widetilde{\Lambda} \simeq M_{\rm PL}$, one finds in most cases that $m_{h^0} \lesssim 150$ GeV, independent of the details of the low-energy supersymmetric model [37]. The NMSSM also permits a tree-level charged Higgs mass below m_W. However, as in the MSSM, the charged Higgs mass becomes large and roughly degenerate with m_{A^0} in the decoupling limit where $m_{A^0} \gg m_Z$.

IV. ESSENTIALS OF HIGGS PHENOMENOLOGY AT FUTURE COLLIDERS

Higgs hunting at future colliders will consist of three phases. Phase one is the initial Higgs boson search in which a Higgs signal is found and confirmed as evidence for new phenomena not described by Standard Model background. Phase two will address the question: should the signal be identified with Higgs physics? Finally, phase three will consist of a detailed probe of the Higgs sector and precise measurements of Higgs sector observables. Further details on the results of this section can be found in Ref. [14].

A. Phase 1 – Demonstrate the Observability of a Higgs Signal

As we plan for future collider facilities, the machine and detector characteristics must be developed in such a way that a Higgs signal can be unambiguously detected above the Standard Model background. In this discussion, we shall focus mainly on the Standard Model Higgs boson ($h^0_{\rm SM}$) and the Higgs bosons of the MSSM (h^0, H^0, A^0, and $H^\pm$). At present, taking into account data from LEP-1 and the most recent LEP-2 data (at $\sqrt{s} = 161$ and 172 GeV), one can exclude a Higgs boson of mass $m_{h^0_{\rm SM}} < 70.7$ GeV [38]. The MSSM bounds are a little more complicated, since they depend primarily on two Higgs sector parameters, but with some dependence on the MSSM spectrum which affects Higgs masses and couplings through virtual loop effects. The current MSSM Higgs mass bounds exclude the mass ranges: $m_{h^0} < 62.5$ GeV (independent of the value of $\tan\beta$) and $m_{A^0} < 62.5$ GeV (assuming $\tan\beta > 1$) [38]. LEP-1 data also excludes charged Higgs masses with $m_{H^\pm} < 44$ GeV in a general two-Higgs-doublet model [39]. (LEP-2 data does not yet improve this bound.) This bound is less interesting in the MSSM, where $m_{H^\pm} \gtrsim m_W$ over most of the MSSM parameter space. The search for $t \to bH^+$ at the Tevatron can, in principle, extend the reach of the charged Higgs search. However, the quoted limits [40] apply only in a very narrow region of parameter space.

Consider the Higgs search at future colliders. The machines we have examined are summarized in Table I. Most work on analyzing the discovery reach of future colliders has focused on the Standard Model Higgs boson and the Higgs bosons of the MSSM. In the latter case, some of the analyses also apply to more general unconstrained versions of the two Higgs doublet model. In the decoupling limit, the discovery limits obtained for $h^0_{\rm SM}$ also apply to the lightest CP-even neutral Higgs boson of a more general non-minimal Higgs sector.

Table V: The $h^0_{\rm SM}$ discovery reach of future colliders. A 5σ signal above background is required for discovery. Note that Run II at the Tevatron complements the LEP Higgs search only for an integrated luminosity well beyond one year at the design luminosity of the Main Injector. For NLC, both $\sqrt{s} = 500$ GeV and 1 TeV cases are shown. The FMC discovery reach is similar to that of the NLC for the same center-of-mass energy and integrated luminosity.

Collider	Integrated Luminosity	Discovery Reach
LEP-2 ($\sqrt{s} = 192$ GeV)	150 pb^{-1}	95 GeV
Tevatron (M.I.)	5–10 fb^{-1}	80–100 GeV
TeV-33	25–30 fb^{-1}	120 GeV
LHC	100 fb^{-1}	800 GeV
NLC-500	50 fb^{-1}	350 GeV
NLC-1000	200 fb^{-1}	800 GeV

1. The Standard Model Higgs Boson

The $h^0_{\rm SM}$ discovery reach of future colliders is summarized in Table V. At LEP-2 running at its maximum energy of $\sqrt{s} = 192$ GeV, the discovery reach of $m_{h^0_{\rm SM}} \simeq 95$ GeV can be

[3] This should be contrasted with the MSSM, where all Higgs self-couplings are related by supersymmetry to gauge couplings. This is the origin of the MSSM bound $m_{h^0} \lesssim \mathcal{O}(m_Z)$ discussed above.

attained by one detector taking data for about one year at design luminosity [41]. With four LEP detectors running, the Higgs mass discovery reach can be achieved sooner (or improve on the significance of any candidate Higgs signal). Additional luminosity cannot significantly extend the Higgs mass reach unless the LEP-2 center-of-mass energy were increased. At Run II of the Tevatron one year of data taking at the Main Injector design luminosity is not sufficient to discover a Standard Model Higgs boson above background. However, two detectors running at design luminosity from three to five years can complement the LEP-2 Higgs search. In particular, the associated production of $Wh^0_{\rm SM}$ with $h^0_{\rm SM} \to b\bar{b}$ may be feasible at the Tevatron, given sufficient integrated luminosity. Assuming a total integrated luminosity of 5 [10] fb^{-1}, a Standard Model Higgs mass discovery reach of 80 [100] GeV is attainable [42,43]. The Tevatron Higgs search technique also applies at higher luminosity. For example, initial studies indicate that at TeV-33, a Standard Model Higgs boson with a mass of 120 GeV can be discovered with an integrated luminosity of 25–30 fb^{-1} [42,43]. The significance of the Higgs signal could be enhanced by the detection of the associated production of $Zh^0_{\rm SM}$, $h^0_{\rm SM} \to b\bar{b}$ [44]. Implicit in these studies is the assumption that the Standard Model contributions are sufficiently well understood that the Higgs signal can be detected as a small excess above background.

The LHC is required if one wants to extend the Higgs mass discovery reach significantly beyond $\mathcal{O}(m_Z)$. Note that according to the discussion of Section IIIB, it only makes sense to consider Standard Model Higgs bosons with mass below 800 GeV.[4] Therefore, Table V implies that the LHC can provide complete coverage of the (weakly-coupled) Standard Model Higgs mass region, assuming that it achieves its design luminosity [14,45,46]. For $m_{h^0_{\rm SM}} \gtrsim 2m_Z$, the "gold-plated mode" $h^0_{\rm SM} \to ZZ \to \ell^+\ell^-\ell^+\ell^-$ provides a nearly background free signature for Higgs boson production until the production rate becomes too small near the upper end of the weakly-coupled Higgs mass regime. In this case, other signatures (*e.g.*, $h^0_{\rm SM} \to ZZ \to \ell^+\ell^-\nu\bar{\nu}$ and $h^0_{\rm SM} \to W^+W^- \to \ell\nu + \text{jets}$) provide additional signatures for Higgs discovery.

The most troublesome Higgs mass range for hadron colliders is the so-called "intermediate Higgs mass regime", which roughly corresponds to $m_Z \lesssim m_{h^0_{\rm SM}} \lesssim 2m_Z$. For 130 GeV$\lesssim m_{h^0_{\rm SM}} \lesssim 2m_Z$, one can still make use of the gold plated mode at the LHC, $h^0_{\rm SM} \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$ (where Z^* is virtual). Standard Model backgrounds begin to be problematical when the branching ratio $\mathrm{BR}(h^0_{\rm SM} \to ZZ^*)$ becomes too small. This occurs for $2m_W \lesssim m_{h^0_{\rm SM}} \lesssim 2m_Z$ where $\mathrm{BR}(h^0_{\rm SM} \to W^+W^-)$ is by far the dominant Higgs decay channel, and for $m_{h^0_{\rm SM}} \lesssim 140$ GeV where the the virtuality of Z^* begins to significantly reduce the $h^0_{\rm SM} \to ZZ^*$ decay rate. A complementary channel $h^0_{\rm SM} \to WW^{(*)} \to \ell^+\nu\ell^-\bar{\nu}$ provides a viable Higgs signature for 155 GeV$\lesssim m_{h^0_{\rm SM}} \lesssim 2m_Z$ [47], and closes a potential hole near the upper end of the intermediate Higgs mass range. For $m_{h^0_{\rm SM}} \lesssim 130$ GeV, the dominant decay channel $h^0_{\rm SM} \to b\bar{b}$ has very large Standard Model two-jet backgrounds. Thus, in this regime, it is necessary to consider rarer production and decay modes with more distinguishing characteristics. Among the signatures studied in the literature are:

(i) $gg \to h^0_{\rm SM} \to \gamma\gamma$,
(ii) $q\bar{q} \to V^* \to Vh^0_{\rm SM}$, $\quad (V = W \text{ or } Z)$,
(iii) $gg \to t\bar{t}h^0_{\rm SM}$,
(iv) $gg \to b\bar{b}h^0_{\rm SM}$, and
(v) $gg \to h^0_{\rm SM} \to \tau^+\tau^-$.

The LHC detectors are being optimized in order to be able to discover an intermediate mass Higgs boson via its rare $\gamma\gamma$ decay mode (with a branching ratio of about 10^{-3}). The other signatures could be used to provide consistency checks for the Higgs discovery as well as provide additional evidence for the expected Higgs-like properties of the Higgs boson candidate. A successful intermediate mass Higgs search via the $\gamma\gamma$ decay mode at the LHC will require maximal luminosity and a very fine electromagnetic calorimeter resolution (at about the 1% level).

In contrast to the Tevatron and LHC Higgs searches, the Standard Model Higgs search at the NLC in the intermediate mass regime is straightforward, due to the simplicity of the Higgs signals, and the relative ease in controlling the Standard Model backgrounds. Higgs production is detected at the NLC via two main signatures. The first involves the extension of the LEP-2 search for

$$e^+e^- \to Zh^0_{\rm SM} \tag{7}$$

to higher energies. In addition, a second process can also be significant: the (virtual) W^+W^- fusion process[5]

$$e^+e^- \to \nu\bar{\nu}W^*W^* \to \nu\bar{\nu}h^0_{\rm SM}. \tag{8}$$

The fusion cross-section grows logarithmically with the center-of-mass energy and becomes the dominant Higgs production process at large $\sqrt{s}/m_{h^0_{\rm SM}}$. For example, at $\sqrt{s} = 500$ GeV, complete coverage of the intermediate Higgs mass regime below $m_{h^0_{\rm SM}} \lesssim 2m_Z$ requires only 5 fb^{-1} of data. The only limitation of the NLC in the Higgs search is the center-of-mass energy of the machine which determines the upper limit of the Higgs boson discovery reach. One would need $\sqrt{s} \simeq 1$ TeV to fully cover the weakly-coupled Standard Model Higgs mass range [48–50].

The techniques for the Standard Model Higgs boson *discovery* at a $\mu^+\mu^-$ collider are, in principle, identical to those employed at the NLC [51,52]. However, one must demonstrate that the extra background resulting from an environment of decaying muons can be tamed. It is believed that sufficient background rejection can be achieved [53]; thus the FMC has

[4] It is possible to imagine theories of electroweak symmetry breaking which produce scalar states heavier than 800 GeV. However, any such scalar is presumably either strongly coupled, and/or composite on the scale of 1 TeV. The consideration of such scalars lies outside the scope of our working group.

[5] The corresponding ZZ fusion process, $e^+e^- \to e^+e^-Z^*Z^* \to e^+e^-h^0_{\rm SM}$ is suppressed by about a factor of ten relative to the W^+W^- fusion process. Nevertheless, at large $\sqrt{s}/m_{h^0_{\rm SM}}$, the $ZZ \to h^0_{\rm SM}$ fusion rate compares favorably to that of $e^+e^- \to Zh^0_{\rm SM}$. As a result, the ZZ fusion process can be used in some cases to study Higgs properties.

the same discovery reach as the NLC at the same center-of-mass energy and luminosity.

2. Higgs Bosons of the MSSM

Next, we turn to the discovery potential at future colliders for the Higgs bosons of the MSSM. If $m_{A^0} \gg m_Z$, then the decoupling limit applies, and the couplings of h^0 to Standard Model particles are identical to those of $h^0_{\rm SM}$. Thus, unless h^0 decays appreciably to light supersymmetric particles, the discussion given above for $h^0_{\rm SM}$ apply without change to h^0. In general, one can consider two types of MSSM Higgs searches at future colliders. First, one can map out the region of MSSM parameter space where at least one MSSM Higgs boson can be discovered in a future collider Higgs search. If no Higgs state is discovered, then the corresponding region of MSSM parameter space would be excluded. (In some cases, the absence of a Higgs discovery would be strong enough to completely rule out the MSSM!) Note that in this approach, one may simply discover one Higgs state—the light CP-even neutral h^0—with properties resembling that of $h^0_{\rm SM}$, which would be consistent with MSSM expectations, but would provide no direct proof that low-energy supersymmetry underlies the Higgs sector dynamics. Second, one can examine the discovery potential for specific states of the non-minimal Higgs sector. As emphasized in Section IIIA, in the decoupling limit, the non-minimal Higgs states are heavy (compared to the Z), nearly degenerate in mass, and weakly-coupled. Discovery of these states at future colliders is far from being assured.

Table VI: MSSM Higgs boson discovery potential

Collider	Comments
LEP-2	Significant but not complete coverage, via $e^+e^- \to H^+H^-$ $e^+e^- \to Zh^0$ $e^+e^- \to h^0A^0$
TeV-33	Limited coverage, complements the LEP-2 search
LHC	(Nearly) complete coverage for the discovery of at least one Higgs boson of the MSSM. Main challenge: the intermediate Higgs mass region $[m_Z \lesssim m_{h^0} \lesssim 2m_Z]$ which requires different search strategies depending on the value of m_{h^0}. Some sensitivity to heavier non-minimal Higgs states.
NLC and FMC	Complete coverage for the discovery of at least one Higgs boson of the MSSM. Sensitivity to to heavier non-minimal states depends on $\sqrt{s}$: $\sqrt{s} \gtrsim 2m_A$ for discovery of $H^\pm, H^0, A^0$ via associated production. $\sqrt{s} \sim m_A$ for $\mu^+\mu^- \to H^0, A^0$ s-channel resonance production.

We summarize the MSSM Higgs boson discovery potential at future colliders in Table VI.[6] Consider first the discovery limits for h^0 of the MSSM at future collider facilities. As described in Section IIIC, the tree-level MSSM predicts that $m_{h^0} \leq m_Z$. Suppose that this predicted bound were unmodified (or reduced) after taking radiative corrections into account. Then the non-observation of h^0 at LEP-2 (which will eventually be sensitive to the mass range $m_{h^0} \lesssim 95$ GeV) would rule out the MSSM. However, for some choices of MSSM parameters, the radiative corrections significantly *increase* the tree-level bound [31]. Consequently, the Higgs searches at LEP-2 (and the Tevatron) cannot completely rule out the MSSM.

On the other hand, considering that the radiatively corrected bound is $m_{h^0} \lesssim 130$ GeV, it would appear that the LHC has access to the full MSSM Higgs sector parameter space. After all, we argued above that the LHC will be able to completely cover the intermediate Standard Model Higgs mass regime. However, when $m_{A^0} \sim \mathcal{O}(m_Z)$, the decoupling limit does not apply, and the properties of h^0 deviate from those of $h^0_{\rm SM}$. Thus, an independent analysis is required to ascertain the discovery potential of the LHC search for MSSM Higgs bosons. In particular, the LHC detector collaborations must demonstrate the feasibility of h^0 discovery in the mass range $m_Z \lesssim m_{h^0} \lesssim 130$ GeV. This is precisely the most difficult region for the LHC Higgs search. At this time, one can argue that the LHC coverage of the MSSM Higgs sector parameter space is nearly complete, although the search strategies sometimes depend on the observation of small signals (above significant Standard Model backgrounds) in more than one channel. Moreover, the present estimates of the statistical significance of the Higgs signal rely on theoretical determinations of both signal and background rates as well as simulations of detector performance. Thus, if no Higgs signal is confirmed by the LHC, it might still be difficult to definitively rule out the MSSM.

The NLC (and FMC) provide complete coverage of the MSSM Higgs sector parameter space once the center-of-mass energy is above 300 GeV. In contrast to the LHC Higgs search, the intermediate Higgs mass regime presents no particular difficulty for the high energy lepton colliders. The associated production

$$e^+e^- \to h^0A^0 \tag{9}$$

provides an addition discovery channel for $m_{A^0} \lesssim \sqrt{s}/2$. If no Higgs signal is seen, then the lepton colliders can unambiguously rule out the MSSM.

3. Higgs Bosons in non-minimal extensions of the MSSM

If no Higgs state is discovered at the LHC and NLC, then the MSSM would cease to be a viable candidate for a theory of

[6] We have not considered the possibility of Higgs decay channels involving supersymmetric particles. This is probably not an issue for the lightest CP-even scalar, h^0. Recall that in the MSSM, $m_{h^0} \lesssim 130$ GeV, and consider the likely constraints on supersymmetric particle masses in the absence of observed supersymmetric particle production at LEP-2. It is then very unlikely that there would be any open supersymmetric channels in h^0 decays. For the heavier Higgs states (H^0, A^0 and $H^\pm$), supersymmetric decay modes can be significant and provide new signatures for Higgs production and decay. This possibility merits further study.

electroweak physics. However, the MSSM is just one model of low-energy supersymmetry. Thus, it is important to consider non-minimal extensions of the MSSM to see whether the low-energy supersymmetric approach could be ruled out in general. Consider the Higgs search at the NLC in the context of a completely general two-Higgs doublet model. Suppose that the non-minimal Higgs states are heavy so that only h^0 is accessible at the NLC. The relevant h^0 production processes are listed in Eqs. (7) and (8). Note that in both cases, the production cross-sections are governed by the strength of the h^0 coupling to vector boson pairs. But, in models with Higgs doublets and singlets (but with no higher Higgs multiplets), these couplings must satisfy a sum rule [54]:

$$\sum_i g^2_{VVh^0_i} = g^2_{VVh^0_{\rm SM}}\,, \tag{10}$$

where $V = W^\pm$ or Z. As an example, the 5σ discovery of the Standard Model Higgs boson with $m_{h^0_{\rm SM}} = 150$ GeV at the NLC running at $\sqrt{s} = 500$ GeV requires only about 2 fb^{-1} of data (see, *e.g.*, Fig. 2.18 of Ref. [49]), corresponding to about 100 Higgs boson events before cuts. Equivalently, the NLC running at $\sqrt{s} = 500$ GeV with an integrated luminosity of 50 fb^{-1} permits the 5σ discovery of a neutral CP-even Higgs boson with 4% of the Standard Model cross section, which corresponds to $g_{VVh^0} \gtrsim 0.2 g_{VVh^0_{\rm SM}}$. Of course, if the VVh^0 coupling were smaller than this, no Higgs state would be discovered in this experiment. However, by raising the center-of-mass energy of the NLC, one must eventually find evidence for at least one of the heavier neutral Higgs states, by virtue of the sum rule [eq. (10)] quoted above.

The situation where the bulk of the VVh^0_i couplings are carried by the heavier Higgs states cannot arise in the MSSM for two reasons. First, the MSSM Higgs mass bound implies that $m_{h^0} \lesssim 130$ GeV, and second, $g_{VVh^0} \lesssim 0.2 g_{VVh^0_{\rm SM}}$ is possible only if $m_{A^0} \lesssim \mathcal{O}(m_Z)$, in which case, the Higgs boson would be discovered via h^0A^0 production [eq. (9)]. In non-minimal extensions of the MSSM Higgs sector, these two objections must be reconsidered.

We reviewed the case of the NMSSM in which one complex Higgs singlet field is added. This model introduces a new independent Higgs self-coupling which *a priori* can take on any value. However, if one imposes the requirement of perturbativity of couplings at all scales below the Planck scale (a requirement motivated by the unification of strong and electroweak couplings near the Planck scale), then one finds that the lightest Higgs boson must satisfy $m_{h^0} \lesssim 150$ GeV. Still, the lightest CP-even Higgs scalar may be very weakly coupled to quarks, leptons and gauge bosons if it is primarily composed of the singlet component. Thus, a detailed analysis is required to see whether the Higgs search at the NLC is sensitive to all regions of the NMSSM Higgs sector parameter space. The analysis of Ref. [55] demonstrated that even for $\sqrt{s} = 300$ GeV, the NLC search would easily detect at least one Higgs state of the NMSSM. Specifically, the minimum Higgs production cross-section in the NMSSM at $\sqrt{s} = 300$ GeV [500 GeV] was found to be 42 fb [17 fb]. Such Higgs production rates are easily detected above background, assuming the NLC luminosity given in Table I.

A similar question can be posed in the case of the LHC Higgs search. As discussed earlier, the LHC search will provide nearly complete coverage of the MSSM Higgs sector parameter space. Nevertheless, the LHC search is operating "at the edge" of its capabilities. By relaxing some of the MSSM constraints to Higgs sector parameters, we expect some holes to develop in region of supersymmetric parameter space accessible to the LHC Higgs search. Ref. [56] examined this question in detail for the case of the NMSSM, and concluded that although the region of inaccessibility is not large, it is possible to find regions of NMSSM Higgs parameter space in which no Higgs boson state could be discovered at the LHC. This analysis does suggest the possibility that future improvements in search strategies and detector capabilities (for example, improved b-tagging) may be able to significanly narrow the region of inaccessibility in the Higgs sector parameter space. Clearly, the supersymmetric Higgs search remains a formidable challenge for future experimentation at LHC.

The above considerations can also be applied to more general non-minimal extensions of the MSSM. Although there is no completely general analysis yet available, under most reasonable model assumptions, the non-observation of a Higgs boson in the intermediate-Higgs-mass regime at the NLC would rule out the low-energy supersymmetric model. Whether this "no-go" theorem can be circumvented by some more exotic approach to low-energy supersymmetry remains to be seen.

4. Observing More Than One Higgs Boson

If only one Higgs boson is discovered, it may closely resemble the $h^0_{\rm SM}$. In this case, one must address the detectability of the non-minimal Higgs states ($H^0, A^0, H^\pm, \cdots$) at future colliders. As emphasized above, all future colliders can provide only incomplete probes of the non-minimal Higgs sector parameter space. Naively, one would expect the masses of all Higgs sector states to be of order the electroweak scale. However, somewhat heavier non-minimal Higgs states often arise in model building. As an example, in low-energy supersymmetric models, the mass scale of the non-minimal Higgs states is controlled by a soft-supersymmetry-breaking parameter which could be as large as 1 TeV. Such heavy states would still be weakly-coupled and difficult to observe at any of the colliders we have examined.

The exploration of the non-minimal Higgs sector parameter space at future colliders could be especially challenging. Detection of heavy non-minimal Higgs states at the LHC is difficult due to the very low signal-to-background ratio of the corresponding Higgs boson signals. In particular, heavy Higgs states couple very weakly to gauge bosons, and would have to be detected via their heavy fermion decays. (At large $\tan\beta$, where the Higgs couplings to down-type fermions is enhanced relative to the Standard Model, it may be possible to observe a heavy neutral Higgs boson via its decay to $\tau^+\tau^-$.) At the NLC, the main obstacle for the discovery of non-minimal Higgs states is

the limit of the center-of-mass energy. For reasons connected to the nature of the decoupling limit, the heavy Higgs states of the MSSM can be produced in sufficient number and detected only if $\sqrt{s} \gtrsim 2m_{A^0}$ [24]. The discovery reach could in principle be somewhat extended by employing the $\gamma\gamma$ collider mode of the NLC. In this mode of operation, the search for $\gamma\gamma \to A^0$ and $\gamma\gamma \to H^0$ can extend the non-minimal Higgs mass discovery reach of the NLC [57].

Finally, the FMC can produce the neutral Higgs states singly via s-channel $\mu^+\mu^-$ annihilation, and would permit the discovery of the heavy neutral Higgs states up to $\sqrt{s} = m_{A^0}$ [51]. The viability of this discovery mode depends on the parameters of the Higgs sector. In the MSSM, the cross-section for $\mu^+\mu^- \to H^0, A^0$ is enhanced for values of $\tan\beta$ above 1. For $m_{H^0}, m_{A^0} \gg m_Z$, H^0 and A^0 are approximately degenerate in mass. Given sufficient luminosity, one can detect H^0 and A^0 (if kinematically accessible) by scanning in $\sqrt{s}$, assuming that $\tan\beta$ is larger than a critical value (which depends on the total luminosity and the Higgs mass). Detection is accomplished via a resonant peak in the Higgs decay to $b\bar{b}$ (and $t\bar{t}$ if allowed). Further details can be found in Ref. [14].

B. Phase 2 – After Discovery: Is It a Higgs Boson?

Suppose that the first candidate Higgs signal is detected. What must one do to prove that the produced state is a Higgs boson? We assume that after the initial discovery is made, further collider running confirms the signal and establishes a useful statistical sample of events. The first step is to ascertain whether the observed state resembles the Standard Model Higgs boson and/or if it is associated with a non-minimal Higgs sector. If $h^0 \simeq h^0_{\rm SM}$, then one must demonstrate that the discovered state has

(i) zero electric and color charge,

(ii) spin zero,

(iii) CP-even quantum number,

(iv) electroweak strength couplings, and

(v) couplings proportional to the mass of the state to which it couples.

Eventually, one would like to make detailed measurements and verify that the Higgs candidate matches all the properties expected of $h^0_{\rm SM}$ to within some precision (small deviations from the $h^0_{\rm SM}$ properties will be addressed in the next section). If the properties of the discovered state are Higgs-like, but differ in detail from those of $h^0_{\rm SM}$, then it is likely that other non-minimal Higgs states are light and may have been produced in the same experiment. Finding evidence for these states will be crucial in verifying the Higgs interpretation of the data.

At an e^+e^- collider (LEP-2 and the NLC), many of the Higgs boson properties can be directly measured due to low backgrounds and simple event structures.[7] One can directly measure the spin and CP-quantum numbers of the Higgs candidate through the angular distributions of production and decay. Specific Higgs decay modes can be separated and individually studied. Accurate measurements of $\sigma(h^0){\rm BR}(h^0 \to X)$ can be made for a number of final states, including $X = b\bar{b}$ and $\tau^+\tau^-$. In this workshop, a breakthrough was reported demonstrating that the detection of $h^0 \to c\bar{c}$ is possible with appreciable efficiency and low mis-identification [58]. Thus, at the lepton colliders, $h^0 \simeq h^0_{\rm SM}$ can be confirmed with some precision.

Table VII: Detectability of the $h^0_{\rm SM}$ at future hadron colliders as a function of the $h^0_{\rm SM}$ mass range. For the Tevatron Higgs search (Run II of the Main Injector and the TeV-33 upgrade), the required integrated luminosity in units of fb^{-1} is indicated in braces. For comparison, the LEP-2 discovery range (via $e^+e^- \to Zh^0_{\rm SM}$) is indicated. For the Tevatron and LHC searches, the Higgs decay modes involved in the primary Higgs discovery signals are shown in parentheses; further details are given in Table VIII.

Mass Range	Observability at Future Colliders
60–80 GeV	LEP-2, Tevatron$\{5\}(b\bar{b})$
80–100 GeV	LEP-2, Tevatron$\{10\}(b\bar{b})$, and LHC$(\gamma\gamma)$
100–120 GeV	Tevatron$\{25$–$30\}(b\bar{b})$ and LHC$(\gamma\gamma)$
120–130 GeV	LHC $(\gamma\gamma)$
130–155 GeV	LHC (ZZ^*)
155–180 GeV	LHC (ZZ^*, W^+W^-)
$\gtrsim 180$ GeV	LHC $(ZZ \to \ell^+\ell^-\ell^+\ell^-, \ell^+\ell^-\nu\bar{\nu})$

The initial Higgs discovery is most likely to occur at either LEP-2 or LHC. Thus, it is important to examine whether it is possible to verify the Higgs interpretation of a Higgs signal discovered at the approved future facilities. A strategy for accomplishing this goal was developed by our working group. We considered what was achievable on the basis of the Higgs searches at LEP-2, Run II at the Tevatron (with some consideration of a possible TeV-33 upgrade[8]) and the LHC. We elucidated all the observables where a Higgs signal could be detected. We considered separately seven specific mass intervals in the range 60 GeV $\lesssim m_{h^0} \lesssim$ 800 GeV, listed in Table VII. We then considered in detail a variety of possible Higgs signatures at each collider (see Table VIII) and evaluated the potential of each channel for supporting the Higgs interpretation of the signal. Taken one by one, each channel provides limited

[7] In principle, the remarks that follow also apply to the FMC. However, it has not yet been demonstrated that the severe backgrounds arising from the constantly decaying muons can be overcome to make precision measurements.

[8] The Higgs discovery reach at the Tevatron depends critically on the total integrated luminosity processed and analyzed by the CDF and D0 detectors. The Tevatron at the Main Injector design luminosity must run with one detector for five years to attain a Higgs discovery reach up to 100 GeV. To extend the Higgs reach further before the start of LHC requires TeV-33.

information. However, taken together, such an analysis might provide a strong confirmation of the Higgs-like properties of the observed state as well as providing a phenomenological profile that could be compared to the predicted properties of the Standard Model Higgs boson. Finally, we considered the limitations of the data from the Higgs searches at the hadron colliders, and examined the possible improvements in the determination of the Higgs properties with new data from the NLC and/or the FMC. A list of the primary Higgs signals at future colliders considered above is given in Table VIII.

Table VIII: Primary $h^0_{\rm SM}$ signatures at future colliders and the corresponding Higgs mass range over which detection of a statistically significant signal is possible. Other Higgs signatures not included in this table are discussed in Ref. [14].

Collider	Signature	Mass Range
LEP-2	$e^+e^- \to Zh^0_{\rm SM}$	$\lesssim 95$ GeV
TeV-33[a]	$W^* \to Wh^0_{\rm SM} \to \ell\nu b\bar{b}$	60–120 GeV
	$Z^* \to Zh^0_{\rm SM} \to \begin{cases} \ell^+\ell^- b\bar{b} \\ \nu\bar{\nu}\ b\bar{b} \end{cases}$	
LHC	$W^* \to Wh^0_{\rm SM} \to \ell\nu b\bar{b}$	80–100 GeV
	$h^0_{\rm SM} + X \to \gamma\gamma + X$	90–140 GeV
	$h^0_{\rm SM} \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$	130–180 GeV
	$h^0_{\rm SM} \to WW^* \to \ell^+\nu\ell^-\bar{\nu}$	155–180 GeV
	$h^0_{\rm SM} \to ZZ \to \ell^+\ell^-\ell^+\ell^-$	180–700 GeV
	$h^0_{\rm SM} \to ZZ \to \nu\bar{\nu}\ell^+\ell^-$	600–800 GeV
	$h^0_{\rm SM} \to W^+W^- \to \ell\nu$+jets	600–800 GeV[b]
NLC	$\left.\begin{array}{l} e^+e^- \to Zh^0_{\rm SM} \\ e^+e^- \to \nu\bar{\nu}h^0_{\rm SM} \\ e^+e^- \to e^+e^-h^0_{\rm SM} \end{array}\right\}$	$\lesssim 0.7\sqrt{s}$
FMC	$\left.\begin{array}{l} \mu^+\mu^- \to Zh^0_{\rm SM} \\ \mu^+\mu^- \to \nu\bar{\nu}h^0_{\rm SM} \\ \mu^+\mu^- \to \mu^+\mu^-h^0_{\rm SM} \end{array}\right\}$	$\lesssim 0.7\sqrt{s}$
	$\mu^+\mu^- \to h^0_{\rm SM}$	up to $\sqrt{s} < 2m_W$

[a] The TeV-33 Higgs signatures listed above are also relevant for lower luminosity Tevatron searches over a more restricted range of Higgs masses, as specified in Table VII.
[b] Ref. [59] argues that the $\ell\nu$+2 jets signal can be detected for Higgs masses up to 1 TeV (although such large Higgs masses lie beyond the scope of this working group).

In order to determine the true identity of the Higgs candidate, it is very important to be able to detect the Higgs signal in at least two different channels. As one can discern from Table VII, the most problematical mass range is 100 GeV$\lesssim m_{h^0_{\rm SM}} \lesssim$ 130 GeV. Higgs bosons in this mass range are not accessible to LEP-2 or Run II of the Tevatron. At the LHC, the most viable signatures in this mass range involve the production of $h^0_{\rm SM}$ followed by $h^0_{\rm SM} \to \gamma\gamma$. However, the Higgs can be produced via a number of different possible mechanisms:

(i) $gg \to h^0_{\rm SM}$,
(ii) $q\bar{q} \to q\bar{q}h^0_{\rm SM}$ via t-channel W^+W^- fusion,
(iii) $q\bar{q} \to Vh^0_{\rm SM}$ via s-channel V-exchange, and
(iv) $gg \to t\bar{t}h^0_{\rm SM}$.

The $gg \to h^0_{\rm SM}$ mechanism dominates, and it will be an experimental challenge to separate out the other production mechanisms. It may be possible to separate $gg \to h^0_{\rm SM}$ and $W^+W^- \to h^0_{\rm SM}$ events using a forward jet tag which would select out the W^+W^- fusion events. It may also be possible to distinguish $Vh^0_{\rm SM}$ ($V = W^\pm$ or Z) and $t\bar{t}h^0_{\rm SM}$ events based on their event topologies. If these other production mechanisms can be identified, then it would be possible to extract information about relative couplings of the Higgs candidate to VV and $t\bar{t}$. Otherwise, one will be forced to rely on matching $\sigma(h^0_{\rm SM})\mathrm{BR}(h^0_{\rm SM} \to \gamma\gamma)$ to Standard Model expectations in order to confirm the Higgs interpretation of $h^0_{\rm SM}$.

In some circumstances, it might be possible to observe the decays $h^0_{\rm SM} \to b\bar{b}$ or $h^0_{\rm SM} \to \tau^+\tau^-$ (after a formidable background subtraction), or identify the Higgs boson produced via $gg \to b\bar{b}h^0_{\rm SM}$. One could then extract the relative coupling strengths of $h^0_{\rm SM}$ to $b\bar{b}$ and/or $\tau^+\tau^-$ final states. These could be compared with the corresponding VV and $t\bar{t}$ couplings (see above), and confirm that the Higgs candidate couples to particles with coupling strengths proportional to the particle masses.

The quantum numbers of the Higgs candidate may be difficult to measure directly at a hadron collider. However, note that if $h^0_{\rm SM} \to \gamma\gamma$ is seen, then the $h^0_{\rm SM}$ cannot be spin-1 (by Yang's theorem). This does not prove that $h^0_{\rm SM}$ is spin-zero, although it would clearly be the most likely possibility. If the coupling $h^0_{\rm SM}VV$ is seen at a tree-level strength, then this would confirm the presence of a CP-even component. Unfortunately, any CP-odd component of the state couples to VV at the loop level, so one would not be able to rule out *a priori* a significant CP-odd component for $h^0_{\rm SM}$.

To summarize, Phase 2 consists of determining whether the Higgs candidate (discovered in Phase 1) can be identified as a Higgs boson. In some Higgs mass ranges, LEP-2, the Tevatron, and/or the LHC will discover the Higgs boson and make a convincing case for the "expected" Higgs-like properties. Ratios of Higgs couplings to different final states may be measured to roughly 20–30%. The NLC (and perhaps the FMC) can make more precise measurements of branching ratios and can directly check the spin and CP-quantum number of the Higgs candidate. The lepton machines (with $\sqrt{s} \gtrsim 300$ GeV) can easily handle the intermediate Higgs mass regime and can provide valuable information in some mass regions that present difficulties to hadron colliders.

C. Phase 3 – Precision Measurements of Higgs Properties

Let us suppose that the Higgs candidate (with a mass no larger than a few times the Z mass) has been confirmed to have the

properties expected of the $h^0_{\rm SM}$ (to within the experimental error). One would then be fairly confident that the dynamics that is responsible for electroweak symmetry breaking is weakly-coupled. Unfortunately, the details of the underlying physics responsible for electroweak symmetry breaking would still be missing. As discussed in Section III, it is not difficult to construct models of the scalar dynamics that produce a light scalar state with the properties of the $h^0_{\rm SM}$. To distinguish among such models, additional properties of the scalar sector must be uncovered. It is the non-minimal Higgs states that encode the structure of the electroweak symmetry breaking dynamics. In order to provide experimental proof of the existence of a non-minimal Higgs sector, one must either demonstrate that the properties of h^0 differ (even if by a small amount) from those of $h^0_{\rm SM}$, or one must directly produce and detect the heavier Higgs states ($H^0, A^0, H^\pm, \cdots$). In general, precision measurements of both light and heavy Higgs properties are essential for distinguishing among models of electroweak symmetry breaking dynamics.

A precision measurement of the lightest Higgs mass could be useful. As noted in Section II, the Higgs mass measurement can provide a non-trivial check of the precision electroweak fits in the context of the Standard Model. This analysis would be sensitive to one-loop (and some two-loop) virtual effects. Any significant discrepancy would indicate the need for new physics beyond the Standard Model. In this context, a Higgs mass measurement with a relative error of about 20% is all that is required. In the MSSM, the light Higgs mass measurement provides an additional opportunity. In Section IIIC, it was noted that the light Higgs mass in the MSSM at tree-level is a calculable function that depends on two Higgs sector parameters. When one-loop effects are included, the Higgs mass becomes dependent upon additional MSSM parameters (the most important of which are the top-squark masses and mixing parameters). Since the radiative corrections to the Higgs mass can be significant, a precision measurement of the Higgs mass could provide a very sensitive test of the low-energy supersymmetric model. Theoretical calculations yield a prediction for the light CP-even neutral Higgs mass (which depends on the MSSM parameters), with an accuracy of about 2 to 3 GeV [35,36]. The anticipated experimental accuracy of the light Higgs mass measurement depends on the Higgs mass range and the collider. Table IX lists the estimated errors in the measurement of the Standard Model Higgs mass, $\Delta m_{h^0_{\rm SM}}$, at future colliders for $m_W \lesssim m_{h^0_{\rm SM}} \lesssim 2m_W$. Note that the numbers quoted in Table IX are considerably *smaller* than the theoretical uncertainties quoted above.

In Table IX, the following assumptions have been made for the various collider runs shown. TeV-33 results assume a total integrated luminosity of $L = 30\ {\rm fb}^{-1}$. LHC results assume $L = 600\ {\rm fb}^{-1}$, which corresponds to running two detectors (ATLAS and CMS) for three years at LHC design luminosity. Three NLC scenarios are listed corresponding to three choices of center-of-mass energy: (i) $\sqrt{s} = 500\,{\rm GeV}$, (ii) $\sqrt{s} = \sqrt{s_{Zh}} \equiv m_Z + m_{h^0_{\rm SM}} + 20\,{\rm GeV}$, and (iii) $\sqrt{s} \simeq m_Z + m_{h^0_{\rm SM}}$ (*i.e.*, threshold for $e^+e^- \to Zh^0_{\rm SM}$). In cases (i) and (ii), we assume $L = 200\ {\rm fb}^{-1}$ and employ the best tracking/calorimetry scenario outlined in Ref. [14]. NLC threshold results [case (iii)] assume $L = 50\ {\rm fb}^{-1}$ and are quoted *before* initial state radiation and beam energy smearing effects are included. In the latter case, including such effects would increase the quoted errors by about 35%. The NLC results are also applicable to the FMC, although with a 15% increase in error in the last case if all the cited effects were included. Finally, the most accurate mass measurements can be obtained by a scan at the FMC for the s-channel Higgs resonance. The FMC scan results listed in Table IX assume that a total luminosity of $L = 200\,{\rm fb}^{-1}$ is devoted to the scan.

Table IX: Anticipated experimental error in the measured value of the Standard Model Higgs mass, $\Delta m_{h^0_{\rm SM}}$, in units of MeV, for various ranges of $m_{h^0_{\rm SM}}$. The notation "?" indicates that a reliable simulation or estimate is not yet available, while "–" means that the corresponding Higgs mass range is not accessible. The assumptions underlying the various collider runs listed below are specified in the text. See Ref. [14] for further details.

	$m_{h^0_{\rm SM}}$ range (GeV)			
Collider	80	m_Z	100–120	120–150
LEP-2 [60]	250	400	–	–
TeV-33	960	?	1500–2700	–
LHC	90	90	95–105	105–90
NLC (500)	370	264	200–120	120–70
NLC ($\sqrt{s_{Zh}}$)	3.6	3.8	4.1–4.8	4.8–6.1
NLC (threshold)	40	70	55–65	65–100
FMC (scan)	0.025	0.35	0.1–0.06	0.06–0.49

Precision measurements of heavy Higgs masses may also play an important role in the study of Higgs phenomena. In the decoupling limit, these mass splittings are of $\mathcal{O}(m_Z^2/m_{A^0})$, which presents a formidable challenge to the design of future Higgs searches. Here is one case where the mass resolution offered by the FMC might be required. For example, it may be possible to resolve the two peaks in a resonance scan for $\mu^+\mu^- \to H^0, A^0$. A measurement of the corresponding mass difference of the two states would probe the structure of the electroweak symmetry breaking dynamics.

Precision measurements of Higgs properties also include branching ratios, cross-sections, and quantum numbers as discussed in Phase 2 above. One must be able to separate cross-sections and branching ratios (instead of simply measuring the product of the two). More challenging will be the measurement of absolute partial widths, which requires a determination of the total Higgs width. Below ZZ threshold, the Standard Model Higgs width is too small to be directly measured, and other strategies must be employed.[9] As an illustration, Table X

[9]The width of $h^0_{\rm SM}$ can be measured directly via $h^0_{\rm SM} \to ZZ \to \ell^+\ell^-\ell^+\ell^-$, if $m_{h^0_{\rm SM}} \gtrsim 190$ GeV. However, in models of non-minimal Higgs sectors, the mass of the Higgs scalar with appreciable couplings to ZZ typically lies below this bound.

presents the anticipated errors in the measurements of some $h^0_{\rm SM}$ branching ratios, the partial decay rate for $h^0_{\rm SM} \to \gamma\gamma$, and the total Higgs width, $\Gamma^{\rm tot}_{h^0_{\rm SM}}$, for $80 \leq m_{h^0_{\rm SM}} \leq 300\,{\rm GeV}$. The quoted errors are determined primarily by considering the data that would be collected by the NLC at $\sqrt{s} = 500\,{\rm GeV}$ with a total integrated luminosity of $L = 200\,{\rm fb}^{-1}$. For BR($h^0_{\rm SM} \to \gamma\gamma$), the NLC analysis has been combined with results from an LHC analysis; while the measurement of $\Gamma(h^0_{\rm SM} \to \gamma\gamma)$ relies on data taken from a 50 fb^{-1} run in the $\gamma\gamma$ collider mode of the NLC (with the corresponding e^+e^- center-of-mass energy of $\sqrt{s} \sim 1.2 m_{h^0_{\rm SM}}$). These quantities also contribute to the net accuracy of the total Higgs width, $\Gamma^{\rm tot}_{h^0_{\rm SM}}$, following the indirect procedure[10] discussed in Ref. [14]. Note that $\Gamma^{\rm tot}_{h^0_{\rm SM}}$ can be measured directly only in the s-channel Higgs production at the FMC. For comparison with the indirect determination of $\Gamma^{\rm tot}_{h^0_{\rm SM}}$, the FMC scan results listed in Table X assume that a total luminosity of $L = 200\ {\rm fb}^{-1}$ is devoted to the scan. With the exception of the case where $m_{h^0_{\rm SM}} \simeq m_Z$, the FMC would provide the most precise measurement of the total Higgs width for values of the Higgs mass below the W^+W^- threshold.

In models of non-minimal Higgs sectors, precision measurements of the branching ratios and partial (and total) decay rates of the lightest CP-even Higgs boson could prove that $h^0 \neq h^0_{\rm SM}$, thereby providing indirect evidence of the non-minimal Higgs states. Once the non-minimal Higgs bosons are directly discovered, detailed measurements of their properties would yield significant clues to the underlying structure of electroweak symmetry breaking. For example, if the Higgs sector arises from a two-doublet model, then precision studies of the heavy Higgs states can provide a direct measurement of the important parameter $\tan\beta$ (the ratio of Higgs vacuum expectation values).[11] The measurement of $\tan\beta$ can also provide a critical self-consistency test of the MSSM, since the parameter $\tan\beta$ also governs the properties of the charginos and neutralinos (and can in principle be determined in precision measurements of supersymmetric processes). Moreover, the couplings of Higgs bosons to supersymmetric particles will provide invaluable insights into both the physics of electroweak symmetry breaking and the structure of low-energy supersymmetry. The possibility that the heavy non-minimal Higgs states have non-negligible branching ratios to supersymmetric partners can furnish an additional experimental tool for probing the Higgs boson–supersymmetry connection.

As in the case of the $h^0_{\rm SM}$ discussed above, the lepton colliders (assuming $\sqrt{s} \gtrsim 2m_{A^0}$ for the NLC and $\sqrt{s} \sim m_{A^0}$ for the FMC) provide the most powerful set of tools for extracting the magnitudes of the Higgs couplings to fermion and vector boson pairs. The Higgs couplings to vector boson pairs directly probe the mechanism of electroweak symmetry breaking [via the sum rule of eq. (10)]. The Higgs coupling to two photons, depends (through their one-loop contributions) on all charged states whose masses are generated by their couplings to the Higgs sector. Precision measurements of the Higgs couplings to fermions are sensitive to other Higgs sector parameters (*e.g.*, $\tan\beta$ and the neutral Higgs mixing parameter α in a two-Higgs-doublet model). Additional information can be ascertained if Higgs self-interactions could be directly measured. This would in principle provide direct experimental access to the Higgs potential. Unfortunately, there are very few cases where the measurement of Higgs self-couplings has been shown to be viable [61].

Finally, one should also consider the possible effects of virtual Higgs interactions [62]. In some models, flavor changing neutral currents mediated by neutral Higgs bosons may be observable. The CP-properties of the heavy Higgs states could be mixed,[12] leading to Higgs mediated CP-violating effects that could be observed in processes with heavy flavor. In some cases, precision measurements of low-energy observables can be quite sensitive to the heavy Higgs states. The canonical example is the process $b \to s\gamma$, which can be significantly enhanced due

Table X: Anticipated experimental errors in the measured values of the $h^0_{\rm SM}$ branching ratios, the partial decay rate, $\Gamma(h^0_{\rm SM} \to \gamma\gamma)$, and total width, $\Gamma^{\rm tot}_{h^0_{\rm SM}}$, in percent, for various ranges of $m_{h^0_{\rm SM}}$. The notation "?" indicates that a reliable simulation or estimate is not yet available or that the number indicated is a very rough guess, while "–" means that the corresponding observable cannot be reliably measured. The results listed below are primarily derived from a multi-year run at the NLC. For $h^0_{\rm SM} \to \gamma\gamma$, data from LHC and the $\gamma\gamma$ collider are also employed to improve the quoted errors. The total Higgs decay rate can be obtained indirectly (by combining measurements of related quantities); the comparison with the direct determination via s-channel Higgs resonance production at the FMC is shown. See the text and Ref. [14] for further details.

	$m_{h^0_{\rm SM}}$ range (GeV)			
Observable	80–130	130–150	150–170	170–300
${\rm BR}(h^0_{\rm SM} \to b\bar{b})$	5–6%	6–9%	20% ?	–
${\rm BR}(h^0_{\rm SM} \to c\bar{c})$	$\sim 9\%$	?	?	–
${\rm BR}(h^0_{\rm SM} \to WW^\star)$	–	16–6%	6–5%	5–14%
${\rm BR}(h^0_{\rm SM} \to \gamma\gamma)$	15%	20–40%	?	–
$\Gamma(h^0_{\rm SM} \to \gamma\gamma)$	12–15%	15–31%	?	13–22%
$\Gamma^{\rm tot}_{h^0_{\rm SM}}$ (indirect)	19–13%	13–10%	10–11%	11–28%
$\Gamma^{\rm tot}_{h^0_{\rm SM}}$ (FMC)	3%[a]	4–7%	–	–

[a] Near the Z peak, the expected FMC uncertainty in $\Gamma^{\rm tot}_{h^0_{\rm SM}}$ is about 30%.

[10] For $m_{h^0_{\rm SM}} \lesssim 130$ GeV, the indirect procedure relies on the $h^0_{\rm SM} \to \gamma\gamma$ measurements. For $m_{h^0_{\rm SM}} \gtrsim 130$ GeV, one may also make use of the $WWh^0_{\rm SM}$ coupling strength extracted from data.

[11] Note that in the decoupling limit (where h^0 cannot be distinguished from $h^0_{\rm SM}$), measurements of processes involving h^0 alone cannot yield any information on the value of $\tan\beta$.

[12] In the decoupling limit, the lightest neutral scalar must be (approximately) a pure CP-even state.

to charged Higgs boson exchange. If there are no other competing non-Standard Model contributions (and this is a big *if*), then present data excludes charged Higgs masses less than about 250 GeV [63]. Eventually, when non-minimal Higgs states are directly probed, it is essential to check for the consistency between their properties as determined from direct observation and from their virtual effects.

V. CONCLUSIONS

This working group has examined the potential of a program of future precision electroweak measurements [13] and the search for weakly-coupled Higgs boson at future colliders [14]. The goal of such a program is to address the outstanding problem of elementary particle physics: what is the origin of electroweak symmetry breaking and the nature of the dynamics responsible for it?

The Higgs search will consist of three phases:

1. discovery of the Higgs signal,
2. verification of the Higgs interpretation, and
3. precision analysis of the Higgs sector properties.

Improvements of precision electroweak measurements can provide an important consistency check of the Higgs interpretation of a Higgs signal, in the same way that the LEP electroweak data provided support for the Tevatron interpretation of the top-quark events. Discovery of a Higgs-like signal alone may not be sufficient to earn a place in the Particle Data Group (PDG) tables. Some basic measurements of the properties of the Higgs candidate will be essential to confirm a Higgs interpretation of the discovery. Higgs searches at LEP-2, Run-II of the Tevatron and/or the LHC will almost certainly discover a Higgs signal if electroweak symmetry breaking dynamics is weakly coupled. Moreover, measurements at these machines will yield evidence for the Higgs interpretation that is sufficient to pass PDG muster in some fraction of the Higgs parameter space. The NLC (and FMC) can discover or definitively rule out the existence of a Higgs boson in the intermediate Higgs mass regime (the mass region most problematical for the Higgs search at hadron colliders). Once a Higgs boson is discovered, the lepton colliders would play a decisive role in the precision measurement of Higgs sector properties.

It is not unlikely that the first Higgs state to be discovered will be experimentally indistinguishable from the Standard Model Higgs boson. This occurs in many theoretical models that exhibit the decoupling of heavy scalar states. In this decoupling limit, the lightest Higgs state, h^0 is a neutral CP-even scalar with properties nearly identical to the $h^0_{\rm SM}$, while the other Higgs bosons of the non-minimal Higgs sector are heavy (compared to the Z) and are approximately mass-degenerate. Thus, discovery of $h^0 \simeq h^0_{\rm SM}$ may shed little light on the dynamics underlying electroweak symmetry breaking. It is then crucial to directly detect and explore the properties of the non-minimal Higgs states. In particular, precision measurements are critical in order to distinguish between h^0 and $h^0_{\rm SM}$ and/or to map out the properties of the non-minimal Higgs states. To accomplish these goals, future colliders of the highest energies and luminosities, considered in this report, are essential.

We have entered a new era in Higgs phenomenology. The methods by which the first Higgs signal will be identified are well known and have been studied in great detail. However, the most outstanding challenge facing the future Higgs searches lies in identifying and exploring in detail the properties of the non-minimal Higgs states. A successful exploration will have a profound effect on our understanding of TeV-scale physics.

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