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BUNCH TRAINS FOR LEP

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Since 1995 LEP has been operated with a bunch train scheme which allows head-on collisions of four trains of up to four bunches within a train. The proposal and its implementation are presented, and the consequences for the beam dynamics are discussed in detail. In particular the side effects due to the separation scheme itself and the parasitic beam-beam encounters are computed. The necessity of a self-consistent treatment is shown and emphasis is placed on a comparison between the expectations and the observations.

Keywords: Bunch train; Luminosity; Beam-Beam

1 INTRODUCTION

The original LEP design¹ assumed that the electron and positron beams each consist of four equidistant bunches. These collide in the four evennumbered pits where the experiments are installed, and are vertically separated by electrostatic separator bumps in the interleaved oddnumbered pits. LEP was operated in this manner from 1989 to 1992. Because of the limitation in the bunch current to about 0.5 mA by the transverse mode-coupling instability, the most promising way of increasing the luminosity is by increasing the number of bunches in each beam. Since 1983, the Cornell Electron Storage Ring CESR had been running with the Pretzel Scheme,² where the electrons and positrons travel on orbits which are distorted in opposite directions over

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practically the whole circumference of the machine by horizontal electrostatic fields. In this case, the maximum number of bunches in each beam is of the order of the horizontal tune Q_x . Parasitic separated beam-beam collisions are arranged to occur approximately every half horizontal betatron wavelength where the separation between the two beams is close to a maximum. In 1988, Rubbia proposed a similar scheme for LEP.³ Jowett, Kalbreier and Myers⁴ implemented it with eight equidistant bunches in each beam and parasitic beam-beam collisions at the centres of the eight arcs. LEP was operated in this manner from 1992 to 1994.⁵ Peak luminosities in excess of $2.0 \times$ 10^{31} cm⁻² s⁻¹ were achieved regularly but the scheme was limited at injection to bunch currents less than 0.5 mA. There was some experimental evidence indicating that this limitation is due to the horizontal beam-beam encounters in mid-arc where the momentum dispersion is maximum. For operation at 45 GeV this did not limit the performance since the beam-beam effects limit the bunch current to about 0.35 mA. However, unlike operation at Z^0 energies, it is unlikely that the beambeam interaction will limit the operation at W^{\pm} energies. Consequently it was necessary to either make a break-through on the pretzel bunch current limitation, or to develop an alternative multi-bunch scheme to permit the intensities needed for LEP2. This required new schemes, which should be designed to avoid this limitation. In 1990, Meller⁶ proposed replacing the equidistant individual bunches in CESR by trains of bunches where the distance between the bunches in a train is much smaller than the distance between the trains, such that whole trains have parasitic encounters where the separation between the two beams is close to a maximum. CESR has been operated with bunch trains since 1994. In 1992, Keil⁷ proposed a similar scheme for LEP, with four equidistant bunch trains in each beam and a horizontal crossing angle in the even-numbered pits for the separation at the parasitic encounters. This scheme was rejected after tests in 1993⁸ because the synchrotron radiation background in the LEP experiments, caused by the horizontal beam offsets in the nearby strong quadrupoles, was too high and could not be controlled. In 1994, Herr⁹ proposed a bunch train scheme which avoids this particular difficulty by having head-on collisions in the even-numbered pits, and starting the vertical separation just beyond the strong quadrupoles closest to the pits. In November 1994, this scheme was tested with single trains of electron and positron bunches, colliding in two diametrically opposite pits.¹⁰ Since May 1995, LEP has been operated with the full scheme of four bunch trains in each beam.¹¹

In this paper, we describe the design and operation of the LEP bunch train scheme in 1995. The technical implementation has been already presented.^{12,13} The remainder of this paper is organized as follows. In Section 2 we describe the electrostatic separator bumps in the even and odd numbered pits, our choices of the number and spacing of the bunches in a train, and the bunch train sextupoles for adjusting the difference of the tunes in the two beams. Section 3 discusses the choice of the amplitude and relative polarities of the electrostatic separator bumps, in order to minimize the undesirable side effects of the parasitic collisions, and compares simulations of the side effects with measurements. In Section 4 we report our results for the observed intensity limits at injection. In Section 5 we compare the results of a selfconsistent theory of several additional side effects with our measurements, including orbit offsets and separations, vertical dispersion, tunes and chromaticities. A comparison of the performance with bunch trains and with the pretzel scheme is given in Section 5.6. Conclusions are given in Section 6.

2 DESIGN CONSIDERATIONS AND IMPLEMENTATION

The bunch train scheme in LEP is constrained to the vertical plane because it is based on the availability of electrostatic separators previously used to provide local separation of electron and positron bunches. The scheme required the addition of eight new vertical electrostatic separators and the displacement of eight existing ones.¹⁴ The sextupoles for correction of the electron–positron tune-split were also modified, and three families of tilted quadrupoles in each experimental interaction point (IP) were moved to locations outside the separator bump, for effective compensation of the coupling from the experimental solenoids.

The maximum length of the bunch train is, in theory, determined by the maximum separation bump length. However, the LEP detectors impose a maximum length of 750 ns, or about 250 m, for reasons discussed elsewhere.^{12,13}

The minimum inter-bunch spacing, and hence the maximum number of bunches in a train, is determined by the magnetic configuration around the experimental interaction regions and the physical position of the innermost separators. The lower limit was set at 74.2 m, corresponding to 247.5 ns or 87 RF wavelengths ($\lambda_{\rm RF}$), which is a spacing which gives adequate separation at the parasitic encounters, and is also favourable for the 1 GHz longitudinal feedback system.¹⁵

With the above constraints the maximum number of bunches per train is limited to four.

2.1 Electrostatic Bumps in Experimental Pits

In the experimental pits the two beams should collide head-on, and should also pass through the centre of the first (strong) quadrupole doublet from the IP. The separator ES2 already installed behind this doublet is used to launch the bunch train bump. The vertical orbit then oscillates freely, crossing the vertical axis every half betatron wavelength. The logical place to close this oscillation is near such a vertical crossing. To avoid vertical orbit offsets in the copper cavities of the RF system and the resulting synchro-betatron resonances,¹⁶ the first such crossing near the seventh quadrupole from the IP was chosen, and new separators were installed at this location (ES7). In principle, any further crossing of the vertical axis could have been chosen for the separator position, permitting more bunches in a train.⁹ A third separator near the fourth quadrupole (ES4) allows all bumps to be closed independently of the machine optics, and ensures that the trains collide head-on.

During injection a separation bump is superimposed on the bunch train bump. When the beams are brought into collision this separation bump is removed by adjusting the fields of the ES2 and ES4 separators.

A typical bunch train bump in the collision configuration in an evennumbered pit is shown in Figure 1. The amplitude of the bumps is a compromise between conflicting requirements. Upper limits are given by the steep rise of the synchrotron radiation background with the bump amplitude in the LEP experiments,¹⁷ by the electric field in the electrostatic separators, and by the aperture of the LEP vacuum chamber. The vertical beam-beam kicks and the beam-beam tune shifts at the parasitic collision points favour a large bump amplitude, while the excitation of the vertical dispersion by the bumps and the



FIGURE 1 Typical electrostatic separator bump in an even-numbered pit. The IP in pit 4 is at the centre of the abscissa. The parasitic collision points are marked by narrow centred vertical bars at $\pm 37 \text{ m}$, $\pm 74 \text{ m}$ and $\pm 111 \text{ m}$ from the IP.

concomitant increase of the vertical emittance favour a small bump amplitude.

At this point, the directions of the bumps in the pits remain as free parameters, and can be used to partially cancel the contributions of different pits to the vertical offsets, the vertical dispersion, and the vertical emittance. These considerations are examined in Section 3.

2.2 Electrostatic Bumps in the Odd-Numbered Pits

The considerations entering into the design of the vertical bumps in the odd-numbered pits are very similar to those for the even-numbered ones. In order to avoid a low vertical separation at one of the parasitic encounters for the bunch spacing $s = 87 \lambda_{\rm RF}$ adopted, the optical layout in the odd-numbered pits uses a high- β with $\beta_x^* \approx 25.2$ m and $\beta_y^* \approx 30$ m at the IP. The outermost separators (designated ES8) were displaced to accommodate the length of the (four bunch) bunch train envisaged. A typical bump in an odd pit is shown in Figure 2.

2.3 Mode of Operation

With bunch trains the method for bringing the beams into collision is to ramp the separator voltage slowly from separation to collision, using



FIGURE 2 Typical electrostatic separator bump in an odd-numbered pit. The IP in pit 1 is at the centre of the abscissa. The parasitic collision points are marked by narrow centred vertical bars at 0 m, \pm 37 m, \pm 74 m and \pm 111 m from the IP.

an exponential interpolation to match the voltage decay constant of the separators. This procedure typically takes two minutes, during which the beam separation changes from fully separated to (nominally) zero. The speed of this procedure is limited by the characteristics of the high voltage circuit, which has a time-constant of around 30 s.

The vertical optimization of the beam collisions (so-called "vernier adjustment") is possible by superimposing a small closed "separation" bump on the bunch train bump using the same separators and main high-voltage generators. The use of the same generators limits the resolution of the vertical adjustment to around $0.4 \,\mu\text{m}$ at $45.6 \,\text{GeV}$; however this is adequate for the vertical beam sizes of the order of $5 \,\mu\text{m}$.

Finally, the basic requirement of bunch separation in the trains at all times means that all 40 separators are operated at high voltage at top beam energy, which increases the probability of a high-voltage break-down, or "spark", occurring during physics conditions. This resulted in a reduced separator system performance at 45.6 GeV beam energy in 1995.^{15,18}

2.4 Bump Closure

A successful operation with bunch trains requires that the separation of the two beams does not propagate outside the desired bumps, i.e. they must be closed. Several mechanisms can be the origin of such a nonclosure:

- Incorrect hardware parameters, e.g. separator voltage or gaps
- Optics mismatch
- Energy mismatch due to asymmetric RF distribution or energy sawtooth

The last point is important for LEP2 where the variation of the particle energy around the circumference is substantial. Asymmetries in the RF distribution cause energy differences between electrons and positrons and therefore a non-closure of the bumps, leading to a residual separation of electrons and positrons around the ring. As long as this residual separation is not too large, it is not harmful but it also implies a residual separation at the collision points. This non-closure can be a few μ m at the interaction point¹⁹ and can be corrected with a vernier adjustment.

2.5 Bunch Train Sextupoles

To allow correction of tune-splits between electron and positron beams, rotated sextupoles are installed in pits 1 and 5, where four such sextupoles were moved to positions where the vertical separation and β_x are both large, and the other four sextupoles into locations where the separation is large and β_y is about twice as large as β_x . This configuration is then sufficient to correct a Q-split of 0.03.¹⁵

3 AMPLITUDES AND DIRECTIONS OF THE BUMPS

While the geometrical shape of the separation bumps is given by the positions of the electrostatic separators and the lattice, and the maximum bump amplitude is determined by hardware constraints, the desired amplitude and the direction of the bumps remain free parameters. These have to be chosen to ensure sufficient separation at the parasitic encounters of the bunches and at the same time to minimize other consequences of the bumps, i.e. (i) residual vertical dispersion which must be kept as small as possible to avoid an increase of the vertical emittance or the excitation of synchro-betatron resonances

caused by a finite dispersion in the RF cavities and (ii) synchrotron radiation background caused by large orbit offsets of particles in the quadrupoles. Both effects are directly related to the amplitude of the separation bumps.

An insufficient separation however, would lead to other effects, i.e. (i) large beam-beam tune shifts and (ii) beam-beam induced orbit effects. Such an insufficient separation would result in low life times and reduced luminosity. A compromise has to be found to meet all requirements simultaneously.

3.1 Vertical Dispersion

Any orbit program, including MAD,²⁰ can compute the vertical dispersion D_y , the relative r.m.s. momentum spread δ in the beams, and the vertical emittance ϵ_y caused by the vertical orbit offsets in the separator bumps. These quantities enter into the vertical r.m.s. beam size σ_y which is given by:

$$\sigma_y = \sqrt{\beta_y \epsilon_y + (D_y \delta)^2}.$$
 (1)

Achieving a good luminosity implies an upper limit on σ_y at the headon collision points IP, and hence upper limits on D_y^* there and on ϵ_y . For typical values, $\beta_y = 0.05 \text{ m}$, $\epsilon_y = 0.5 \text{ nm}$, $\delta = 0.001$, the two terms in Eq. (1) become equal at $D_y^* = 5 \text{ mm}$.

The magnitude of the residual vertical dispersion depends on the amplitude of the vertical orbit and its direction at the bumps. Since the vertical orbit offsets in the separator bumps have opposite sign for electrons and positrons, the vertical dispersions D_y for electrons and positrons have opposite sign. Furthermore, the dispersions created by each of the local orbit distortions in the eight interaction regions can accumulate or cancel. The relative amplitudes and directions of the eight bumps can be chosen such as to minimize the residual vertical dispersion. The calculations of the residual vertical dispersion are summarized in Table I. The maximum value, the r.m.s. and the values of the dispersion at the four collision points are shown and alternating bump directions are assumed. The measurements of the vertical dispersion performed by measuring the closed orbits with slightly different beam energies confirmed the calculations very well. The vertical

Separation	D _{max} (m)	D _{r.m.s.} (m)	D_y^{IP2} (mm)	D_y^{IP4} (mm)	D_y^{IP6} (mm)	D_y^{IP8} (mm)
All eight points	0.100	0.038	+2.21	-0.48	+ 2.22	-0.01
Four even points	0.077	0.038	+2.68	-0.01	+2.70	-0.48
Four odd points	0.077	0.029	- 0.47	-0.47	- 0.48	-0.48

TABLE I First order calculation for residual vertical dispersion from bunch train bumps, collision optics, 45.6 GeV

dispersion at the collisions points is well below the above mentioned value of $D_v^* = 5 \text{ mm}$.

3.2 Effects from Parasitic Beam-Beam Interactions

Further insight into the side effects of the parasitic encounters can be gained by a first-order calculation, starting with the vertical orbits caused by the electrostatic separator bumps. The vertical orbit kick, $\Delta y'$, the horizontal and vertical beam-beam tune shifts, ξ_x and ξ_y , at a parasitic encounter are given by:

$$\Delta y' = -\frac{Nr_e}{\gamma y}, \qquad \xi_x = \frac{Nr_e\beta_x}{8\pi\gamma y^2}, \qquad \xi_y = -\frac{Nr_e\beta_y}{8\pi\gamma y^2}.$$
 (2)

Here we have assumed that the vertical r.m.s. beam radius is much smaller than the vertical orbit offset y at the parasitic encounter, $\sigma_y \ll y$; N is the intensity of the opposite bunch, r_e is the classical electron radius, and γ is the usual relativistic factor. The total separation between the beams at the parasitic encounter is 2y. Any vertical orbit kick $\Delta y'_k$ causes a vertical orbit distortion y_o and a vertical orbit slope y'_o at any observation point around LEP which can be calculated using the standard equations.²² The closed orbit position and slope of a particular bunch are obtained by adding the contributions of all parasitic encounters with the bunches of the counter-rotating beam.

The horizontal and vertical tunes in LEP are $Q_x \approx 90.3$ and $Q_y \approx 76.2$. The phase advance in an arc is about one eighth of the tunes. The optical functions in all even-numbered pits are very similar, and the same is true in the odd-numbered pits. Since Q_y is just above a multiple of four, the vertical phase advance between two neighbouring even-numbered pits is just above an integer. If the bump directions are chosen appropriately, the contributions of the two sets of beam-beam kicks in two neighbouring even-numbered pits to the vertical orbit distortion y_0 and the vertical orbit slope y'_0 nearly cancel. The same argument applies to the odd-numbered pits. We therefore have chosen the directions of the bumps such that they alternate between neighbouring even-numbered and neighbouring odd-numbered pits. At this point, there remains one free parameter, the relative directions of the bumps in two neighbouring pits, say pit 1 and pit 2. After numerical studies, we chose opposite directions, resulting in the relative directions +--++--+ for all eight pits. A positron deflected upwards at the outmost separator of the bump defines positive direction +. This results in a typical vertical positron orbit in LEP as shown in Figure 3.

In general, different bunches in different trains meet the bunches in the trains of the opposite beam at different parasitic encounters. Therefore, different bunches travel on different vertical orbits and have different vertical slopes around LEP. Hence, different vertical collision offsets δy and different slopes $\delta y'$ exist between any two bunches colliding at the head-on interaction points. In order to avoid a drop in luminosity and the excitation of synchro-betatron resonances by the beam-beam collisions¹⁰ the conditions $\delta y \ll \sigma_y$ and $\delta y' \ll \sigma_y/2\sigma_s$ should be satisfied, where σ_s is the bunch length. With typical parameters, $\beta_y = 0.05$ m, $\epsilon_y = 0.5$ mm, $\sigma_s = 15$ mm, we need $\delta y \ll 5$ µm and $\delta y' \ll 167$ µrad.

From symmetry arguments it is evident²¹ that, for an ideal machine without imperfections and equal bunch populations, the first bunch of



FIGURE 3 Typical vertical positron orbit in LEP. Pit 1 is at the left edge of the graph.

a train has an orbit offset of the same magnitude and opposite sign as the last bunch of the equivalent counter-rotating bunch train. Similar arguments hold for each bunch of a train, resulting in an asymmetric orbit for bunches along a train. It is easy to remove the average vertical offset by vernier adjustments, but it is impossible to remove the spread in the vertical offset between the bunches. For trains of only two bunches the symmetry allows a vernier adjustment to collide both bunches of a train head on, although not on the same orbit.

3.3 Calculation Results and Measurements

The computation of many side effects is embedded in a computer program orbit9.²³ This reads an optics file from MAD with the optical functions at the parasitic and head-on collision points, and generates tables of several quantities. Tables II and III show results for typical even-numbered and odd-numbered pits respectively. All quantities are given for the injection energy of 20 GeV and the maximum bump amplitudes. The interaction regions 4 and 8 are very similar and the same is true for 2 and 6. We have therefore chosen points 2 and 4 for this study. The parasitic encounters are numbered by the pit number and the label of the bunch which encounters bunch a of the opposing beam in that location, e.g. 4b is the first and 4d is the third encounter from the interaction point 4. They are symmetrical with respect to the interaction point and therefore only the encounters on one side of the bump are given in Table II. For smaller bump amplitudes the beambeam kicks and the tune shift scale with the vertical orbit offset y as shown in Eq. (2). In a typical even-numbered pit, the vertical offset y is

TABLE II First-order calculation results at 20 GeV with bunch intensities of 0.2 mA for the vertical orbit offset y in mm, vertical parasitic beam–beam kicks $\Delta y'$ in µrad, horizontal and vertical beam–beam tune shifts ξ_x and ξ_y in units of 10^{-3} at the encounters in the neighbourhood of pits 2 and 4. For comparison the relative amplitudes (y^{\lim}/y) and tune shifts ξ^{\lim} are given where the life time became low in the experiment

Encounter	y (mm)	$\Delta y'$ (µrad)	$10^3 \xi_x$	$10^3 \xi_y$	y^{\lim}/y	$10^3 \xi_x^{\lim}$	$10^3 \xi_y^{\rm lim}$
4d	5.74	3.50	0.36	-1.86	0.67	1.00	-6.7
4c	15.82	1.27	0.08	-0.36	0.34	1.11	-5.2
4b	12.07	1.67	0.69	-0.24	0.34	11.00	-3.5
2d	6.63	3.02	0.78	-0.42	0.29	13.40	-7.2
2c	19.73	1.01	0.08	-0.22	0.23	2.60	-7.2
2b	10.85	1.85	0.85	-0.22	0.23	17.60	-4.6

TABLE III First-order calculation results at 20 GeV with bunch intensities of 0.2 mA for the vertical orbit offset y in mm, vertical parasitic beam-beam kicks $\Delta y'$ in µrad, horizontal and vertical beam-beam tune shifts ξ_x and ξ_y in units of 10^{-3} at the encounters in the neighbourhood of a typical odd-numbered pit, pit 1. For comparison the relative amplitudes (y^{\lim}/y) and tune shifts ξ^{\lim} are given where the life time became low in the experiment

Encounter	y (mm)	$\Delta y'$ (µrad)	$10^3 \xi_x$	$10^3 \xi_y$	y^{\lim}/y	$10^3 \xi_x^{\lim}$	$10^3 \xi_y^{\text{lim}}$
ld	-7.40	-2.70	1.01	-0.55	0.21	9.00	-4.9
1c	7.22	2.77	0.26	-1.74	0.40	0.57	-3.8
1b	5.60	3.58	2.57	-0.59	0.21	23.10	-5.3
1a	7.20	2.78	0.39	-0.46	0.21	3.40	-4.0

largest at the second encounter, and hence, the beam-beam kick $\Delta y'$ is smallest there. The horizontal beam-beam tune shift ξ_x is largest at the nearest encounter, while the vertical beam-beam tune shift ξ_y is largest at the farthest encounter.

To study the effect of the finite separation, we brought a single positron bunch into collision with selected electron bunches from different trains and bunch positions in the trains.^{24,25} Starting at the maximum bump amplitudes, we systematically lowered the amplitudes of the separation at the parasitic encounter under study and measured the tune and the lifetime of the bunches until a lifetime of less than one hour was found. The corresponding vertical orbit offset is called y^{\lim} The relative reduction of the separation y^{\lim}/y and the corresponding beam-beam tune shifts ξ^{lim} are given in Table II together with the calculation for the full bump amplitudes. For all even-numbered pits the third encounter (2d and 4d) where the first bunch of a train encounters the last of the opposing train, is the worst²⁴ corresponding to the encounter where the separated vertical beam-beam tune shift is largest (Table II). In particular this is true for pit 4 and the separation needed for good lifetime is much larger in pits 4 and 8 compared to pits 2 and 6 (cf. Table II). The pits 4 and 8 are therefore less favourable for the parasitic collision of the fourth bunch. For the other parasitic encounters it was possible to reduce the separation by a factor three or more without effect on the beam. In all cases the vertical tune shift ξ_{ν}^{\lim} reached values of approximately 0.005 when the lifetimes became low.

In a typical odd-numbered pit, shown in Table III, the vertical offset y is smallest at the nearest encounter, and hence the beam-beam kick $\Delta y'$ is largest there. The horizontal beam-beam tune shift ξ_x is largest at the nearest encounter, while the vertical beam-beam tune shift ξ_y is

largest at the second encounter on either side of the IP. The numbering of the parasitic encounters is the same as for even-numbered pits except that the central encounter is included (1a). Another study reducing the separation in the odd points showed that bad life times are reached much earlier when the bunches are colliding in the second parasitic encounter (1c) where the vertical beam–beam tune shift is largest. Again the value of the vertical tune shift ξ_{y}^{lim} is remarkably constant around values of approximately 0.005 for the cases with low life times. All odd number interaction regions are identical and therefore give the same results.

Table IV shows the vertical offsets δy and the vertical slopes $\delta y'$ for the four bunches in a typical bunch train at the interaction point IP in a rather bad even-numbered pit, pit 4. The offsets shown are those before the vernier adjustment of the vertical separator bumps to optimize the luminosity. The luminosity assumes a maximum when the quadratic form of the bunch distances is minimized. Therefore the optimum shift \bar{y} can be exactly calculated as:

$$\bar{y} = \frac{1}{2N} \sum_{i=1}^{N} (\delta y_i^{e} - \delta y_i^{p}), \qquad (3)$$

where δy_i^e and δy_i^p are the offsets of the *i*th bunch for electrons and positrons respectively (with $\delta y_i^e = -\delta y_{N-i+1}^p$) and N is the number of bunches in a train. For the values in Table IV this gives $\bar{y} = 2.59 \,\mu\text{m}$. This value has to be subtracted from the offsets in Table IV. The main contribution is due to a single bunch of the train.

TABLE IV First-order calculation results for the vertical orbit offsets δy in μ m and vertical orbit slopes $\delta y'$ in μ rad for the four positron bunches in a typical bunch train at the interaction point IP in an unfavourable even-numbered pit, pit 4. The first bunch is labeled a, the last one d. The energy is 45.6 GeV and the assumed current 0.2 mA per bunch

Bunch	$\delta y \text{ (mm)}$	$\delta y'$ (µrad)
a	0.70	9.49
b	-0.52	-13.50
c	0.77	26.00
d	3.20	8.04

For fewer bunches in a train, the smaller number of parasitic encounters and the symmetries reduce the effects. In the special case of only two bunches per train the collision offsets become zero when the optimum adjustment is applied, and provided the symmetry between the bunches is not broken due to unequal intensities.

The observed life time problems of the fourth bunch in a train caused by insufficient separation led to the decision to operate LEP with only three bunches per train through most of the running period 1995. This allowed the reduction of the separation bump amplitudes, thus further reducing the vertical dispersion. Separation bumps as small as 50% of the maximum (hardware limited) amplitude were regularly used for operation. The smaller bump amplitudes also significantly reduced the background in the experiments.

4 INTENSITY LIMITATIONS AT INJECTION

The bunch intensity may be limited by several mechanisms. Starting from the single bunch limit, the achievable intensity is decreased when trains of closely spaced bunches are accumulated. It is further reduced by the presence of the second beam. We have studied the effect of these mechanisms on the intensity for several configurations and parameters and compared them to the intensities achieved previously.

4.1 Single Beam

The single beam single bunch intensity is limited by the transverse mode coupling instability and was measured to be 0.610 mA in $1995.^{26}$ This result is in good agreement with the expected value, due to the increased LEP impedance over the years.^{27,28}

The intensity accumulated into the first bunch of a train was close to the single beam limit.²⁶ However, the intensity accumulated in the second bunch was lower than this limit by about 10-15%. The accumulation into the second bunch was always limited by a loss of intensity each time the first bunch was injected.

The effects of the bunch train bumps on the single beam limit were small (5-8%). In the design phase of the bunch train project, multibunch beam break up (MBBU)²⁹ was considered a possible mechanism limiting the intensities. A small oscillation of the leading bunch can lead to large oscillations and losses in later bunches coupled through the wakefields.³⁰ It was indeed observed that later bunches in a train have much larger oscillations than the first bunch, which seems to indicate the presence of MBBU. Measurements with three bunches per train showed that the second bunch in a train has the strongest oscillation while the oscillation of the third bunch was slightly smaller. The amplitude and strength of this oscillation was independent of a vertical offset in superconducting RF cavities.

4.2 Two Beams

The bunch intensity which can be obtained at injection with two counter-rotating beams in LEP is always smaller than the bunch intensity obtained with a single beam. In 1993, in the case of four against four bunches, with local vertical separation in the eight interaction points, this intensity reduction was 12%. With eight on eight bunches (the beams are separated in the additional crossing points by horizontal pretzel) the reduction was between 20% and 25%.³¹ With four trains of two bunches the measured intensity reduction was 22%.

For trains of three bunches, up to 0.35 mA per bunch were regularly achieved in daily operation of the machine during September 1995 but the intensity was deliberately limited for background considerations.³²

For the case of trains of four bunches, 0.25 mA per bunch were accumulated during regular operation in June 1995.³³ Operational difficulties (during the ramp, squeeze and collide) were encountered with trains of four bunches and only two fills with all 32 bunches could be put into collision. An amplitude reduction to 80% of the nominal amplitude had no effect on the accumulated intensity.²⁶ In 1994, one train of two bunches against one train of two bunches at 60% of the bump amplitude gave the same maximum intensity as for 100%.³⁴

For LEP2 it is foreseen to operate only with two bunches per train and the highest possible bunch current. In tests of the maximum attainable intensity into two bunches per train, we found that the accumulation into two bunches per train was slightly better for $87 \lambda_{RF}$ than for 174 λ_{RF} bunch spacing. However both spacings are not ideal for two bunches per train and in 1996 a spacing of 118 λ_{RF} was used. The maximum intensity at injection was studied for different RF configurations³⁵ and no bunch train related problems were found up to about 0.550 mA per bunch, where the intensity could not be further increased due to RF limitations.

5 COMPARISON OF SELF-CONSISTENT THEORY AND MEASUREMENTS

The first-order calculation is essentially valid for vanishing bunch current, and therefore does not include the consequences of the nearly head-on and parasitic encounters. These effects are obtained by a selfconsistent computation which is embedded in a computer program train.³⁶ It reads the same optics file from MAD as the orbit9 program, and two files with the second-order TRANSPORT³⁷ maps for the sectors between the parasitic and head-on collision points for the forward and backward beam, respectively. It then finds the individual closed orbits of all bunches, as well as vertical dispersion, tunes and chromaticities. The understanding and evaluation of the sideeffects via the self-consistent calculations was vital in understanding the limitations of the scheme. In this section comparisons between measurements and the predictions of the calculations are made.

Combining as it did, the push for high luminosity and the commissioning of the bunch train scheme essentially from scratch, the first year's operation with bunch trains was difficult.¹¹ Essentially four bunches per train proved extremely difficult to manage, and a fully satisfactory set of measurements under standard conditions is hard to establish. However, in the second running period a dedicated energy scan took place which included an extended period of steady running with three bunches per train. Most of the measurements quoted below come from this period. In order to isolate the effects predicted by the self-consistent calculations, we concentrate on differences between bunches in a train rather than absolute values.

5.1 Separation

Table V shows the results of calculations of the separations s_y in μ m for the three bunches, labeled a, b and c in a typical bunch train in the evennumbered pits. The vertical separation s_y is symmetrical between the

TABLE V Self-consistent theory results for the separation s_y in μ m for the three bunches in a typical bunch train. The first bunch is labeled a, the last one c. The bunch current is I = 0.25 mA, the beam energy is E = 45.6 GeV

Bunch	IP2	IP4	IP6	IP8
a	1.42	-1.59	1.82	0.15
b	0.32	0.07	0.61	2.09
c	1.42	-1.59	1.82	0.15
$(s_y(a) - s_y(b))/2$	0.55	-0.76	0.61	-0.97

TABLE VI Measured family shape at different energies during the energy scan. The error on each value is around 10%

	Energy	IP2	IP4	IP6	IP8
$y_a - y_b [\mu m]$	44 GeV 46 GeV	0.55 0.70	-0.55 -0.69	0.58 0.45	$-1.20 \\ -0.89$
$y_c - y_b [\mu \mathrm{m}]$	44 GeV 46 GeV	0.60 0.76	$-0.66 \\ -0.84$	0.50 0.48	$-1.03 \\ -0.90$

leading and trailing bunches in a train; $(s_y(a) - s_y(b))/2$ is shown to facilitate comparison with the measured results. The factor of two allows for the fact that vernier settings are quoted in offset at the IP rather than separation. The measurement of the vertical separation between bunches in a train is a by-product of the luminosity optimization by vernier scans. The average over the scans performed at different energies is shown in Table VI with a sizable range of bunch currents with an average of 0.25 mA. In spite of this, very good agreement between the calculated and measured results can be seen in Table V.

5.2 Crossing Angle

The calculated vertical crossing angles s'_{y} between the pairs of bunches in two typical trains are shown in Table VII. The contribution to the crossing angle from bunch train effects is minimal in points 2 and 6, but significant in 4 and 8. The angle and position of the beam at the IP are measured systematically by extrapolation from two position monitors either side of the IP. The average measured crossing angle for the first bunch at point 4 during the last running period of 1995 was 114 µrad.

TABLE VII Self-consistent theory results for the vertical crossing angle s'_y in mrad for the three bunches in a typical bunch train. The bunch current is I=0.25 mA, the beam energy is E=45.6 GeV

Bunch	IP2	IP4	IP6	IP8
a	0.006	0.082	0.007	0.074
b	0.000	0.000	0.000	0.000
c	-0.006	-0.082	-0.007	-0.074

TABLE VIII Self-consistent theory results for the vertical dispersion D_{y}^{\pm} in mm at the even-numbered interaction points for typical e⁺ and e⁻ bunches. The bunch current is I = 0.25 mA, the beam energy is E = 45.6 GeV

Bunch	D_{y2}^{+}	D_{y4}^{+}	D_{y6}^+	D_{y8}^{+}	D_{y2}^{-}	D_{y4}^{-}	D_{y6}^{-}	D_{y8}^-
a b c	0.875 0.898 0.877	-1.197 -1.190 -1.176	0.924 0.934 0.910	-0.967 -0.965 -0.961	-0.877 -0.898 -0.875	1.172 1.190 1.197	-0.910 -0.934 -0.924	0.961 0.965 0.967
Average	0.883	-1.186	0.923	-0.964	-0.883	1.186	-0.923	0.964

A figure of this order is observed in all points and is almost certainly dominated by bump non-closure. Some very limited analysis of BOM readings from all three bunches in a train was performed. Effects similar to those of Table VIII are not apparent.

5.3 Vertical Dispersion

Table VIII shows the calculated vertical dispersion D_y^{\pm} at the evennumbered interaction points for the electron and positron bunches. The small differences in D_y^{\pm} between the bunches cause differences in the c.m. energies between bunch collisions. They cannot be obtained with a first-order perturbation theory. The values of the dispersion are less than 2 mm, and small enough for the drop in luminosity to be insignificant. The predicted dispersion, with these bunch currents, for each bunch is effectively the same. This justifies taking the average for bunches a, b and c. Only crude measurements of the absolute dispersion at the IP were possible. However, the difference dispersion is measured by observing the shift in the optimal vernier setting due to a shift in the beam energy via shifts in the RF frequency. A summary of such measurements made during the scan is shown in Table IX. Averaging for each energy a direct comparison with the calculated results is possible (see Table X). The measured results are in good agreement with the self-consistent calculations.

5.4 **Tunes and Chromaticities**

Tables XI and XII show the calculated fractional parts of the horizontal and vertical tunes, q_x^{\pm} and q_y^{\pm} and the calculated horizontal and

 TABLE IX
 Difference dispersion measurements in mm at each interaction point and at different energies

Energy	$\Delta D_{y2}^* \text{ (mm)}$	$\Delta D_{y4}^* \text{ (mm)}$	ΔD_{y6}^* (mm)	ΔD_{y8}^* (mm)
44 GeV 45 GeV 46 GeV	$\begin{array}{c} 2.04 \pm 0.09 \\ 2.05 \pm 0.15 \\ 1.36 \pm 0.28 \end{array}$	$\begin{array}{c} -2.27 \pm 0.11 \\ -2.58 \pm 0.19 \\ -2.35 \pm 0.25 \end{array}$	$\begin{array}{c} 1.61 \pm 0.18 \\ 2.60 \pm 0.15 \\ 3.61 \pm 0.49 \end{array}$	-1.53 ± 0.11 -1.52 ± 0.11 -1.30 ± 0.29

TABLE X Calculated and measured difference dispersion in mm at each IP, the beam energy is E = 45.6 GeV

	ΔD_{y2}^* (mm)	$\Delta D_{y4}^{*}(\text{mm})$	ΔD_{y6}^* (mm)	ΔD_{y8}^* (mm)
Calculation Measured	1.77	-2.37 -2.4	1.85 2.61	-1.93 -1.45
Measured	1.82	-2.4	2.61	

TABLE XI Self-consistent theory results for the fractional parts q_x^{\pm} and q_y^{\pm} of the horizontal and vertical tunes, and horizontal and vertical chromaticities Q'_x^{\pm} and Q'_y^{\pm} for typical e⁺ and e⁻ bunches. The bunch current is I=0.2 mA, the beam energy is E=22.0 GeV

Bunch	q_x^+	q_y^+	Q'_x^+	Q'_y^+	q_x^-	q_y^-	Q'_x^-	Q'_y^-
a	0.3117	0.1930	1.00	1.01	0.312	0.1924	0.87	1.23
b	0.3196	0.1973	0.95	1.03	0.320	0.1973	0.95	1.03
c	0.3120	0.1924	0.87	1.23	0.312	0.1930	1.00	1.01
a — b	-0.008	-0.0043	0.05	-0.02	-0.008	-0.005	-0.08	0.2

TABLE XII Self-consistent theory results for the fractional parts q_x^{\pm} and q_y^{\pm} of the horizontal and vertical tunes, and horizontal and vertical chromaticities Q'_x^{\pm} and Q'_y^{\pm} for typical e⁺ and e⁻ bunches. The bunch current is I=0.2 mA, the beam energy is E=45.6 GeV

Bunch	q_x^+	q_y^+	Q'_x^+	Q'_y^+	q_x^-	q_y^-	Q'_x^-	Q'_y^-
a	0.3363	0.2133	0.63	0.14	0.3364	0.2132	0.62	0.14
b	0.3456	0.2161	0.68	-0.08	0.3456	0.2161	0.68	-0.08
с	0.3364	0.2132	0.62	0.15	0.3363	0.2133	0.63	0.14
a – b	-0.009	-0.003	-0.06	0.22	-0.008	-0.003	0.04	0.22

vertical chromaticities Q'^{\pm}_x and Q'^{\pm}_y at 22 and 45.6 GeV, respectively. The values of q^{\pm}_x and q^{\pm}_y are the coherent tunes, shifted from the machine tunes, $q_{x0} = 0.290$ and $q_{v0} = 0.195$ by the beam-beam forces, calculated by averaging over both the exciting and the target bunch.³⁸ Both coherent tunes are higher than the machine tunes because of the positive beam-beam tune shifts from the head-on collisions. The difference between the coherent and machine tunes is higher in the horizontal direction, because the horizontal parasitic tune shifts are positive and the vertical ones negative, as can be seen in Eq. (2). The tunes are symmetrical between leading and trailing bunches, and there is a tune spread. The chromaticities Q'^{\pm}_x and Q'^{\pm}_y are caused by the vertical separation s_v of the head-on collisions in the even-numbered pits. They are also calculated by averaging over both the exciting and the target bunch, and also symmetrical between leading and trailing bunches. Typical measured tune and chromaticities splits are shown in Tables XIII and XIV. Two results are clear, a large tune split between electrons and positrons, and a relatively small split between the bunches in a train. The former is explained by effects other than those caused by bunch trains, e.g. orbit distortions, the RF sawtooth and solenoid edge effects. The consistent results show that bunches a and c should be split with respect to bunch b. This is rather nicely mirrored in the measured results in Table XIII where the same pattern is clearly observable. The order of magnitude of the predicted effect is also confirmed.

Bunch	q_x^+	q_y^+	Q'_x^+	Q'_y^+	q_x^-	q_y^-	Q'_x^-	Q'_y^-	
a	0.2843	0.1964	1.3	1.5	0.2999	0.1788	3.2	0.5	
b	0.2880	0.2011	0.8	2.1	0.3083	0.1825	2.9	1.8	
с	0.2853	0.1964	1.3	1.9	0.3000	0.1788	2.5	1.2	
Diff	-0.004	-0.005	0.5	-0.6	-0.009	-0.004	-0.3	-1.3	

TABLE XIII Measured tunes and chromaticities: 22 GeV, 0.2 mA/bunch

TABLE XIV Measured tune and chromaticity: 45.6 GeV, 0.2 mA/bunch

Bunch	q_x^+	q_y^+	Q'_x^+	Q'_y^+	q_x^-	q_y^-	Q'_x^-	Q'_y^-
a	0.2680	0.1668	5.24	0.71	0.2786	0.1525	5.99	1.28
b	0.2708	0.1710	5.80		0.2864	0.1634	4.27	
с	0.2679	0.1668	5.20	0.79	0.2787	0.1525	5.90	1.94
Diff	-0.003	-0.0042	-0.6	un et de daar	-0.008	-0.011	-1.7	—

5.5 Synchrotron Tune Differences between Bunches

The RF voltage seen by the bunches in a train and therefore the synchrotron tune, which is proportional to the square root of the RF voltage, will be different for two reasons:

- 1. For the two-frequency copper RF system the RF waveform is modulated. The RF voltage changes therefore between the passage of two bunches of a train. This is not the case for superconducting (SC) cavities.
- 2. The cavity filling time is long compared to the time between the passage of two bunches. Due to beam loading the RF voltage decreases therefore for each bunch passage. For the SC cavities the stored energy is much higher than for the room temperature cavities. Therefore this effect can be neglected when LEP is operated with SC cavities only.

The synchrotron tune Q_s is calculated for a particular bunch by taking into account the position of the bunch on the RF envelope and the decrease in voltage caused by the energy lost to preceding bunches. Phase changes due to beam loading are not taken into account. For a train with three bunches of 0.4 mA the calculated difference in Q_s between bunches a and c is 1.9% when bunch b is in the nominal position (the nominal position is defined as the position which produces maximum total RF voltage, i.e. the position of the bunches when LEP is operated without bunch trains) and only copper cavities are used. With a bunch current of 0.2 mA the difference is 0.8%.¹³

During the LEP running period in 1995 Q_s was logged for all the bunches in a train. On average the synchrotron tune for electrons was 0.6% lower for bunch c than for bunch a. For positrons the difference was 1.0%.³⁹ These values agree with expectations taking into account that in this running period both SC and copper cavities were used.

5.6 Luminosity and Beam–Beam Tune Shifts

Having abandoned the fourth bunch in the trains, the hoped for doubling of the luminosity became impossible and a maximum increase of 50% was the new aim. When LEP was operated with three bunches per train, the luminosity was still not fully up to the expectations and

particularly the beam-beam tune shift achieved was lower than expected. Values between $\xi_y = 0.025$ and 0.030 were the best found during the year. This should be compared with tune shifts of $\xi_y = 0.03$ -0.04 regularly obtained with four bunches and the Pretzel scheme with 8 bunches, for which best values around $\xi_y \approx 0.045$ were obtained. As a consequence, the peak luminosities with bunch trains of three bunches per train (i.e. twelve bunches per beam) eventually only slightly exceeded the peak luminosity for the eight bunch pretzel scheme and reached best values around 2.5×10^{31} cm⁻² s⁻¹.

It was already demonstrated (Table V) that with three bunches per train the bunches do not collide head-on. It was believed that the lower beam-beam tune shift was caused by the offset collision. In a run with only two bunches per train where all bunches can be collided head-on, the beam-beam tune shift quickly reached values above 0.040 with a maximum at 0.045.

6 CONCLUSIONS

Throughout 1995 LEP was operated with bunch trains. The commissioning of the new scheme was relatively straightforward, and the implementation was successful both from the conceptual and the hardware viewpoint. The principle of operation with up to four bunches per train was demonstrated; however the actual luminosity performance obtained was worse than had been hoped for. The current which could be accumulated into trains of one particle type was virtually unaffected by the presence of the bunch train bumps, although the intensity of the second bunch in a train was 10-15% lower than that of the first. The injection of counter-rotating trains of the second particle reduced the intensity limit by around 20%, although the separation could be reduced by up to 40% without further loss. Bunch currents above 0.5 mA were accumulated easily and it can be hoped that the bunch intensities needed for LEP2 can be achieved.

At 45.6 GeV, operation with four bunches per train proved to be virtually impossible, due to the spread of tunes and chromaticities between the bunches in a train. The bunches which crossed at the unfavourable parasitic encounters had very poor life times in collision, and were generally lost. Some of these effects had not been predicted by

the simulation techniques available in 1994, but were evident from the self-consistent calculation subsequently developed. With three bunches per train the magnitude of the effects was significantly reduced, and steady operating conditions could be established. Measured tune spreads were below 0.01 in both planes, and chromaticity spreads below 0.6 units, in good agreement with the self-consistent simulations. The difference in parasitic encounters between bunches means that the bunches experience different beam-beam kicks, have different orbits and therefore different positions and angles at the IP. With three bunches per train, the maximum inter-bunch spread was about 1 µm in vertical position, and about 100 µrad in angle. This meant that the individual bunches could not be collided head-on, resulting in a small reduction in luminosity and a more serious error for the centre-of-mass energy, due to the non-zero, opposite signed vertical dispersion of the electron and positron bunches at the IP. The correct choice of bunch train bump directions was essential to minimize residual vertical dispersion, which was measured at 2.6 mm at the IP in agreement with the simulations; frequent vernier scans were also necessary to minimize the vertical miscrossings. With two bunches per train head-on collisions could be obtained, boding well for an operation of this scheme at LEP2 energies.

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