

Non-local generalization of the axial anomaly and x -dependence of the anomalous gluon contribution

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Abstract

The generalization of the axial anomaly is considered. It is shown that bilocal axial quark operators on the light cone possess, beside the point-like anomaly, also a light-like anomaly. The consequences for the definition of anomaly-free quark distribution functions and the effect of both the gluon coefficient function and splitting kernels in polarized deep inelastic scattering are discussed.

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I. INTRODUCTION

In the last years much effort was put, on both experimental and theoretical sides, into understanding the spin structure of the proton (for reviews of the EMC spin crisis, see [1]). New data for the polarized deep inelastic scattering (DIS) structure function g_1 from SLAC [2] and the SMC [3] are consistent with the previous EMC [4] result. Two different scenarios can explain the data, large anomalous sea-quark or large anomalous gluon interpretation [5–7], which are connected with the problem of the anomalous gluon contribution to g_1 . In fact, a renormalization group transformation can shift the anomalous contribution to the quark sector or to the gluon sector. In this way, either the anomaly manifested in the matrix element of the axial quark current is responsible for the large polarization of the sea or it can be perturbatively taken into account in the gluon coefficient function, while the quark matrix element is anomaly-free. The dependence of this renormalization group transformation on the gluon momentum fraction x is still controversial.

From the physical point of view, it is more natural that the anomaly is attributed to the gluons. The gluonic coefficient function then absorbs only the short-distance contributions, while the quark distribution corresponds to a conserved operator and may be invoked in the low-energy description of the proton. The corresponding picture of the nucleon spin structure is most popular and the gluon polarization is considered to be its most important unknown ingredient. The problem of its x -dependence is the “physical” counterpart of the above-mentioned renormalization group transformation.

The accounting for the anomaly by the finite renormalization transformation is effectively resulting in the substitution in the partonic expression for the structure function g_1 ,

$$\Delta q \rightarrow \Delta \tilde{q} = \Delta q - \frac{\alpha_s}{2\pi} \Delta g, \quad (1.1)$$

for the first moment of the spin-dependent quark distribution for each flavour. The over-simplified “naive” x -dependent analogue is just

$$\Delta \tilde{q}(x) = \Delta q(x) - \frac{\alpha_s}{2\pi} \Delta G(x). \quad (1.2)$$

However, such an expression is not compatible, in principle, with the fact that the gluon contribution starts at one-loop level only. The simplest, consistent approach was explored in Ref. [8] a few years ago, by taking the infrared (IR) finite part E_{IR} of the box diagram, responsible for the photon–gluon interaction:

$$\Delta\tilde{q}(x) = \Delta q(x) - \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} E_{\text{IR}}\left(\frac{x}{z}\right) \Delta g(z). \quad (1.3)$$

However, the choice of the finite part is ambiguous and the specific role of the axial anomaly remains unclear. This is why the attempt followed in Ref. [9] to perform the decomposition of the box diagram, referring to its kinematical structure. The anomaly then contributes to the singlet structure, associated with the spin structure function $g_T = g_1 + g_2$ of the photon-gluon scattering¹:

$$g_T^n = \frac{1}{2n(n+1)} \Delta g^n. \quad (1.4)$$

Attributing the anomalous gluon contribution to this structure function resulted in its $1-x$ -dependence. Later, this expression was obtained and exploited in different approaches [11,12]. However, both derivations and interpretations of this result do not seem complete and some more solid ground is desirable.

The anomalous gluon contribution plays an exceptional role in the general classification of perturbative QCD contributions. Formally, it is a part of the next-to-leading order (NLO) contribution. However, due to the well-known growth of the first moment of the spin-dependent gluon distribution, compensating one power of α_s makes it essential also at leading order. The recently calculated two-loop, spin-dependent anomalous dimensions [13,14] make it now possible to perform the complete NLO analysis of the experimental data [15–18]. These NLO calculations were performed in the dimensional regularization using the 't Hooft–Veltman–Breitenlohner–Maison (HVBM) scheme [19] in which γ^5 does not anticommute in

¹The recent analysis [10] confirms that the zero first moment of this structure function is manifested only for regularization schemes, providing the zero moment of g_1 as well.

the unphysical space-time dimensions. In this minimal subtraction (MS) scheme both chiral invariance in the non-singlet sector and the one-loop character of the singlet axial anomaly are explicitly broken and their restoration requires an additional finite renormalization.

As the HVBM scheme is used, a further renormalization group transformation is performed [16,20] in order to make the result compatible with the standard factorization prescription and the low-energy intuitive description of the proton. However, only the value of the first moment was fixed in that procedure, so that there remains an ambiguity in this transformation. Note that the function $1 - x$ was also proposed for this purpose [12].

In the present article, we are suggesting the non-local generalization of the axial anomaly based on the canonical Ward identities for light-ray operators. This allows us to give the natural description of both anomaly-free singlet quark distribution and anomalous gluon contribution, which fix the x -dependence of the renormalization group transformation. The final result is a rigorous proof of the $1 - x$ -behaviour of the anomalous contribution in NLO. Here, we do not address the issue of higher loop corrections to the generalized axial anomaly.

The paper is organized as follows. Section II discusses the chiral invariance breaking of flavour non-singlet light-ray operators due to the renormalization, and their restoration by a finite renormalization. Then we derive Ward identities for light-ray operators and compute the non-local singlet axial anomaly, which can be expressed as a divergence from a generalized topological current. In Section III we use our results to define the anomaly-free singlet quark distribution and show the consequences for coefficient functions and evolution kernels in NLO approximation.

II. AXIAL ANOMALY OF LIGHT-RAY OPERATORS

As mentioned before, chiral invariance is broken in the HVBM scheme; however, as it is known from the renormalization of the axial current, it can be restored by a finite renormalization, so that the non-singlet Ward identity will be fulfilled. This finite renormalization constant z^{NS} can be computed for massless QCD from the requirement that the anticom-

mutativity of γ^5 is effectively restored [21], i.e.

$$z^{\text{NS}} \langle [j_\mu^{5,a}] \psi \bar{\psi} \rangle = \langle [j_\mu^a] \psi \bar{\psi} \rangle \gamma^5, \quad \text{with} \quad z^{\text{NS}} = 1 - \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2), \quad (2.1)$$

where $j_\mu^{5,a} = \frac{1}{2} \bar{\psi} [\gamma_\mu, \gamma^5] \lambda^a \psi$ with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $j_\mu^a = \bar{\psi} \gamma_\mu \lambda^a \psi$ are the axial vector and the vector current, respectively, λ^a is a flavour matrix, the symbol [...] denotes minimal subtraction, and $C_F = \frac{4}{3}$ is the usual QCD colour factor. Flavour and colour indices are suppressed for simplicity.

The one-loop character of the singlet axial anomaly can also be ensured by some finite renormalization of the current. This required a two-loop calculation of the Ward identities sandwiched between the two-gluon state. The α_s correction to the corresponding z factor, $z^{\text{S}} = 1 - \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2)$, coincides with the non-singlet factor z^{NS} . Because of the appearance of so-called light-to-light subdiagrams, this coincidence is spoiled beyond the one-loop level. Note that the one-loop approximation of z^{S} can also be calculated by the requirement that the singlet Ward identities are fulfilled for the two-quark state. Since at leading order the axial anomaly does not appear in this Ward identity, it follows that the finite renormalization factor is the same as that for the non-singlet channel.

The same problems as discussed above also occur for composite operators on the light-cone, which appear in the definition of spin-dependent quark distribution functions. For technical reasons we start with a more general definition of bi-local operators, which are not necessarily on the light-cone:

$$O_\mu^{5,a}(x, y) = \frac{1}{2} \bar{\psi}(x) U_s(x, y) [\gamma_\mu, \gamma^5] \lambda^a \psi(y), \quad (2.2)$$

$$U_s(x, y) = P \exp \left\{ -ig \int_0^1 d\tau A_k^\mu(x\tau + y[1-\tau]) t_k(x_\mu - y_\mu) \right\}.$$

Here, $U_s(x, y)$ ensures gauge invariance, where the gauge field A_k^μ is path-ordered along a straight line connecting the fermion fields. For light-like distances, i.e. $(x - y)^2 = 0$, these operators can be expanded in terms of local twist-2 and twist-3 operators. We set $x = \kappa_1 \tilde{x}, y = \kappa_2 \tilde{x}$, where \tilde{x} is a light-cone vector and, after contraction with \tilde{x}^μ , we get the

leading twist-2 light-ray operators [22]²:

$$\begin{aligned} O^{5,a}(\kappa_1, \kappa_2; \tilde{x}) &= \tilde{x}^\mu O_\mu^{5,a}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}), \\ &= \bar{\psi}(\kappa_1 \tilde{x}) U(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \not{\tilde{x}} \gamma^5 \lambda^a \psi(\kappa_2 \tilde{x}). \end{aligned} \quad (2.3)$$

To determine the finite non-singlet renormalization constant in the forward case, we require (for massless QCD), in analogy to Eq. (2.1), the validity of

$$\int_0^1 dx z^{\text{NS}}(x) \langle [O^{5,a}(0, \kappa x; \tilde{x})] \psi \bar{\psi} \rangle = \langle [O^a(0, \kappa; \tilde{x})] \psi \bar{\psi} \rangle \gamma^5, \quad (2.4)$$

where O^a is analogous to the definition of $O^{5,a}$ in Eq. (2.3), but without γ^5 matrix. In the dimensional regularization using the HVBM scheme, the leading-order result (restricted to the forward case) is

$$z^{\text{NS}}(x) = \delta(1-x) - \frac{\alpha_s}{\pi} 2C_F(1-x) + O(\alpha_s^2), \quad (2.5)$$

while in the Pauli–Villars regularization, chiral invariance holds true without finite renormalization.

As in the case of the axial current one expects, in leading order, that the same finite renormalization as in Eq. (2.5) has to be performed for the singlet light-ray operator. Beyond the leading order this is no longer true and the question is: How can we fix this finite renormalization constant?

After we have seen that the restoration of chiral invariance requires an x -dependent finite renormalization, a second question arises: Does the axial anomaly also depend on x ? To answer this question we use the equation of motion to derive Ward identities for the divergence of the non-local operators (2.2). A straightforward calculation provides that the divergence of this operator is

² Since \tilde{x} is an external four-vector that can be kept four dimensional in the dimensional-regularized operator vertex, it follows that in the HVBM scheme the relation $[\not{\tilde{x}}, \gamma^5] = 2 \not{\tilde{x}} \gamma^5$ is valid, so that indeed Eq. (2.3) comes from the definition (2.2).

$$\begin{aligned}
(\partial_x^\mu + \partial_y^\mu) O_\mu^{5,a}(x, y) &= \Omega^{\text{EOM}}(x, y) + i\bar{\psi}(x)U_s(x, y)\gamma^5 m^a \psi(y) - \\
&ig \int_0^1 d\tau \bar{\psi}(x)U_s(x, x\tau)\gamma_\alpha F^{\alpha\beta}(x\tau + y[1 - \tau])(x_\beta - y_\beta) \times \\
&U_s(y[1 - \tau], y)\gamma^5 \lambda^a \psi(y), \tag{2.6}
\end{aligned}$$

$$\Omega^{\text{EOM}}(x, y) = \bar{\psi}(x)([\not{D}(x) - im]U_s(x, y) + U_s(x, y)[\not{D}(y) - im])\gamma^5 \lambda^a \psi(y), \tag{2.7}$$

where $\Omega^{\text{EOM}}(x, y)$ denotes the equation of motion operators, $F_{\alpha\beta} = F_{\alpha\beta}^a t^a$ is the field strength tensor, and $m_{ij}^a = (m_i + m_j)\lambda_{ij}^a$ is a mass matrix.

For $(x - y)^2 \neq 0$ the operator is ultraviolet-finite and the Ward identity (2.6) should be satisfied. However, the situation will be changed if we go to a light-like (not only short, as usually discussed) distance. Then the operator has to be renormalized, and anomalous terms can occur. For instance, if we naively anticommute $\not{D}(y)$ with γ^5 in the second expression in the r.h.s. of Eq. (2.7) then the use of the equation of motion will provide only contact terms. However, in the HVBM scheme the non-anticommutativity of γ^5 gives an anomalous term, which should be cancelled by a finite renormalization (see the discussion in [23]), so that the anticommutativity of γ^5 is effectively restored as discussed above. We calculated this anomalous term and after an appropriate definition of the appearing operators the finite renormalization constant (2.5) was extracted from the Ward identity. As discussed above for the local case, because of the absence of the singlet axial anomaly in the Ward identities for the Green functions of quark fields at one-loop order, it follows that $z^{\text{S}}(x)$ coincides with $z^{\text{NS}}(x)$ at this order.

For the singlet case ($a = 0, \lambda_{ij}^0 = \delta_{ij}, m_{ij}^0 = 2m_i \delta_{ij}$), we expect an anomalous term of the form $\epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta}(\dots)F_{\gamma\delta}(\dots)$, which can be computed from the difference of the l.h.s. and r.h.s. of the Ward identity (2.6) sandwiched between the two gluon states (see diagrams in Fig. 1). We now set $x = \kappa_1 \tilde{x}, y = \kappa_2 \tilde{x}$ with $\tilde{x}^2 = 0$ and choose the light-cone gauge, i.e. $\tilde{x}A = 0$. After a straightforward calculation, it turns out that in both Pauli–Villars regularization and HVBM scheme the same anomaly appears, so that the singlet Ward identity is actually given by

$$\left(\partial_{(\kappa_1\tilde{x})}^\mu + \partial_{(\kappa_2\tilde{x})}^\mu\right) O_\mu^{5,0}(\kappa_1\tilde{x}, \kappa_2\tilde{x}) = O_F(\kappa_1, \kappa_2; \tilde{x}) + \frac{\alpha_s}{4\pi} N_f \int_0^1 dx \int_0^{1-x} dy 2 \times \\ F_{\mu\nu}^a([\kappa_1(1-x) + \kappa_2x]\tilde{x}) \tilde{F}^{a\mu\nu}([\kappa_2(1-y) + \kappa_1y]\tilde{x}), \quad (2.8)$$

where

$$O_F(\kappa_1, \kappa_2; \tilde{x}) = -ig \int_{\kappa_1}^{\kappa_2} d\tau \bar{\psi}(\kappa_1\tilde{x}) \gamma_\alpha F^{\alpha\beta}(\tau\tilde{x}) \tilde{x}_\beta \gamma^5 \psi(\kappa_2\tilde{x}). \quad (2.9)$$

Here, $\tilde{F}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}^a$, with $\epsilon_{0123} = +1$, $\alpha_s = \frac{g^2}{4\pi}$, and N_f is the number of flavours. Here we applied the equation of motion and neglected the quark masses. In the local case, which follows from setting $\kappa_1 = \kappa_2 = \kappa$, the operator $O_\mu^{5,0}(\kappa\tilde{x}, \kappa\tilde{x})$ coincides with the local singlet axial current $j_\mu^{5,0}(\kappa\tilde{x})$ and from Eq. (2.8) we recover the well-known expression for the local anomaly: $\frac{\alpha_s}{4\pi} N_f F_{\mu\nu}^a(\kappa\tilde{x}) \tilde{F}^{a\mu\nu}(\kappa\tilde{x})$.

Finally, we introduce a non-local generalization of the topological current

$$K^\mu(x, y) = \frac{\alpha_s}{4\pi} N_f \epsilon^{\mu\alpha\beta\gamma} \left[A_\alpha^a(x) \partial_\beta A_\gamma^a(y) - \frac{g}{3} f_{abc} A_\alpha^a(x) A_\beta^b(y) A_\gamma^c(y) + \{x \leftrightarrow y\} \right], \quad (2.10)$$

with $x = \kappa_1\tilde{x}$ and $y = \kappa_2\tilde{x}$, so that the anomaly in Eq. (2.8) can be written as a divergence

$$\left(\partial_{(\kappa_1\tilde{x})}^\mu + \partial_{(\kappa_2\tilde{x})}^\mu\right) K_\mu(\kappa_1\tilde{x}, \kappa_2\tilde{x}) = \frac{\alpha_s}{4\pi} N_f F_{\mu\nu}^a(\kappa_1\tilde{x}) \tilde{F}^{a\mu\nu}(\kappa_2\tilde{x}). \quad (2.11)$$

We are now able to define the anomaly-free operator

$$\tilde{O}_\mu^{5,0}(\kappa_1\tilde{x}, \kappa_2\tilde{x}) = O_\mu^{5,0}(\kappa_1\tilde{x}, \kappa_2\tilde{x}) - \\ 2 \int_0^1 dx \int_0^{1-x} dy K_\mu([\kappa_1(1-x) + \kappa_2x]\tilde{x}, [\kappa_2(1-y) + \kappa_1y]\tilde{x}). \quad (2.12)$$

III. CONSEQUENCES FOR POLARIZED DISTRIBUTION FUNCTIONS

As shown in the previous Section light-ray operators possess anomalous contributions. For the non-singlet case these anomalies are a pure artefact of choosing a non-invariant chiral renormalization scheme. The polarized quark distribution functions should be defined in a chiral invariant manner, so that in a general renormalization scheme an additional finite renormalization is necessary

$$\Delta q^{\text{NS}}(x, Q^2) = \int_x^1 \frac{dy}{y} z_5^{\text{NS}}(y) \int \frac{d\kappa}{2\pi(\tilde{x}S)} \langle P, S | [O^{5,\text{NS}}(0, \kappa; \tilde{x})] | P, S \rangle e^{i(x/y)\kappa(\tilde{x}P)}, \quad (3.1)$$

where S^ρ denotes the polarization vector of the nucleon and the renormalization point square μ^2 is set equal to the momentum transfer squared Q^2 . The leading-order approximation of $z_5^{\text{NS}}(y)$ in the HVBM scheme is given in Eq. (2.5). This finite renormalization affects the NLO approximation of both quark coefficient functions and splitting kernels and agrees with the additional renormalization group transformation performed in the NLO calculation [13,14].

Because of the anomaly, the naive definition of the singlet distribution function

$$\begin{aligned} \Delta \tilde{\Sigma}(x, Q^2) &= \sum_{i=u,d,\dots} \Delta q_i(x, Q^2) + \Delta \bar{q}_i(x, Q^2), & \Delta \bar{q}_i(x, Q^2) &= \Delta q_i(-x, Q^2) \\ &= \int_x^1 \frac{dy}{y} z_5^{\text{S}}(y) \int \frac{d\kappa}{2\pi(\tilde{x}S)} \langle P, S | [O^{5,0}(0, \kappa; \tilde{x})] + \{\kappa \rightarrow -\kappa\} | P, S \rangle e^{i(x/y)\kappa(\tilde{x}P)} \end{aligned} \quad (3.2)$$

(please note that $\Delta \tilde{\Sigma}$ is defined in terms of the operator $O^{5,0}$) cannot be interpreted as probability for finding a polarized quark flavour singlet configuration with given longitudinal momentum fraction x . In addition to the finite multiplicative renormalization³, it is also necessary to remove the axial anomaly from the definition of this function [5–7]. Thus, one has to define the singlet distribution function in terms of the anomaly-free operator (2.12), which provides

$$\Delta \Sigma(x, Q^2) = \Delta \tilde{\Sigma}(x, Q^2) - k(x, Q^2), \quad (3.3)$$

where

$$k(x, Q^2) = 2 \int_x^1 \frac{dy}{y} (1-y) \int \frac{d\kappa}{2\pi(\tilde{x}S)} \langle P, S | \tilde{x}^\mu K_\mu(0, \kappa \tilde{x}) + \{\kappa \rightarrow -\kappa\} | P, S \rangle e^{i(x/y)\kappa(\tilde{x}P)}, \quad (3.4)$$

and the generalized topological current K_μ is defined in Eq. (2.10).

The problem that K_μ is gauge-variant, and thus that $k(x, Q^2)$ contains also unphysical components, can somehow be resolved by the choice of a physical gauge. In the light-cone

³This renormalization is due to the diagrams without two-gluon intermediate states, which are the same for the non-singlet and the singlet case, so that $z_5^{\text{S}}(y) = z_5^{\text{NS}}(y)$, as discussed in Section II.

gauge, the gauge-invariant twist-2 gluon operator

$$G(\kappa_1, \kappa_2; \tilde{x}) = i\tilde{x}^\alpha \tilde{F}_{\alpha\beta}(\kappa_1\tilde{x}) F_a^{\beta\gamma}(\kappa_2\tilde{x}) \tilde{x}_\gamma \quad (3.5)$$

can be expressed in the forward case by

$$G(0, \kappa; \tilde{x}) = - \left(\frac{\alpha_s}{2\pi} N_f \right)^{-1} i\tilde{x}^\alpha \partial_\kappa \tilde{x}^\mu K_\mu(0, \kappa\tilde{x}). \quad (3.6)$$

Furthermore, from the definition of the gluon distribution function

$$\Delta g(x, Q^2) = \frac{1}{x(\tilde{x}P)} \int \frac{d\kappa}{2\pi(\tilde{x}S)} \langle P, S | G(0, \kappa; \tilde{x}) + \{\kappa \rightarrow -\kappa\} | P, S \rangle e^{ix\kappa(\tilde{x}P)}, \quad (3.7)$$

and from Eqs. (3.4) and (3.6), and after performing a partial integration, we find that $k(x, Q^2)$ is actually given in terms of the gauge-invariant gluon distribution function. Thus, to remove the axial anomaly of the polarized quark distribution function it is sufficient to subtract a certain amount of the gluon distribution function:

$$\Delta\Sigma(x, Q^2) = \Delta\tilde{\Sigma}(x, Q^2) - K(x) \otimes \Delta g(x, Q^2), \quad K(x) = -\frac{\alpha_s}{\pi} N_f (1-x), \quad (3.8)$$

where the convolution is defined as

$$A(x) \otimes B(x) = \int_0^1 dy \int_0^1 dz \delta(x - yz) A(y) B(z). \quad (3.9)$$

Indeed, removing the axial anomaly is equivalent to the following (additive) renormalization group transformation:

$$\begin{aligned} \Delta C_g(x) &= \Delta\tilde{C}_g(x) + K \otimes \Delta\tilde{C}_q(x), & \Delta C_q(x) &= \Delta\tilde{C}_q(x), \\ \Delta P_{gg}(x) &= \Delta\tilde{P}_{gg}(x) + K \otimes \Delta\tilde{P}_{gq}(x), & \Delta P_{gq}(x) &= \Delta\tilde{P}_{gq}(x), \\ \Delta P_{qq}(x) &= \Delta\tilde{P}_{qq}(x) - K \otimes \Delta\tilde{P}_{gq}(x), \\ \Delta P_{qg}(x) &= \Delta\tilde{P}_{qg}(x) - \frac{\beta}{g} K(x) + K \otimes [\Delta\tilde{P}_{qq} - \Delta\tilde{P}_{gg} - \Delta\tilde{P}_{gq} \otimes K](x), \end{aligned} \quad (3.10)$$

where ΔC_i are the coefficient functions, ΔP_{ij} are the spin-dependent splitting kernels, and $\beta = \mu \frac{d}{d\mu} g(\mu)$ is the renormalization group coefficient of the running coupling constant.

Because of gauge invariance the operator product analysis suggests that the zero moment of ΔC_g vanishes. After the transformation (3.10) is performed, the gauge-variant axial anomaly contributes to the gluonic sector, so that the zero moment is now given by

$$\int_0^1 dx \Delta C_g(x) = \int_0^1 dx K(x) + O(\alpha_s^2) = -\frac{\alpha_s}{2\pi} N_f + O(\alpha_s^2). \quad (3.11)$$

For completeness we give the difference of the splitting kernels in NLO:

$$\begin{aligned} \Delta P_{gg}(x) - \Delta \tilde{P}_{gg}(x) &= -(\Delta P_{qq}(x) - \Delta \tilde{P}_{qq}(x)) \\ &= \left(\frac{\alpha_s}{2\pi}\right)^2 2C_F N_F [3(1-x) + (2+x) \ln x], \\ \Delta P_{qg}(x) - \Delta \tilde{P}_{qg}(x) &= \left(\frac{\alpha_s}{2\pi}\right)^2 N_f \{C_F [(1-x)(1-4 \ln(1-x) + 2 \ln x)] + \\ &\quad C_A [(1-x)(-16 + 4 \ln(1-x)) - 4(2+x) \ln x]\}, \end{aligned} \quad (3.12)$$

where the Casimir operator C_A is equal to the number of colours. Note, that these differences will vanish in the limit $x \rightarrow 1$. For small x they contribute only to the subleading behaviour of the NLO result (the leading terms are given by $\ln^2 x$).

Another opportunity to perform the evolution coincides with the one proposed by Cheng [12]. Namely, one should perform the evolution in the gauge-invariant (say, $\overline{\text{MS}}$) scheme and *afterwards* restore the anomaly-free distribution by applying Eq. (3.8). The $1-x$ behaviour in this approach is actually coming from the mass term in the box graph. The contact with our derivation may be achieved by noting that the cancellation of normal and anomalous divergence, resulting in the effective conservation of axial current in the limit of infinite quark mass, is valid for the non-local anomaly as well. It is especially clear for the Pauli-Villars regularization, when the contribution of regulator fermions (calculated in Section II) is looking, up to the sign, precisely such as that of the quark masses, which is the starting point of the approach of Cheng.

For practical purposes, irrespective of the used evolution scheme, it is possible to define an effective gluon distribution, which is just the combination appearing in g_1 , i.e. the convolution of $(1-x)$ with $\Delta g(x)$:

$$\Delta g^{\text{eff}}(x, Q^2) = 2(1-x) \otimes \Delta g(x, Q^2), \quad (3.13)$$

so that the first moments of effective gluon distribution coincide with the “original” one, while the structure function g_1 has at leading order the simple partonic form, suitable for the extraction of partonic distributions from the experimental data [24]:

$$g_1(x, Q^2) = \frac{1}{2} \Delta \tilde{\Sigma}(x, Q^2) = \frac{1}{2} \left[\Delta \Sigma(x, Q^2) - \frac{\alpha_s}{2\pi} N_f \Delta g^{\text{eff}}(x, Q^2) \right]. \quad (3.14)$$

It is convenient to have the evolution equation directly for effective distribution functions. While the diagonal kernels are not changed, the moments of the off-diagonal kernels are changed in a straightforward manner:

$$\Delta P_{gq}^{\text{eff}}(n) = \frac{2}{n(n+1)} \Delta P_{gq}(n), \quad \Delta P_{qg}^{\text{eff}}(n) = \frac{n(n+1)}{2} \Delta P_{qg}(n). \quad (3.15)$$

Note, especially, that the influence of the effective gluon distribution to the quark evolution, governed by $\Delta P_{qg}^{\text{eff}}$, appears to be much less singular in n . We are not presenting the explicit form for effective NLO anomalous dimensions, which are rather lengthy. At leading order, when the x dependence is easily restored, Eq. (3.15) results in the equations

$$\Delta P_{gq}^{\text{eff}}(x) = \frac{\alpha_s}{\pi} C_F [3(x-1) - (x+2) \ln x], \quad \Delta P_{qg}^{\text{eff}} = -\frac{\alpha_s}{4\pi} N_f \frac{d}{dx} \delta(1-x). \quad (3.16)$$

The first moments of the splitting kernels provide an important check for the normalization of the non-local contribution and determine the evolution in the corresponding sum-rules. So we summarize the consequences for the zero moment of the splitting kernels, which come from general renormalization arguments of the axial vector current and the topological current, which are verified up to two- and three-loop order, respectively [25]. From current conservation of $j_\mu^{5,0} = \tilde{j}_\mu^{5,0} - k_\mu$ (here, $\tilde{j}_\mu^{5,0}$ refers to the original definition in terms of quark fields), the Adler–Bardeen theorem, and the renormalization properties of gauge-invariant operators, it follows that

$$\begin{aligned} \int_0^1 dx \Delta P_{gg}(x) &= \int_0^1 dx \Delta \tilde{P}_{gg}(x) + \gamma_j = -\frac{\beta}{g} + \gamma_j, \\ \int_0^1 dx \Delta P_{gq}(x) &= \int_0^1 dx \Delta \tilde{P}_{gq}(x) = -\frac{\gamma_j}{N_f [\alpha_s / (2\pi)]}, \\ \int_0^1 dx \Delta P_{qg}(x) &= \int_0^1 dx \Delta \tilde{P}_{qg}(x) = 0, \\ \int_0^1 dx \Delta P_{qq}(x) &= \int_0^1 dx \Delta \tilde{P}_{qq}(x) - \gamma_j = 0, \end{aligned} \quad (3.17)$$

where the three-loop order approximation for the anomalous dimension of the axial vector current γ_j and for the β -function are known⁴ [25,26]:

$$\gamma_j = - \left(\frac{\alpha_s}{4\pi} \right)^2 6C_F N_f + \left(\frac{\alpha_s}{4\pi} \right)^3 \left(-\frac{142}{3}C_A + \frac{4}{3}N_F + 18C_F \right) C_F N_f, \quad (3.18)$$

$$\begin{aligned} \frac{\beta}{g} = & -\frac{\alpha_s}{4\pi} \left(\frac{11}{3}C_A - \frac{2}{3}N_f \right) - \left(\frac{\alpha_s}{4\pi} \right)^2 \left(\frac{34}{3}C_A^2 - 2C_f N_f - \frac{10}{3}C_A N_f \right) - \\ & \left(\frac{\alpha_s}{4\pi} \right)^3 \left(\frac{2857}{54}C_A^3 + C_F^2 N_f - \frac{205}{18}C_F C_A N_f - \frac{1415}{54}C_A^2 N_f + \frac{11}{9}C_F N_f^2 + \frac{79}{54}C_A N_f^2 \right). \end{aligned} \quad (3.19)$$

These results allow [27] the extraction of the anomaly equation renormalization for a number of loops exceeding that of γ_j by 1. For the leading three-loop contribution, this coincides with the calculation of Anselm and Johansen [28], while the four-loop correction require a one-loop finite correction to the gluon matrix element of the topological current [29]. As a result, the four-loop correction takes the form

$$Z_j = - \left(\frac{\alpha_s}{4\pi} \right)^3 6C_F N_f^2 + \left(\frac{\alpha_s}{4\pi} \right)^4 \left(-\frac{214}{3}C_A + \frac{4}{3}N_F + 18C_F \right) C_F N_f. \quad (3.20)$$

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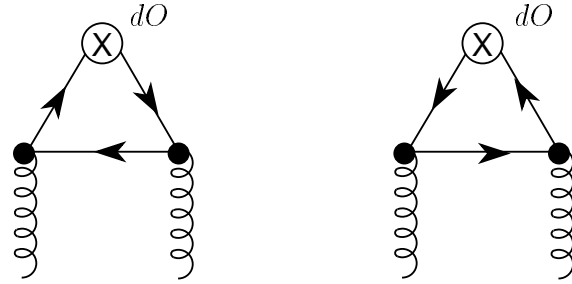
⁴ Recently, the four-loop approximations for the β -function and for $\bar{\gamma}_j$ were calculated. Here, $\bar{\gamma}_j$ denotes the anomalous dimension in the $\overline{\text{MS}}$ scheme and the finite renormalization constant z^S is known up to the order α_s^2 . To obtain γ_j and, therefore, also the first moment of $\Delta P_{gq}(x)$ at three-loop order it is necessary to know z^S up to the order α_s^3 . As proposed above, this α_s^3 correction can be obtained by the requirement that the singlet Ward identities for pure quark Green functions are fulfilled at three-loop order. The four-loop calculation of both sides of the Ward identities for the two-gluon state gives then the consistency check for the Adler-Bardeen theorem.

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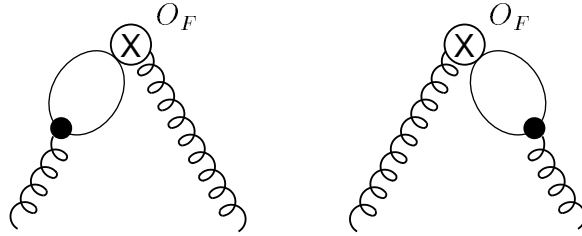
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(a)



(b)

Figure 1: Diagrammatical representation for the l.h.s. (a) and for the r.h.s. (b) of the non-local singlet Ward identity (2.8) in the one-loop approximation. The symbol dO refers to the divergence $(\partial_{(\kappa_1 \tilde{x}}^\mu + \partial_{(\kappa_2 \tilde{x}}^\mu) O_\mu^{5,0}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})$ and O_F denotes the operator $O_F(\kappa_1, \kappa_2; \tilde{x})$ defined in Eq. (2.9).