Scale dependence of the chiral-odd twist-3 distributions $h_L(x)$ and $e(x)$.

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Abstract

We evaluate the complete leading order evolution kernels for the chiral-odd twist-3 distributions $e(x)$ and $\widetilde{h}_L(x)$ of the nucleon. We establish the connection between the evolution equations in light-cone position and the light-cone fraction representations which makes correspondence between the nonlocal string operator product expansion and QCD inspired parton model. The compact expression obtained for the local anomalous dimension matrix coincides with previous calculations. In the multicolour QCD as well as in the large-x limit the twist-3 distributions obey simple DGLAP equations. Combining these two limits we propose improved DGLAP equations and compare them numerically with the solutions of the exact evolution equations.

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1 Introduction.

The factorization theorems [1] provides us with powerful tools for study the processes with large momentum transfers. They give the possibility for separation of the contributions responsible for physics of large and small distances. The former are parametrized by the hadron's parton distribution functions, or in general sense by parton correlators, which are uncalculable at the moment from the first principles of the theory, while the second ones — hard-scattering subprocesses, can be dealt perturbatively. The parton distributions are defined in QCD by the target matrix elements of the light-cone correlators of field operators [2]. This representation allows for the estimation of these quantities by the nonperturbative methods presently available which are closed to the fundamental QCD lagrangian [3].

The increasing accuracy of the experimental measurements requires the unravelling of the twist-3 effects in the hard processes which manifest the quantum mechanical interference of partons in the interacting hadrons. The most important advantage of the twist-3 structure functions is that, while being important for understanding of long-range quark-gluon dynamics, they contribute at leading order in $1/Q$ (Q being the momentum of the probe) to certain asymmetries [4, 5] and therefore may be directly extracted from experiments [6]. To confront the theory with high precision data the knowledge of the scale dependence of measurable quantities is needed. The Q^2-evolution of the parton distributions [7, 8] can be predicted by exploiting the powerful methods of renormalization group (RG) and QCD perturbation theory. The evolution equation of the twist-3 polarized chiral-even nucleon structure function $g_2(x)$ was extensively discussed in the literature [9, 10, 11, 12, 13] as well as its solution in the multicolour QCD and asymptotical values of orbital momentum $n \to \infty$ ($x \to 1$ region) [14].

In this paper we address to the study of the evolution of other twist-3 structure functions of the nucleon: chiral-odd distributions $e(x)$ and $h_L(x)$ [5] which open a new window to explore the nucleon content. The present study is provoked by several reasons and pursues various goals. First, while the anomalous dimensions of the local operators corresponding to the moments of the chiral-odd distributions were computed for the real QCD case [15], the kernels of the evolution equations were derived only in the multicolour limit [16]. From the point of view of experimental measurements the knowledge of the evolution of the whole x-dependent distribution is welcome. However, an exact equations with account of $\mathcal{O}(1/N_c^2)$ effects are lost at present, though we may expect the sizable $1/N_c^2$ effects for the small x behaviour of the distribution functions provided the nonleading in N_c-terms yields the rightmost singularity in the complex n-plane of the angular momentum as compared to the leading order result. Second, up to now the relation between different formulation of the evolution equations in the light-cone fraction [9, 10] and light-cone position [11, 17] representations was obscure. So, the aim of the present paper is two-fold: to complete the gap in our knowledge of the exact (with account for the $1/N_c^2$ effects) higher twist evolution equations for chiral-odd distributions and to provide the relation between different calculational approaches. We attempt here to clarify these issues.

The outline of the paper is the following. We start with the construction of the basis of the twist-3 chiral-odd polarized and nonpolarized correlation functions mixed under the renormalization group evolution and find the equations which govern their scale dependence in the momentum fraction as well as in the light-cone position representations in section 2. We present the Fourier transformation of evolution kernels which makes the transition from one representation to another simple in section 3. Section 4 is devoted to the construction of the generalized DGLAP equation in mixed representation for the twist-3 distributions which is the most useful for solving the RG equations in the large-N_c limit as well as for asymptotical values of the orbital momentum n. Corresponding analytical solution as well as numerical study of exact equations is performed in section 5. The final section contains discussion and concluding remarks.

To make the discussion complete and more transparent we include several appendixes. In appendix A we present definitions and some properties of Θ-functions appeared in the formulation of the evolution equations in the momentum space. In appendix B the evolution equations for the redundant basis of operators are found in the abelian gauge theory. This shows the self-consistency of the whole approach. The anomalous dimension matrix of local twist-3 operators obtained from the evolution equations we have derived in the main text are written in appendix C. It coincides with results known in the literature [15].

2 Evolution of chiral-odd twist-3 correlation functions.

As we have mentioned in the Introduction the parton distribution functions in QCD are defined by the Fourier transforms along the null-plane of the forward matrix element of the parton field operators product separated by interval λ on the light cone:

$$\mathcal{F}(\lambda) \equiv \mathcal{F}(\lambda, 0) = \phi^*(0)\phi(\lambda n), \tag{1}$$

where ϕ denotes quark ψ or gluon field B_μ. We suppress the dependence on the renormalization scale label, μ_R, necessary to render this equation well defined in field theory. Throughout the paper we use the ghost-free $B_+ \equiv B_\mu n^\mu = 0$ gauge. Here n is a light-cone vector $n^2 = 0$ normalized with respect to four-vector $P = p + \frac{1}{2}M_h^2 n$ of the parent hadron h of mass M_h, *i.e.* $nP = 1$, and p is a null vector along opposite tangent to the light cone such that $p^2 = 0$, $np = 0$. In any covariant gauge a path-ordered link factor should be inserted in between the ϕ-fields to maintain the gauge invariance of the physical quantities. The Fourier transformations from the coordinate to the momentum space and vice versa are given by

$$F(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h|\mathcal{F}(\lambda)|h\rangle, \quad \langle h|\mathcal{F}(\lambda)|h\rangle = \int dx e^{-i\lambda x} F(x). \tag{2}$$

Both of these representations display the complementary aspects of the factorization. The light-cone position representation is suitable to make contact with operator product expansion (OPE) approach, while the light-cone fraction representation is appropriate for establishing the language of the parton model. Throughout the paper we will use the light-cone position and the light-cone fraction representations in parallel.

The multiparton distributions corresponding to the interference of higher Fock components in the hadron wave functions which emerge at the twist-3 level are the generalizations of (1) to three parton fields

$$\mathcal{F}(\lambda, \mu) \equiv \mathcal{F}(\lambda, 0, \mu) = \phi^*(\mu n)\phi(0)\phi(\lambda n). \tag{3}$$

We do not display the quantum numbers of the field operators since they are not of relevance at the moment. The direct and inverse Fourier transforms are

$$F(x, x') = \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{i\lambda x - i\mu x'} \langle h|\mathcal{F}(\lambda, \mu)|h\rangle, \quad \langle h|\mathcal{F}(\lambda, \mu)|h\rangle = \int dx dx' e^{-i\lambda x + i\mu x'} F(x, x'). \tag{4}$$

The variables x and x' are the momentum fractions of incoming ϕ and outgoing ϕ^* partons, respectively. The restrictions on their physically allowed values come from the support properties of the multiparton distribution functions discussed at length in Ref. [19], namely, $F(x, x')$ vanishes unless $0 \leq x \leq 1$, $0 \leq x' \leq 1$.

Beyond the leading twist level the intuitive parton-like picture is not so immediate as one usually starts with overcomplete set of correlation functions. However, the point is that the equations of motion for field operators imply several relations between correlators and the problem of construction of the simpler operator basis is reduced to appropriate exploiting of these equalities. The guiding line to disentangle the twist structure is clearly seen in the light-cone formalism of Kogut and Soper [18]. Consider, for instance, the correlators containing two quarks $\bar{\psi}\psi$. Then decomposing the Dirac field into "good" and "bad" components with Hermitian projection operators $\mathcal{P}_\pm = \frac{1}{2}\gamma_\mp\gamma_\pm$: $\psi_\pm = \mathcal{P}_\pm\psi$, we have three possible combinations $\psi_+^\dagger\psi_+$, $\psi_+^\dagger\psi_- \pm \psi_-^\dagger\psi_+$ and $\psi_-^\dagger\psi_-$, which are of the twist two, three and fore, respectively. The origin of this counting lies in the dynamical dependence of the "bad" components of the Dirac fermions

$$\psi_- = -\frac{i}{2}\partial_+^{-1}\left(i \not{D}_\perp + m\right)\gamma_+\psi_+. \tag{5}$$

These components depend on the underlying QCD dynamics, that is, they are implicitly involve extra partons and thus correspond to the generalized off-shell partons which possess the transverse momentum. For this reason we come back to the on-shell massless collinear partons of the naive parton model but supplied with multiparton correlations through the constraint (5). The operators constructed from the "good" components only were named quasipartonic [10]. The advantage of handling them is that they endow the theory with parton like interpretation for higher twists.

At the twist-3 level the nucleon has two chiral-odd distributions e and h_L [2] which can be measured in the polarized Drell-Yan and semi-inclusive DIS processes. Under QCD evolution they couple to complicated quark-gluon operators and correlation functions depending on the quark mass and intrinsic transverse momentum. We will treat below unpolarized and polarized cases separately.

2.1 Unpolarized distributions.

In the unpolarized case we define the following redundant basis chiral-odd twist-3 correlation functions:

$$e(x) = \frac{x}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h|\bar{\psi}(0)\psi(\lambda n)|h\rangle, \tag{6}$$

$$M(x) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h|\bar{\psi}(0) m\gamma_+ \psi(\lambda n)|h\rangle, \tag{7}$$

$$D_1(x, x') = \frac{1}{2}\int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{i\lambda x - i\mu x'} \langle h|\bar{\psi}(\mu n) \mathrm{g}\gamma_+ \not{B}^{\perp}(0)\psi(\lambda n)|h\rangle, \tag{8}$$

$$D_2(x', x) = \frac{1}{2}\int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{i\mu x' - i\lambda x} \langle h|\bar{\psi}(\lambda n) \mathrm{g}\not{B}^{\perp}(0)\gamma_+ \psi(\mu n)|h\rangle. \tag{9}$$

The functions D_1 and D_2 are related by complex conjugation $[D_1(x, x')]^* = D_2(x', x)$. The quantities determined by these equations form a closed set under renormalization, however, they are not independent since there is relation between them due to the equation of motion for the Heisenberg fermion field operator

$$e(x) - M(x) - \int dx' D(x, x') = 0, \tag{10}$$

where we have introduced the convention

$$D(x, x') = \frac{1}{2}\left[D_1(x, x') + D_2(x', x)\right]. \tag{11}$$

This function is real valued and antisymmetric with respect to the exchange of its arguments:

$$[D(x, x')]^* = D(x, x'), \qquad D(x, x') = -D(x', x). \tag{12}$$

In appendix B we present a set of RG equations for the correlation functions determined by Eqs. (6)-(9) derived in abelian gauge theory. The relation (10) provides a strong check of our

[2] In the language of OPE the local twist-3 as well as twist-2 operators contribute to the matrix elements of the distribution function h_L. Eq. (27) is a consequence of this fact.

calculations[3]. It allows to reduce the RG analysis to the study of scale dependence of the three-parton D and mass dependent M correlators only.

The function $D(x, x')$ is gauge variant provided we use the gauge other than light-cone, therefore, we are forced to introduce the gauge invariant quantity

$$Z(x, x') = (x - x')D(x, x'). \tag{13}$$

Using the advantages of the light-cone gauge, where the gluon field is expressed in terms of the field strength tensor (the residual gauge degrees of freedom are fixed by imposing an antisymmetric boundary conditions on the field which allows unique inversion):

$$B_\mu(\lambda n) = \partial_+^{-1} G_{+\mu}(\lambda n) = \frac{1}{2}\int_{-\infty}^{\infty} dz \epsilon(\lambda - z) G_{+\mu}(z), \tag{14}$$

and taking into account the relation

$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{\pm i\lambda x} \epsilon(\lambda - z) = \pm \frac{i}{2\pi} \mathrm{PV}\frac{1}{x} e^{\pm izx}, \tag{15}$$

we can easily obtain from Eqs. (8)-(9) the definition of the gauge invariant quantities in terms of three-particle string operators. Generically

$$Z(x, x') = \frac{1}{2}\int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{i\lambda x - i\mu x'} \langle h | \mathcal{Z}(\lambda, \mu) + \mathcal{Z}(-\mu, -\lambda) | h \rangle, \tag{16}$$

where

$$\mathcal{Z}(\lambda, \mu) \equiv \mathcal{Z}(\lambda, 0, \mu) = \frac{1}{2}\bar{\psi}(\mu n) \mathrm{g} G_{+\rho}(0) \sigma^{\perp}_{\rho +} \psi(\lambda n). \tag{17}$$

And in the same way for mass dependent nonlocal string operator

$$\mathcal{M}^j(\lambda) \equiv \mathcal{M}^j(\lambda, 0) = \frac{m}{2}\bar{\psi}(0)\gamma_+ (iD_+(\lambda))^j \psi(\lambda n) \tag{18}$$

the Fourier transform is

$$M^j(x) \;=\; x^j M(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h | \mathcal{M}^j(\lambda) | h \rangle. \tag{19}$$

For the spin dependent scattering discussed below the only difference is that one should insert also γ_5-matrix between the fields in the definitions of the string operators (17), (18).

[3]This fact follows from general renormalization properties of gauge invariant operators as one expects that the counter term for the equation of motion operator can be given only by the operator itself. Its matrix element being taken with respect to the physical state decouples completely from the renormalization group evolution.

2.2 Polarized distributions.

Analogously, the set of correlation functions for the polarized case is as follows:

$$h_1(x) = \frac{1}{2} S_\sigma^\perp \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h|\bar{\psi}(0) i\sigma_{+\sigma}^{\perp} \gamma_5 \psi(\lambda n)|h\rangle, \tag{20}$$

$$h_L(x) = \frac{x}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h|\bar{\psi}(0) i\sigma_{+-} \gamma_5 \psi(\lambda n)|h\rangle, \tag{21}$$

$$\widetilde{M}(x) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h|\bar{\psi}(0) m\gamma_+ \gamma_5 \psi(\lambda n)|h\rangle, \tag{22}$$

$$K(x) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h|\bar{\psi}(0) i\gamma_+ \not{\partial}_\perp \gamma_5 \psi(\lambda n)|h\rangle, \tag{23}$$

$$\widetilde{D}_1(x, x') = \frac{1}{2} \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{i\lambda x - i\mu x'} \langle h|\bar{\psi}(\mu n) \mathrm{g}\gamma_+ \not{B}^\perp(0) \gamma_5 \psi(\lambda n)|h\rangle, \tag{24}$$

$$\widetilde{D}_2(x', x) = \frac{1}{2} \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{i\mu x' - i\lambda x} \langle h|\bar{\psi}(\lambda n) \mathrm{g}\gamma_+ \not{B}^\perp(0) \gamma_5 \psi(\mu n)|h\rangle, \tag{25}$$

where $S_\sigma^\perp$ denotes the transverse polarization vector of the hadron h ($S^2 = -M_h^2$). The derivative in the correlation function $K(x)$ acts on the quark field before setting its argument on the light cone.

Besides the identity arising from the equation of motion

$$h_L(x) - \widetilde{M}(x) - K(x) - \int dx' \widetilde{D}(x, x') = 0, \tag{26}$$

there is an equation provided by Lorentz invariance

$$2x h_1(x) = 2h_L(x) - x \frac{\partial}{\partial x} K(x) - 2x \int dx' \frac{\widetilde{D}(x, x')}{(x' - x)}. \tag{27}$$

It means that both parts of this equality are expressed in terms of matrix elements of different components of one and the same twist-2 tensor operator. Again we have introduced the C-even quantity $\widetilde{D}$ which has the properties

$$\left[\widetilde{D}(x, x')\right]^* = \widetilde{D}(x, x'), \quad \widetilde{D}(x, x') = \widetilde{D}(x', x). \tag{28}$$

Combining the two equations (26), (27) we can obtain the following relation between correlators

$$\left(2 - x\frac{\partial}{\partial x}\right) h_L(x) = 2x h_1(x) - x\frac{\partial}{\partial x} \widetilde{M}(x) + \int dx' \frac{x}{x' - x} \left[\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right] \widetilde{Z}(x, x'), \tag{29}$$

where $\widetilde{Z}(x, x')$ is a gauge-invariant quantity introduced by Eq. (13), but for function $\widetilde{D}(x, x')$. Solving the differential equation with respect to $h_L(x)$ the integration constant can be found from the support properties of the distribution: $h_L(x) = 0$ for $|x| \geq 1$. The solution is

$$h_L(x) = 2x^2 \int_x^1 \frac{d\beta}{\beta^2} h_1(\beta) + \widetilde{M}(x) - 2x^2 \int_x^1 \frac{d\beta}{\beta^3} \widetilde{M}(\beta) + x^2 \int_x^1 \frac{d\beta}{\beta^2} \int \frac{d\beta'}{\beta' - \beta} \left[\frac{\partial}{\partial \beta} - \frac{\partial}{\partial \beta'}\right] \widetilde{Z}(\beta, \beta'). \tag{30}$$

Similar relation has been found by Jaffe and Ji[4] in Ref. [5]. Here the dynamical twist-3 contribution is explicitly related to the particular integral of the three-parton correlation function $\widetilde{Z}$. In terms local operators it looks like

$$(n+3)[h_L]_n = 2[h_1]_{n+1} + (n+1)\widetilde{M}_n + \sum_{l=1}^{n}(n-l+1)\widetilde{Z}_n^l, \tag{31}$$

and the definition of moments of distribution functions is given in appendix C by Eq. (C.1).

As before excluding the functions (21), (23) using the relations (26)-(27) we can chose the basis of independent functions in the form: $h_1(x)$, $\widetilde{M}(x)$, $\widetilde{D}(x, x')$.

2.3 Evolution equations.

Note that in leading logarithmic approximation the evolution equations that govern the Q^2-dependence of the three-particle correlation functions are the same discarding the mixing with the quark mass operator. Therefore we omit the "tilde"-sign in what follows. We evaluate the evolution equations using different approaches described in [9, 10] and [13], respectively, and apart from general remarks we will not go into details.

The peculiar feature of the light-like gauge is the presence of spurious IR pole $1/k_+$ in the density matrix of the gluon propagator:

$$\begin{aligned} D_{\mu\nu}(k) &= \frac{d_{\mu\nu}(k)}{k^2 + i0} \\ d_{\mu\nu} &= g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{k_+}. \end{aligned} \tag{32}$$

The central question is how to handle this unphysical pole when $k_+ = 0$. In the calculation of the relevant Feynman diagrams shown in Fig. 1 (and trivial self-energy insertions into external legs) we assume two different approaches which employ the principal value (PV) and Mandelstam-Leibbrandt prescriptions (ML) [20]:

$$\mathrm{PV}\frac{1}{k_+} = \frac{1}{2}\left\{\frac{1}{(kn)+i0} + \frac{1}{(kn)-i0}\right\}, \tag{33}$$

$$\mathrm{ML}\frac{1}{k_+} = \frac{(kn^*)}{(kn)(kn^*)+i0} \tag{34}$$

with arbitrary four-vector n^* satisfying $n^{*2} = 0$, $nn^* = 1$ (without lost of generality we can put it equal to p). The first prescription will be used in the momentum space [9, 10, 21] while the second one in the coordinate space formulation [13]. As a byproduct we verify that both of them do lead to the same result.

[4]Corresponding expressions in Ref. [5] contain misprints.

In the light-cone fraction representation we get for the correlation function $D(x,x')$:

$$\dot{D}\,(x,x') = -\frac{\alpha}{2\pi}\Bigg\{-C_F\frac{(x-x')}{xx'}\left[x'M(x)\Theta^0_{11}(x',x'-x) \pm xM(x')\Theta^0_{11}(x,x-x')\right]$$
$$+\int d\beta\Bigg(C_F D(\beta,x')\frac{x}{x'}\Theta^0_{11}(x,x-x') + \frac{C_A}{2}\Bigg([D(\beta,x')-D(x,x')]\frac{x}{(x-\beta)}\Theta^0_{11}(x,x-\beta)$$
$$+[D(\beta+x',x')-D(x,x')]\frac{(x-x')}{(x-x'-\beta)}\Theta^0_{11}(x-x',x-x'-\beta)$$
$$+\frac{(\beta+x-x')}{x'}\left(D(\beta,x')\frac{x}{(x'-x)}\Theta^0_{11}(x,x-\beta) + D(\beta+x',x')\Theta^0_{11}(x-x',x-x'-\beta)\right)\Bigg)$$
$$+\left(C_F-\frac{C_A}{2}\right)\Bigg(D(\beta,x')\frac{(\beta+x-x')}{(x'-x)}\Theta^0_{111}(x,x-x',x-x'+\beta)$$
$$+[D(\beta,x'-x+\beta)-D(x,x')]\frac{x}{x-\beta}\Theta^0_{11}(x,x-\beta)\Bigg)\Bigg)$$
$$+\int d\beta'\Bigg(C_F D(x,\beta')\frac{x'}{x}\Theta^0_{11}(x',x'-x) + \frac{C_A}{2}\Bigg([D(x,\beta')-D(x,x')]\frac{x'}{(x'-\beta')}\Theta^0_{11}(x',x'-\beta')$$
$$+[D(x,\beta'+x)-D(x,x')]\frac{(x'-x)}{(x'-x-\beta')}\Theta^0_{11}(x'-x,x'-x-\beta')$$
$$+\frac{(\beta'+x'-x)}{x}\left(D(x,\beta')\frac{x'}{(x-x')}\Theta^0_{11}(x',x'-\beta') + D(x,\beta'+x)\Theta^0_{11}(x'-x,x'-x-\beta')\right)\Bigg)$$
$$+\left(C_F-\frac{C_A}{2}\right)\Bigg(D(x,\beta')\frac{(\beta'+x'-x)}{(x'-x)}\Theta^0_{111}(x',x'-x,x'-x+\beta')$$
$$+[D(x-x'+\beta',\beta')-D(x,x')]\frac{x'}{x'-\beta'}\Theta^0_{11}(x',x'-\beta')\Bigg)\Bigg) - \frac{3}{2}C_F D(x,x')\Bigg\}, \qquad (35)$$

and for mass dependent correlation function we have

$$\dot{M}(x) = -C_F\frac{\alpha}{2\pi}\int d\beta M(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + \frac{\beta+x}{\beta}\Theta^0_{11}(x,x-\beta)\right\}, \qquad (36)$$

where we have used the dot as shorthand for the logarithmic derivative with respect to the renormalization scale $\dot{} = \mu_R^2\partial/\partial\mu_R^2$ and the standard plus-prescription fulfilling $\int dx[...]_+ = 0$ (for definition see Eq. (B.4) of appendix B). The kernel of the last equation resembles the nonsinglet splitting function of (non)polarized scattering up to the additional term $\frac{\alpha}{2\pi}\frac{3}{2}C_F\delta(x-\beta)$ which is the one-loop renormalization constant of the quark mass taken with minus sign. An explicit form of the Θ-functions is given in the appendix A. Throughout the paper the plus and minus signs in the mass-operator term correspond to the functions D (e) and $\widetilde{D}$ (h_L), respectively.

For the string operators (or their matrix elements) we obtain the following compact RG equation:

$$\dot{\mathcal{Z}}(\lambda,\mu) \;=\; \frac{\alpha}{2\pi}\int_0^1 dy\int_0^{\bar{y}} dz\Big\{C_F\bar{y}^2\delta(z)\left[\mathcal{M}^1(\lambda-\mu y) \pm \mathcal{M}^1(\lambda y-\mu)\right]$$

$$
\begin{aligned}
&+ \frac{C_A}{2}\left[2\bar{z} + [N(y,z)]_+ - \frac{7}{4}\delta(\bar{y})\delta(z)\right]\left[\mathcal{Z}(\lambda y, \mu - \lambda z) + \mathcal{Z}(\lambda - \mu z, \mu y)\right] \\
&+ \left(C_F - \frac{C_A}{2}\right)\left[\left[[L(y,z)]_+ - \frac{1}{2}\delta(y)\delta(z)\right]\mathcal{Z}(\lambda\bar{z} + \mu z, \mu\bar{y} + \lambda y)\right. \\
&- \left.\left. 2z\left[\mathcal{Z}(-\lambda y, \mu - \lambda\bar{z}) + \mathcal{Z}(\lambda - \mu\bar{z}, -\mu y)\right]\right]\right\}, \qquad (37)
\end{aligned}
$$

with

$$
\begin{aligned}
&[N(y,z)]_+ = N(y,z) - \delta(\bar{y})\delta(z)\int_0^1 dy' \int_0^{\bar{y}'} dz' N(y',z'), \quad N(y,z) = \delta(\bar{y} - z)\frac{y^2}{\bar{y}} + \delta(z)\frac{y}{\bar{y}}, \\
&[L(y,z)]_+ = L(y,z) - \delta(y)\delta(z)\int_0^1 dy' \int_0^{\bar{y}'} dz' L(y',z'), \quad L(y,z) = \delta(y)\frac{\bar{z}}{z} + \delta(z)\frac{\bar{y}}{y}. \qquad (38)
\end{aligned}
$$

The equation written so far should be supplied by the following

$$
\dot{\mathcal{M}}^1(\lambda) = \frac{\alpha}{2\pi}C_F\int_0^1 dy\left\{\left[\frac{2}{\bar{y}}\right]_+ - 2 - y - y^2\right\}\mathcal{M}^1(\lambda y), \qquad (39)
$$

$$
\dot{h}_1(\lambda) = \frac{\alpha}{2\pi}C_F\int_0^1 dy\left\{\left[\frac{2}{\bar{y}}\right]_+ - 2 + \frac{3}{2}\delta(\bar{y})\right\}h_1(\lambda y). \qquad (40)
$$

The last one when transformed to the momentum space using the formulae of the next section coincides with the result obtained in Ref. [22].

3 From light-cone position to light-cone fraction representations.

Having at hand the evolution equations in different representation for the same quantities it is instructive to relate the kernels in both cases. Such a bridge can be easily established using the Fourier transformation for the parton distribution functions given by Eqs. (2), (4).

First, we come to more simple case of the two-particle correlation functions $\mathcal{F}$. The general form of the evolution equation in the light-cone position space is of the following generic form

$$
\dot{\mathcal{F}}(\lambda) = \int_0^1 dy \mathcal{K}(y)\mathcal{F}(\lambda y), \qquad (41)
$$

where $\mathcal{K}(y)$ is a evolution kernel in the coordinate space. By exploiting the definitions (2) we can recast the Fourier transform on the language of two-particle evolution kernels. In this way we find the direct transformation

$$
K(x,\beta) = \int_0^1 dy \mathcal{K}(y)\delta(x - y\beta). \qquad (42)
$$

And using the general formula

$$
\int_0^1 dy f(y)\delta(x - y\beta) = f\left(\frac{x}{\beta}\right)\Theta^0_{11}(x, x - \beta). \qquad (43)
$$

with familiar Θ-function of the momentum space formulation, we can observe that the RG equations for two-parton correlators derived in previous section are indeed coincide. The inverse transformation can be done

$$\int \frac{dx d\beta}{2\pi} e^{-i\lambda x + i\mu\beta} K(x,\beta) = \int_0^1 dy \mathcal{K}(y)\delta(\mu - y\lambda) \tag{44}$$

with the help of the formula

$$\int \frac{dx d\beta}{2\pi} f\left(\frac{x}{\beta}\right) \Theta^0_{11}(x, x-\beta) e^{-i\lambda x + i\mu\beta} = \int_0^1 dy f(y)\delta(\mu - y\lambda). \tag{45}$$

Corresponding transformation for three-particle correlators is a little bit more involved. General form of the evolution equation for the light-cone string operator $\mathcal{Z}(\lambda,\mu)$ reads

$$\dot{\mathcal{Z}}(\lambda,\mu) = \int_0^1 dy \int_0^{\bar{y}} dz \mathcal{K}(y,z)\mathcal{Z}(\eta_{11}\lambda + \eta_{12}\mu, \eta_{21}\lambda + \eta_{22}\mu), \tag{46}$$

where η_{ij} are linear functions of the variables y, z. In the momentum fraction representation the evolution equation looks like

$$\dot{Z}(x,x') = \int d\beta d\beta' K(x,x',\beta,\beta')Z(\beta,\beta'). \tag{47}$$

Specifying the particular form of the functions η_{ij} we list below the corresponding conversion formulae.

For $C_A/2$-part of the evolution equation the Fourier transformation gives

$$K(x,x',\beta,\beta') = \delta(\beta' - x') \int_0^1 dy \int_0^{\bar{y}} dz \mathcal{K}(y,z)\delta(x - x'z - \beta y). \tag{48}$$

The particular contributions are

$$\mathcal{K}_1(y,z) = \delta(\bar{y})\delta(z) \stackrel{FT}{\to} K_1(x,x',\beta,\beta') = \delta(\beta - x)\delta(\beta' - x'), \tag{49}$$

$$\begin{aligned}\mathcal{K}_2(y,z) = \delta(z)\frac{y}{\bar{y}} + \delta(\bar{y} - z)\frac{y^2}{\bar{y}} \stackrel{FT}{\to} K_2(x,x',\beta,\beta') &= \delta(\beta' - x')\Big\{\frac{x}{(\beta - x)}\Theta^0_{11}(x, x-\beta) \\ &+ \frac{(x-x')^2}{(\beta - x)(\beta - x')}\Theta^0_{11}(x - x', x - \beta)\Big\},\end{aligned} \tag{50}$$

$$\begin{aligned}\mathcal{K}_3(y,z) = 1 \stackrel{FT}{\to} K_3(x,x',\beta,\beta') &= \delta(\beta' - x')\Xi_1(x, x-x', x-\beta) \\ &= -\delta(\beta' - x')\Theta^0_{111}(x, x-x', x-\beta),\end{aligned} \tag{51}$$

$$\begin{aligned}\mathcal{K}_4(y,z) = z \stackrel{FT}{\to} K_4(x,x',\beta,\beta') &= \delta(\beta' - x')\Big\{\frac{(x-\beta)}{x'}\Xi_1(x, x-x', x-\beta) \\ &+ \frac{\beta}{x'}\Xi_2(x, x-x', x-\beta)\Big\}.\end{aligned} \tag{52}$$

Where we have used (43) and the following useful formula

$$\begin{aligned}&\Xi_n(x, x-x', x-\beta) \equiv \int_0^1 dy y^n \Theta^0_{11}((x-\beta) + y\beta, (x-\beta) - y(x'-\beta)) \\ &= \frac{1}{n}\left[1 - \left(\frac{\beta - x}{\beta - x'}\right)^n\right]\Theta^0_{11}(x, x-x') + \frac{1}{n}\frac{\beta}{x'}\left[\left(\frac{\beta - x}{\beta - x'}\right)^n - \left(\frac{\beta - x}{\beta}\right)^n\right]\Theta^0_{11}(x, x-\beta).\end{aligned} \tag{53}$$

The analogous results for $(C_F - C_A/2)$-part of the kernel read

$$K(x,x',\beta,\beta') = \delta(\beta' - x') \int_0^1 dy \int_0^{\bar{y}} dz \mathcal{K}(y,z) \delta(x - x'\bar{z} + \beta y), \tag{54}$$

and in particular

$$\mathcal{K}_1(y,z) = z \overset{FT}{\to} K_1(x,x',\beta,\beta') = \delta(\beta' - x') \Big\{ \frac{(x' - x - \beta)}{x'} \Xi_1(x' - x, -x, x' - x - \beta) + \frac{\beta}{x'} \Xi_2(x' - x, -x, x' - x - \beta) \Big\}. \tag{55}$$

In addition we have

$$K(x,x',\beta,\beta') = \int_0^1 dy \int_0^{\bar{y}} dz \mathcal{K}(y,z) \delta(x - \beta\bar{z} + \beta' y) \delta(x' - \beta'\bar{y} + \beta z), \tag{56}$$

and

$$\mathcal{K}_1(y,z) = \delta(z) \frac{\bar{y}}{y} \overset{FT}{\to} K_1(x,x',\beta,\beta') = \delta(\beta - (\beta' - x' + x)) \frac{x'}{(\beta' - x')} \Theta^0_{11}(x', x' - \beta'). \tag{57}$$

These formulae complete the list of transformations. Collecting the particular contributions we can easily verify that the evolution equations given by Eqs. (35) and (37) agree with each other. It should be noted that it is enough to have at hand Eqs. (43), (53) only to perform the conversion from one representation to another.

4 Generalized DGLAP-type equations.

To study the large-N_c limit and to establish the relation to the evolution equation for the partially Mellin transformed operators introduced in Ref. [11, 14, 16] we proceed further to the generalized DGLAP representation of the evolution equation for tree-particles distributions first given in the second paper of Ref. [13]. For this purpose we define a new function Fourier transformed with respect to λ-variable only

$$z(x,u) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle h | \mathcal{Z}(\bar{u}\lambda, -u\lambda) \pm (u \to \bar{u}) | h \rangle, \tag{58}$$

which is even under charge conjugation and depends on the variables x and u. The latter has the meaning of the relative position of the gluon field on the light cone. For $0 \leq u \leq 1$ the gluon field lies between the two quark fields. Because of the support property $|x| \leq \mathrm{Max}(1, |2u - 1|)$, the variable x is then restricted to $|x| \leq 1$ and can be interpreted as an effective momentum fraction.

The evolution equation for $z(x,u)$ can be derived in a straightforward way from the RG equation (37) for the nonlocal string operator $\mathcal{Z}$. It can be presented in a form of the generalized

DGLAP-type equation:

$$\dot{z}(x,u) = \frac{\alpha_s}{2\pi}\int\frac{dy}{y}\int dv\Big\{P_{zz}(y,u,v)z\left(\frac{x}{y},v\right)+P_{zm}(y,u,v)m\left(\frac{x}{v}\right)\Big\}, \tag{59}$$

$$\dot{m}(x) = \frac{\alpha_s}{2\pi}\int\frac{dy}{y}P_{mm}(y)m\left(\frac{x}{y}\right). \tag{60}$$

Here, $m(x) = xM(x)$ and the integration region is determined by both the support of $z(x,u)$ and by the kernels

$$\begin{aligned} &P_{zz}(x,u,v) \\ &= \left(C_F - \frac{C_A}{2}\right)\left[\Theta_1(x,u,v)[L(x,u,v)]_+ - \Theta_2(x,u,v)M(x,u,v) - \frac{1}{4}\delta(u-v)\delta(\bar{x})\right] \\ &+\frac{C_A}{2}\Theta_3(x,u,v)\left[M(x,u,v)+[N(x,u,v)]_+ - \frac{7}{4}\delta(u-v)\delta(\bar{x})\right] + \begin{pmatrix} u\to\bar{u} \\ v\to\bar{v}\end{pmatrix}, \end{aligned} \tag{61}$$

$$P_{zm}(x,u,v) = C_F\bar{x}^2\theta(x)\theta(\bar{x})\frac{x}{v}\left[\delta(v-\bar{u}-xu)\pm\delta(v-u-x\bar{u})\right], \tag{62}$$

$$P_{mm}(x) = C_F\left[\left[\frac{2}{\bar{x}}\right]_+ - 2 - x - x^2\right], \tag{63}$$

where the auxiliary functions are defined by:

$$\begin{aligned} \Theta_1(x,u,v) &= \theta(x)\theta(u-xv)\theta(\bar{u}-x\bar{v}), \\ \Theta_2(x,u,v) &= \theta\left(-\frac{x\bar{v}}{\bar{u}}\right)\theta\left(\frac{1-xv}{\bar{u}}\right)\theta\left(\frac{x-u}{\bar{u}}\right), \\ \Theta_3(x,u,v) &= \theta\left(\frac{\bar{x}}{\bar{u}}\right)\theta\left(\frac{x\bar{v}}{\bar{u}}\right)\theta\left(\frac{xv-u}{\bar{u}}\right), \\ L(x,u,v) &= \frac{u^2}{v(v-u)}\delta(u-xv), \\ M(x,u,v) &= \frac{2x(1-xv)}{\bar{u}^3}, \\ N(x,u,v) &= \frac{\bar{v}\epsilon(\bar{u})}{\bar{u}(v-u)}\left[\frac{\bar{v}}{\bar{u}}\delta(\bar{x}) + \frac{u^2}{v}\delta(u-xv)\right]. \end{aligned} \tag{64}$$

The plus-prescription for arbitrary function A is defined by the equation

$$\begin{aligned} &\Theta_i(x,u,v)[A(x,u,v)]_+ \\ &= \Theta_i(x,u,v)A(x,u,v) - \delta(\bar{x})\delta(u-v)\int_0^1 dx'\int dv'\,\Theta_i(x',u,v')A(x',u,v'). \end{aligned} \tag{65}$$

Note that, due to the evolution, the variable u is no longer restricted to the region $0 \leq u \leq 1$.

Going further we introduce the Mellin transforms

$$z^n(u) = \int dx x^{n-1}z(x,u) \quad \text{and} \quad m^n = \int dx x^{n-1}m(x) = \int dx\, x^n M(x), \tag{66}$$

where n is complex angular momentum. Operators with different n do not mix with each other and satisfy the evolution equations

$$\dot{z}^n(u) = \frac{\alpha_s}{2\pi}\int dv \left\{ P^n_{zz}(u,v) z^n(v) + \delta(u-v)\left[P^n_{zm}(v) \pm P^n_{zm}(\bar{v})\right] m^n \right\}, \tag{67}$$

$$\dot{m}^n = \frac{\alpha_s}{2\pi} P^n_{mm} m^n. \tag{68}$$

The kernels are given by

$$P^n_{zz}(u,v) = \left(C_F - \frac{C_A}{2}\right)\left[\Theta_1(u,v)[L^n(u,v)]_+ - \Theta_2(u,v)M^n_1(u,v) - \frac{1}{4}\delta(u-v)\right]$$
$$+\frac{C_A}{2}\Theta_3(u,v)\left[M^n_2(u,v) + [N^n(u,v)]_+ - \frac{7}{4}\delta(u-v)\right] + \begin{pmatrix} u \to \bar{u} \\ v \to \bar{v} \end{pmatrix},$$

$$P^n_{zm}(v) = C_F \frac{2 - \bar{v}^n[2 + n(2 + (n+1)v)v]}{n(n+1)(n+2)v^3},$$

$$P^n_{mm} = -C_F\left(S_n + S_{n+2}\right), \tag{69}$$

where the auxiliary functions read

$$\Theta_1(u,v) = \theta(v-u), \quad \Theta_2(u,v) = \theta(-\bar{v})\theta(1-vu), \quad \Theta_3(u,v) = \theta(\bar{v})\theta(v-u). \tag{70}$$

$$L^n(u,v) = \frac{\epsilon(v)}{v-u}\left(\frac{u}{v}\right)^{n+1},$$
$$M^n_1(u,v) = \frac{2}{\bar{u}^3}\left\{\frac{1}{n+1}\left[\frac{1}{v^{n+1}} - u^{n+1}\right] - \frac{v}{n+2}\left[\frac{1}{v^{n+2}} - u^{n+2}\right]\right\},$$
$$M^n_2(u,v) = \frac{2}{\bar{u}^3}\left\{\frac{1}{n+1}\left[1 - \left(\frac{u}{v}\right)^{n+1}\right] - \frac{v}{n+2}\left[1 - \left(\frac{u}{v}\right)^{n+2}\right]\right\},$$
$$N^n(u,v) = \frac{\bar{v}\epsilon(\bar{u})}{\bar{u}(v-u)}\left\{\frac{\bar{v}}{\bar{u}} + \epsilon(v)\left(\frac{u}{v}\right)^{n+1}\right\}. \tag{71}$$

And the plus-prescription is defined in the following way

$$\Theta_i(u,v)[A^n(u,v)]_+ = \Theta_i(u,v)A^n(u,v) - \delta(u-v)\int dv' \Theta_i(u,v')A^n(u,v'). \tag{72}$$

It is not difficult to observe that in multicolour limit (67) is exactly reduced to the equation of Ref. [16] which was the starting point of their analysis. However, we will start from Eq. (59) in the mixed representation and show in the next section that in the multicolour limit the generalized splitting functions can be diagonalized.

5 Solution of the evolution equations in multicolour QCD.

This section is devoted to the solution of the evolution equations we have derived in the previous sections for the twist-3 correlation functions. First of all, we perform an extensive numerical study of the exact equation (67). For simplicity we restrict ourselves to the homogeneous case only, *i.e.* we discard the quark-mass operator which is certainly justified assumption for the light u- and d-quark species. The solution we are interested in is given in terms of the eigenvalues and eigenfunctions of the anomalous dimension matrix ${}_{\mathcal{Z}\mathcal{Z}}\gamma^l_n$ of the local operators $\mathcal{Z}^l_n$ calculated in appendix C. The eigenvalue problem we have attacked has no analytical solution, however, the diagonalization can be done numerically for moderately large orbital momentum n, *e.g.* $n \leq 100$, which is quite enough for practical purposes. Second, we provide the analytical solution of the generalized DGLAP equation in the multicolour limit of QCD. In this case it reduces to the familiar ladder-type equation which holds for the twist-2 operators. It will be shown that such a reduction also occurs in the limit $x \to 1$ too. Assembling both results allows us to construct the improved equations for two-quark twist-3 distributions. Exploiting some examples of the evolution for the moments we confront these two approaches. In particular, we study the accuracy of improved DGLAP equation as compared to exact evolution supplied with different models of gluon light-cone position distribution for the three-particle correlation function at low momentum scale.

5.1 Evolution of the moments.

To obtain the solution of the evolution equation (67) we choose n as a positive integer. In this case, as follows from the definition (58) of $\mathcal{Z}^n(u)$ the n-th moment is actually given by the following linear combination of local operators $\mathcal{Z}^l_n$ (see Eq. (C.1)):

$$\mathcal{Z}^n(u) = \sum_{l=1}^{n} C^{l-1}_{n-1}\, u^{n-l}\bar{u}^{l-1}\mathcal{Z}^l_n, \qquad (73)$$

so that $\mathcal{Z}^n(u)$ is a polynomial of degree $n-1$ in u. Thus the kernel $P^n_{\mathcal{Z}\mathcal{Z}}(u,v)$ possesses n polynomial eigenfunctions $e^n_l(v)$:

$$\int dv\, P^n_{\mathcal{Z}\mathcal{Z}}(u,v) e^n_l(v) = -\lambda^n_l e^n_l(u), \quad l = 1, \dots, n, \qquad (74)$$

where $-\lambda^n_l$ denotes the eigenvalues. These eigenfunctions can be constructed by diagonalization

$$C^{k-1}_{n-1} \int dv\, P^n_{\mathcal{Z}\mathcal{Z}}(u,v) v^{n-k}\bar{v}^{k-1} \;=\; \sum_{l=1}^{n} C^{l-1}_{n-1}\, {}_{\mathcal{Z}\mathcal{Z}}\gamma^n_{lk}\, u^{n-l}\bar{u}^{l-1}, \qquad (75)$$

where the anomalous dimension matrix ${}_{\mathcal{Z}\mathcal{Z}}\gamma^n_{lk}$ of the local operators is given by Eq. (C.8) in Appendix C. Actually, this is a purely algebraic task and we find

$$e^n_k(u) = \sum_{l=1}^{n} C^{l-1}_{n-1}\, u^{n-l}\bar{u}^{l-1} E^n_{lk}, \text{ with } \left\{ (E^n)^{-1}\, {}_{\mathcal{Z}\mathcal{Z}}\gamma^n E^n \right\}_{kl} = -\lambda^n_k \delta(k-l), \qquad (76)$$

where $\delta(k-l)$ is a Kronequer symbol defined by eq. (C.9). The spectrum of the eigenvalues λ_l^n up to $n=50$ is shown in Fig. 2 (a) together with some examples of the eigenfunctions of the kernel $P^n_{\mathcal{Z}\mathcal{Z}}$ for particular orbital momenta. The solution for the moments $\mathcal{Z}^n(u)$ (in the massless case) is then expressed in terms of the eigenfunctions and eigenvalues we have found:

$$\mathcal{Z}^n\left(u,Q^2\right)=\sum_{l=1}^{n}c_l^n\left(Q_0^2\right)e_l^n(u)\exp\left\{-\int_{Q_0^2}^{Q^2}\frac{dt}{t}\frac{\alpha_s(t)}{2\pi}\lambda_l^n\right\}. \tag{77}$$

The coefficients $c_l^n\left(Q_0^2\right)$ at the reference momentum squared Q_0^2 have to be determined from the nonperturbative input $\mathcal{Z}^n\left(u,Q_0^2\right)$:

$$c_l^n\left(Q_0^2\right)=\sum_{k=1}^{n}(E_{lk}^n)^{-1}\frac{(n-k)!}{(n-1)!}\frac{d^{k-1}}{dw^{k-1}}(1+w)^{n-1}\mathcal{Z}^n\left(\frac{1}{1+w},Q_0^2\right)_{|w=0}. \tag{78}$$

5.2 Reduction of the evolution equations.

In the large-N_c limit only the planar diagrams (Fig. 1 (a, d)) survive and the kernel $P^n_{\mathcal{Z}\mathcal{Z}}(u,v)$ has two known dual eigenfunctions: 1 and $1-2u$, so that $\int_0^1 du\, e_l^n(u)=\delta_{l1}+\mathcal{O}(1/N_c)$ and $\int_0^1 du\,(1-2u)e_l^n(u)=\delta_{l2}+\mathcal{O}(1/N_c)$, where $l=1,2$ correspond to the lowest two eigenvalues of the spectrum shown in Fig. 2 (a). A straightforward calculation provides for the following DGLAP evolution kernels

$$\int_0^1 du\left\{\begin{array}{c}1\\ \frac{1-2u}{1-2v}\end{array}\right\}P_{\mathcal{Z}\mathcal{Z}}(x,u,v)=N_c\,\theta(\bar{x})\theta(x)\left\{\begin{array}{c}\left[\frac{x^2}{\bar{x}}\right]_+ +\frac{1}{2}x^2-\frac{5}{4}\delta(\bar{x})\\ \left[\frac{x^2}{\bar{x}}\right]_+ -\frac{3}{2}x^2-\frac{5}{4}\delta(\bar{x})\end{array}\right\}+\mathcal{O}\left(\frac{1}{N_c}\right), \tag{79}$$

and for the mass-mixing kernels the exact results read

$$\begin{aligned}
&\int_0^1 du\left\{\begin{array}{c}1\\ \frac{1-2u}{1-2v}\end{array}\right\}P_{\mathcal{Z}m}(x,u,v)=C_F\,\theta(\bar{x})\theta(x)\left\{\begin{array}{c}x(2-x)\\ 0\end{array}\right\}\quad\text{for e},\\
&\int_0^1 du\left\{\begin{array}{c}1\\ \frac{1-2u}{1-2v}\end{array}\right\}P_{\mathcal{Z}m}(x,u,v)=C_F\,\theta(\bar{x})\theta(x)\left\{\begin{array}{c}0\\ x(2-3x)\end{array}\right\}\quad\text{for }\widetilde{h}_L.
\end{aligned} \tag{80}$$

As has been first observed in Ref. [14] in the context of the chiral-even distribution $g_2(x)$ similar equations hold true also for the $\frac{1}{N_c}$-suppressed terms in the $x\to 1$ limit for flavour nonsinglet twist-3 evolution kernels. In the present chiral-odd case we find

$$\int_0^1 du\left\{\begin{array}{c}1\\ \frac{1-2u}{1-2v}\end{array}\right\}P_{\mathcal{Z}\mathcal{Z}}(x,u,v)=-\frac{1}{N_c}\theta(\bar{x})\theta(x)\left\{\begin{array}{c}\left[\frac{1}{\bar{x}}\right]_+ +\frac{5}{4}\delta(\bar{x})+\mathcal{O}(\bar{x}^0)\\ \left[\frac{1}{\bar{x}}\right]_+ +\frac{19}{12}\delta(\bar{x})+\mathcal{O}(\bar{x}^0)\end{array}\right\}+N_c\cdots, \tag{81}$$

where the $N_c\cdots$ symbolize the $x\to 1$ limit of Eq. (79).

The eigenfunctions we have obtained coincide precisely with the coefficients which appear in the decomposition of $e(x,Q^2)$ and $\widetilde{h}_L(x,Q^2)$ in terms of three-particle correlation functions. To observe this explicitly we need the relations similar to ones given by Eqs. (10) and (30) transformed to the mixed representation (58). Namely, we have

$$e(x) = \frac{1}{x} m(x) - \frac{1}{2} \frac{d}{dx} \int_0^1 du\, z(x,u), \qquad (82)$$

$$\bar{h}_L(x) = \frac{1}{x} \widetilde{m}(x) - \frac{1}{2} \frac{d}{dx} \int_0^1 du\, (1-2u) \widetilde{z}(x,u), \qquad (83)$$

where we introduce for convenience a new function $\bar{h}_L(x)$, so that $h_L(x)$ reads:

$$h_L(x) = 2 \int_x^1 dy\, \frac{x^2}{y^2} h_1(y) - x \frac{d}{dx} \int_x^1 \frac{dy}{y} \frac{x^2}{y^2} \bar{h}_L(y). \qquad (84)$$

For the massless case the last term on the RHS coincides with the twist-3 part $\widetilde{h}_L$.

From the observations we have made above it follows that in the large-N_c as well as in the large-x limit the twist-3 distributions satisfy the DGLAP evolution equations. By combining the large-N_c evolution together with the large-x result for the $\frac{1}{N_c}$-suppressed terms we can improve the accuracy of such an approximation within the factor 5-10 as compared with multicolour limit taken alone (see Fig. 2 (b)). Thus, the functions $e(x, Q^2)$ and $\bar{h}_L(x, Q^2)$ obey the following improved evolution equations

$$\left\{ \begin{array}{c} \dot{e}(x) \\ \dot{\bar{h}}_L(x) \end{array} \right\} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left\{ \begin{array}{c} P_{ee}(y)\, e\left(\frac{x}{y}\right) + \left[\frac{1}{x}\{P_{mm}(y) - P_{ee}(y)\} - \frac{1}{2} P_{em}(y) \frac{d}{dx}\right] m\left(\frac{x}{y}\right) \\ P_{\bar{h}\bar{h}}(y)\, \bar{h}_L\left(\frac{x}{y}\right) + \left[\frac{1}{x}\{P_{mm}(y) - P_{\bar{h}\bar{h}}(y)\} - \frac{1}{2} P_{\bar{h}m}(y) \frac{d}{dx}\right] m\left(\frac{x}{y}\right) \end{array} \right\}, \qquad (85)$$

with

$$P_{ee}(y) = 2C_F \left[\frac{y}{\bar{y}}\right]_+ + \frac{C_A}{2} y + \left(\frac{C_F}{2} - C_A\right) \delta(\bar{y}) + \mathcal{O}\left(\bar{y}^0/N_c\right),$$

$$P_{\bar{h}\bar{h}}(y) = 2C_F \left[\frac{y}{\bar{y}}\right]_+ - \frac{3C_A}{2} y + \left(\frac{7C_F}{6} - \frac{4C_A}{3}\right) \delta(\bar{y}) + \mathcal{O}\left(\bar{y}^0/N_c\right), \qquad (86)$$

$$P_{em}(y) = C_F x(2-x),$$

$$P_{\bar{h}m}(y) = C_F x(2-3x). \qquad (87)$$

The evolution kernel for the mass-dependent correlator was already given by Eq. (63). In Eq. (86) we have added subleading terms of order $\mathcal{O}\left(\bar{y}^0/N_c\right)$ (which we do not specify here) so that the first moment of each kernel coincides with the corresponding eigenvalue of the kernel $P^n_{\mathcal{Z}\mathcal{Z}}(u,v)$. This guarantees that the solution for the lowest moments given below in Eq. (94) will be reproduced exactly. Note that in the massless case $\widetilde{h}_L(x)$ fulfills the same evolution equation as $\bar{h}_L(x)$. The simplest way to verify this is to make the Mellin transform of the corresponding evolution equations.

Let us add a few remarks on the momentum space formulation. As we have seen above the solution of the evolution equations in the asymptotical regimes is the most straightforward in the light-cone position representation. And it is by no means trivial to observe the appearance of the DGLAP equations in momentum fraction representation. However, inspired by our knowledge acquired from the coordinate space where the asymptotic solution was given by the convolution of the three-particle correlation function with certain weight function which is essentially the same as entering into the equation that gives rise to the twist-3 contribution to the two-parton correlators at the tree level, we are able to check that the integrals

$$e(x) = \int d\beta' D(x,\beta'), \tag{88}$$

$$\widetilde{h}_L(x) = x^2 \int_x^1 \frac{d\beta}{\beta^2} \int \frac{d\beta'}{\beta'-\beta} \left\{ 2 + (\beta-\beta') \left[\frac{\partial}{\partial\beta} - \frac{\partial}{\partial\beta'} \right] \right\} \widetilde{D}(\beta,\beta') \tag{89}$$

arisen from Eqs. (10), (30) with quark-mass as well as twist-2 effects neglected satisfy the DGLAP equations, namely

$$\dot{e}(x) = -\frac{\alpha}{4\pi} N_c \int d\beta e(\beta) \left\{ 2 \left[\frac{\beta}{(x-\beta)} \Theta^0_{11}(x, x-\beta) \right]_+ + \left(2 - \frac{x}{\beta} \right) \Theta^0_{11}(x, x-\beta) - \frac{1}{2}\delta(\beta - x) \right\}, \tag{90}$$

$$\dot{\widetilde{h}}_L(x) = -\frac{\alpha}{4\pi} N_c \int d\beta \widetilde{h}_L(\beta) \left\{ 2 \left[\frac{\beta}{(x-\beta)} \Theta^0_{11}(x, x-\beta) \right]_+ + \left(2 + 3\frac{x}{\beta} \right) \Theta^0_{11}(x, x-\beta) - \frac{1}{2}\delta(\beta - x) \right\}. \tag{91}$$

The corresponding anomalous dimensions equal

$$\dot{[e]}_n = \frac{\alpha}{4\pi} N_c \left\{ -2\psi(n+2) - 2\gamma_E + \frac{1}{2} + \frac{1}{n+2} \right\} [e]_n, \tag{92}$$

$$\dot{[\widetilde{h}_L]}_n = \frac{\alpha}{4\pi} N_c \left\{ -2\psi(n+2) - 2\gamma_E + \frac{1}{2} - \frac{3}{n+2} \right\} [\widetilde{h}_L]_n. \tag{93}$$

Which are exactly the anomalous dimensions $\gamma_n^\pm$ found in Ref. [16] for e and h_L, respectively, with replacement $n \to j-1$[5].

5.3 Examples of the evolution.

For the lowest few moments the evolution equation (67) can be solved exactly. Taking into account the symmetry properties of the quark-gluon correlation function we obtain the following results for the (nonvanishing) first two moments

$$z^1\left(u, Q^2\right) = z^1\left(Q_0^2\right) \exp\left\{ -\frac{55}{18} \int_{Q_0^2}^{Q^2} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \right\},$$

[5]The difference in the anomalous dimensions is due to an extra power of the momentum fraction x included in the definition of the twist-3 correlation functions.

$$
\begin{aligned}
z^2\left(u,Q^2\right) &= z^2\left(Q_0^2\right)\exp\left\{-\frac{73}{18}\int_{Q_0^2}^{Q^2}\frac{dt}{t}\frac{\alpha_s(t)}{2\pi}\right\} \\
\widetilde{z}^2\left(u,Q^2\right) &= \widetilde{z}^2\left(Q_0^2\right)(1-2u)\exp\left\{-\frac{52}{9}\int_{Q_0^2}^{Q^2}\frac{dt}{t}\frac{\alpha_s(t)}{2\pi}\right\}, \\
\widetilde{z}^3\left(u,Q^2\right) &= \widetilde{z}^3\left(Q_0^2\right)(1-2u)\exp\left\{-\frac{1099}{180}\int_{Q_0^2}^{Q^2}\frac{dt}{t}\frac{\alpha_s(t)}{2\pi}\right\},
\end{aligned}
\tag{94}
$$

where $z^n\left(Q_0^2\right)$ are related to the following matrix elements of the local operators (up to normalization)

$$
\begin{aligned}
z^1\left(Q_0^2\right) &= Z_1^1\left(Q_0^2\right) = \langle h|\mathcal{Z}_1^1|h\rangle_{|\mu^2=Q_0^2}, \quad z^2\left(Q_0^2\right) = Z_2^1\left(Q_0^2\right) = \langle h|\mathcal{Z}_2^1|h\rangle_{|\mu^2=Q_0^2}, \\
\widetilde{z}^2\left(Q_0^2\right) &= \widetilde{Z}_2^1\left(Q_0^2\right) = \langle h|\widetilde{\mathcal{Z}}_2^1|h\rangle_{|\mu^2=Q_0^2}, \quad \widetilde{z}^3\left(Q_0^2\right) = \widetilde{Z}_3^1\left(Q_0^2\right) = \langle h|\widetilde{\mathcal{Z}}_3^1|h\rangle_{|\mu^2=Q_0^2}.
\end{aligned}
\tag{95}
$$

Thus the Q^2-dependence for the first moments of $e(x)$ and $\widetilde{h}_L(x)$ can be predicted uniquely since the initial values at the low momentum scale are given by these moments themselves. For larger n the evolution is sensitive to the shape of the gluon distribution between the quark fields. However, as can be traced from Figs. 3 and 4 the dependence on the assumed different toy models of the light-cone position distribution at $Q_0^2 = 1\mathrm{GeV}^2$ is quite small: the typical relative deviations for the moments of the twist-3 non- and polarized structure functions are 2% and 5%, respectively, at $Q^2 = 100\mathrm{GeV}^2$. In the calculations we have set the number of flavours $N_f = 3$ and $\Lambda_{QCD} = 0.25\mathrm{GeV}$. For the "gap"-type (end-point concentrated) distributions (see Figs. 3, 4 dashed line) the deviation is small as compared to the "coefficient function"-type model (1 and $1-2u$, solid lines) predictions, while for the "hump" (end-point suppressed) distributions (dashed-dotted line) it is a little bit larger. The accuracy of the multicolour approximation is about 15-20% at a scale $Q^2 = 100\mathrm{GeV}^2$. These numbers are natural and can be expected from the discrepancy between the DGLAP anomalous dimensions and the exact lowest two eigenvalues of the spectrum (see Fig. 2 (b)). Nevertheless, there is one very important exception from the naive expectation. If the initial gluon distribution is strongly suppressed in the end-point region, for instance, as $u^{[(n-1)/2]}$ for $u \to 0$, then the large-N_c approximation breaks down for large n (polarized moments are more sensitive than unpolarized ones). In this case the evolution is not smooth. The shape of this function will be turned immediately into the end-point concentrated one. So, we can get rid of this "hump"-type model for a momentum transfer $Q \gtrsim 1$ GeV and argue that such distribution could not occur in the nonperturbative domain too. It is the most likely to assume that the momentum fraction function $Z(x,x')$ is rather smooth. From the equation

$$
z^n(u) = \int dx \int dx' (xu + x'\bar{u})^{n-1} Z(x,x') \tag{96}
$$

it follows then that $z^n(u)$ can not be strongly suppressed in the end-point region. For instance, if $Z(x,x')$ is a positive definite and concentrated in the region $0 \leq x, x'$ then $z^n(u)$ can not vanish at $u = 0, 1$ unless it is identically zero.

6 Discussion and conclusion.

In the present paper we have investigated the Q^2-dependence of the chiral-odd distributions of the nucleon $e(x)$ and $h_L(x)$. Using the constraint equalities coming from equation of motion and Lorentz invariance which provide a certain sum rules for structure function, the problem is reduced to the study of the renormalization of the multiparton correlators in the lowest order of the perturbation theory. As a result we construct an exact (with account of $1/N_c^2$ effects) one-loop evolution in light-cone fraction as well as in light-cone position representations. For these purposes we have used two techniques which employ the light-like gauge for the gluon field. Accepting different prescription on the spurious pole in the gluon propagator we were able to verify that they do lead to the same results. From the calculational point of view the momentum space technique is much more easy to treat. However, the coordinate space makes apparent the symmetries involved and as a byproduct diagonalization of evolution kernels is easy to handle. We establish the bridge between different formulations of the QCD evolution. It is straightforward to obtain the evolution kernels in light-cone fraction representation starting form the coordinate space and vice versa using the Fourier transform. Using this transformation it is straightforward to obtain the coordinate space two-particle evolution kernels for Faddeev-type equations with pair-wise particle interaction which govern the scale dependence for higher twist (more than 3) quasi-partonic operators [10]. This representation will be the most useful for studying the eigenvalue problem. We hope to return to this question in future.

In the multicolour limit as well as for $x \to 1$ we obtain the ladder-type evolution equations for the twist-3 part of the distribution functions. Joining these asymptotics together we construct an improved DGLAP equation which generally has very good accuracy on the level of few percents. We argue that the observed discrepancy between these reduced equations and the exact evolution could occur only for unphysical initial conditions for the latter. To clarify the situation completely, it would be helpful to have the low energy model predictions for the distribution of the gluon field in the quark-gluon correlator.

Acknowledgements. We would like to thank V.M. Braun for discussion at an early stage of the work. The authors are grateful to the CERN Theory Division for its hospitality during their visit where this work has been started. A.B. was supported by the Russian Foundation for Fundamental Research, grant N 96-02-17631. D.M. was financially supported by the Deutsche Forschungsgemeinschaft (DFG).

A Definition and some properties of Θ-functions.

The Θ-functions entering into the evolution equations in the momentum fraction representation are given by the formula

$$\Theta^m_{i_1 i_2 \dots i_n}(x_1, x_2, ..., x_n) = \int_{-\infty}^{\infty} \frac{d\alpha}{2\pi i} \alpha^m \prod_{k=1}^{n} (\alpha x_k - 1 + i0)^{-i_k} . \quad \text{(A.1)}$$

For our practical purposes it is enough to have an explicit form of the functions

$$\Theta^0_1(x) = 0, \quad \text{(A.2)}$$

$$\Theta^0_2(x) = \delta(x), \quad \text{(A.3)}$$

$$\Theta^0_{11}(x_1, x_2) = \frac{\theta(x_1)\theta(-x_2) - \theta(x_2)\theta(-x_1)}{x_1 - x_2}, \quad \text{(A.4)}$$

since the other ones are expressed in their terms via relations

$$\Theta^0_{21}(x_1, x_2) = \frac{x_2}{x_1 - x_2} \Theta^0_{11}(x_1, x_2), \quad \text{(A.5)}$$

$$\Theta^1_{21}(x_1, x_2) = \frac{1}{x_1 - x_2} \Theta^0_{11}(x_1, x_2) - \frac{1}{x_1 - x_2} \Theta^0_2(x_1), \quad \text{(A.6)}$$

$$\Theta^0_{22}(x_1, x_2) = - \frac{2x_1 x_2}{(x_1 - x_2)^2} \Theta^0_{11}(x_1, x_2), \quad \text{(A.7)}$$

$$\Theta^0_{111}(x_1, x_2, x_3) = \frac{x_2}{x_1 - x_2} \Theta^0_{11}(x_2, x_3) - \frac{x_1}{x_1 - x_2} \Theta^0_{11}(x_1, x_3), \quad \text{(A.8)}$$

$$\Theta^1_{111}(x_1, x_2, x_3) = \frac{1}{x_1 - x_2} \Theta^0_{11}(x_2, x_3) - \frac{1}{x_1 - x_2} \Theta^0_{11}(x_1, x_3). \quad \text{(A.9)}$$

In the main text we have used the relations

$$\int d\beta \beta^n \Theta^0_{11}(\beta, \beta - x') \Theta^0_{11}(x, x - \beta) = x'^n \Xi_n(x, x - x', x) = \frac{1}{n} \left[x'^n - x^n \right] \Theta^0_{11}(x, x - x'). \quad \text{(A.10)}$$

$$\mathrm{PV} \int d\beta \frac{x}{(x - \beta)} \left[\Theta^0_{11}(\beta, \beta - x) + \Theta^0_{11}(x, x - \beta) \right] = 0. \quad \text{(A.11)}$$

B Evolution equations in abelian gauge theory.

In this appendix we present a pedagogical illustration of the renormalization group mixing problem for the redundant basis of correlation functions defined by Eqs. (6)-(9) and Eqs. (20)-(25) for the non- and polarized cases, respectively, in the framework of the abelian gauge theory. Its aim is to show the selfconsistency of the whole approach we have used as the equations derived below satisfy the constraint equalities given by Eqs. (10), (26), (27) which are employed further to reduce the overcomplete set of correlators to the independent basis of functions.

In the calculations of the corresponding evolution kernels we follow the methods developed in Ref. [9] (see Ref. [21] for more recent discussion of the RG equations for the time-like twist-3 cut vertices). The one-loop Feynman diagrams giving rise to the transition amplitudes of two-particle correlation functions into the two- and three-parton ones are shown in Fig. 5 (a, b). The last figure (c) on this picture is specific for the vertices having non-quasi-partonic form [10], that is for $e(x)$ and $h_L(x)$, and displays the addendum due to contact term which results from the cancellation of the propagator adjacent to the quark-gluon and bare vertices. As an output the vertex acquires the three-particle piece. The radiative correction to the three-parton correlators are presented in Fig. 1 (a, b, c).

The straightforward calculation yields the evolution equations for spin-independent case in the form

$$\dot{M}(x) = -\frac{\alpha}{2\pi}\int d\beta M(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + \frac{\beta+x}{\beta}\Theta^0_{11}(x,x-\beta)\right\}, \tag{B.1}$$

$$\begin{aligned}\dot{e}(x) &= \frac{\alpha}{2\pi}\int d\beta\Bigg(e(\beta)\left\{\frac{x}{\beta}\Theta^0_{11}(x,x-\beta)+\frac{1}{2}\delta(\beta-x)\right\}\\ &- M(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + x\Theta^1_{21}(x,x-\beta)+2\Theta^0_{11}(x,x-\beta)\right\}\\ &- \int d\beta' D(\beta,\beta')\Bigg\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + \frac{x}{x-\beta}\Theta^0_{111}(x,x-\beta,x-\beta+\beta')\\ &+ \delta(\beta-x)\int d\beta''\frac{\beta}{\beta''}\Theta^0_{111}(\beta'',\beta''-\beta,\beta''-\beta')+2\Theta^0_{11}(x,x-\beta)\Bigg\}\Bigg),\end{aligned} \tag{B.2}$$

$$\begin{aligned}\dot{D}(x,x') &= -\frac{\alpha}{2\pi}\Bigg\{\left[\frac{x'}{x}e(x)-M(x)\right]\Theta^0_{11}(x',x'-x)-\left[\frac{x}{x'}e(x')-M(x')\right]\Theta^0_{11}(x,x-x')\\ &+ \int d\beta'\Bigg(D(x,\beta')\frac{(\beta'-x+x')}{(x-x')}\Theta^0_{111}(x',x'-x,x'-x+\beta')\\ &+ \frac{x'}{x'-\beta'}[D(x-x'+\beta',\beta')-D(x,x')]\Theta^0_{11}(x',x'-\beta')\Bigg)\\ &+ \int d\beta\Bigg(D(\beta,x')\frac{(\beta-x'+x)}{(x'-x)}\Theta^0_{111}(x,x-x',x-x'+\beta)\\ &+ \frac{x}{x-\beta}[D(\beta,x'-x+\beta)-D(x,x')]\Theta^0_{11}(x,x-\beta)\Bigg)-\frac{3}{2}D(x,x')\Bigg\},\end{aligned} \tag{B.3}$$

where the dot denotes the derivative with respect to the UV cutoff $\dot{} = \Lambda^2\partial/\partial\Lambda^2$ and plus-prescription is defined by the equation

$$\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ = \frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)-\delta(\beta-x)\int d\beta''\frac{\beta}{(\beta''-\beta)}\Theta^0_{11}(\beta'',\beta''-\beta). \tag{B.4}$$

In the same way we can immediately obtain the set of evolution equations for the spin-dependent distributions (20)-(25)

$$\dot{\widetilde{M}}(x) = -\frac{\alpha}{2\pi}\int d\beta \widetilde{M}(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + \frac{\beta+x}{\beta}\Theta^0_{11}(x,x-\beta)\right\}, \tag{B.5}$$

$$\dot{h}_1(x) = -\frac{\alpha}{2\pi}\int d\beta h_1(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + 2\Theta^0_{11}(x,x-\beta) - \frac{3}{2}\delta(x-\beta)\right\}, \tag{B.6}$$

$$\begin{aligned}
\dot{h}_L(x) &= \frac{\alpha}{2\pi}\int d\beta\Bigg(h_L(\beta)\left\{\frac{x}{\beta}\Theta^0_{11}(x,x-\beta) + \frac{1}{2}\delta(\beta-x)\right\} \\
&- \widetilde{M}(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + \left(2+\frac{x}{\beta}\right)\Theta^0_{11}(x,x-\beta)\right\} \\
&- K(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + \left(2+3\frac{x}{\beta}\right)\Theta^0_{11}(x,x-\beta) - \delta(x-\beta)\right\} \\
&- \int d\beta' \widetilde{D}(\beta,\beta')\Bigg\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + \frac{x}{x-\beta}\Theta^0_{111}(x,x-\beta,x-\beta+\beta') \\
&+ \delta(\beta-x)\int d\beta''\frac{\beta}{\beta''}\Theta^0_{111}(\beta'',\beta''-\beta,\beta''-\beta') + 2\left(1+\frac{x}{\beta}\right)\Theta^0_{11}(x,x-\beta)\Bigg\}\Bigg),
\end{aligned} \tag{B.7}$$

$$\begin{aligned}
\dot{K}(x) &= \frac{\alpha}{2\pi}\int d\beta\Bigg(2h_L(\beta)\frac{x}{\beta}\Theta^0_{11}(x,x-\beta) - 2\widetilde{M}(\beta)\Theta^0_{11}(x,x-\beta) \\
&- K(\beta)\left\{2\left[\frac{\beta}{(x-\beta)}\Theta^0_{11}(x,x-\beta)\right]_+ + 2\left(1+2\frac{x}{\beta}\right)\Theta^0_{11}(x,x-\beta) - \frac{3}{2}\delta(x-\beta)\right\} \\
&- \int d\beta'\widetilde{D}(\beta,\beta')\Bigg\{2\frac{(x-\beta+\beta')}{(x-\beta)}\Theta^0_{111}(x,x-\beta,x-\beta+\beta') + 2\frac{x}{\beta}\Theta^0_{11}(x,x-\beta)\Bigg\}\Bigg),
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
\dot{\widetilde{D}}(x,x') &= -\frac{\alpha}{2\pi}\Bigg\{\left[\frac{x'}{x}[h_L(x)-K(x)] - \widetilde{M}(x)\right]\Theta^0_{11}(x',x'-x) \\
&+ \left[\frac{x}{x'}[h_L(x')-K(x')] - \widetilde{M}(x')\right]\Theta^0_{11}(x,x-x') \\
&+ \int d\beta'\Bigg(\widetilde{D}(x,\beta')\frac{(\beta'-x+x')}{(x-x')}\Theta^0_{111}(x',x'-x,x'-x+\beta') \\
&+ \frac{x'}{x'-\beta'}[\widetilde{D}(x-x'+\beta',\beta') - \widetilde{D}(x,x')]\Theta^0_{11}(x',x'-\beta')\Bigg) \\
&+ \int d\beta\Bigg(\widetilde{D}(\beta,x')\frac{(\beta-x'+x)}{(x'-x)}\Theta^0_{111}(x,x-x',x-x'+\beta) \\
&+ \frac{x}{x-\beta}[\widetilde{D}(\beta,x'-x+\beta) - \widetilde{D}(x,x')]\Theta^0_{11}(x,x-\beta)\Bigg) - \frac{3}{2}\widetilde{D}(x,x')\Bigg\}.
\end{aligned} \tag{B.9}$$

The anomalous dimensions calculated from the evolution equation for the distribution $h_1(x)$ coincide (up to the colour group factor C_F) with the result of Ref. [22].

By exploiting the relation provided by equation of motion and Lorentz invariance we can easily verify that the RG equations thus constructed are indeed correct and the renormalization program can be reduced to the study of logarithmic divergences of the three-parton ($Z(x,x')$, $\widetilde{Z}(x,x')$) and quark mass ($M(x)$, $\widetilde{M}(x)$) correlators in perturbation theory.

C Local anomalous dimensions.

In this appendix we pass from the evolution equations for correlators to the equations for their moments and find this way the anomalous dimension matrix for local twist-3 operators.

We define the moments as follows

$$\begin{aligned} F_n &= \int dx x^n F(x) \quad \text{for any two-particle correlator,} \\ Z_n^l &= \int dx dx' x^{n-l} x'^{l-1} Z(x,x'). \end{aligned} \tag{C.1}$$

In the language of operator product expansion these equalities specify the expansion of nonlocal string operators in towers of local ones, namely:

$$\begin{aligned} \mathcal{Z}_n^l &= i^{n-1}(-1)^{l-1}\frac{\partial^{l-1}}{\partial\mu^{l-1}}\frac{\partial^{n-l}}{\partial\lambda^{n-l}}\,\mathcal{Z}(\lambda,\mu)|_{\lambda=\mu=0} \\ &\qquad = \frac{1}{2}\bar{\psi}(0)(iD_+)^{l-1}\mathrm{g}G_{+\rho}(0)\sigma^{\perp}_{\rho+}\begin{pmatrix} I \\ \gamma_5 \end{pmatrix}(iD_+)^{n-l}\psi(0), \\ \mathcal{M}_n &= i^n\frac{\partial^n}{\partial\lambda^n}\,\mathcal{M}(\lambda)|_{\lambda=0} = \frac{m}{2}\bar{\psi}(0)\gamma_+\begin{pmatrix} I \\ \gamma_5 \end{pmatrix}(iD_+)^n\psi(0). \end{aligned} \tag{C.2}$$

The inverse transformations to the nonlocal representation is given by

$$\mathcal{Z}(\lambda,\mu) = \sum_{n=0,\,m=0}^{\infty}(-i)^{n+m}(-1)^m\frac{\mu^m}{m!}\frac{\lambda^n}{n!}\,\mathcal{Z}^{m+1}_{n+m+1}, \qquad \mathcal{M}(\lambda) = \sum_{n=0}^{\infty}(-i)^n\frac{\lambda^n}{n!}\,\mathcal{M}_n. \tag{C.3}$$

Now it is a simple task to derive the algebraic equations for the mixing of local operators under the change of the renormalization scale from the evolution equations (35)-(39). They are

$$\dot{M}_n = \frac{\alpha}{2\pi}\,{}_{MM}\gamma^n M_n, \tag{C.4}$$

$$\dot{Z}_n^l = \frac{\alpha}{2\pi}\left\{\left[{}_{ZM}\gamma^n_{n-l+1} \pm {}_{ZM}\gamma^n_l\right]M_n + \sum_{k=1}^{n}{}_{ZZ}\gamma^n_{lk}Z^k_n\right\}, \tag{C.5}$$

where the anomalous dimensions are given by the expressions

$${}_{MM}\gamma^n = -C_F\left(S_n + S_{n+2}\right), \tag{C.6}$$

$${}_{ZM}\gamma^n_l = \frac{2C_F}{l(l+1)(l+2)}, \tag{C.7}$$

$$
\begin{aligned}
{}_{zz}\gamma^{n}_{lk} &= \frac{3}{4}C_F\delta(l-k) + \frac{C_A}{2}\left\{\theta(l-k-1)\frac{(k+1)(k+2)}{(l-k)(l+1)(l+2)} - \delta(l-k)\left[S_{k-1}+S_{k+2}\right]\right\} \\
&+ \left(C_F - \frac{C_A}{2}\right)\left\{\theta(l-k-1)\left[\frac{2(-1)^k C^k_l}{l(l+1)(l+2)} + \frac{(-1)^{l-k}}{(l-k)}\frac{C^{k-1}_n}{C^{l-1}_n}\right]\right. \\
&+ \left.\delta(l-k)\left[\frac{2(-1)^k}{k(k+1)(k+2)} - S_k\right]\right\} + \begin{pmatrix} k\to n-k+1 \\ l\to n-l+1 \end{pmatrix}. \qquad \text{(C.8)}
\end{aligned}
$$

Here we have used the following step-functions

$$
\theta(i-j) = \begin{cases} 1, & i\geq j \\ 0, & i<j \end{cases} \qquad \delta(i-j) = \begin{cases} 1, & i=j \\ 0, & i\neq j \end{cases} \qquad \text{(C.9)}
$$

and conventions $S_n = \sum_{k=1}^{n}\frac{1}{k}$ and $C^m_n = \frac{n!}{m!(n-m)!}$ are the binomial coefficients. The plus and minus signs in this equation corresponds to the functions e and h_L, respectively. These analytical expressions coincide with result of Ref. [15].

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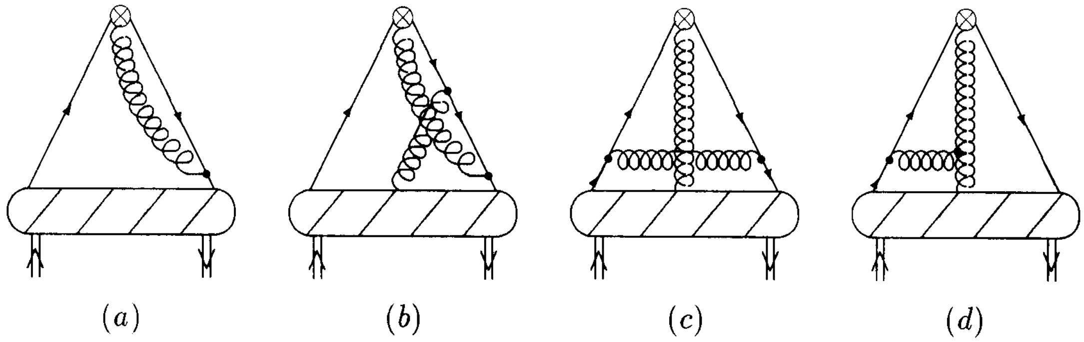

Figure 1: The one-loop renormalization of the three-parton correlation functions.

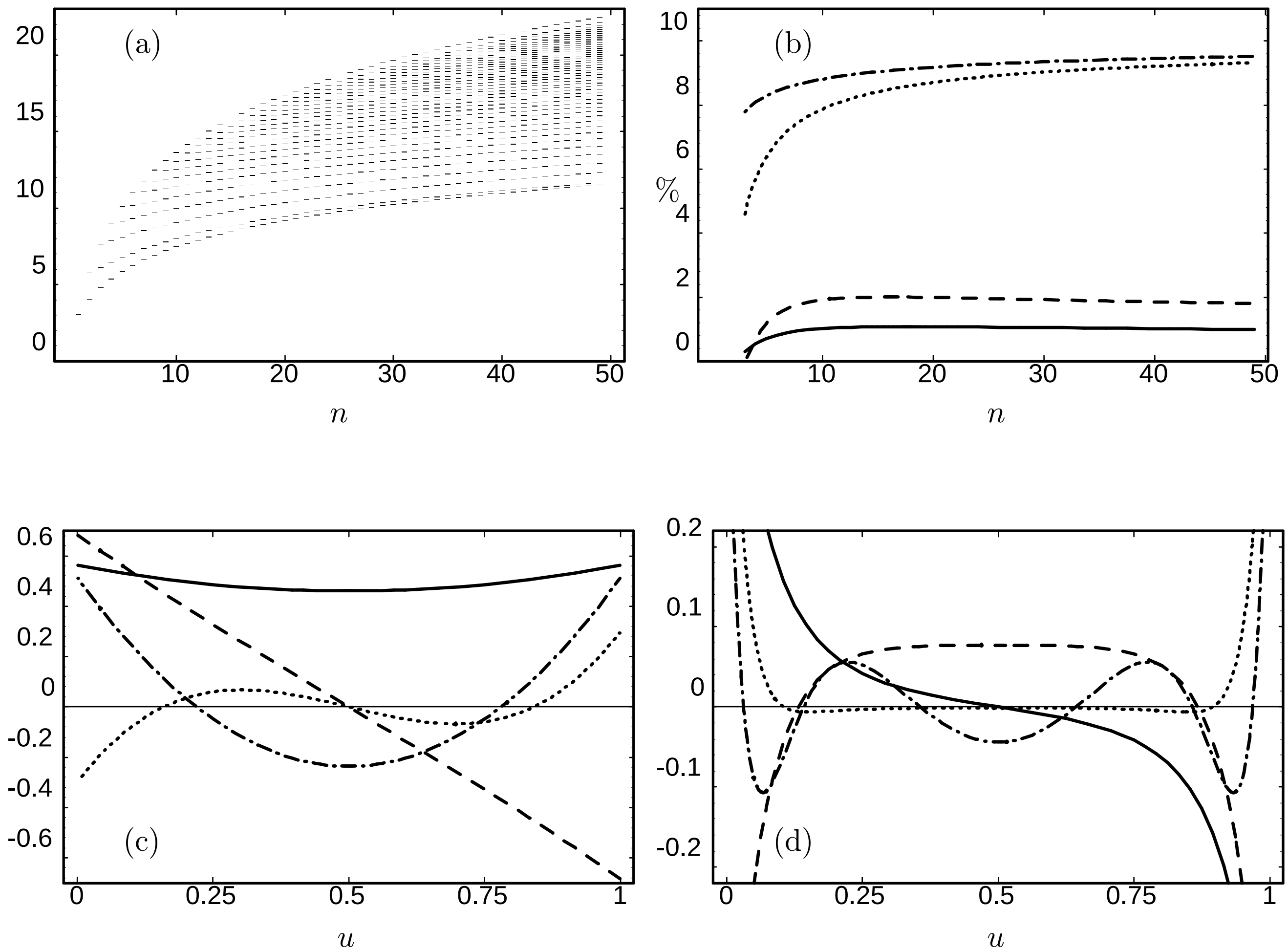

Figure 2: The spectrum of the eigenvalues λ_l^n for the evolution kernel $P^n_{\mathcal{Z}\mathcal{Z}}$ defined in (69) is shown in (a). In (b) the relative deviation $1 - \lambda_l^n/(-\gamma_l^n)$ (in %) for the two lowest eigenvalues of the spectrum, *i.e.* $l = 1, 2$, is plotted: the solid (dashed) line γ_n^1 (γ_n^2) corresponds to the n-th moments of the improved kernel P_{ee} ($P_{\bar{h}\bar{h}}$) defined by Eq. (86); the dashed-dotted (dotted) line is the multicolour approximation for P_{ee} ($P_{\bar{h}\bar{h}}$). In the improved approximations subleading terms were taken into account to reproduce the first two eigenvalues exactly. Eigenfunctions of the kernel $P^n_{\mathcal{Z}\mathcal{Z}}$ are shown for $n = 4$ in (c) and for $n = 30$ with $l = 2, 3, 6, 29$ in (d).

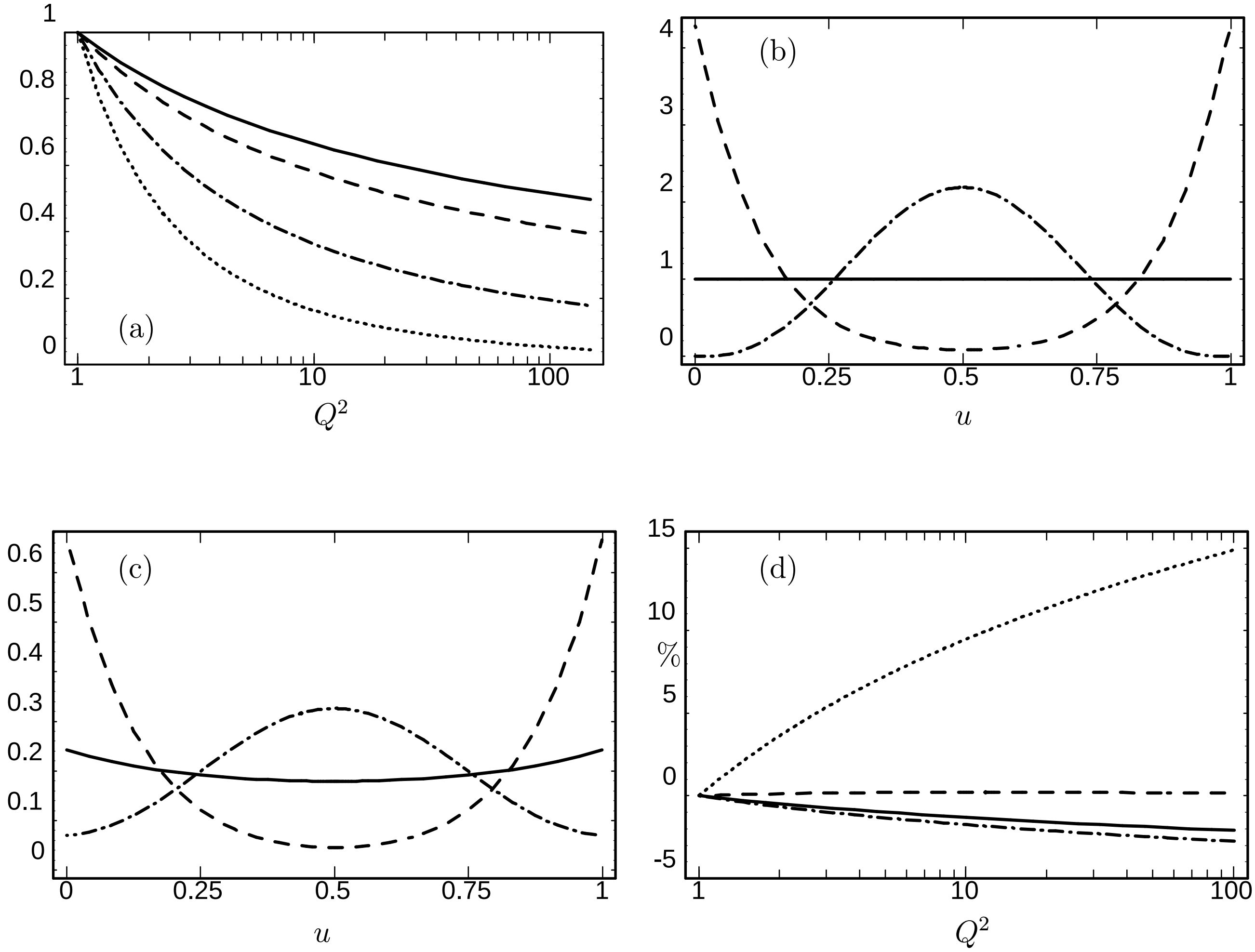

Figure 3: The predictions of the improved evolution equation for the moments $[e]_n$, normalized to 1 at $Q_0^2 = 1\text{GeV}^2$, are shown in (a) for $n = 1$ (solid line), $n = 2$ (dashed line), $n = 10$ (dashed-dotted line) and $n = 100$ (dotted line). In (b) three different models for the gluon light-cone position distributions at $Q_0^2 = 1\text{GeV}^2$ for $n = 10$ are presented, which are evolved up to the scale $Q^2 = 100\text{GeV}^2$ in (c). The relative deviation $1 - [e(Q^2)]_n^{\rm ex}/[e(Q^2)]_n^{\rm im}$ (in %) of the exact evolution for the assumed gluon distributions from the improved DGLAP equation is shown in (d). The dotted line corresponds to the relative deviation $1 - [e(Q^2)]_n^{N_c}/[e(Q^2)]_n^{\rm coef}$ with respect to the multicolour approximation, where "coef" refers to the gluon distribution which is equivalent to the corresponding coefficient function.

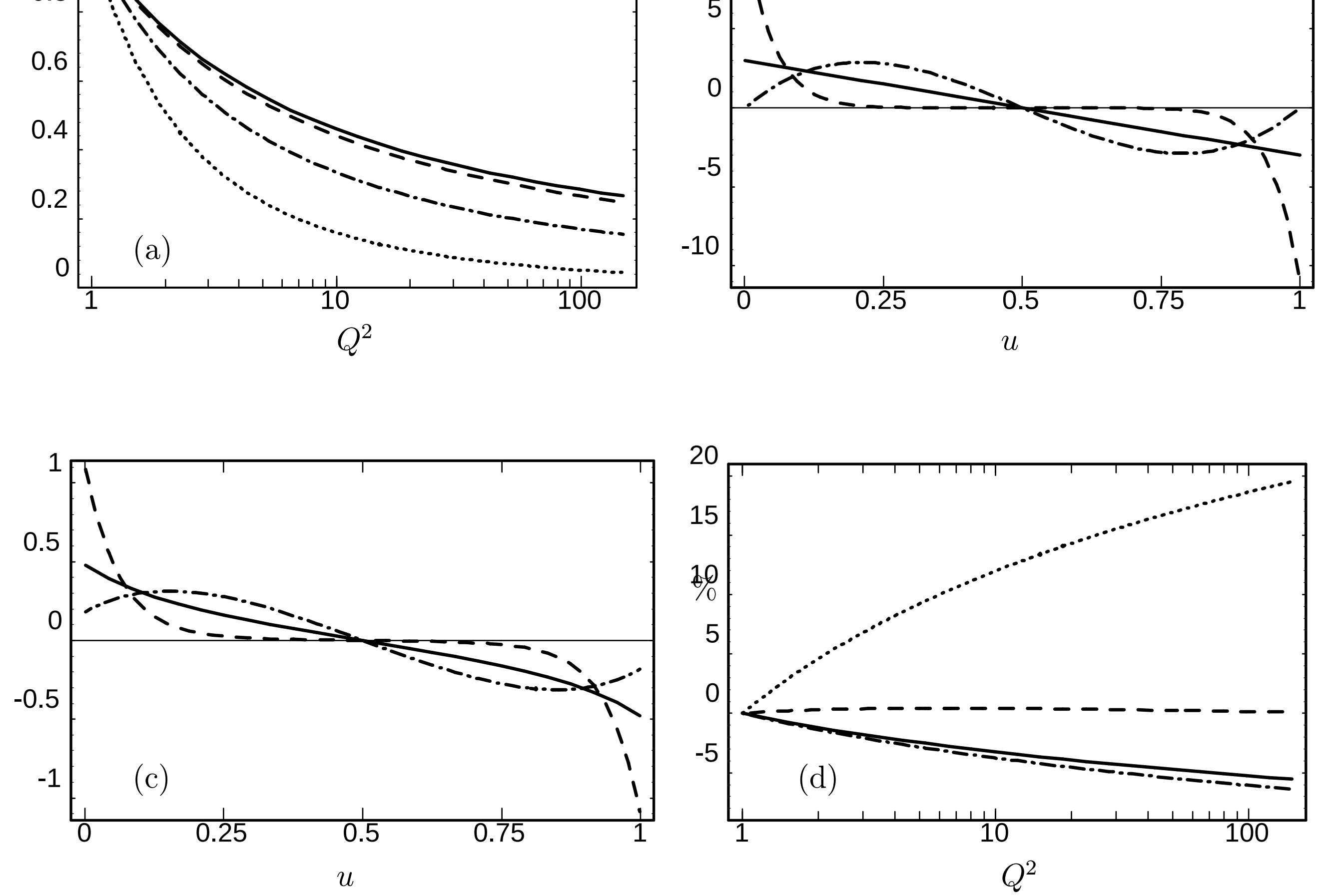

Figure 4: The predictions of the improved evolution equation for the moments $[\widetilde{h}_L]_n$, normalized to 1 at $Q_0^2 = 1\text{GeV}^2$, are shown in (a). Here the solid line and dashed line represent $n = 2$ and $n = 3$, respectively. The further description is the same as in Fig. 3, except that $n = 20$ in (b)-(d).

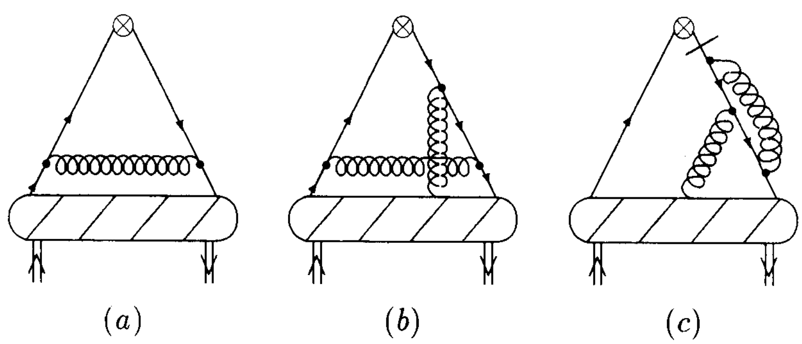

Figure 5: One-loop radiative corrections to the two-particle correlators in the abelian gauge theory. The fermion propagator crossed with a bar on diagram (c) means the contraction of the corresponding line into the point.