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## Theory of Inclusive B Decays

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### Abstract

We present the theory of inclusive decays of hadrons containing a heavy quark and discuss its most important applications to the decays of  $B$  mesons. We also review the theoretical understanding of the hadronic parameters  $\lambda_1$  and  $\lambda_2$  (or  $\mu_\pi^2$  and  $\mu_G^2$ ) entering the heavy-quark expansion.

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## 1. INTRODUCTION

Hadronic bound states of a heavy quark with light constituents (quarks, antiquarks and gluons) are characterized by a large separation of mass scales: the heavy-quark mass  $m_Q$  is much larger than the mass scale  $\Lambda_{\text{QCD}}$  associated with the light degrees of freedom. Equivalently, the Compton wave length of the heavy quark ( $\lambda_Q \sim 1/m_Q$ ) is much smaller than the size of the hadron containing it ( $R_{\text{had}} \sim 1/\Lambda_{\text{QCD}}$ ). In such a situation, it is appropriate to separate the physics associated with these two scales, so that all dependence on the heavy-quark mass becomes explicit. The framework in which to perform this separation is the heavy-quark (or  $1/m_Q$ ) expansion [1]–[8], which is a specific realization of the operator product expansion (OPE) [9, 10].

There are (at least) two important reasons why it is desirable to separate short- and long-distance phenomena: A technical reason is that after this separation we can calculate a big portion of the relevant physics (i.e. all short-distance effects) using perturbation theory and renormalization-group techniques; in particular, in this way we are able to control all logarithmic dependence on the heavy-quark mass. An important physical reason is that, after the short-distance physics has been separated, the long-distance physics may simplify due to the realization of approximate symmetries, which relate the long-distance properties of many observables to a small number of hadronic matrix elements. The second point is particularly exciting, since it allows us to make statements beyond the range of applicability of perturbation theory.

In our case, an approximate spin–flavour symmetry is realized in systems in which a single heavy quark interacts with light degrees of freedom by the exchange of soft gluons [11]–[15]. The origin of this symmetry is not difficult to understand. In a heavy hadron, the heavy quark is surrounded by a most complicated, strongly interacting cloud of light quarks, antiquarks and gluons. However, the fact that the Compton wavelength of the heavy quark is much smaller than the size of the hadron leads to simplifications. To resolve the quantum numbers of the heavy quark would require a hard probe; the soft gluons exchanged between the heavy quark and the light constituents can only resolve distances much larger than  $1/m_Q$ . Therefore, the light degrees of freedom are blind to the flavour (mass) and spin orientation of the heavy quark. They experience only its colour field, which extends over large distances because of confinement. In the rest frame of the heavy quark, it is in fact only the electric colour field that is important; relativistic effects such as colour magnetism vanish as  $m_Q \rightarrow \infty$ . Since the heavy-quark spin participates in interactions only through such relativistic effects, it decouples. It follows that, in the limit  $m_Q \rightarrow \infty$ , hadronic systems which differ only in the flavour or spin quantum numbers of their heavy quark have the same configuration of their light degrees of freedom. Although this observation still does not allow us to calculate what this configuration is, it provides relations between the properties of such particles as the heavy mesons  $B$ ,  $D$ ,  $B^*$  and  $D^*$ , or the heavy baryons  $\Lambda_b$  and  $\Lambda_c$ .

Heavy-quark symmetry is an approximate symmetry, and corrections arise since the quark masses are not infinitely heavy. Nevertheless, results derived on the basis of heavy-quark symmetry are model-independent consequences of QCD in a well-defined limit. The symmetry-breaking corrections can, at least in principle, be studied in a systematic way. A convenient framework for analysing these corrections is given by the heavy-quark effective theory (HQET) [16], which provides a systematic expansion around the limit  $m_Q \rightarrow \infty$ . In the HQET, a heavy quark inside a hadron moving with velocity  $v$  is described by a velocity-dependent field  $h_v$  subject to the constraint  $\not{v} h_v = h_v$ . This field is related to the original heavy-quark field by a phase redefinition, so that it carries the “residual momentum”  $k = p_Q - m_Q v$ , which characterizes the interactions of the heavy quark with gluons. The effective Lagrangian of the HQET is [16]–[19]

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{C_{\text{kin}}}{2m_Q} \bar{h}_v (iD_{\perp})^2 h_v + \frac{C_{\text{mag}} g_s}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + O(1/m_Q^2), \quad (1)$$

where  $D_{\perp}^{\mu} = D^{\mu} - (v \cdot D) v^{\mu}$  contains the components of the gauge-covariant derivative orthogonal to the velocity, and  $g_s G^{\mu\nu} = i[D^{\mu}, D^{\nu}]$  is the gluon field-strength tensor. The leading term in the effective Lagrangian, which gives rise to the Feynman rules of the HQET, is invariant under a global  $SU(2n_h)$  spin-flavour symmetry group, where  $n_h$  is the number of heavy-quark flavours. This symmetry is explicitly broken by the higher-dimensional operators arising at order  $1/m_Q$ , whose origin is most transparent in the rest frame of the heavy hadron: the first operator corresponds to (minus) the kinetic energy resulting from the motion of the heavy quark inside the hadron (in the rest frame,  $(iD_{\perp})^2$  is the operator for  $-\mathbf{k}^2$ ), and the second operator describes the chromo-magnetic interaction of the heavy-quark spin with the gluon field. The coefficients  $C_{\text{kin}}$  and  $C_{\text{mag}}$  result from short-distance effects and, in general, depend on the scale at which the operators are renormalized. At the tree level,  $C_{\text{kin}} = C_{\text{mag}} = 1$ .

At this point it is instructive to recall a more

familiar example of how approximate symmetries relate the long-distance properties of several observables. The strong interactions of pions are severely constrained by the approximate chiral symmetry of QCD. In a certain kinematic regime, where the momenta of the pions are much less than 1 GeV (the scale of chiral-symmetry breaking), the long-distance physics of scattering amplitudes is encoded in a few “reduced matrix elements”, such as the pion decay constant. An effective low-energy theory called chiral perturbation theory provides a systematic expansion of scattering amplitudes in powers of the pion momenta, and thus helps to derive the relations between different scattering amplitudes imposed by chiral symmetry [20]. A similar situation holds for the case of heavy quarks. Heavy-quark symmetry implies that, in the limit where  $m_Q \gg \Lambda_{\text{QCD}}$ , the long-distance properties of several observables is encoded in few hadronic parameters, which can be defined in terms of operator matrix elements in the HQET.

In particular, the forward matrix elements of the two dimension-five operators in (1),

$$\begin{aligned} O_{\text{kin}} &= -\bar{h}_v (iD_{\perp})^2 h_v, \\ O_{\text{mag}} &= \frac{g_s}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v, \end{aligned} \quad (2)$$

play a most significant role in many applications of the HQET. They appear, for instance, in the spectroscopy of heavy hadrons and in the description of inclusive weak decays [1]. For the  $B$  meson, we define two parameters  $\lambda_1$  and  $\lambda_2$  (or, equivalently,  $\mu_{\pi}^2$  and  $\mu_G^2$ ) by [21]

$$\begin{aligned} -\lambda_1 &= \mu_{\pi}^2 = \frac{1}{2m_B} \langle B | O_{\text{kin}} | B \rangle, \\ 3\lambda_2 &= \mu_G^2 = \frac{1}{2m_B} \langle B | O_{\text{mag}} | B \rangle. \end{aligned} \quad (3)$$

Whereas  $\lambda_2$  is directly related to the mass splitting between vector and pseudoscalar mesons through

$$m_{B^*}^2 - m_B^2 = 4\lambda_2 + O(1/m_b), \quad (4)$$

the parameter  $\lambda_1$  cannot be determined from hadron spectroscopy. It is, however, related to the difference of the pole masses of two heavy

quarks through

$$m_b - m_c = (\overline{m}_B - \overline{m}_D) + \lambda_1 \left( \frac{1}{2\overline{m}_B} - \frac{1}{2\overline{m}_D} \right) + \dots \quad (5)$$

Here  $\overline{m}_B = \frac{1}{4}(m_B + 3m_{B^*})$  and  $\overline{m}_D = \frac{1}{4}(m_D + 3m_{D^*})$  denote the spin-averaged meson masses, defined such that they do not receive a contribution from the chromo-magnetic interaction. In the following section, we discuss the theoretical status of the parameters  $\lambda_1$  and  $\lambda_2$  and their properties under renormalization.

## 2. OPERATOR RENORMALIZATION AND THE VIRIAL THEOREM

In quantum field theory, local composite operators such as  $O_{\text{kin}}$  and  $O_{\text{mag}}$  require renormalization to be well defined. The matrix elements of these operators (i.e.  $\lambda_1$  and  $\lambda_2$ ) depend on the subtraction scheme and on the renormalization scale  $\mu$  introduced in the process of removing ultraviolet divergences. This dependence is cancelled by an opposite scale dependence of the Wilson coefficient functions  $C_{\text{kin}}$  and  $C_{\text{mag}}$  in the effective Lagrangian (1). The most common regularization scheme in QCD is dimensional regularization [22, 23] with modified minimal subtraction ( $\overline{\text{MS}}$  scheme [24]). Ultraviolet divergences are regulated by working in  $d = 4 - 2\epsilon$  space-time dimensions, and are subtracted by removing the poles in  $1/\epsilon$ . The scale dependence of the Wilson coefficients is governed by renormalization-group equations of the form

$$\mu \frac{d}{d\mu} C(\mu) = \gamma C(\mu), \quad (6)$$

where  $\gamma$  is called the anomalous dimension.

The anomalous dimension of the kinetic operator  $O_{\text{kin}}$  vanishes to all orders in perturbation theory,  $\gamma_{\text{kin}} \equiv 0$ . This is a consequence of the reparametrization invariance of the HQET, i.e. and invariance under infinitesimal changes of the velocity  $v$  used in the construction of the effective Lagrangian [25]–[27]. It follows that in dimensional regularization  $C_{\text{kin}} \equiv 1$ . The anomalous dimension of the chromo-magnetic operator does not vanish, however. It has been known at the

one-loop order since many years [17, 18], but only very recently the two-loop coefficient has been calculated [28, 29]. For  $N_c = 3$  colours, the result is

$$\gamma_{\text{mag}} = \frac{3\alpha_s}{2\pi} + \left(17 - \frac{13}{6} n_f\right) \left(\frac{\alpha_s}{2\pi}\right)^2 + O(\alpha_s^3), \quad (7)$$

where  $n_f$  is the number of light-quark flavours. Given this result, together with the one-loop matching condition [17]

$$C_{\text{mag}}(m_Q) = 1 + \frac{13}{6} \frac{\alpha_s(m_Q)}{\pi} + O(\alpha_s^2), \quad (8)$$

one can work out the next-to-leading order expression for  $C_{\text{mag}}(\mu)$ , starting from the general solution to (6) [24, 30]:

$$C(\mu) = C(m_Q) \exp \int_{\alpha_s(m_Q)}^{\alpha_s(\mu)} d\alpha_s \frac{\gamma(\alpha_s)}{\beta(\alpha_s)}, \quad (9)$$

where  $\beta(\alpha_s) = d\alpha_s(\mu)/d \ln \mu$  is the  $\beta$  function. The product of the Wilson coefficient  $C_{\text{mag}}(\mu)$  and the scale-dependent matrix element  $\lambda_2(\mu)$  is renormalization-group invariant. Hence, in the presence of renormalization effects, what can be determined from the vector–pseudoscalar mass splitting in (4) is the combination

$$\lambda_2 \equiv C_{\text{mag}}(m_b) \lambda_2(m_b) \approx 0.12 \text{ GeV}^2. \quad (10)$$

The numerical value is specific for the  $B$  system, since according to our definition the renormalized parameter  $\lambda_2$  depends logarithmically on the  $b$ -quark mass.

Unfortunately, the question about the value of the kinetic-energy parameter  $\lambda_1$  is more difficult to answer, even from a conceptual point of view. We have already mentioned that spectroscopic relations involving  $\lambda_1$ , such as (5), depend on the heavy-quark pole masses, which are not physical parameters. In the remainder of this section, we shall argue that indeed the parameter  $\lambda_1$  is not physical, but has an intrinsic ambiguity of order  $\Lambda_{\text{QCD}}^2$ . (However, the difference of the values of  $\lambda_1$  in two different hadrons is a physical parameter and can be extracted from spectroscopy.) This means that  $\lambda_1$  must be defined in a non-perturbative way; in other words, different definitions cannot be simply related to each other using

Table 1

Theoretical estimates of the parameter  $\lambda_1$  (QCDSR: QCD sum rules, HQSR: heavy-quark sum rules, Exp.: experimental data on inclusive decays, QM: quark models)

Reference	Method	$-\lambda_1$ [GeV <sup>2</sup> ]
Eletsky, Shuryak [31]	QCDSR	$0.18 \pm 0.06$
Ball, Braun [32]	QCDSR	$0.52 \pm 0.12$
Neubert [33]	QCDSR	$0.10 \pm 0.05$
Giménez et al. [34]	Lattice	$-0.09 \pm 0.14$
Bigi et al. [35]	HQSR	$> 0.36$
Gremm et al. [36]	Exp.	$0.19 \pm 0.10$
Falk et al. [37]	Exp.	$\approx 0.1$
Chernyak [38]	Exp.	$0.14 \pm 0.03$
Hwang et al. [39]	QM	$0.5 \pm 0.1$
De Fazio [40]	QM	$0.66 \pm 0.13$

perturbation theory. The wide spread in the theoretical predictions for the parameter  $\lambda_1$ , shown in Tab. 1, is partially a reflection of this problem. Future efforts should concentrate on understanding better the relations between the various definitions underlying these estimates.

The problem of the ambiguity in the value of the heavy-quark kinetic energy is closely related to the well-known ambiguity in the definition of the pole mass of a heavy quark [41, 42]. The reason lies in the divergent behaviour of perturbative expansions in large orders, which is associated with the existence of singularities along the real axis in the Borel plane, the so-called renormalons [43]–[51]. When the pole mass  $m_Q$  is related to a short-distance mass (such as the running mass in the  $\overline{\text{MS}}$  scheme), which is a well-defined quantity in perturbation theory, then the corresponding perturbation series

$$m_Q = m_Q^{\text{SD}} \left\{ 1 + c_1 \alpha_s(m_Q) + c_2 \alpha_s^2(m_Q) + \dots + c_n \alpha_s^n(m_Q) + \dots \right\}, \quad (11)$$

contains numerical coefficients  $c_n$  that grow as  $n!$  for large  $n$ , rendering the series divergent and

not Borel summable. The best one can achieve is to truncate the series at the minimal term, but this leads to an unavoidable arbitrariness of order  $\Delta m_Q \sim \Lambda_{\text{QCD}}$  (the size of the minimal term). This observation, which at first sight seems a serious problem, should not come as a surprise. We know that because of confinement quarks do not appear as physical states in nature. Hence, there is no unique way to define their on-shell properties such as a pole mass. In view of this, it is actually remarkable that QCD perturbation theory “knows” about its incompleteness and indicates, through the appearance of renormalon singularities, the presence of non-perturbative effects. We must first specify a scheme how to truncate the QCD perturbation series before non-perturbative parameters such as  $m_Q$  and  $\lambda_1$  become well-defined quantities. The actual values of these parameters will depend on this scheme.

In the difference of the pole masses on the left-hand side in (5), the leading renormalon ambiguity of order  $\Lambda_{\text{QCD}}$  cancels. However, there remains a residual ambiguity of order  $\Lambda_{\text{QCD}}^2/m_Q$  [52]. This point was unclear for some time, since the corresponding renormalon singularity does not appear in the so-called bubble approximation, in which most explicit calculations in the Borel plane are performed [41, 53]. However, it is now known that this singularity must be present beyond the bubble approximation, and the ambiguity in the difference  $m_b - m_c$  is then absorbed by a corresponding ambiguity of order  $\Lambda_{\text{QCD}}^2$  in the parameter  $\lambda_1$  in (5) [52].

It is instructive to consider the same problem from a different point of view, by studying the properties of the kinetic operator under renormalization in regularization schemes with a dimensionful cutoff parameter  $\mu$ . In such schemes, the operator  $O_{\text{kin}}$  can mix with the lower-dimensional operator  $\bar{h}_v h_v$  (the “identity”), because the two operators have the same quantum numbers. If such a mixing is present, it leads to an additive contribution to the parameter  $\lambda_1$  of the form  $\mu^2 C[\alpha_s(\mu)]$ , which one would like to subtract in order to define a renormalized parameter. The coefficient  $C$  can be calculated order by order in an expansion in the small coupling constant  $\alpha_s(\mu)$ , and it appears at first sight that

the quadratically divergent term could be subtracted using perturbation theory. This impression is erroneous, however, because  $C$  may contain non-perturbative contributions of the form  $\exp[-8\pi/\beta_0\alpha_s(\mu)] = (\Lambda_{\text{QCD}}/\mu)^2$ , which cannot be controlled in perturbation theory [54]. Such terms contribute an amount of order  $\Lambda_{\text{QCD}}^2$  to the parameter  $\lambda_1$ , which is of the same order as the renormalized parameter itself. Hence, if the kinetic operator mixes with the identity, it is necessary that the quadratically divergent contribution to  $\lambda_1$  be subtracted in a non-perturbative way, and hence the heavy-quark kinetic energy by itself is not directly a physical quantity.

The question whether there is a mixing of the kinetic energy with the identity, and whether there exists a corresponding renormalon singularity, has been addressed by several authors, with seemingly controversial conclusions. At the one-loop order, such a mixing has indeed been observed when the HQET is regularized on a space-time lattice [54]. Likewise, a “physical” definition of a parameter  $\lambda_1(\mu)$  has been suggested, which absorbs certain  $O(\alpha_s)$  corrections appearing in the zero-recoil sum rules for heavy-quark transitions [35]. This definition is such that  $d\lambda_1(\mu)/d\mu^2 \propto \alpha_s(\mu)$ , indicating again a one-loop mixing of the kinetic energy with the identity. On the other hand, this mixing has not been observed at the one-loop order in two Lorentz-invariant cutoff regularization schemes [53], which use a Pauli–Villars regulator or a cutoff on the virtuality of the gluon in one-loop Feynman diagrams [55]. This observation appeared as a puzzle, which was resolved when it was shown that the mixing between the kinetic energy and the identity is forbidden at the one-loop order in all regularization schemes with a Lorentz-invariant UV cutoff,<sup>1</sup> but that in general there is no symmetry that protects the matrix elements of the kinetic operator from quadratic divergences, and so a mixing with the identity occurs from the two-loop order on [52].

In understanding this situation, the virial theorem of the HQET is of great help. This theorem provides a relation between the kinetic energy of a heavy quark inside a hadron and its chromo-electric interactions with gluons [56]. For the  $B$  meson, it can be written in the form

$$\lim_{v' \rightarrow v} \frac{\langle B(v') | \bar{h}_{v'} v_\mu v'_\nu i g_s G^{\mu\nu} h_v | B(v) \rangle}{(v \cdot v')^2 - 1} = -\frac{1}{3} \langle B | O_{\text{kin}} | B \rangle. \quad (12)$$

Note that in the rest frame of the initial or the final meson, the operator appearing on the left-hand side of this relations only contains the chromo-electric field  $E_c^i = -G^{0i}$ . The virial theorem is a most useful relation. Not only is it intuitive, generalizing a well-known concept of classical physics; it also helps in understanding better the properties of the kinetic operator. For instance, it has been used to estimate the hadronic parameter  $\lambda_1$  using QCD sum rules [33]. More important, however, is the fact that the virial theorem can be employed to analyse the properties of the kinetic operator under renormalization [52]. (Parenthetically, we note that this theorem has also been used in the calculation of the two-loop anomalous dimension of the chromo-magnetic operator [28, 57].) In the limit  $v' \rightarrow v$ , the properties of the two operators  $O_{\text{kin}}$  and  $O_{\text{el}}^{\mu\nu} = \bar{h}_{v'} i g_s G^{\mu\nu} h_v$  are related to each other. The chromo-electric operator  $O_{\text{el}}^{\mu\nu}$  can mix with the lower-dimensional operator  $(v^\mu v'^\nu - v^\nu v'^\mu) \bar{h}_{v'} h_v$ . However, in any regularization scheme with a Lorentz-invariant ultraviolet cutoff, such a mixing can only come from Feynman diagrams involving gluons attached to both heavy-quark lines, since otherwise there is no way to get the factor  $(v^\mu v'^\nu - v^\nu v'^\mu)$ . Such diagrams appear first at the two-loop order, as shown in Fig. 1. The virial theorem thus provides for a simple explanation of the fact that the mixing of the kinetic operator with the identity was not observed at the one-loop order in Lorentz-invariant regularization schemes. On the other hand, an explicit calculation confirms that the mixing is present at the two-loop order, and thus there is indeed a quadratic divergence (and thus a renormalon problem) affecting the matrix elements of

<sup>1</sup>The fact that a mixing at the one-loop order was observed in Refs. [35, 54] is a consequence of the explicit breaking of Lorentz invariance by the regularization schemes adopted in these calculations.

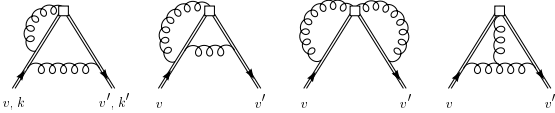


Figure 1. Two-loop diagrams contributing to the mixing of the chromo-electric operator with lower-dimensional operators. Not shown are two copies of the first two diagrams with the “outer” gluon attached to the other heavy-quark line.

the kinetic operator [52].

At the end of this discussion, we stress that the “renormalon ambiguities” are not a conceptual problem for the heavy-quark expansion. It can be shown quite generally that these ambiguities cancel in all predictions for physical quantities [58]–[60]. The way the cancellations occur is intricate, however. The generic structure of the heavy-quark expansion for an observable is of the form:

$$\text{Observable} \sim C[\alpha_s(m_Q)] \left( 1 + \frac{\Lambda}{m_Q} + \dots \right), \quad (13)$$

where  $C[\alpha_s(m_Q)]$  represents a perturbative coefficient function, and  $\Lambda$  is a dimensionful non-perturbative parameter. The truncation of the perturbation series defining the coefficient function leads to an arbitrariness of order  $\Lambda_{\text{QCD}}/m_Q$ , which cancels against a corresponding arbitrariness of order  $\Lambda_{\text{QCD}}$  in the definition of the non-perturbative parameter  $\Lambda$ . Thus, only when the short- and long-distance contributions are combined in the heavy-quark expansion, an unambiguous result is obtained.

### 3. INCLUSIVE B DECAY RATES

Inclusive decay rates determine the probability of the decay of a particle into the sum of all possible final states with a given set of global quantum numbers. An example is provided by the inclusive semileptonic decay rate of the  $B$  meson,  $\Gamma(B \rightarrow X_c \ell \bar{\nu})$ , where the final state consists of a lepton–neutrino pair accompanied by any number of hadrons with total charm-quark number  $n_c = 1$ . Here we shall discuss the theoretical

description of inclusive decays of  $B$  mesons (an analogous description holds for all hadrons containing a heavy quark) [61]–[70]. From the theoretical point of view, such decays have two advantages: first, bound-state effects related to the initial state (such as the “Fermi motion” of the heavy quark inside the hadron [68, 69]) can be accounted for in a systematic way using the heavy-quark expansion; secondly, the fact that the final state consists of a sum over many hadronic channels eliminates bound-state effects related to the properties of individual hadrons. This second feature is based on a hypothesis known as quark–hadron duality, which is an important concept in QCD phenomenology. The assumption of duality is that cross sections and decay rates, which are defined in the physical region (i.e. the region of time-like momenta), are calculable in QCD after a “smearing” or “averaging” procedure has been applied [71]. In semileptonic decays, it is the integration over the lepton and neutrino phase space that provides a “smearing” over the invariant hadronic mass of the final state (so-called global duality). For non-leptonic decays, on the other hand, the total hadronic mass is fixed, and it is only the fact that one sums over many hadronic states that provides an “averaging” (so-called local duality). Clearly, local duality is a stronger assumption than global duality. It is important to stress that quark–hadron duality cannot yet be derived from first principles, although it is a necessary assumption for many applications of QCD. The validity of global duality has been tested experimentally using data on hadronic  $\tau$  decays [72]. Some more formal attempts to address the problem of quark–hadron duality can be found in Refs. [73, 74].

Using the optical theorem, the inclusive decay width of  $B$  mesons can be written in the form

$$\Gamma(B \rightarrow X) = \frac{1}{m_B} \text{Im} \langle B | \mathbf{T} | B \rangle, \quad (14)$$

where the transition operator  $\mathbf{T}$  is given by

$$\mathbf{T} = i \int d^4x T \{ \mathcal{L}_{\text{eff}}(x), \mathcal{L}_{\text{eff}}(0) \}. \quad (15)$$

In fact, inserting a complete set of states inside the time-ordered product, we recover the stan-



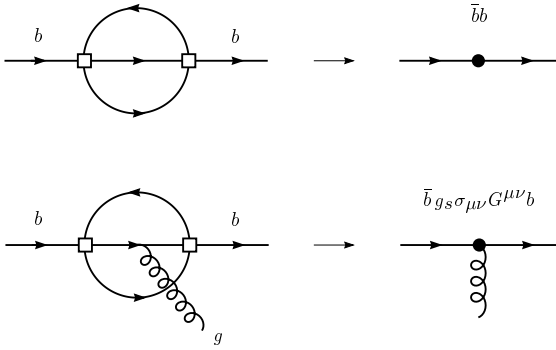


Figure 2. Perturbative contributions to the transition operator  $\mathbf{T}$  (left), and the corresponding operators in the OPE (right). The open squares represent a four-fermion interaction of the effective Lagrangian  $\mathcal{L}_{\text{eff}}$ , while the black circles represent local operators in the OPE.

standard expression

$$\Gamma(B \rightarrow X) = \frac{1}{2m_B} \sum_X (2\pi)^4 \delta^4(p_B - p_X) \times |\langle X | \mathcal{L}_{\text{eff}} | B \rangle|^2 \quad (16)$$

for the decay rate. Here  $\mathcal{L}_{\text{eff}}$  is the effective weak Lagrangian corrected for short-distance effects arising from the exchange of gluons with virtualities between  $m_W$  and  $m_b$  [75]–[79]. If some quantum numbers of the final states  $X$  are specified, the sum over intermediate states is to be restricted appropriately. In the case of the inclusive semileptonic decay rate, for instance, the sum would include only those states  $X$  containing a lepton–neutrino pair.

In perturbation theory, some contributions to the transition operator are given by the two-loop diagrams shown on the left-hand side in Fig. 2. Because of the large mass of the  $b$  quark, the momenta flowing through the internal propagator lines are large. It is thus possible to construct an OPE for the transition operator, in which  $\mathbf{T}$  is represented as a series of local operators containing the heavy-quark fields. The operator with the lowest dimension,  $d = 3$ , is  $\bar{b}b$ . It arises by contracting the internal lines of the first diagram.

The only gauge-invariant operator with dimension 4 is  $\bar{b}i\not{D}b$ ; however, the equations of motion imply that between physical states this operator can be replaced by  $m_b\bar{b}b$ . The first operator that is different from  $\bar{b}b$  has dimension 5 and contains the gluon field. It is given by  $\bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b$ . This operator arises from diagrams in which a gluon is emitted from one of the internal lines, such as the second diagram shown in Fig. 2. For dimensional reasons, the matrix elements of such higher-dimensional operators are suppressed by inverse powers of the heavy-quark mass.

In the next step, the hadronic matrix elements of the local operators in the OPE are expanded in powers of  $1/m_b$ , using the technology of the HQET. The result is [21, 64, 65]

$$\begin{aligned} \frac{\langle B | \bar{b}b | B \rangle}{2m_B} &= 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} + O(1/m_b^3), \\ \frac{\langle B | \bar{b}g_s\sigma_{\mu\nu}G^{\mu\nu}b | B \rangle}{2m_B} &= 6\lambda_2 + O(1/m_b). \end{aligned} \quad (17)$$

Thus, any inclusive decay rate can be written in the form [62]–[64]

$$\begin{aligned} \Gamma(B \rightarrow X_f) &= \frac{G_F^2 m_b^5}{192\pi^3} \left\{ c_3^f \left( 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \right) \right. \\ &\quad \left. + c_5^f \frac{\lambda_2}{m_b^2} + O(1/m_b^3) \right\}, \end{aligned} \quad (18)$$

where the prefactor arises naturally from the loop integrations, and  $c_n^f$  are calculable coefficient functions (which also contain the relevant CKM matrix elements) depending on the quantum numbers  $f$  of the final state. It is instructive to understand the appearance of the kinetic-energy contribution  $\lambda_1/2m_b^2$ . This is nothing but the field-theory analogue of the Lorentz factor  $(1 - \mathbf{v}_b^2)^{1/2} \simeq 1 - \mathbf{k}^2/2m_b^2$ , in accordance with the fact that the lifetime,  $\tau = 1/\Gamma$ , for a moving particle (the  $b$  quark) increases due to time dilation.

The main result of the heavy-quark expansion for inclusive decay rates is the observation that the free quark decay (i.e. the parton model) provides the first term in a systematic  $1/m_b$  expansion [61]. For dimensional reasons, the corresponding rate is proportional to the fifth power of

the  $b$ -quark mass. The non-perturbative corrections, which arise from bound-state effects inside the  $B$  meson, are suppressed by two powers of the heavy-quark mass, i.e. they are of relative order  $(\Lambda_{\text{QCD}}/m_b)^2$ . Note that the absence of first-order power corrections is a consequence of the equations of motion, as there is no independent gauge-invariant operator of dimension 4 that could appear in the OPE. The fact that bound-state effects in inclusive decays are strongly suppressed explains a posteriori the success of the parton model in describing such processes [80, 81].

The hadronic parameters  $\lambda_1$  and  $\lambda_2$  appearing in the heavy-quark expansion (18) have been discussed in the previous section. For a given inclusive decay channel, what remains to be calculated is the coefficient functions  $c_n^f$ . This can be done using perturbation theory. We shall now discuss three important applications of this general formalism.

### 3.1. Determination of $|V_{cb}|$ from Inclusive Semileptonic Decays

The extraction of  $|V_{cb}|$  from the inclusive semileptonic decay rate of  $B$  mesons is based on the general expression (18), with the short-distance coefficients [62]–[64]

$$\begin{aligned} c_3^{\text{SL}} &= |V_{cb}|^2 \left[ 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \ln x^2 \right. \\ &\quad \left. + O(\alpha_s) \right], \\ c_5^{\text{SL}} &= -6|V_{cb}|^2 (1 - x^2)^4. \end{aligned} \quad (19)$$

Here  $x = m_c/m_b$ , and  $m_b$  and  $m_c$  are the pole masses of the  $b$  and  $c$  quarks, defined to a given order in perturbation theory [82]. The  $O(\alpha_s)$  terms in  $c_3^{\text{SL}}$  are known exactly [83], while only partial calculations of higher-order corrections exist [84, 85]. The main sources of theoretical uncertainties are the dependence on the heavy-quark masses, unknown higher-order perturbative corrections, and the assumption of global quark–hadron duality. A conservative estimate of the total theoretical error on the extracted value of  $|V_{cb}|$  is  $\delta|V_{cb}|/|V_{cb}| \approx 10\%$  [86]. Taking the result of Ball et al. [85] for the central value, and using  $\tau_B = (1.60 \pm 0.03)$  ps for the average  $B$ -meson

lifetime [87], we find

$$\begin{aligned} |V_{cb}| &= (0.040 \pm 0.004) \left( \frac{B_{\text{SL}}}{10.8\%} \right)^{1/2} \\ &= (40 \pm 1_{\text{exp}} \pm 4_{\text{th}}) \times 10^{-3}. \end{aligned} \quad (20)$$

In the last step, we have used  $B_{\text{SL}} = (10.8 \pm 0.5)\%$  for the semileptonic branching ratio of  $B$  mesons (see below). The value of  $|V_{cb}|$  extracted from the inclusive semileptonic width is in excellent agreement with that obtained from the analysis of the exclusive decay  $B \rightarrow D^* \ell \bar{\nu}$  using heavy-quark symmetry [88]–[93]. This agreement is gratifying given the differences of the methods used, and it provides an indirect test of global quark–hadron duality. Combining the two measurements gives  $|V_{cb}| = 0.039 \pm 0.002$  [86]. After  $V_{ud}$  and  $V_{us}$ , this is now the third-best known entry in the Cabibbo–Kobayashi–Maskawa (CKM) matrix.

### 3.2. Semileptonic Branching Ratio for Decays into $\tau$ Leptons

Semileptonic decays of  $B$  mesons into  $\tau$  leptons are of particular importance, since they are sensitive probes of physics beyond the Standard Model [94]. From the theoretical point of view, the ratio of the semileptonic rates (or branching ratios) into  $\tau$  leptons and electrons can be calculated reliably. This ratio is independent of the factor  $m_b^5$ , the hadronic parameter  $\lambda_1$ , and CKM matrix elements. To order  $1/m_b^2$ , one finds [67, 95, 96]

$$\begin{aligned} \frac{B(B \rightarrow X \tau \bar{\nu}_\tau)}{B(B \rightarrow X e \bar{\nu}_e)} &= f(x_c, x_\tau) + \frac{\lambda_2}{m_b^2} g(x_c, x_\tau) \\ &= 0.22 \pm 0.02, \end{aligned} \quad (21)$$

where  $f$  and  $g$  are calculable coefficient functions depending on the mass ratios  $x_c = m_c/m_b$  and  $x_\tau = m_\tau/m_b$ , as well as on  $\alpha_s(m_b)$ . Two new measurements of the semileptonic branching ratio of  $b$  quarks,  $B(b \rightarrow X \tau \bar{\nu}_\tau)$ , have been reported by the ALEPH and OPAL Collaborations at LEP [97, 98]. The weighted average is  $(2.68 \pm 0.28)\%$ . Normalizing this result to the LEP average value  $B(b \rightarrow X e \bar{\nu}_e) = (10.95 \pm 0.32)\%$  [87], we obtain

$$\frac{B(b \rightarrow X \tau \bar{\nu}_\tau)}{B(b \rightarrow X e \bar{\nu}_e)} = 0.245 \pm 0.027, \quad (22)$$

in good agreement with the theoretical prediction (21) for  $B$  mesons.

### 3.3. Semileptonic Branching Ratio and Charm Counting

The semileptonic branching ratio of  $B$  mesons is defined as

$$B_{\text{SL}} = \frac{\Gamma(B \rightarrow X e \bar{\nu})}{\sum_{\ell} \Gamma(B \rightarrow X \ell \bar{\nu}) + \Gamma_{\text{had}} + \Gamma_{\text{rare}}}, \quad (23)$$

where  $\Gamma_{\text{had}}$  and  $\Gamma_{\text{rare}}$  are the inclusive rates for hadronic and rare decays, respectively. The main difficulty in calculating  $B_{\text{SL}}$  is not in the semileptonic width, but in the non-leptonic one. As mentioned previously, the calculation of non-leptonic decay rates in the heavy-quark expansion relies on the strong assumption of local quark-hadron duality.

Measurements of the semileptonic branching ratio have been performed by various experimental groups, using both model-dependent and model-independent analyses. The status of the results is controversial, as there is a discrepancy between low-energy measurements performed at the  $\Upsilon(4s)$  resonance and high-energy measurements performed at the  $Z^0$  resonance. The situation has been reviewed recently by Richman [87], whose numbers we shall use in this section. The average value at low energies is  $B_{\text{SL}} = (10.23 \pm 0.39)\%$ , whereas high-energy measurements give  $B_{\text{SL}}(b) = (10.95 \pm 0.32)\%$ . The label ( $b$ ) indicates that this value refers not to the  $B$  meson, but to a mixture of  $b$  hadrons (approximately 40%  $B^-$ , 40%  $B^0$ , 12%  $B_s$ , and 8%  $\Lambda_b$ ). Assuming that the corresponding semileptonic width  $\Gamma_{\text{SL}}(b)$  is close to that of  $B$  mesons,<sup>2</sup> we can correct for this fact and find  $B_{\text{SL}} = (\tau_B/\tau_b) B_{\text{SL}}(b) = (11.23 \pm 0.34)\%$ , where  $\tau_b = (1.56 \pm 0.03)$  ps is the average lifetime corresponding to the above mixture of  $b$  hadrons. The discrepancy between the low- and high-energy measurements of the semileptonic branching ratio is therefore larger than three standard deviations. If we take the average and inflate the error to account for this fact, we obtain

$$B_{\text{SL}} = (10.80 \pm 0.51)\%. \quad (24)$$

An important aspect in interpreting this result is charm counting, i.e. the measurement of the

<sup>2</sup>Theoretically, this is expected to be a very good approximation.

average number  $n_c$  of charm hadrons produced per  $B$  decay. Theoretically, this quantity is given by

$$n_c = 1 + B(B \rightarrow X_{c\bar{c}s'}) - B(B \rightarrow X_{\text{no}c}), \quad (25)$$

where  $B(B \rightarrow X_{c\bar{c}s'})$  is the branching ratio for decays into final states containing two charm quarks, and  $B(B \rightarrow X_{\text{no}c}) \approx 0.02$  is the Standard Model branching ratio for charmless decays [99]–[101]. The average value obtained at low energies is  $n_c = 1.12 \pm 0.05$  [87], whereas high-energy measurements give  $n_c = 1.23 \pm 0.07$  [102]. The weighted average is

$$n_c = 1.16 \pm 0.04. \quad (26)$$

The naive parton model predicts that  $B_{\text{SL}} \approx 15\%$  and  $n_c \approx 1.2$ ; however, it has been known for some time that perturbative corrections could change these predictions significantly [99]. With the establishment of the heavy-quark expansion, the non-perturbative corrections to the parton model could be computed, and their effect turned out to be very small. This led Bigi et al. to conclude that values  $B_{\text{SL}} < 12.5\%$  cannot be accommodated by theory [103]. Later, Bagan et al. have completed the calculation of the  $O(\alpha_s)$  corrections including the effects of the charm-quark mass, finding that they lower the value of  $B_{\text{SL}}$  significantly [104]. Their original analysis has recently been corrected in an erratum. Here we shall present the results of an independent numerical analysis using the same theoretical input (for a detailed discussion, see Ref. [105]). The semileptonic branching ratio and  $n_c$  depend on the quark-mass ratio  $m_c/m_b$  and on the ratio  $\mu/m_b$ , where  $\mu$  is the scale used to renormalize the coupling constant  $\alpha_s(\mu)$  and the Wilson coefficients appearing in the non-leptonic decay rate. The freedom in choosing the scale  $\mu$  reflects our ignorance of higher-order corrections, which are neglected when the perturbative expansion is truncated at order  $\alpha_s$ . We allow the pole masses of the heavy quarks to vary in the range

$$\begin{aligned} m_b &= (4.8 \pm 0.2) \text{ GeV}, \\ m_b - m_c &= (3.40 \pm 0.06) \text{ GeV}, \end{aligned} \quad (27)$$

corresponding to  $0.25 < m_c/m_b < 0.33$ . The value of the difference  $m_b - m_c$  is obtained

from (5) using  $\lambda_1 = -(0.4 \pm 0.2) \text{ GeV}^2$ . Non-perturbative effects appearing at order  $1/m_b^2$  in the heavy-quark expansion are described by the single parameter  $\lambda_2$  defined in (10); the dependence on the parameter  $\lambda_1$  is the same for all inclusive decay rates and cancels out in the predictions for  $B_{\text{SL}}$  and  $n_c$ . For the two choices  $\mu = m_b$  and  $\mu = m_b/2$ , we obtain [105]

$$\begin{aligned} B_{\text{SL}} &= \begin{cases} 12.0 \pm 1.0\%; & \mu = m_b, \\ 10.9 \pm 1.0\%; & \mu = m_b/2, \end{cases} \\ n_c &= \begin{cases} 1.20 \mp 0.06; & \mu = m_b, \\ 1.21 \mp 0.06; & \mu = m_b/2. \end{cases} \end{aligned} \quad (28)$$

The uncertainties in the two quantities, which result from the variation of  $m_c/m_b$  in the range given above, are anticorrelated. Notice that the semileptonic branching ratio has a stronger scale dependence than  $n_c$ . By choosing a low renormalization scale, values  $B_{\text{SL}} < 12\%$  can easily be accommodated. This is indeed not unnatural. Using the BLM scale-setting method [106], it has been estimated that  $\mu \gtrsim 0.32m_b$  is an appropriate scale to use in this case [84].

The combined theoretical predictions for the semileptonic branching ratio and charm counting are shown in Fig. 3. They are compared with the experimental results obtained from low- and high-energy measurements. It has been argued that the combination of a low semileptonic branching ratio and a low value of  $n_c$  would constitute a potential problem for the Standard Model [101]. However, with the new experimental and theoretical numbers, only for the low-energy measurements a discrepancy remains between theory and experiment. Note that, with (25), our results for  $n_c$  can be used to calculate the branching ratio  $B(B \rightarrow X_{c\bar{c}s'})$ , which is accessible to a direct experimental determination. Our prediction of  $(22 \pm 6)\%$  for this branching ratio agrees well with the preliminary result reported by the CLEO Collaboration, which is  $B(B \rightarrow X_{c\bar{c}s'}) = (23.9 \pm 3.8)\%$  [107].

#### 4. SUMMARY

We have presented the theory of inclusive decays of hadrons containing a heavy quark, and discussed some of its most important applications

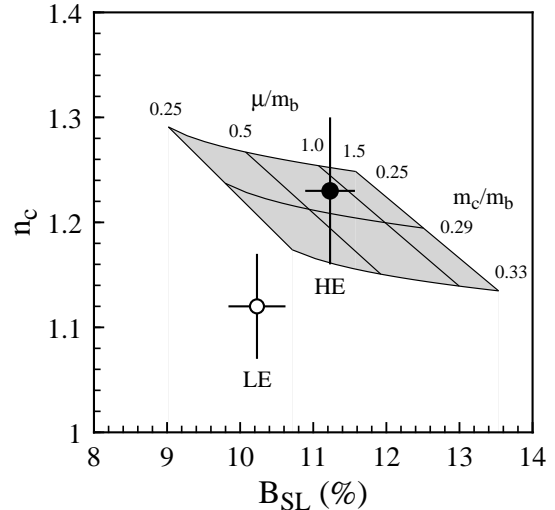


Figure 3. Theoretical prediction for the semileptonic branching ratio and charm counting as a function of the quark-mass ratio  $m_c/m_b$  and the renormalization scale  $\mu$ . The data points show the average experimental values obtained in low-energy (LE) and high-energy (HE) measurements, as discussed in the text.

to the decays of  $B$  mesons: the determination of  $|V_{cb}|$  from inclusive semileptonic decays, semileptonic decays into  $\tau$  leptons, and the semileptonic branching ratio. The theoretical tools that allow us to perform quantitative calculations are the heavy-quark symmetry, the heavy-quark effective theory, and the  $1/m_Q$  expansion. In the case of inclusive decay rates, the non-perturbative information entering the theoretical description is encoded in two hadronic parameters,  $\lambda_1$  and  $\lambda_2$  (or  $\mu_\pi^2$  and  $\mu_G^2$ ). We have reviewed the theoretical understanding of these parameters, concerning both their numerical values and their properties under renormalization. The parameter  $\lambda_2$ , renormalized at the scale  $\mu = m_b$ , can be extracted from the spectroscopy of  $B$  mesons. The parameter  $\lambda_1$ , on the other hand, suffers from a renormalon ambiguity problem and thus needs a non-perturbative subtraction to be well defined.

## REFERENCES

1. M. Neubert, Phys. Rep. **245**, 259 (1994); Int. J. Mod. Phys. A **11**, 4173 (1996).
2. M. Shifman, in: QCD and Beyond, Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, June 1995, edited by D.E. Soper (World Scientific, Singapore, 1996), p. 409.
3. A. Falk, Preprint JHU-TIPAC-96017 [hep-ph/9610363], to appear in: The Strong Interaction, from Hadrons to Partons, Proceedings of the 24th SLAC Summer Institute on Particle Physics, Stanford, California, August 1996.
4. H. Georgi, in: Perspectives in the Standard Model, Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, Boulder, Colorado, 1991, edited by R.K. Ellis, C.T. Hill, and J.D. Lykken (World Scientific, Singapore, 1992), p. 589.
5. N. Isgur and M.B. Wise, in: Heavy Flavours, edited by A.J. Buras and M. Lindner (World Scientific, Singapore, 1992), p. 234.
6. B. Grinstein, Ann. Rev. Nucl. Part. Sc. **42**, 101 (1992).
7. T. Mannel, Chinese J. Phys. **31**, 1 (1993).
8. A.G. Grozin, Preprint Budker INP 92-97 (1992), unpublished.
9. K. Wilson, Phys. Rev. **179**, 1499 (1969); D **3**, 1818 (1971).
10. W. Zimmermann, Ann. Phys. **77**, 536 and 570 (1973).
11. E.V. Shuryak, Phys. Lett. B **93**, 134 (1980); Nucl. Phys. B **198**, 83 (1982).
12. J.E. Paschalis and G.J. Gounaris, Nucl. Phys. B **222**, 473 (1983); F.E. Close, G.J. Gounaris and J.E. Paschalis, Phys. Lett. B **149**, 209 (1984).
13. S. Nussinov and W. Wetzel, Phys. Rev. D **36**, 130 (1987).
14. M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; **47**, 801 (1988) [**47**, 511 (1988)].
15. N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
16. H. Georgi, Phys. Lett. B **240**, 447 (1990).
17. E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); **243**, 427 (1990).
18. A.F. Falk, B. Grinstein and M.E. Luke, Nucl. Phys. B **357**, 185 (1991).
19. T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B **368**, 204 (1992).
20. H. Leutwyler, Preprint [hep-ph/9609465], to appear in: Effectives Theories and Fundamental Interactions, Proceedings of the 34th International School of Subnuclear Physics, Erice, Italy, July 1996.
21. A.F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 and 2982 (1993).
22. G. 't Hooft, Nucl. Phys. B **61**, 455 (1973); G. 't Hooft and M. Veltman, Nucl. Phys. B **44**, 189 (1972).
23. C.G. Bollini and J.J. Giambiagi, Phys. Lett. B **40**, 566 (1972); Nuovo Cim. B **12**, 20 (1972).
24. W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. D **18**, 3998 (1978); A.J. Buras, Rev. Mod. Phys. **52**, 199 (1980).
25. M. Luke and A.V. Manohar, Phys. Lett. B **286**, 348 (1992).
26. Y.-Q. Chen, Phys. Lett. B **317**, 421 (1993).
27. M. Finkemeier, H. Georgi and M. McIrvin, Preprint HUTP-96-A053 [hep-ph/9701243].
28. G. Amorós, M. Beneke and M. Neubert, Preprint CERN-TH/97-3 [hep-ph/9701375].
29. A. Czarnecki and A.G. Grozin, Preprint TTP-97-04 [hep-ph/9701415].
30. E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B **129**, 66 (1977) [E: **139**, 545 (1978)].
31. V. Eletsky and E. Shuryak, Phys. Lett. B **276**, 191 (1992).
32. P. Ball and V.M. Braun, Phys. Rev. D **49**, 2472 (1994).
33. M. Neubert, Phys. Lett. B **389**, 727 (1996).
34. V. Giménez, G. Martinelli and C.T. Sachrajda, Preprint CERN-TH/96-175 [hep-lat/9607055].
35. I. Bigi, M. Shifman, N.G. Uraltsev and A. Vainshtein, Phys. Rev. D **52**, 196 (1995); Int. J. Mod. Phys. A **9**, 2467 (1994).
36. M. Gremm, A. Kapustin, Z. Ligeti and M.B. Wise, Phys. Rev. Lett. **77**, 20 (1996).
37. A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D **53**, 6316 (1996).

38. V. Chernyak, Nucl. Phys. B **457**, 96 (1995); Phys. Lett. B **387**, 173 (1996).
39. D.S. Hwang, C.S. Kim and W. Namgung, Phys. Rev. D **54**, 5620 (1996); Preprint KEK-TH-492 [hep-ph/9608392].
40. F. De Fazio, Mod. Phys. Lett. A **11**, 2693 (1996).
41. M. Beneke and V.M. Braun, Nucl. Phys. B **426**, 301 (1994).
42. I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. D **50**, 2234 (1994).
43. G. 't Hooft, in: The Whys of Subnuclear Physics, Proceedings of the 15th International School on Subnuclear Physics, Erice, Sicily, 1977, edited by A. Zichichi (Plenum Press, New York, 1979), p. 943.
44. B. Lautrup, Phys. Lett. B **69**, 109 (1977).
45. G. Parisi, Phys. Lett. B **76**, 65 (1978); Nucl. Phys. B **150**, 163 (1979).
46. F. David, Nucl. Phys. B **234**, 237 (1984); **263**, 637 (1986).
47. A.H. Mueller, Nucl. Phys. B **250**, 327 (1985).
48. V.I. Zakharov, Nucl. Phys. B **385**, 452 (1992); M. Beneke and V.I. Zakharov, Phys. Rev. Lett. **69**, 2472 (1992).
49. M. Beneke, Phys. Lett. B **307**, 154 (1993); Nucl. Phys. B **405**, 424 (1993).
50. D. Broadhurst, Z. Phys. C **58**, 339 (1993).
51. A.H. Mueller, in: QCD – 20 Years Later, edited by P.M. Zerwas and H.A. Kastrup (World Scientific, Singapore, 1993), p. 162; Phys. Lett. B **308**, 355 (1993).
52. M. Neubert, Preprint CERN-TH/96-282 [hep-ph/9610471], to appear in Phys. Lett. B.
53. G. Martinelli, M. Neubert and C.T. Sachrajda, Nucl. Phys. B **461**, 238 (1996).
54. L. Maiani, G. Martinelli and C.T. Sachrajda, Nucl. Phys. B **368**, 281 (1992).
55. M. Neubert, Phys. Rev. D **51**, 5924 (1995).
56. M. Neubert, Phys. Lett. B **322**, 419 (1994).
57. G. Amorós and M. Neubert, Preprint CERN-TH/96-336 [hep-ph/9612298], to appear in Phys. Lett. B.
58. M. Neubert and C.T. Sachrajda, Nucl. Phys. B **438**, 235 (1995).
59. M. Beneke, V.M. Braun and V.I. Zakharov, Phys. Rev. Lett. **73**, 3058 (1994).
60. M. Luke, A.V. Manohar and M.J. Savage, Phys. Rev. D **51**, 4924 (1995).
61. J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B **247**, 399 (1990).
62. I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B **293**, 430 (1992) [E: **297**, 477 (1993)]; I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); I.I. Bigi et al., in: Proceedings of the Annual Meeting of the Division of Particles and Fields of the APS, Batavia, Illinois, 1992, edited by C. Albright et al. (World Scientific, Singapore, 1993), p. 610.
63. B. Blok, L. Koyrakh, M.A. Shifman and A.I. Vainshtein, Phys. Rev. D **49**, 3356 (1994) [E: **50**, 3572 (1994)].
64. A.V. Manohar and M.B. Wise, Phys. Rev. D **49**, 1310 (1994).
65. M. Luke and M.J. Savage, Phys. Lett. B **321**, 88 (1994); A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D **49**, 3367 (1994).
66. T. Mannel, Nucl. Phys. B **413**, 396 (1994).
67. A.F. Falk, Z. Ligeti, M. Neubert and Y. Nir, Phys. Lett. B **326**, 145 (1994).
68. M. Neubert, Phys. Rev. D **49**, 3392 and 4623 (1994).
69. I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Int. J. Mod. Phys. A **9**, 2467 (1994).
70. T. Mannel and M. Neubert, Phys. Rev. D **50**, 2037 (1994).
71. E.C. Poggio, H.R. Quinn and S. Weinberg, Phys. Rev. D **13**, 1958 (1976).
72. M. Girone and M. Neubert, Phys. Rev. Lett. **76**, 3061 (1996).
73. M. Shifman, Minneapolis Preprint TPI-MINN-95/15-T [hep-ph/9505289], in: Proceedings of the Joint Meeting of the International Symposium on Particles, Strings and Cosmology and the 19th Johns Hopkins Workshop on Current Problems in Particle Theory, Baltimore, Maryland, March 1995, p. 69; B. Chibisov, R.D. Dikeman, M. Shifman and N. Uraltsev, Preprint TPI-MINN-96-05-T [hep-ph/9605465].
74. C.G. Boyd, B. Grinstein and A.V. Manohar, Phys. Rev. D **54**, 2081 (1996).
75. G. Altarelli and L. Maiani, Phys. Lett. B **52**,

- 351 (1974).
76. M.K. Gaillard and B.W. Lee, *Phys. Rev. Lett.* **33**, 108 (1974).
  77. F.G. Gilman and M.B. Wise, *Phys. Rev. D* **20**, 2392 (1979).
  78. G. Altarelli, G. Curci, G. Martinelli and S. Petrarca, *Phys. Lett. B* **99**, 141 (1981); *Nucl. Phys. B* **187**, 461 (1981).
  79. A.J. Buras and P.H. Weisz, *Nucl. Phys. B* **333**, 66 (1990).
  80. G. Altarelli et al., *Nucl. Phys. B* **208**, 365 (1982).
  81. C.H. Jin, W.F. Palmer and E.A. Paschos, *Phys. Lett. B* **329**, 364 (1994); A. Bareiss and E.A. Paschos, *Nucl. Phys. B* **327**, 353 (1989).
  82. R. Tarrach, *Nucl. Phys. B* **183**, 384 (1981).
  83. Y. Nir, *Phys. Lett. B* **221**, 184 (1989).
  84. M. Luke, M.J. Savage and M.B. Wise, *Phys. Lett. B* **343**, 329 (1995); **345**, 301 (1995).
  85. P. Ball, M. Beneke and V.M. Braun, *Phys. Rev. D* **52**, 3929 (1995).
  86. M. Neubert, in: *Proceedings of the 17th International Symposium on Lepton-Photon Interactions*, Beijing, P.R. China, August 1995, edited by Z. Zhi-Peng and C. He-Sheng (World Scientific, Singapore, 1996), p. 298.
  87. J.D. Richman, Preprint [hep-ex/9602009], to appear in: *Proceedings of the 28th International Conference on High Energy Physics*, Warsaw, Poland, July 1996.
  88. M. Neubert, *Phys. Lett. B* **264**, 455 (1991); **338**, 84 (1994).
  89. H. Albrecht et al. (ARGUS Collaboration), *Z. Phys. C* **57**, 533 (1993).
  90. B. Barish et al. (CLEO Collaboration), *Phys. Rev. D* **51**, 1041 (1995).
  91. D. Buskulic et al. (ALEPH Collaboration), *Phys. Lett. B* **359**, 236 (1995).
  92. P. Abreu et al. (DELPHI Collaboration), *Z. Phys. C* **71**, 539 (1996).
  93. K. Ackerstaff et al. (OPAL Collaboration), Preprint CERN-PPE-96-162 (1996).
  94. Y. Grossman and Z. Ligeti, *Phys. Lett. B* **332**, 373 (1994); **347**, 399 (1995).
  95. S. Balk, J.G. Körner, D. Pirjol and K. Schilcher, *Z. Phys. C* **64**, 37 (1994).
  96. L. Koyrakh, *Phys. Rev. D* **49**, 3379 (1994).
  97. ALEPH Collaboration, Paper PA10-019, contributed to the International Conference on High Energy Physics, Warsaw, Poland, July 1996.
  98. OPAL Collaboration, Paper PA05-038, contributed to the International Conference on High Energy Physics, Warsaw, Poland, July 1996.
  99. G. Altarelli and S. Petrarca, *Phys. Lett. B* **261**, 303 (1991).
  100. H. Simma, G. Eilam and D. Wyler, *Nucl. Phys. B* **352**, 367 (1991).
  101. G. Buchalla, I. Dunietz and H. Yamamoto, *Phys. Lett. B* **364**, 188 (1995).
  102. D. Buskulic et al. (ALEPH Collaboration), *Phys. Lett. B* **388**, 648 (1996).
  103. I. Bigi, B. Blok, M.A. Shifman and A. Vainshtein, *Phys. Lett. B* **323**, 408 (1994).
  104. E. Bagan, P. Ball, V.M. Braun and P. Gosdzinsky, *Nucl. Phys. B* **432**, 3 (1994); *Phys. Lett. B* **342**, 362 (1995) [E: **374**, 363 (1996)]; E. Bagan, P. Ball, B. Fiol and P. Gosdzinsky, *Phys. Lett. B* **351**, 546 (1995).
  105. M. Neubert and C.T. Sachrajda, *Nucl. Phys. B* **483**, 339 (1997).
  106. S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, *Phys. Rev. D* **28**, 228 (1983); G.P. Lepage and P.B. Mackenzie, *Phys. Rev. D* **48**, 2250 (1993).
  107. K. Honscheid, to appear in: *Proceedings of the 20th Johns Hopkins Workshop on Current Problems in Particle Theory*, Heidelberg, Germany, June 1996.