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Flavour-Dependent and Basis-Independent Measures of R Violation

Sacha Davidson

Max Planck Institut fur Physik, Föhringer Ring 6, 80805 München, Germany

and

John Ellis

Theory Division, CERN, CH-1211, Genève 23, Switzerland

Abstract

We construct lepton flavour-dependent and basis-independent measures of R-parity violation in the minimal supersymmetric extension of the Standard Model (MSSM). We work in the context of exact supersymmetry, neglecting the effects of Higgs vacuum expectation values and soft supersymmetry-breaking terms. We devote particular attention to appropriate choices of flavour eigenstates, and to the counting and enumeration of R-violating invariants in two- and three-generation models. We also make an indicative application of our results to derive possible basis-independent cosmological upper bounds on flavour-dependent violation of R parity.

1   Introduction

One of the most attractive possible extensions of the Standard Model is supersymmetry (SUSY) [1]. The Lagrangian for the minimal supersymmetric extension of the Standard Model (MSSM) could in principle contain renormalisable baryon- ($B-$) and lepton- ($L-$) number-violating interactions. These are constrained by the fact that neither B nor L violation has yet been observed experimentally. It is possible to remove these interactions by imposing a multiplicatively-conserved global symmetry called R parity [2], whose action can be defined at the level of the MSSM superpotential such that matter superfields (L, Q, E^c, U^c, D^c) are multiplied by -1, but the Higgs superfields are invariant. In component notation, R parity multiplies fields by $(-1)^{3B+L+2S}$, where S is the spin.

Alternatively, one can merely require that possible R-violating couplings be sufficiently small that their effects would not (yet) have been seen by experiment [3]. One might also want to bear in mind the drastic cosmological implications of possible R violation, even at levels below the current experimental upper limits, for example for the survival of any primordial baryon or lepton asymmetry [4, 5]. However, there is problem with the formulation of experimental or cosmological upper bounds on the magnitudes of R-violating couplings: L violation can be moved around the superpotential by basis transformations, so the apparent magnitudes of these coupling constants are basis dependent. In a previous paper [6] we have therefore derived basis-independent expressions that parametrize R violation in the MSSM: we denote [6] subsequently as DE.

The R-violating invariants of DE have various limitations: for example, there we neglected soft SUSY-breaking interactions and the Higgs vacuum expectation values, which we do here also. Another shortcoming of DE was that we summed over lepton flavours, since otherwise the invariants would have depended on the basis chosen for the singlet right-handed leptons. This means that the basis-independent expressions of DE only measured the sum of the violation of the lepton flavours L_e, L_μ and L_τ. In this paper, we determine the appropriate basis for the quarks and singlet right-handed leptons, obviating the need to sum over the quark and singlet lepton flavours. In this way, we obtain basis-independent measures of the violation of individual lepton flavours. Since there are more terms in the flavour sums of DE than there are R-violating couplings, not all the terms in our previous sums can be independent invariants. We therefore discuss the number of distinct, basis-independent combinations of coupling constants that measure R-parity breaking if one does *not* sum over lepton flavours, and express the "superfluous" invariants in terms of an independent basis set. We then use these to extend our previous discussion of the possible cosmological constraints on R-parity violation to include the case where additional flavour symmetries are present.

2   Review of Flavour- and Basis-Independent Measures of R Violation

The problem that DE sought to address is that the magnitude of possible R-violating superpotential couplings is basis dependent. The MSSM superpotential, including possible R-violating

terms, is:

$$W = \mu_H \bar{H}H + h_u^{ij}\bar{H}Q_iU_j^c + h_d^{ij}HQ_iD_j^c + h_e^{ij}HL_iE_j^c + \epsilon_i\bar{H}L_i + \lambda_1^{ijk}L_iL_jE_k^c + \lambda_2^{ijk}L_iQ_jD_k^c + \lambda_3^{ijk}U_i^cD_j^cD_k^c$$
(1)

Hereon, we focus on the violation of lepton number, and so neglect the possible B-violating coupling constant λ_3. It is well known that ϵ in (1) can be rotated away by a basis transformation. This is because the doublet Higgs and lepton superfields H, L_i have the same gauge quantum numbers, and SUSY removes the Standard Model distinction between H and the L_i that is based on their spins. Once we allow for R violation, there is no unambiguous distinction between H and the L_i. If it is unclear how to define a lepton, then it is also unclear which coupling constants violate L.

If the Higgs and doublet lepton superfields are assembled into a vector

$$\phi_I = (H, L_i)$$
(2)

whose index I runs from 0 to 3, then the superpotential (1) can be rewritten as:

$$W = \mu^I\bar{H}\phi_I + \lambda_e^{IJr}\phi_I\phi_J E_k^c + \lambda_d^{Ipq}\phi_I Q_p D_q^c + h_u^{ij}\bar{H}Q_iU_j^c$$
(3)

where $\mu_I = (\mu, \epsilon_i)$, $\frac{1}{2}h_e^{jk} = \lambda_e^{0jk}$, $\lambda_1^{ijk} = \lambda_e^{ijk}$, $\lambda_d^{0jk} = h_d^{jk}$, and $\lambda_2^{ijk} = \lambda_d^{ijk}$. This superpotential contains R violation if different coupling constants, as vectors and matrices in ϕ space, choose different directions to be the Higgs. The differences between the various directions chosen to be the Higgs parametrize the amount of R violation present, and are independent of basis. Therefore, to obtain basis-independent measures of R violation [6], one should first consider a lepton Yukawa interaction, which defines a plane spanned by a Higgs and lepton in ϕ space, then use another interaction to fix one direction in that plane as the Higgs, with the orthogonal direction being defined as the lepton. If some other interaction chooses as the Higgs a direction that has a non-zero projection onto the direction previously chosen to be a lepton, this yields a measure of R violation that is independent of the basis chosen in ϕ space. Note that at least one Yukawa coupling must enter into any such measure of R violation, because, in the absence of the Yukawa couplings, all the directions in ϕ space are equally good choices to be the Higges, and it is not possible to define R violation.

A number of algebraic expressions, which correspond to sums of closed supergraphs, were computed in DE using this algorithm:

$$\delta_1 = \frac{(\mu^{I*}\lambda_e^{IJp}\lambda_d^{Jqr*})(\mu^K\lambda_e^{KLp*}\lambda_d^{Lqr})}{|\mu|^2|\lambda_e|^2|\lambda_d|^2}$$
(4)

$$\delta_2 = \frac{\mu^{I*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_e^{KLq}\lambda_e^{LMp*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJp}\lambda_e^{JKq*}\mu^K)(\lambda_e^{LMp*}\lambda_e^{LMq})}{|\mu|^2 Tr[\lambda_e^{p*}\lambda_e^p\lambda_e^{q*}\lambda_e^q]}$$
(5)

$$\delta_3 = \frac{\lambda_d^{Irs*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_e^{KLq}\lambda_e^{LMp*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_d^{Krs})(\lambda_e^{LMp*}\lambda_e^{LMq})}{|\lambda_d|^2 Tr[\lambda_e^{p*}\lambda_e^p\lambda_e^{q*}\lambda_e^q]}$$
(6)

$$\delta_4 = \frac{Tr[\lambda_e^p\lambda_e^{q*}\lambda_e^r\lambda_e^{r*}\lambda_e^q\lambda_e^{p*}] + Tr[\lambda_e^p\lambda_e^{q*}\lambda_e^r\lambda_e^{p*}\lambda_e^q\lambda_e^{r*}]}{Tr[\lambda_e^p\lambda_e^{p*}\lambda_e^r\lambda_e^{r*}\lambda_e^q\lambda_e^{q*}]}$$
$$- \frac{1}{2}\frac{Tr[\lambda_e^p\lambda_e^{q*}\lambda_e^r\lambda_e^{p*}]Tr[\lambda_e^q\lambda_e^{r*}] + Tr[\lambda_e^r\lambda_e^{q*}\lambda_e^p\lambda_e^{r*}]Tr[\lambda_e^q\lambda_e^{p*}]}{Tr[\lambda_e^p\lambda_e^{p*}\lambda_e^r\lambda_e^{r*}\lambda_e^q\lambda_e^{q*}]}$$
(7)

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$$\delta_5 = \frac{(\lambda_d^{Ist*}\lambda_e^{IJp}\lambda_d^{Jqr*})(\lambda^{Kst}\lambda_e^{KLp*}\lambda_d^{Lqr}) + (\lambda_d^{Ist*}\lambda_e^{IJp}\lambda_d^{Jqr*})(\lambda_d^{Kqt}\lambda_e^{KLp*}\lambda_d^{Lsr})}{|\lambda_e|^2|\lambda_d|^4} \quad . \tag{8}$$

Here the traces are over the capitalized ϕ indices, and the lower-case indices correspond to quark and right-handed lepton generations, which are also summed [1].

Some of the couplings in eqn (3) yield interactions of particles in the ϕ subspace with particles in other multi-dimensional spaces, *e.g.* E^c, Q, D^c. The magnitudes of the couplings therefore depend on the choice of basis in these other spaces. In DE, this was resolved by summing over all the indices: those of ϕ, of the quarks, and of the right-handed leptons. This produced basis-independent expressions that measured the total L violation. However, expressions that measure the magnitude of R violation for a given quark or lepton generation would have additional phenomenological applications. It would also be interesting to know how many independent invariants there are: it is clear that, if one *did not* do the quark and right-handed lepton sums in the expressions (4) –(8), there would be more "invariants" than R-violating couplings, so some of these invariants must be redundant.

3 Specification of the Superfield Bases

There are various possible choices for the Q, D^c and E^c bases. The most useful ones are presumably those that diagonalise the propagators in flavour space. In the absence of soft SUSY breaking, this diagonalisation can be performed at the superfield level. Moreover, we are neglecting the Higgs vacuum expectation values, as is appropriate for conditions in the early Universe. We may therefore choose the Q basis such that

$$\sum_{Ip} \lambda_d^{Iqp}\lambda_d^{Irp*} + \sum_s h_u^{qs}h_u^{rs*} \propto \delta^{qr} \;, \tag{9}$$

the D^c basis such that

$$\sum_{Ip} \lambda_d^{Ipq}\lambda_d^{Ipr*} \propto \delta^{qr} \;, \tag{10}$$

and the E^c basis such that

$$\sum_{IJ} \lambda_e^{IJq}\lambda_e^{IJr*} \propto \delta^{qr} \;. \tag{11}$$

These conditions ensure that the sums of the vacuum polarization supergraphs are flavour diagonal for each of the corresponding types of particle:Q, D^c, E^c. For example, the conditions (9, 10, 11) ensure that the one-loop propagators are flavour-diagonal at high temperatures in the early universe, when soft SUSY breaking and Higgs vacuum expectation values can be neglected. In the absence of R-parity breaking, the above prescription yields the Standard Model mass-eigenstate basis for the E^c and D^c: writing $\lambda_e^{0jk} = \frac{1}{2}h_e^{jk}$ (the Yukawa coupling matrix of the leptons) and $\lambda_e^{ijk} = 0$, equation (11) is equivalent to $h_e^\dagger h_e$ being diagonal. This

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[1] Note that we have rewritten slightly δ_4, as compared to the expression in DE. If one sums over all the right-handed lepton indices, the expression above is equivalent that in to DE. However, if one does *not* perform the right-handed index sum, the above expression vanishes in the absence of R violation, whereas the version in DE does not.

is the usual mass-eigenstate basis for the singlet right-handed charged leptons in the Standard Model, and a similar argument applies to D^c.

4 Counting the Flavour-Dependent and Basis-Independent Invariants in Two Generations

Having chosen a well-motivated basis for the quarks and right-handed leptons, we can now construct invariants that measure the violation of individual lepton flavours, where these are defined to be associated with the right-handed leptons E_i^c. To illustrate how this may be done, we first consider as a warm-up exercise a simplified model with just two generations, whose geometry is easier to visualise than the fully realistic three-generation case. If one simply considers all the expressions listed in equations (4) - (8), and does not perform the quark and singlet-lepton sums, one obtains 42 invariants in the two-generation model: eight from δ_1, two from δ_2, eight from δ_3, none from δ_4, and 24 from δ_5 [2]. It is clear that this is overcounting, because in two generations there are only twelve possible R-violating couplings: two from each of μ and the four λ_d, and one from each λ_e [3]. The reason that there are more invariants than R-violating couplings is as follows. The different couplings $\mu, \lambda_d, \lambda_e, ...$ choose different directions in ϕ space to be the Higgs and the leptons, as seen in Fig. 1. The various invariants measure the differences between these vectors, which is a geometric invariant independent of the basis chosen in the ϕ space. However, it is geometrically clear that if one knows the vectors in ϕ space corresponding to the differences between μ and λ_d^{11} and between μ and λ_d^{12}, then the invariant measuring the difference between λ_d^{11} and λ_d^{12} is not providing any new information.

Given the excess of invariants, we now analyze the number of independent invariants, and express the extra dependent ones in terms of an independent basis set. This is simpler in two generations, because the ϕ space is three-dimensional, and hence easier to visualize. We expect to find 10 independent invariants in two generations, because, if one chooses to work in the basis where $\mu = (\mu, 0, 0)$, two L-violating couplings arise from each of the four λ_d^{pq}, and one L-violating coupling from each of the λ_e. This expectation of finding ten independent invariants is indeed confirmed by our analysis.

The λ_e have antisymmetric indices, so they have a zero eigenvector in this two-dimensional model. This means that they can be pictured as a plane in ϕ space, spanned by the Higgs and a lepton:

$$\lambda_e^{IJk} = |\lambda_e^k|(\hat{H}^I \hat{L}_k^J - \hat{L}_k^I \hat{H}^J) \tag{12}$$

where $|\lambda_e^i| = Tr[\lambda_e^i \lambda_e^{i\dagger}]$. The intersection of the two planes chooses a direction for the Higgs, which is here labelled $\hat{H}$. The directions orthogonal to $\hat{H}$ in the planes defined by the λ_e^i are labelled by $\hat{L}_i$ ($i = 1, 2$). The lepton directions $\hat{L}_i$ are orthogonal by eqn (11):

$$0 = \lambda_e^{IJ1} \lambda_e^{JI2} \propto (\hat{H}^I \hat{L}_1^J - \hat{L}_1^I \hat{H}^J)(\hat{H}^J \hat{L}_2^I - \hat{L}_2^J \hat{H}^I) = -2|\hat{H}|^2 \hat{L}_1 \cdot \hat{L}_2 \tag{13}$$

These definitions of $\hat{H}$ and $\hat{L}_i$ provide the axes in Fig. 1.

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[2]The corresponding number in the full three-generation model would be 321 invariants.
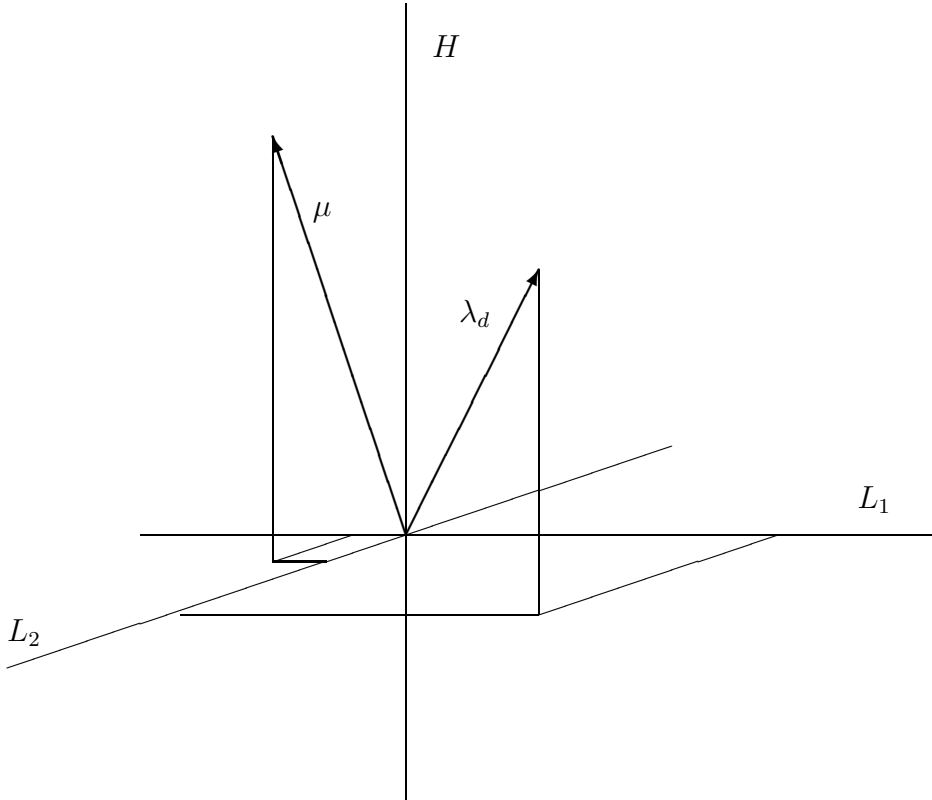
[3]There are 39 such couplings in the three-generation model.

Figure 1: Directions chosen by the interactions μ, λ_d^{pq} and λ_e^i in the two-generation ϕ space: $i, p, q = 1, 2$. The lepton Yukawa couplings λ_e^1 and λ_e^2 each choose a plane which is spanned by the Higgs and a lepton. The λ_e choice of the Higgs is therefore the intersection of the two planes, here labelled $\hat{H}$. In general, μ and the λ_d^{pq} each choose a preferred direction to identify as the Higgs: their projections onto the choice of L_1 and L_2 made by the λ_e are basis-independent measures of R-parity violation. Note that there are in principle four distinct λ_d^{pq}: for simplicity, we show only one.

There is R violation in this two-generation model if the direction chosen to be the Higgs by the λ_e couplings differs from the direction chosen by μ or the λ_d couplings. The lepton-number violation for each flavour can be parametrized by taking the inner product between μ or λ_d - directions in ϕ space that should correspond to the Higgs - and $\hat{L}_i$ as defined by the λ_e. This gives the following 10 independent invariants

$$\delta_2^{12} = \frac{|\mu \cdot \hat{L}_1|^2}{|\mu|^2} = \frac{\mu^{I*}\lambda_e^{IJ1}\lambda_e^{JK2*}\lambda_e^{KL2}\lambda_e^{LM1*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJ1}\lambda_e^{JK2*}\mu^K)(\lambda_e^{LM1*}\lambda_e^{ML2})}{|\mu|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{14}$$

$$\delta_2^{21} = \frac{|\mu \cdot \hat{L}_2|^2}{|\mu|^2} = \frac{\mu^{I*}\lambda_e^{IJ2}\lambda_e^{JK1*}\lambda_e^{KL1}\lambda_e^{LM2*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJ2}\lambda_e^{JK1*}\mu^K)(\lambda_e^{LM2*}\lambda_e^{ML1})}{|\mu|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{15}$$

$$\delta_3^{12rs} = \frac{|\lambda_d^{pq} \cdot \hat{L}_1|^2}{|\lambda_d^{pq}|^2} = \frac{\lambda_d^{Irs*}\lambda_e^{IJ1}\lambda_e^{JK2*}\lambda_e^{KL2}\lambda_e^{LM1*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJ1}\lambda_e^{JK2*}\lambda_d^{Krs})(\lambda_e^{LM1*}\lambda_e^{ML2})}{|\lambda_d^{rs}|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{16}$$

$$\delta_3^{21rs} = \frac{|\lambda_d^{pq} \cdot \hat{L}_2|^2}{|\lambda_d^{pq}|^2} = \frac{\lambda_d^{Irs*}\lambda_e^{IJ2}\lambda_e^{JK1*}\lambda_e^{KL1}\lambda_e^{LM2*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJ2}\lambda_e^{JK1*}\lambda_d^{Krs})(\lambda_e^{LM2*}\lambda_e^{ML1})}{|\lambda_d^{rs}|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{17}$$

where the first expression in each case indicates the geometric interpretation of the invariant as indicated in Fig. 1, and the $\lambda_e\lambda_e$ traces in the second terms in all of these formulae are zero in the basis chosen by (11). The subscripts are derived from the numbering of the δ invariants in equations (4) to (8), and the superscripts are the generation indices of the right-handed leptons and quarks. One can also construct other invariants, corresponding to δ_1 and δ_5 without their generation sums. However, these can be expressed in terms of the invariants (14) - (17), as shown below, and so are not independent. We therefore find the expected 10 invariant measures of R violation in this two-generation model. We can express other possible invariants in terms of the independent basis set (14) — (17). The combination of coupling constants that appears in the relation between one set of invariants and the other is $\sqrt{\delta}$, which may be interpreted as the length of a vector in ϕ space, see (14) — (17). For instance, the vector corresponding to the difference between λ_d^{pq} and λ_d^{rs}, is the vector sum of the differences between λ_d^{pq} and μ, and between μ and λ_d^{rs}.

In the basis chosen by the λ_e, the invariants can written as:

$$\sqrt{\delta_1^{ipq}} = \frac{(\lambda_d^{pq})_i\mu_0 - \mu_i(\lambda_d^{pq})_0}{|\mu||\lambda_d^{pq}|} \tag{18}$$

$$\sqrt{\delta_2^{ij}} = \frac{\mu_i}{|\mu|} \quad , j \neq i \tag{19}$$

$$\sqrt{\delta_3^{ijpq}} = \frac{(\lambda_d^{pq})_i}{|\lambda_d^{pq}|} \quad , j \neq i \tag{20}$$

$$\sqrt{\delta_5^{pqirs}} = \frac{(\lambda_d^{pq})_i(\lambda_d^{rs})_0 - (\lambda_d^{rs})_i(\lambda_d^{pq})_0}{|\lambda_d^{pq}||\lambda_d^{rs}|} \quad . \tag{21}$$

It is clear from (18) that one can express

$$\sqrt{\delta_1^{ipq}} = \sqrt{\delta_3^{ijpq}(1 - \delta_2^{ij} - \delta_2^{ji})} - \sqrt{\delta_2^{ij}(1 - \delta_3^{ijpq} - \delta_3^{jipq})} \quad , j \neq i \tag{22}$$

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and from (21) one finds

$$\sqrt{\delta_5^{pqirs}} = \sqrt{\delta_3^{ijpq}(1 - \delta_3^{ijrs} - \delta_3^{jirs})} - \sqrt{\delta_3^{ijrs}(1 - \delta_3^{ijpq} - \delta_3^{jipq})} \quad , \ j \neq i \qquad (23)$$

where we have written $\mu_0 = |\mu|[1 - \delta_2^{ij} - \delta_2^{ji}]^{1/2}$, for instance.

5 Extension to the Three-Generation Case

The full three-generation model is more complicated to analyze, both because it is more difficult to visualize - the ϕ space is four-dimensional, and because the λ_e can contain R violation independently of the other interactions. We therefore count invariants by making a particular basis choice that is motivated by the coupling constants. The couplings μ and λ_d are still vectors in ϕ space that would like to define the direction corresponding to the Higgs, but each λ_e is now geometrically represented by *two* non-intersecting planes. To see this, we recall that, in three generations, any λ_e (say λ_e^1) is a four-dimensional anti-symmetric matrix whose eigenvalues can be labelled as $\pm i h_1$ and $\pm i c_1$. This means that there is a basis in which

$$\lambda_e^1 = \begin{bmatrix} 0 & h_1 & 0 & 0 \\ -h_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_1 \\ 0 & 0 & -c_1 & 0 \end{bmatrix} . \qquad (24)$$

Each of the two non-intersecting planes in ϕ space represented by (24) would like to be spanned by a Higgs and a lepton. However, if a pair of the λ_e^1 eigenvalues vanish, then λ_e corresponds to a single plane, as in the two-generation model discussed in the previous section, and lepton flavour is conserved. In the general non-degenerate case, both of these planes should be spanned by a Higgs and a lepton field. This means that even a single λ_e coupling does not in general conserve lepton flavour, because one right-handed lepton can interact with two flavours of left-handed lepton. Moreover, it would also like two directions in ϕ space to be the Higgs.

In a similar fashion, a second coupling λ_e^2 can also be pictured as two planes. The directions in which the λ_e^2 planes intersect with the λ_e^1 planes can be chosen as the Higgs and, for instance, L_3. The other leptons L_1 and L_2 can be taken as the orthogonal directions in the λ_e^1 planes. With this choice, the Higgs and L_3 fields do not interact, and nor do L_1 and L_2, so λ_e^2 takes the form

$$\lambda_e^2 = \begin{bmatrix} 0 & e_1 & e_2 & 0 \\ -e_1 & 0 & 0 & -b_2 \\ -e_2 & 0 & 0 & b_1 \\ 0 & b_2 & -b_1 & 0 \end{bmatrix} \qquad (25)$$

The third coupling of this type, λ_e^3, is of generic form in the basis chosen in this way by λ_e^1 and λ_e^2. The counting of lepton-flavour-violating couplings therefore yields one from λ_e^1, three from λ_e^2 and five from λ_e^3. However, there are also three conditions imposed on these couplings by the choice of right-handed lepton basis (11), so we expect 6 independent invariant measures of R violation. We could take, for instance, $\delta_4^{iii} : i = 1, 2, 3$ and $\delta_4^{ijk}, i \neq j \neq k$. This choice has

the feature that the δ_4^{iii} parametrize L_i violation in the λ_e^i, and the remaining three invariants then measure the differences between the bases chosen by the various λ_e^i.

As in the two-generation case, μ and the λ_d^{pq} choose directions in ϕ space that should be identified with the Higgs. The components of these vectors along the directions chosen by the λ_e to be leptons are invariant measures of R violation. We therefore need three further invariants for each of μ and the λ_d^{pq}, which can be taken to be:

$$\delta_2^{12} = \frac{\mu^{I*}\lambda_e^{IJ1}\lambda_e^{JK2*}\lambda_e^{KL2}\lambda_e^{LM1*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJ1}\lambda_e^{JK2*}\mu^K)(\lambda_e^{LM1*}\lambda_e^{ML2})}{|\mu|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{26}$$

$$\delta_2^{21} = \frac{\mu^{I*}\lambda_e^{IJ2}\lambda_e^{JK1*}\lambda_e^{KL1}\lambda_e^{LM2*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJ2}\lambda_e^{JK1*}\mu^K)(\lambda_e^{LM2*}\lambda_e^{ML1})}{|\mu|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{27}$$

$$\delta_2^{31} = \frac{\mu^{I*}\lambda_e^{IJ3}\lambda_e^{JK1*}\lambda_e^{KL1}\lambda_e^{LM3*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJ3}\lambda_e^{JK1*}\mu^K)(\lambda_e^{LM3*}\lambda_e^{ML1})}{|\mu|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{3*}\lambda_e^3]} \tag{28}$$

$$\delta_3^{12rs} = \frac{\lambda_d^{Irs*}\lambda_e^{IJ1}\lambda_e^{JK2*}\lambda_e^{KL2}\lambda_e^{LM1*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJ1}\lambda_e^{JK2*}\lambda_d^{Krs})(\lambda_e^{LM1*}\lambda_e^{ML2})}{|\lambda_d^{rs}|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{29}$$

$$\delta_3^{21rs} = \frac{\lambda_d^{Irs*}\lambda_e^{IJ2}\lambda_e^{JK1*}\lambda_e^{KL1}\lambda_e^{LM2*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJ2}\lambda_e^{JK1*}\lambda_d^{Krs})(\lambda_e^{LM2*}\lambda_e^{ML1})}{|\lambda_d^{rs}|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{2*}\lambda_e^2]} \tag{30}$$

$$\delta_3^{31rs} = \frac{\lambda_d^{Irs*}\lambda_e^{IJ3}\lambda_e^{JK1*}\lambda_e^{KL1}\lambda_e^{LM3*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJ3}\lambda_e^{JK1*}\lambda_d^{Krs})(\lambda_e^{LM3*}\lambda_e^{ML1})}{|\lambda_d^{rs}|^2 Tr[\lambda_e^{1*}\lambda_e^1\lambda_e^{3*}\lambda_e^3]} \tag{31}$$

Note that $|\mu\lambda_e^1\lambda_e^3|^2$ is not included in the above list, although $|\mu\lambda_e^1\lambda_e^2|^2$ is. This is because the rôle of the expressions (26) - (31) is to determine whether μ and/or λ_d share the same definition of the Higgs with the λ_e, whereas the differences among the λ_e have already been accounted for by the δ_4 invariants. The purpose of the second λ_e factor in expressions (26) - (31) is only to choose a direction in the plane of the first λ_e to be the Higgs. The orthogonal direction is then the lepton, and one projects μ or λ_d onto the direction of this lepton field. If the three λ_e happen to conserve R parity, which occurs if they each represent a single plane, and the three planes intersect in a common direction, which is then defined unambiguously to be the Higgs, then $\mu\lambda_e^1\lambda_e^3 \sim \mu_1 h_1 h_3$ and $\mu\lambda_e^1\lambda_e^2 \sim \mu_1 h_1 h_2$. Thus one measures the same R-violating coupling, whether the second matrix is taken to be λ_e^2 or λ_e^3. We therefore have $6 + 30 = 36$ invariants in the three-generation case, as expected.

Another way to count invariants in the three-generation case, which may seem more transparent, is to choose the Higgs to be the direction corresponding to μ, and the leptons to be $L_i = \mu \cdot \lambda_e^i$. The right-handed lepton basis must then be chosen such that

$$\mu^\dagger \lambda_e^i \lambda_e^{j\dagger} \mu \propto \delta^{ij} \tag{32}$$

so that the L_i are orthogonal in ϕ space. The λ_e are then of the form

$$\begin{bmatrix} 0 & h_1 & 0 & 0 \\ -h_1 & 0 & \lambda_e^{121} & \lambda_e^{131} \\ 0 & -\lambda_e^{121} & 0 & \lambda_e^{231} \\ 0 & -\lambda_e^{131} & -\lambda_e^{231} & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & h_2 & 0 \\ 0 & 0 & \lambda_e^{122} & \lambda_e^{132} \\ -h_2 & -\lambda_e^{122} & 0 & \lambda_e^{232} \\ 0 & -\lambda_e^{132} & -\lambda_e^{232} & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & h_3 \\ 0 & 0 & \lambda_e^{123} & \lambda_e^{133} \\ 0 & -\lambda_e^{123} & 0 & \lambda_e^{233} \\ -h_3 & -\lambda_e^{133} & -\lambda_e^{233} & 0 \end{bmatrix}.$$

$$\tag{33}$$

The $9 \times 3 = 27$ invariants

$$\delta_1^{\ell pq} = \frac{|\mu^{I*}\lambda_e^{IJ\ell}\lambda_d^{Jpq*}|^2}{|\mu|^2|\lambda_e^{\ell}|^2|\lambda_d^{pq}|^2} \tag{34}$$

then measure L_ℓ violation in the λ_d^{pq}, and the nine invariants

$$\delta_2^{jl} = \frac{\mu^{I*}\lambda_e^{IJj}\lambda_e^{JKl*}\lambda_e^{KLl}\lambda_e^{LMj*}\mu^M - .5\mu^I\lambda_e^{IJj*}\lambda_e^{JKl}\mu^{K*}Tr[\lambda_e^j\lambda_e^{l*}]}{|\mu|^2 Tr[\lambda_e^{j*}\lambda_e^j\lambda_e^{l*}\lambda_e^l]} \tag{35}$$

measure L_j violation in λ_e^l:

$$
\begin{array}{llll}
\not{L}_1 & \text{in } \lambda_e^1: & \delta_2^{11} \sim |\lambda_e^{321}|^2 \\
\not{L}_1 & \text{in } \lambda_e^2: & \delta_2^{12} \sim |\lambda_e^{122}|^2 + |\lambda_e^{132}|^2 \\
\not{L}_1 & \text{in } \lambda_e^3: & \delta_2^{13} \sim |\lambda_e^{123}|^2 + |\lambda_e^{133}|^2 \\
\not{L}_2 & \text{in } \lambda_e^1: & \delta_2^{21} \sim |\lambda_e^{211}|^2 + |\lambda_e^{231}|^2 \\
\not{L}_2 & \text{in } \lambda_e^2: & \delta_2^{22} \sim |\lambda_e^{312}|^2 \\
\not{L}_2 & \text{in } \lambda_e^3: & \delta_2^{23} \sim |\lambda_e^{213}|^2 + |\lambda_e^{233}|^2 \\
\not{L}_3 & \text{in } \lambda_e^1: & \delta_2^{31} \sim |\lambda_e^{321}|^2 + |\lambda_e^{311}|^2 \\
\not{L}_3 & \text{in } \lambda_e^2: & \delta_2^{32} \sim |\lambda_e^{312}|^2 + |\lambda_e^{322}|^2 \\
\not{L}_3 & \text{in } \lambda_e^3: & \delta_2^{33} \sim |\lambda_e^{213}|^2
\end{array} \tag{36}
$$

completing our second enumeration of a basis of independent flavour-dependent measures of R violation [4].

6 Application to Lepton Flavour Violation in Early Cosmology

In the hot plasma of the early Universe at temperatures above the electroweak phase transition (EPT): $T \gtrsim T_c \simeq 100$ GeV, non-perturbative processes that violate $B + L$ [7] are in thermal equilibrium. If R-parity non-conserving interactions that violate all the L_i are also in thermal equilibrium above the EPT, then any asymmetry in B or the L_i gets washed out. This means that either the baryon asymmetry of the Universe (BAU) observed today was not present in the thermal plasma above the electroweak phase transition, motivating scenarios in which it was made at the EPT, or the R-violating couplings are sufficiently small to be out of equilibrium above the EPT. This argument has been used to place strong constraints on R-violating Yukawa couplings [4, 5] of order

$$\lambda_R \lesssim 10^{-7} \tag{37}$$

where λ_R is some generic R-violating Yukawa coupling. In this section we review these constraints in the basis-independent approach of DE, with the added ingredient that we discuss

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[4]Note that, in the two-generation case, λ_e^j can not violate L and L_j simultaneously. This is because λ_e^2 couples E_2^c to either HL_1, HL_2 or L_1L_2. Therefore, in the two-generation case it is easy to check that $\delta_2^{jj} = 0$. However, in the three-generation case, λ_e^{ijk} can violate L and L_k simultaneously via the couplings λ_e^{ijk} $i, j, \neq k$, and this is measured by δ_2^{jj}.

carefully the bounds on each lepton flavour separately, using the flavour-dependent invariants isolated in the previous sections [5].

We first expand on the previous paragraph's discussion of the origin of the bounds. The minimal supersymmetric extension of the Standard Model (MSSM) has four global quantum numbers: B, L_1, L_2, L_3 that are conserved at the perturbative level, in the absence of the R-violating couplings (1). However, anomalous non-perturbative processes that are in thermal equilibrium above the electroweak phase transition, but irrelevantly weak today, violate the combination $B + L$, so the global conserved quantum numbers in the MSSM are $B/3 - L_i$: $i : 1, 2, 3$. If one then introduces the R-violating interactions of (1), there are no conserved global quantum numbers left. This means that, with the MSSM particle content, all asymmetries in the plasma are washed out before the electroweak phase transition if the R-violating interactions are in thermal equilibrium. As has already been mentioned, one possible origin for the baryon asymmetry observed in the Universe today, is to make the asymmetry at the transition, which may be possible for certain areas of SUSY parameter space [8]. One can also create the asymmetry from particles decaying after the EPT [9, 10], or store it in particles not in thermal equilibrium with the MSSM while the R-parity violating interactions are present [12]. For the rest of this paper, we will neglect these scenarios, and assume that the BAU was present in the plasma above the EPT, and must be protected from the depredations of the R-violating interactions.

We assume that non-perturbative $B + L$ violation is in thermal equilibrium before the EPT, and consider only the renormalisable R-violating interactions of equation (1) [6]. A sufficient condition for the BAU to survive is that all the R-parity violating interactions should be out of thermal equilibrium before the EPT, which gives the bound (37). However, this is too restrictive, because it means that the asymmetries in all three of the $B/3 - L_i$ are preserved, whereas keeping one of them would be sufficient to preserve the BAU.

More precise bounds can be obtained by using our invariants in an analysis based on chemical potentials [7]. We assume an interaction to be in effective chemical equilibrium in the early Universe if the interaction rate exceeds the expansion rate. If this condition is satisfied, the sum of the chemical potentials of the particles participating in the interaction is zero [15] [8]. In the Standard Model and the MSSM, all the interactions are in equilibrium once the temperature has dropped to $\sim 1 - 10$ TeV, imposing [14] a set of linear equations on the chemical potentials that we now review. Since the gauge bosons have zero chemical potential, all the members of a gauge multiplet share the same chemical potential. The Higgs interactions in the Standard Model imply:

$$q^i + h - u_R^j = 0 \tag{38}$$
$$q^i - h - d_R^j = 0 \tag{39}$$
$$\ell^i - h - e_R^i = 0 \tag{40}$$

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[5] For a more extensive discussion in the mass-eigenstate basis, see [5].

[6] Non-renormalisable interactions are different [13], in that they may be out of equilibrium before all the Standard Model interactions come into equilibrium, in which case the asymmetries are not neccessarily washed out.

[7] We follow her the naive kinetic theory discussion of [5, 14], which is adequate for our purpose. For a more complete analysis using the grand canonical ensemble, see [11].

[8] We work in a mass-eigenstate basis.

where the chemical potential of a particle is labelled by the particle's name: q is the left-handed quark doublet, h the Higgs doublet, and so on. The flavour non-diagonal quark couplings to the Higgs imply that all the left-handed quarks have the same chemical potential, so $q^i = q^j \equiv q$, and similarly for the different flavours of u_R and d_R quarks. The anomalous $B + L$ violation transforms left-handed quarks into left-handed anti-leptons:

$$3N_g q + \sum_i \ell^i = 0 \tag{41}$$

where N_g is the number of quark generations. We therefore have six linear equations for 10 unknowns. The four free parameters correspond to the four conserved charges in the plasma: Q_{em} and the $B/3 - L_i$. The plasma can contain asymmetries associated with the three conserved global quantum numbers $B/3 - L_i$, but is required not to have an electric charge [9] asymmetry. The electric charge density can be written as

$$n_{em} = \sum_a Q_a (n_a - n_{\bar{a}}) \tag{42}$$

where the sum over a runs over all the particles in the plasma, Q_a is the particle electric charge, and $n_a (n_{\bar{a}})$ is the (anti-)particle number density. For masses and chemical potentials much smaller than the temperature, the particle asymmetry is :

$$n - \bar{n} \simeq \frac{g\mu T^2}{3} \left[1 + \mathcal{O}\left(\frac{m^2}{T^2}\right) \right] \quad \text{for bosons,} \tag{43}$$

and

$$n - \bar{n} \simeq \frac{g\mu T^2}{6} \left[1 + \mathcal{O}\left(\frac{m^2}{T^2}\right) \right] \quad \text{for fermions,} \tag{44}$$

where g is the number of internal degrees of freedom of the particle, and m is the zero-temperature mass [10]. If we neglect the mass contributions in (44) and (43), since their contribution to the asymmetry is reduced with respect to the first terms by $m^2/T^2 \ll 1$, the electric charge density in the Standard Model is

$$n_{em} = N_g(q + 2u_R - d_R) - \sum_i \ell^i - \sum_i e_R^i + 2h = 0 \tag{45}$$

(This equation must be multiplied by three in the MSSM at $T > m_{SUSY}$.) In the MSSM, there are more than twice as many new particles, and therefore many new chemical potentials, but these turn out all to be equal to those of the Standard Model particles. The gauginos are majorana particles, so can not store any asymmetry and must have zero chemical potential. The gaugino-scalar-fermion interactions then imply that the chemical potentials of the spartners ($\equiv \phi$) are equal to those of the Standard Model particles($\equiv \psi$):

$$\phi = \psi \tag{46}$$

---

[9]Or hypercharge - it amounts to the same constraint [14].

[10]See [10, 5, 16] for an analysis including these mass effects, and [11] for an careful discussion of the relation of the chemical potentials and masses to the asymmetries. We use here μ as a generic chemical potential, and hope that the distinction between the chemical potential and the μ term in the MSSM superpotential will be clear from the context.

The only remaining new chemical potential is that of the second Higgs $\bar{H}$, which is required by the mass term $\mu H \bar{H}$ to have equal and opposite chemical potential to the Higgs H.

The solution of the equations of chemical equilibrium in the absence of R violation is easily obtained. One can represent the chemical potentials in the MSSM as a vector in a ten-dimensional space: $\vec{\mu} \equiv (\ell^1, \ell^2, \ell^3, e_R^1, e_R^2, e_R^3, h, q, u_R, d_R)$. The equilibrium conditions (38 - 41) and (45) force this vector to be orthogonal to the directions associated with Q_{em} and the interactions: for instance, the interaction (38)corresponds to the direction (0,0,0,0,0,0,1,1,-1,0). The conservation of the three $B/3 - L_i$ implies that the projections of the chemical potential vector $\vec{\mu}$ onto the directions corresponding to $B/3 - L_i$ are constant, fixing $\vec{\mu}$. The direction corresponding to $B/3 - L_1$ is

$$\vec{v}_{\frac{B}{3}-L_1} \equiv (-2, 0, 0, -1, 0, 0, 2, 1, 1) \tag{47}$$

and there are similar expressions for $\vec{v}_{\frac{B}{3}-L_2}$ and $\vec{v}_{\frac{B}{3}-L_3}$. it is important to note that these vectors are *not* orthogonal to the interaction vectors (38 - 41), nor to the electric charge direction (45). This is to be expected, since the Yukawa interactions can change the ratio of left- and right-handed quarks, although they cannot change the total quark number. One cannot simply express $\vec{\mu}$ in terms of the $\vec{v}_{\frac{B}{3}-L_i}$. Instead, one can subtract from the $\vec{v}_{\frac{B}{3}-L_i}$ the components along the directions corresponding to (38 - 41) and (45), leaving three vectors $\vec{V}_i$ that are the projections of the $\vec{v}_{\frac{B}{3}-L_i}$ onto the space allowed for $\vec{\mu}$. Then $\vec{\mu}$ can be expressed in terms of the $\vec{V}_i$, as:

$$\vec{\mu} = \eta_{\frac{B}{3}-L_i} \vec{V}_i \tag{48}$$

where $\eta_{\frac{B}{3}-L_i}$ is the conserved ratio of $B/3 - L_i$ to photons. The baryon-to-photon ratio is therefore [11]

$$\eta_B = \vec{\mu} \cdot \vec{B} = \sum_i \eta_{\frac{B}{3}-L_i} \vec{V}_i \cdot \vec{B} = \frac{28}{79} \sum_i \eta_{\frac{B}{3}-L_i} \tag{49}$$

where $\vec{B} = (0, 0, 0, 0, 0, 0, 0, 6, 3, 3)$.

Now suppose we include the R-violating interaction $\lambda_3^{ijk} U_i^c D_j^c D_k^c$ of equation (1) to the thermal soup above the electroweak phase transition. We assume for the moment that all the L_i-violating couplings are zero or small enough to satisfy (37), and neglect them. If λ_3 is in thermal equilibrium, i.e., at least one $\lambda_{ijk} > 10^{-7}$ in the thermal mass-eigenstate basis, then there is an additional constraint on the chemical potentials

$$u_R + d_R + d_R = 0 \quad . \tag{50}$$

In conjunction with (38 - 41) and (45), this constraint forces $q = 0$, so there is no baryon asymmetry. An alternative way to formulate this is to note that the direction in chemical potential space corresponding to $\vec{B}$ can be written as a linear combination of the directions (38 - 40), (50) and (45), to which $\vec{\mu}$ must be orthogonal. So, in the presence of the non-perturbative $B+L$ violation and even one of the λ_3^{ijk}, the baryon asymmetry will be washed out, up to lepton mass effects [10, 16] [11]. The baryon-number-violating couplings λ_3^{ijk} should therefore satisfy

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[11]This caveat applies when there are different asymmetries in the various lepton flavours, but is unlikely to

(37). This is unsurprising, since any one of the λ_3 interactions would violate all the $B/3 - L_i$, at least one of which needs to be conserved [12].

We now consider the L_i-violating couplings present in (1), assuming that the λ_3^{ijk} satisfy (37). We wish to find the minimal bounds on the various lepton-flavour-violating rates that ensure that an asymmetry in at least one of the $B/3 - L_i$ is preserved. If we include interactions that violate all of L_1, L_2 and L_3, they imply the chemical potential relations

$$h = \ell^i \tag{51}$$

for $i = 1, 2, 3$, implying that all the asymmetries are washed out. Even the lepton mass effects cannot help in this case, because all the lepton flavours would have the same asymmetry by (51). However, if the interactions in equilibrium conserve a lepton flavour, say $L_1 \equiv L_e$, then the asymmetry in $B/3 - L_1$ is conserved, and a baryon asymmetry

$$\eta_B = \frac{28}{79}\eta_{\frac{B}{3}-L_1} \tag{52}$$

will remain. Therefore, to preserve the BAU, the interactions violating B and at least one of the L_i must be out of equilibrium. In our basis-independent formulation, requiring that the asymmetry in $B/3 - L_1$ be conserved will translate into a bound on the invariants δ involving the Yukawa coupling λ_e^1.

To set a bound on the L_e-violating couplings or invariants, we need an estimate of the L_e-violating rates in the early Universe at $T \gtrsim 100$ GeV. If one worked in a random basis, one would identify μ^1, $(\lambda_d^{pq})^1$ - the ϕ^1 element of the μ and λ_d^{pq} vectors, $(\lambda_e^1)^{1j} : j : 1, 2, 3$ and $(\lambda_e^2)^{1J}, (\lambda_e^3)^{1J}, J : 0, ..., 3$ as L_e-violating couplings, and require that the rates for the processes mediated by these couplings to be out of equilibrium. The rate associated with any R-violating Yukawa coupling λ_R can be estimated as

$$\Gamma_{yuk} \simeq 10^{-2}\lambda_R^2 T \quad . \tag{53}$$

and the rate for the mass corrections to gauge interaction rates can be taken as

$$\Gamma_{mass} \simeq 10^{-1}\frac{m^2}{T^2}g^2 T \tag{54}$$

where the inverse powers of ten account for factors of 4π, etc.. Requiring that all the "L_e-violating" rates listed above be less than the expansion rate at $T \sim 100$ GeV would give the bounds

$$\lambda_R < 10^{-7} \tag{55}$$

preserve a large enough asymmetry, since it would be proportional to $(m_\tau/T)^2$, see equations (43) and (44). A similar argument can be made using the slepton mass differences in the MSSM [5], which could preserve a larger asymmetry if their mass differences are large and the sleptons are light enough to be still present in the thermal bath when the sphalerons go out of equilibrium.

[12]Note that the λ_3^{ijk} alone, in the absence of $B + L$ violation, is not sufficient to take the BAU to zero in the presence of a lepton asymmetry [5]. This is because the lepton asymmetry carries electric charge, which must be cancelled by an asymmetry in the higgs and quarks. In the presence of λ_3, the electric charge asymmetry in the quarks also carries baryon number.

$$\mu^1 < 100 \text{ keV} \tag{56}$$

However, in the absence of a careful discussion of the basis dependence of this analysis, it is unclear to which elements of the coupling constant vectors and matrices (55) applies, because it is not obvious which direction in ϕ space is L_1, and (56) is wrong because the gauge interactions are diagonal in any basis for ϕ space, and so cannot violate R parity. The correct bounds can be calculated by working in the $T = 100$ GeV mass-eigenstate basis, or by estimating the rates in some basis-independent way. The drawback of the thermal mass-eigenstate basis is that it is not the same as the zero-temperature Standard Model mass-eigenstate basis we live in today. It would is therefore desirable to find some basis-independent way of estimating rates associated with the invariants that parametize L_e violation.

We first consider which invariants must be zero for L_e to be conserved. We would like to find a direction in ϕ space that can be chosen as the left-handed lepton associated with E_1^c. For this to be possible, λ_e^1 must correspond to only one plane in ϕ space, spanned by E_L and the Higgs, and the other two λ_e can only intersect that plane in one direction, which is then the Higgs. This then means that these other two λ_e can also only consist of one plane each, because if either of these consisted of two orthogonal planes, they would intersect the λ_e plane twice. We conclude that the three λ_e^i must each consist of a single plane, and that λ_e^1 intersects at least one of the λ_e^2, λ_e^3 planes, which chooses the direction for the Higgs: L_1 is then the perpendicular direction in the λ_e^1 plane. If λ_e^1 intersects both λ_e^2 and λ_e^3, it must do so in the same direction [13]. The λ_d and μ must have no components in the direction E_L, to ensure that L_e is conserved.

This geometry must now be translated into statements about the vanishing of certain invariants. First, we need $\delta_1^{1pq} \sim |\mu \lambda_e^1 \lambda_d^{pq}|^2 = 0$ for all p, q. Recall that λ_e^1 represents a single plane, as in the two generation case, so is of the form (12). Setting this invariant to zero means that μ and λ_d^{pq} do not have components along a direction in the λ_e^1 plane. Secondly, for similar reasons we also need $\delta_5^{pq1rs} = 0, \forall\, p, q, r, s$, though this invariant is not independent, as previously discussed. Thirdly, we need $\delta_2^{1j} = 0$ for all j, to ensure that the direction in the λ_e^1 plane chosen to be the Higgs by μ is the same as the direction chosen by the λ_e^j, and there is again a similar argument for δ_3^{1pq}. Finally, we need $\delta_4^{i1j} = 0$. This implies that there is no L_1 violation among the λ_e: as previously argued, each λ_e must represent a single plane, and if both λ_e^2 and λ_e^3 intersect the λ_e^1 plane, it must be in the same direction. Writing λ_e^1 in the form (12), where $\hat{H}$ is the direction in which the three λ_e intersect, it is clear that $\lambda_e^2 \lambda_e^1 \lambda_e^3 = 0$. The δ_4^{11j} must also be zero, because the last two terms cancel the first two in the expression (8), in the absence of L_1 violation.

We now know which invariants must be zero to conserve L_e exactly. However, this is not necessary to preserve the baryon asymmetry: we only need L_e violation to be out of thermal equilibrium at $T \sim 100$ GeV. We therefore need to be able to estimate a rate for each invariant. It is possible to visualise any given invariant (e.g., δ_1^{1pq}), made up of three coupling constants $(\mu, \lambda_e^1, \lambda_d^{pq}$ in this case) as representing the squared amplitude for the process mediated by the "middle" interaction (λ_e^1 in this case), with thermal masses due to the "outside interactions" $(\mu, \lambda_d$ in this case) on the external ϕ legs in one of the amplitudes: see figure 2. For the

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[13]Note that R violation in the λ_e is still possible: e.g., λ_e^1 could choose the plane (0-1), λ_e^2 the plane (0-2) and λ_e^3 the plane (2-3). In this case, the invariants $\delta_4^{123}, \delta_4^{132}$ and δ_4^{ijk} with $i, j, k = 2, 3$ could still be non-zero.

invariant to represent a process taking place in the early Universe, the interactions appearing in it would have to be in thermal equilibrium. In the case of δ_1^{1pq}, this would require $|\mu|$ to be in equilibrium, so that it gives a mass to a direction in ϕ space that could be the Higgs. Also, the row of the matrix λ_e^1 that transforms this "Higgs" into a lepton would have to be in equilibrium, and we would need the component of λ_d^{pq} coupling to this lepton direction to be in equilibrium. We can estimate self-consistently the coupling constant for the λ_e^1-mediated process to be $|\lambda_e^1|$, as will be explained below, and the relevant component squared of λ_d^{pq} has magnitude $\delta_1^{1pq} \times |\lambda_d^{pq}|^2$. We could just as well have made this argument starting with λ_d^{pq}, in which case the relevant R-violating coupling squared would be the mass $\delta_1^{1pq} \times |\mu|^2$, so the basis-independent bound on L_e violation associated with the invariant δ_1^{1pq} is

$$\delta_1^{1pq} \times \min\{\Gamma_{|\lambda_d^{pq}|}, \Gamma_{|\mu|}\} < H \tag{57}$$

The rate associated with $|\lambda_e^1|$ must also be in equilibrium, because otherwise no direction in ϕ space would be defined as E_L.

Consider now the invariants δ_2^{1j}. These can also be imagined as corresponding to Feynman diagrams with thermal masses that should belong to the Higgs on both ϕ lines, and a Yukawa interaction that converts a Higgs into E_L internally. Since one of the ϕ lines is supposed to be E_L, one of the thermal masses that should belong to a Higgs but is on the E_L line must be out of equilibrium. So we obtain the similar bound

$$\delta_2^{1j} \times \min\{\Gamma_{|\lambda_e^j|}, \Gamma_{|\mu|}\} < H \tag{58}$$

provided that $|\lambda_e^1|$ is in thermal equilibrium [14].

Our previous assertion that one can estimate the coupling constant for Higgs-lepton conversion due to λ_e as $|\lambda_e^1|$ merits some further discussion. To see why this statement might be suspect, suppose that the Higgs is ϕ_0 and E_L is ϕ_1, so that the coupling constant for the HE_LE^c interaction is $(\lambda_e^1)^{01}$. If $(\lambda_e^1)^{23}$ was much larger, it would make a greater contribution to $|\lambda_e^1|$, and one could overestimate the rate for the HE_LE^c process. However, this does not happen, because any element of the matrix λ_e^1 that does not mediate the HE_LE^c process is an L_1-violating coupling, and is therefore required to be smaller than the L_1-conserving one.

We conclude that in basis-independent notation, the appropriate cosmological bounds are

$$\delta_1^{1pq} \times \min\{\Gamma_{|\mu|}, \Gamma_{|\lambda_d^{pq}|}\} < H \tag{59}$$

$$\delta_2^{1j} \times \min\{\Gamma_{|\mu|}, \Gamma_{|\lambda_e^j|}\} < H \tag{60}$$

$$\delta_3^{1jpq} \times \min\{\Gamma_{|\lambda_e^j|}, \Gamma_{|\lambda_d^{pq}|}\} < H \tag{61}$$

$$\delta_4^{j1k} \times \min\{\Gamma_{|\lambda_e^j|}, \Gamma_{|\lambda_e^k|}\} < H \tag{62}$$

$$\delta_5^{rs1pq} \times \min\{\Gamma_{|\lambda_d^{rs}|}, \Gamma_{|\lambda_d^{pq}|}\} < H \tag{63}$$

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[14]It may be noted that the λ_e give thermal masses to both the Higgs and the leptons, so that the ϕ leg with the thermal mass induced by λ_e does not have to be a Higgs, but could just as well be a lepton. This is true, but in the invariant δ_2^{11} this contribution is cancelled by the second term in the invariant. So $\delta_2^{11} \times \Gamma_{|\lambda_e^1|} < H$ means that the R-violating contribution is out of equilibrium.
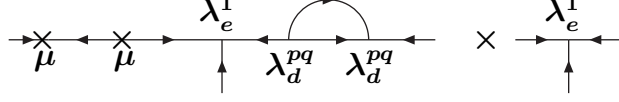
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Figure 2: Representation of the invariant $\delta_1^{1pq} \propto |\mu^\dagger \lambda_e^1 \lambda_d^{pq*}|^2$ as the Feynman supergraph for a scattering process mediated by λ_e^1, with thermal masses on the external legs induced by the interactions μ and λ_d^{pq}.

where the rates Γ and H are to be evaluated at $T \sim T_c \sim 100$ GeV, using the rate estimates (55) and (56). This provides the following bounds on the invariants:

$$\delta_1^{1pq}|\lambda_d^{pq}|^2 < 10^{-14}, \quad p, q : 1..3 \tag{64}$$

$$\delta_2^{1j}|\lambda_e^j|^2 < 10^{-14}, \quad j : 1..3 \tag{65}$$

$$\delta_3^{1jpq} \times \min\{|\lambda_e^j|^2, |\lambda_d^{pq}|^2\} < 10^{-14}, \quad j, p, q : 1..3 \tag{66}$$

$$\delta_4^{k1j} \times \min\{|\lambda_e^j|^2, |\lambda_e^k|^2\} < 10^{-14}, \quad j, k : 1..3 \tag{67}$$

$$\delta_5^{rs1pq} \times \min\{|\lambda_d^{rs}|^2, |\lambda_d^{pq}|^2\} < 10^{-14}, \quad r, s, p, q : 1..3 \tag{68}$$

where i, j, k are lepton family indices, and p, q, r, s are quark family indices. We can conservatively estimate the magnitudes of the various coupling constant vectors and matrices as taking their Standard Model values. If the couplings satisfy these bounds, any asymmetry in $B/3 - L_1$ present in thermal equilibrium in the plasma before the EPT will be preserved. Of course, it could just as well be an asymmetry in some other lepton flavour that is conserved, in which case one would obtain identical bounds, but with the lepton-flavour index "1" replaced by "2" or "3".

The results (64) — (68) can be translated into basis-dependent bounds on various individual coupling constants, that may be more intuitive. For instance, to preserve an asymmetry in $B/3 - L_1$, the R-violating coupling μ^1 should satisfy

$$\frac{\mu^1}{|\mu|} h_b < 10^{-7} \tag{69}$$

in the thermal mass-eigenstate basis, where h_b is the bottom Yukawa coupling. This can be seen by writing out the invariant δ_1^{133} in the thermal mass-eigenstate basis, and requiring that each term in the sum making up δ_1^{133} satisfy the bound (64). Similarly, the other L_e-violating couplings should satisfy

$$(\lambda_d^{pq})^1 < 10^{-7} \tag{70}$$

$$(\lambda_e^1)^{1j}, (\lambda_e^1)^{j1} < 10^{-7} \quad j : 1..3 \tag{71}$$

$$(\lambda_e^{2,3})^{1J}, (\lambda_e^{2,3})^{J1} < 10^{-7} \quad J : 0..3 \tag{72}$$

in the thermal mass-eigenstate basis.

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7   Conclusions

In the Standard Model, there are no renormalisable lepton-number-violating operators, and lepton flavour number is conserved because the neutrinos are massless. However, with the MSSM particle content, numerous renormalisable lepton-number-violating operators can be constructed. This is because the leptons have the same gauge quantum numbers as a Higgs field, so the sleptons can interact with the same particles as the Higgs. If any of these lepton-number non-conserving operators are present in the Lagrangian, there is no longer a global symmetry defining lepton number, so a "lepton" and a "lepton-number-violating interaction" are no longer uniquely defined. If the baryon asymmetry we see today was present in the thermal soup above the electroweak phase transition, in the presence of the anomalous non-perturbative $B + L$-violating electroweak interactions, then at least one lepton flavour must have been conserved at the time, as we have briefly reviewed. Whether or not an asymmetry remains in the plasma is a physical question, which cannot depend on how one chooses to define a "lepton" or "lepton-number violation".

We have therefore identified basis-independent combinations of coupling constants that parametrize lepton-flavour violation in the superpotential. The various couplings of the Higgs and the leptons can be visualised as vectors and planes in a space spanned by the Higgs and lepton superfields. The vectors μ and λ_d^{pq} are directions that should be the Higgs, and the λ_e^i correspond to one or two planes that should be spanned by the Higgs and the ith lepton. It therefore takes three couplings to construct a geometric quantity that measures R violation. One needs a Yukawa coupling, to provide a plane that would like to be spanned by a Higgs and a lepton, and then one needs two further interactions: one to choose a direction in the plane to be the Higgs - the perpendicular direction in the plane is now the lepton, and another to conflict with this choice. Our invariants correspond to the projection of a vector that should be the Higgs onto a direction chosen to be a lepton by other interactions. This is a geometric, basis-independent measure of R violation.

We have listed all the invariants that we can construct out of three coupling constants, which is the minimum required, as discussed above. It is geometrically clear that these invariants are not all independent: indeed, there are 321 of them with three generations of leptons. Therefore, we have worked out how many independent invariants there are, finding the 36 expected in the three-generation case. In the two-generation case, we have also demonstrated explicitly how to express the remaining invariants in terms of the independent ones.

In the early Universe, we can associate a lepton-flavour-violating rate with each of the invariants. This allows us to discuss the survival of the baryon asymmetry of the Universe in a basis-independent way. We have therefore determined the bounds on our invariants that would allow a baryon asymmetry to remain in the plasma before the electroweak phase transition. These bounds reduce to those of [6] if flavour dependence is neglected. In the basis-dependent formulation that we would prefer to avoid, but which is more intuitive, we find that a baryon asymmetry would remain in the plasma if all the dimensionless Yukawa couplings violating some lepton flavour i are less than 10^{-7}, and the superpotential mass term μ^i mixing the Higgs and the ith lepton satisfies $h_b \times \mu^i < 10^{-7}\mu^0$, where h_b is the bottom yukawa, and μ^0 mixes the two Higges. These are the same bounds on the Yukawa couplings as were found in previous work [4, 5], but the bound on μ^i is weaker.

As noted in the introduction, we have not included the soft SUSY-breaking masses or the Higgs vacuum expectation value in the construction of our invariants. These would both be required for a complete discussion of phenomenological bounds on the R-violating couplings in our basis-independent approach. It is at present unclear to us whether there is any possible confusion that could arise from the basis dependence of such experimental bounds, which are calculated in the zero-temperature mass-eigenstate basis. We expect that our invariants should be useful for comparing bounds on R-violating couplings calculated at different energy scales, e.g., the GUT scale, the electroweak phase transition temperature, and at low energy, but such an analysis is left to a future paper.

Acknowledgements

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